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LINE PROFILES FOR CLASSICAL CEPHEID SV VULPECULAE AND FOR SUPERGIANTS BETA AQUARII AND 9 PEGASI*

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ABSTRACT

Curves of growth constructed at phases near maximum light, maximum radius, minimum light, and minimum radius gave the damping and Doppler parameters needed to study the line profiles of the classical cepheid SV Vul [(P = 45.1 days; F7 I*ab* (max. light) to K0 I*ab* (min. light)]. Values of the ratio of scattering to total absorption and limb darkening were assumed, and profiles of λ 4508 of Fe II were calculated for ME atmospheres using the IBM 650 program of M. H. Wrubel. It was found impossible to fit the profiles unless some macroscopic motion, either rotation or large-scale turbulence, was introduced. An identical method of analysis led to the same conclusion regarding the line profiles of the MK standards β Aqr (G0 I*b*) and 9 Peg (G5 I*b*). Visual estimates of the rotational velocities of seven other cepheids were made. It is concluded that the line profiles are too wide to assume that the cepheids conserved their angular momentum in shells, if they evolved from the main sequence to their present positions in the H-R diagram. It is therefore suggested that either (1) the cepheids rotate as rigid bodies or (2) macroturbulence is largely responsible for the line profiles. We suggest that the latter alternative is more likely correct than the former.

I. INTRODUCTION

Broadly speaking, the line profiles of classical cepheids may be used to study two basic problems. The first of these is evolutionary in character and deals with the following question: Are the present line profiles those that would be expected if the cepheids had originated from main-sequence stars and had evolved along some generally accepted evolutionary paths, with mass and angular momentum conserved? Closely related to this is a second question, which may be regarded as essentially astrophysical in character: How does the variation in the line broadening, if any, as well as the variation in temperature, pressure, and opacity through the cepheid cycle, bear on the running-wave hypothesis?

The classical cepheid SV Vul is of particular interest in connection with these questions because of its long period (P = 45.1 days) and large spectral-type variation (F7 I*ab* to K0 I*ab*). Line profiles have been obtained at four phases—maximum light, maximum radius, minimum light, and minimum radius—and have been compared with those of the standard supergiants α Per (F5 I*b*), β Aqr (G0 I*b*), and 9 Peg (G5 I*b*). We have been able to obtain a fairly definitive answer to the first question; however, our study indicates that a complete discussion of the second question would require the construction of detailed model atmospheres for cepheids; since this does not seem altogether feasible, in principle, at the present time, we shall defer discussion of the second question for a later paper.

II. THE OBSERVED AND COMPUTED LINE PROFILES

The spectrograms of SV Vul were obtained by the late Dr. R. F. Sanford with the coudé spectrograph of the 100-inch reflector. One of these (Table 1, Ce 8045) has a dispersion of 4.5 A/mm; the balance is 10.2 A/mm. The MK standard spectra of β Aqr and 9 Peg were obtained with the same equipment at the latter dispersion by Dr. G. H. Herbig.

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Line profiles for a ME atmosphere may be constructed, provided that we know four parameters: (1) the microturbulent velocity, V; (2) the damping parameter, $\log a$; (3) the ratio ϵ of true absorption to total absorption (including line scattering); and (4) the limb darkening, $B^{(0)}/B^{(1)}$. The first two can be given roughly by curve-of-growth analysis; the latter two have to be assumed. However, our calculations show that the limb darkening has little effect on the mechanism of line broadening, and only a small range of ϵ can be permitted and still maintain a good fit to the observed profiles.

In general, we have followed the procedure of Abt (1957, 1958) in constructing the lines profiles, with a few modifications. The line λ 4508 of Fe II was analyzed. This line was chosen because (1) it is relatively free from blending even in the G spectral-type range; (2) it is a subordinate line arising from an energy level at 2.84 volts and therefore is comparatively free from the line splitting that affects many of the resonance lines in cepheids (Kraft 1959); (3) its strength is practically constant over the cycle because of the near-constancy of the ionization of iron; (4) it is the same line as that used by Abt (1957, 1958) in his study of the A and F stars of classes II and I*b*. ME curves of growth were constructed, following Greenstein (1948) for SV Vul and the two standard stars. Because of the great complexity of the spectrum, especially in the G-type range, equivalent widths were not measured shortward of λ 4375. This had the effect of rather severely limiting the number of Fe I lines upon which to base the analysis. However, we were fortunate in having an exposure of SV Vul, obtained in the first order of the grating by Sanford (at phase $0^{\mathrm{P}}.99$) in the region between the D lines and Hα, concurrent with the normal second-order photograph in the blue region of the spectrum. Thus our curve of growth at phase $0^{\mathrm{P}}99$ was based on 48 lines of Fe I, Fe II, and Ti II in the blue region, and 44 lines of Fe I and Fe II in the red. Many of the latter lay on the linear portion of the curve of growth, and thus all regions of the curve were well covered, with the exception of the square-root branch. We were thus able to obtain at this phase the relative abundances of Ti II and Fe II. At other phases, when serious blending in the photographic region drastically reduced the number of Fe I lines available for analysis, we assumed the Ti versus Fe abundance that had been derived at phase $0^{\mathrm{P}}.99$; an excellent basic curve of growth could be made from these two elements alone. The elements Fe and Ti remained essentially in singly ionized form during the entire cycle of SV Vul. Our curve-of-growth parameters for β Aqr and 9 Peg are less good than those derived for SV Vul because of the lack of red plates from which to supplement the equivalent widths obtained in the blue.

In Table 1, we list the parameters T_{exc}, T_{ion}, $\log \kappa$, $\log P_e$, $\log \rho$, and V derived from the curves of growth, along with the same parameters given by Greenstein (1948) for α Per (F5 I*b*). The microturbulent velocities of β Aqr and 9 Peg are very similar to that of α Per; however, V for SV Vul runs between 20 and 60 per cent larger, depending on phase. A rather surprising result is that T_{ion} for all stars in the late F- and G-type range comes out larger than T_{eff}, when the latter is assigned according to spectral type (Keenan and Morgan 1951). (Since these authors give no effective temperatures for I*ab* stars, we have resorted to assigning the values of I*b* stars to SV Vul.) We can think of no really satisfactory explanation for this anomaly, since, in general, we would expect departure from thermodynamic equilibrium to produce the opposite effect, as is the case among earlier-type stars. However, the following points may be noted: (1) Among the early F stars of class I*b*, Abt (1958) finds $T_{\mathrm{eff}} > T_{\mathrm{ion}}$ by about 1000° K, but in HR 690 (F7 I*b*), T_{eff} is actually 50° K less than T_{ion}. While the latter difference is probably not significant, it does indicate a trend which may extend into the G supergiants. (2) From model-atmosphere considerations Whitney (1955*a*, *b*) has reproduced four of the six colors measured by Stebbins, Kron, and Smith (1952) for η Aql at various phases. The necessary effective temperatures are found to be a few hundred degrees higher at maximum and minimum light than would be assigned by the Keenan-Morgan calibration of Code's (1947) spectral types. (3) The Keenan-Morgan calibration is based, in part, on Hall's

(1941) photoelectric colors for supergiants. It is now known that even the brightest supergiants of late spectral type are reddened to some extent. For example, from the photometry of the α Per group, Harris (1956) finds a color excess of 0.12 mag. $(B - V)$ for α Per. If the rate of change of color with spectral class were the same among F supergiants as among F dwarfs, this would correspond to an increase of 400° K in T_{eff}.

On the other hand, it is quite possible that some stratification effect between the ionized and neutral forms of Fe might lead to an error in T_{ion}, though we were unable to detect any difference in the curves of growth based on Fe I, Fe II, or Ti II. The determination of T_{ion} is also tied up with the determination of κ, and an error in the absorption coefficient of H^- might also play a role in the value of T_{ion}. At present, we suggest that some of the discrepancy may be accounted for by revising upward slightly the effective temperature scale for supergiants.

Because of the paucity of unblended strong lines, our curves of growth do not have a square-root branch; thus we have calculated log a from ρ, using a van der Waals type

TABLE 1

LINE PROFILE AND CURVE-OF-GROWTH PARAMETERS FOR α PER, β AQR, 9 PEG, AND SV VUL

Star	α Per*	β Aqr	9 Peg	SV Vul	SV Vul	SV Vul	SV Vul
Plate No.	..	Ce 6648	Ce 6712	Ce 5452	Ce 4752	Ce 8045	Ce 5331
Sp.	F5 I*b*	G0 I*b*	G5 I*b*	G5 I*ab*	G8 I*ab*	G0 I*ab*	F7 I*ab*
Phase (P)†	.....	....	....	0.47	0.69	0.95	0.99
log κ (λ 4300)	−1 64	−1.55	−2.39	−2 18	−3.28	−1.42	−1 48
T_{exc} (° K)	5150	4900±300	4000±400	4600±300	4000±300	5200±300	4900±200
T_{eff}	6200	5000	4290	4290	4000	5000	5550
T_{ion}	6140	5370	4810	4950	4630	5820	5900
log P_e	+0 1	−0 07	−1.05	−0 81	−1 99	+0.37	+0 12
log P_g	+2 3	+3 10	+2.35	+2 45	+1.35	+3.35	+2 65
log ρ	−9.4	−8 55	−9.28	−9.19	−10.26	−8.37	−9 06
log a	−2 7	−1 86	−2 59	−2 60	−3.96	−1.83	−2 42
$v \sin i$ (km/sec)	19 5	19	21	22	23	27	23
$v \sin i$ (predicted)	13	10	8	7	7	7	7
V (km/sec)	6.3	6 2	6.2	8.0	9.4	9.9	7 6

* Data, except for $v \sin i$, taken from Greenstein (1948)

† Sanford (1956).

of interaction with neutral H atoms (cf. Aller 1953). With these values of log a and the microturbulence V from the curves of growth, line profiles were calculated for a ME atmosphere from an IBM 650 program generously made available to us by Dr. M. H. Wrubel. For no choice of either ϵ or limb darkening was it found possible to fit the observed line profiles of λ 4508 in any of the stars; this conclusion is in complete agreement with Abt's results for A and F supergiants. For consistency with Abt's work, we have taken $\epsilon = 0$; however, in what follows, values of ϵ up to 0.05 would give about as good agreement as $\epsilon = 0$.

The calculated profiles were rotationally broadened by Unsöld's (1938) graphical method; the results are shown in Figure 1. Abt's procedure was more nearly exact than ours; he calculated the rotational velocity by broadening the ME profile in such a way as to take into account the limb darkening explicitly in the formula for the residual intensity rather than the integrated flux. Our less nearly exact procedure evidently gives satisfactory results: in panel *1* of Figure 1, we can fit Abt's profile of λ 4508 in α Per with a rotational broadening of 19.5 km/sec, compared with Abt's value of 20 km/sec. The mean projected rotational velocity for the two G-type I*b* standards was found to be 20 km/sec, and the mean value for the four phases of SV Vul was 24 km/sec.

Abt found that his rotational velocities for A and F bright giants and supergiants agreed well with the visual estimates made by Herbig and Spalding (1955), based on the Fe I lines λλ 4404, 4472, and 4476. For β Aqr and 9 Peg, these authors gave $v \sin i <$ 15 km/sec for both stars, and these values are not in very good agreement with our results. We have not investigated whether the formal rotational velocities might be different for different lines; however, none of the above-mentioned Fe I lines were measured by us for equivalent width because we found them to be somewhat blended, with a depression of the continuum in this region due to overlapping line wings. We suggest that these effects may account for the difference between our values of $v \sin i$ and those given by Herbig and Spalding.

III. ROTATIONAL INTERPRETATION OF THE PROFILES

We have found satisfactory agreement with the observed profiles of λ 4508 in all cases, provided that the computed profiles are broadened by a macroscopic motion formally like rotation. The question is whether the values of $v \sin i$ so derived are consistent with evolution from the main sequence with mass and angular momentum conserved. Abt (1958) has concluded that the present rotational velocities of A and F stars

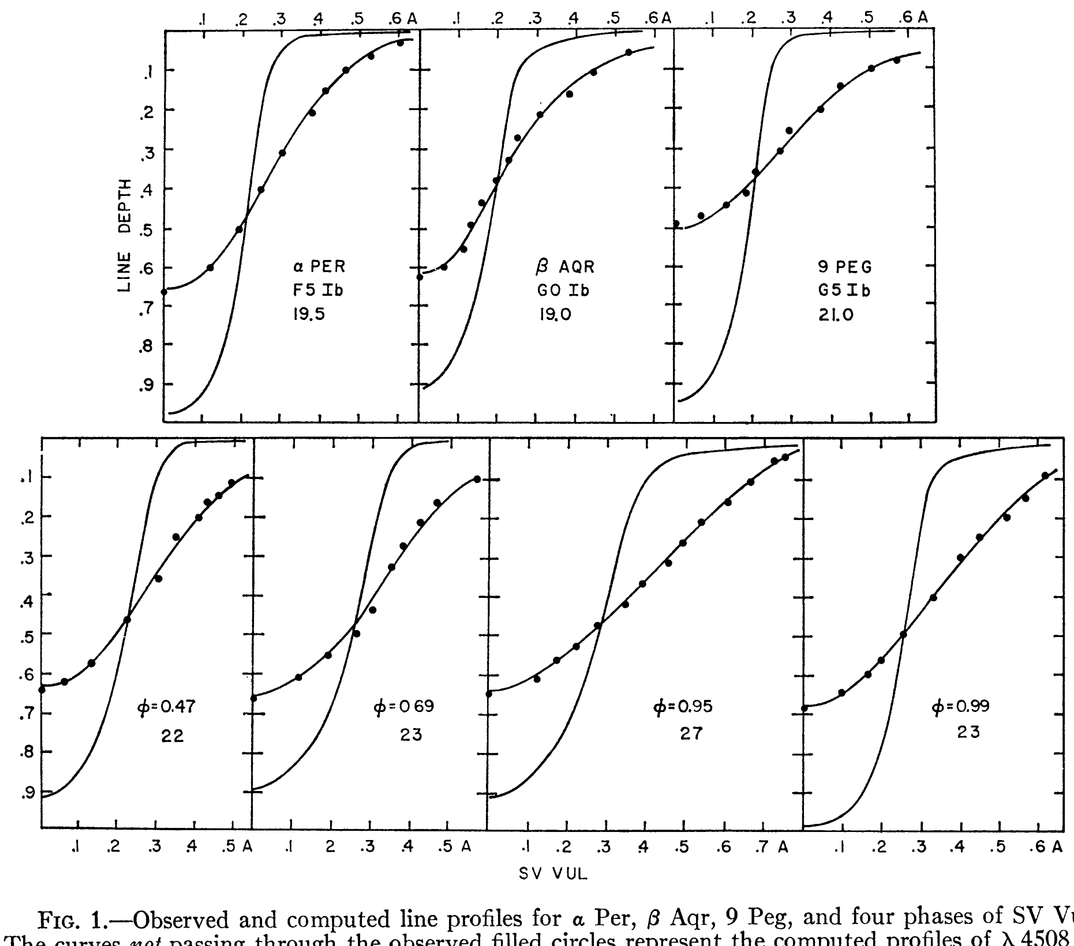

FIG. 1.—Observed and computed line profiles for α Per, β Aqr, 9 Peg, and four phases of SV Vul. The curves *not* passing through the observed filled circles represent the computed profiles of λ 4508 of Fe II based on the curve-of-growth parameters of Table 1. The computed profiles have been broadened by various amounts of rotation; the values of $v \sin i$ giving the best fit are entered in each panel. The "observed" profiles contain various amounts of instrumental broadening corresponding to Voigt profiles. The profile for α Per is taken from the work of Abt (1958).

of class I*b* are consistent with evolution from the main sequence along the Sandage-Schwarzschild (1952) tracks, provided that the angular momentum is conserved in shells, i.e., there is no radial transfer of angular momentum.

Following Sandage (1958*a*), we have assumed evolutionary tracks for cepheids consisting of a 1-mag. rise from the zero-age main sequence followed by horizontal evolution in the M_{bol}, log T_{eff} plane. The cepheid instability strip given by Sandage was entered with the appropriate period; this implies that SV Vul was originally a main-sequence star of spectral type B1. The mean projected rotational velocity for such a star is 157.5 km/sec (Slettebak and Howard 1955). If SV Vul has evolved from the main sequence with no radial transfer of angular momentum, we predict its present projected rotational velocity to be 7 km/sec; if angular momentum is exchanged freely, $v \sin i$ would be about 14 km/sec (cf. Oke and Greenstein 1954). These values are decidedly smaller than the mean value of $v \sin i$, derived from Figure 1, of 24 km/sec. In a similar manner, we have assigned absolute magnitudes to β Aqr and 9 Peg on the basis of the Wilson-Bappu (1957) correlation; with angular momentum conserved in shells, we predict $v \sin i = 10$ and 8 km/sec, respectively, for these stars; rigid-body rotation would produce more satisfactory agreement with the profiles exhibited in Figure 1.

These results appear to be at variance with Abt's conclusion that, among A and F class II and I*b* supergiants, evolution from the main sequence with angular momentum conserved in shells will account for the observed line profiles. Considering SV Vul alone, for the moment, we can look for means of rectifying the disagreement. First, we might suppose that the evolutionary track is wrong. For example, if SV Vul started as an O5 star with a rotational velocity of 215 km/sec, it would now have the observed value of $v \sin i$ for the case of angular momentum conserved in shells. The rotational velocity of 9 Sgr (O5), for example, is given by Slettebak (1949) as 210 km/sec. However, from the mass-luminosity law (Kuiper 1938) we would then have to accept a mass of more than 40$\odot$ for SV Vul, corresponding to a dynamical gravity of 40 cm/sec^2.

Following Schwarschild, Schwarzschild, and Adams (1948), we can estimate the spectroscopic gravity from

$$g_{sp} \sim \frac{X}{2\tau} \frac{P_k}{P_g} \frac{P_g}{P_e} P_e \bar{\kappa} ,$$

where P_k is the kinetic pressure resulting from turbulence, X is the hydrogen abundance by weight, and the other symbols have their usual meanings. For $X = 0.75$ and $\tau = 0.3$, the maximum value of g_{sp} is obtained by assuming that the profile is entirely a result of macroturbulence (no rotation) with a characteristic velocity σ of $\frac{2}{3} v \sin i$ (see Sec. IV). At phase $0^{\text{P}}47$ in SV Vul, corresponding to maximum radius, when the accelerations are a minimum, we have $\sigma = 14$ km/sec; whence, since the kinetic and gas pressures are proportional to the squares of their respective velocities, we have $P_k/P_g = 3.7$, and $g_{sp} = 14$ cm/sec^2. If the line profiles are due entirely or partially to rotation, g_{sp} will be still smaller. If the progenitor of SV Vul were, in fact, a star at B1 V, its mass (Kuiper 1938) would be 16$\odot$; this would produce much better agreement between the spectroscopic and dynamical gravities than would be the case if the main-sequence progenitor were of type O5.

Furthermore, the evidence that exists from the study of cepheids in open clusters (Irwin 1957; Arp 1958; Kraft 1958; Sandage 1958*b*) suggests that the evolution of stars across the upper part of the H-R diagram through the cepheid region proceeds in a nearly horizontal fashion. It must be admitted, however, that these studies are confined to cepheids with periods under 10 days; we are certain, however, that SV Vul cannot have had a progenitor later than B3 V. Thus it does not appear that the evolutionary track can be in sufficient error to bring agreement with the non-rigid-body rotation.

In order to check the generality of this conclusion, we have made visual estimates of $v \sin i$ from λ 4508 in seven other cepheids, well distributed in period. All spectrograms were taken with the 100-inch coudé at a dispersion of 10 A/mm and are listed in Table 2. We have found that, by using our profiles for the four phases of SV Vul and those for the two standards β Aqr and 9 Peg as calibration, satisfactory visual estimates of $v \sin i$ can be made for the other cepheids, using a spectrocomparator, provided that we make the assumption that the line profiles of all cepheids are due essentially to (formal) rotation. We estimate that a well-exposed plate will yield in this way a value of $v \sin i$ with an error of ± 3 km/sec. We list in Table 3 the mean values of $v \sin i$ for the various cepheids; the mean is taken over all the plates measured for a given star without regard to distribution with respect to phase. Since the profiles are not of constant width during the cycle (they are wider on the ascending, as compared with the descending, branch of the light-curve), the mean values are not internally quite consistent, but, since the variation in $v \sin i$ over the cycle is small (7 km/sec in X Cyg, the "largest" variation), we can neglect this for statistical purposes. Table 3 gives, as well, the predicted values

TABLE 2

ESTIMATES OF v SIN i FOR SEVEN CLASSICAL CEPHEIDS

Star	Period (days)	Plate No. (Ce)	Phase (P)	$v \sin i$ (km/sec)	Observer and Reference
EV Sct.... .	3.1	11164	0 97	20::	Kraft (1958)
S Sge... .	8.4	6647	0.41	19	Herbig (1952)
		6653	0.52	21:	Herbig
ζ Gem...	10.1	8896	0.34	19:	Herbig
TT Aql...	13.8	10380	0.15	23	Kraft
		10565	0.54	21	Kraft
		10566	0.60	21	Kraft
		10568	0.62	22	Kraft
		10572	0.69	22	Kraft
		10574	0 75	23.5	Kraft
		10577	0.76	23	Kraft
		10578	0.82	25	Kraft
		10581	0.84	22	Kraft
X Cyg...	16.4	5218	0.32	18.5	Sanford
		5237	0.08	20	Babcock
		5263	0.45	19	Sanford
		5332	0.62	23	Sanford
		8887	0.90	23	Herbig
		8890	0.95	24	Herbig
		8917	0.06	20:	Struve
		9217	0.82	24	Struve
		9233	0.94	23:	Struve
		9417	0.45	20	Struve
		10409	0.51	21	Kraft (1957)
		10412	0.57	21	Kraft
		10419	0.70	21:	Kraft
		10495	0.80	24	Kraft
		10496	0.81	24 5	Kraft
WZ Sgr.......	21 8	10485	0.01	23	Kraft
		10491	0.05	23	Kraft
		10611	0.79	27:	Kraft
T Mon........	27.0	4111	0.68	22	Sanford (1956)
		4211	0.15	21	Sanford
		4218	0.22	20	Sanford
		4230	0.30	19.5	Sanford
		4598	0.37	22:	Sanford
		4626	0.48	21	Sanford
		5112	0.58	20	Sanford
		6206	0.86	25:	Sanford

of $v \sin i$, using the horizontal evolutionary tracks, if we assume angular momentum conserved, as in our two extreme cases.

Since the plates of TT Aql and WZ Sgr were confined largely to the ascending branch of the light-curve, our average value of $v \sin i$ for each of these stars is probably a little too large; on the other hand, the average value for T Mon is probably a little too small. The measured values of Table 3 give a mean value of $v \sin i$ of 22 km/sec; the run with period is essentially flat, with perhaps a slight increase with increasing period. The predicted mean $v \sin i$ for the case of rigid-body rotation is 23 km/sec and is in better agreement with the observed value than for the case of angular momentum conserved in shells; however, in both these cases the run of $v \sin i$ decreases with increasing period, and this is different from that which is observed.

TABLE 3

MEAN ESTIMATED AND PREDICTED VALUES OF v SIN i FOR EIGHT CEPHEIDS

Star	P (days)	$v \sin i$ (km/sec) (est.)	No Plates	$v \sin i$ (km/sec) (shells)	$v \sin i$ (km/sec) (rigid)
EV Sct......	3 1	20::	1	22	44
S Sge....	8 4	20:	2	13	26
ζ Gem......	10 1	19:	1	12	24
TT Aql....	13.8	22 4	9	10	20
X Cyg......	16 4	21.7	15	9	18
WZ Sgr.....	21.8	24	3	9	18
T Mon.......	27.0	21.4	8	8	16
SV Vul.......	45.1	24	4	7	14
Average....	....	22	....	11	23

IV. MACROTURBULENCE

We suggest that large-scale turbulence is more likely primarily responsible for broadening the line profiles of cepheids than is rotation. Schwarzschild *et al.* (1948), Unsöld and Struve (1949), and more recently (and by a more rigorous procedure) Abt (1957, 1958) have shown that radial macroturbulence in supergiants cannot be distinguished from rotation in its effect on the line profiles; it is found that if the macroturbulent motions are represented by a Gaussian distribution with a mean velocity σ, then approximately $\sigma = \frac{2}{3} v \sin i$. In particular, Abt (1958) has calculated (including the effect of limb darkening on the specific intensity) a line profile for α Per with $\sigma = 13$ km/sec that cannot be distinguished from $v \sin i = 20$ km/sec. A simpler, though naturally less nearly exact, graphical procedure was invented by Unsöld and Struve (1949) and applied by them to the study of δ CMa. Although no account of limb darkening is taken, with this method we get a good fit to Abt's observed line profile of α Per with $\sigma = 14$ km/sec. Satisfactory representations for the profiles in SV Vul, β Aqr, and 9 Peg can also be made; one of these is illustrated in Figure 2.

Though we believe that the run of values of $v \sin i$ with period speaks in favor of macroturbulence rather than rotation, there is a physical factor which tends to cloud the distinction and make an "either/or" decision impossible at present. Let us suppose that the immediate predecessors of the cepheids were A and F supergiants like those analyzed by Abt. Present ideas of stellar interiors (Hoyle and Schwarzschild 1955) suggest that such stars are in radiative equilibrium throughout the greater part of their radii, whereas G-type supergiants, such as cepheids, *may* have extensive regions in convective equilibrium extending downward from the surface. If the A and F supergiants are rotating differentially, as suggested by Abt's study, they would presumably have difficulty maintaining that mode when they became cepheids if convection currents should transport angular momentum as well as flux. Thus the same physical conditions that increase macroturbulence might also tend to favor rigid-body rotation.[1]

The main question would seem to be whether the angular momentum can be transported by convection through a distance of the order of the radius in a time significantly shorter than the lifetime of the star as a cepheid.

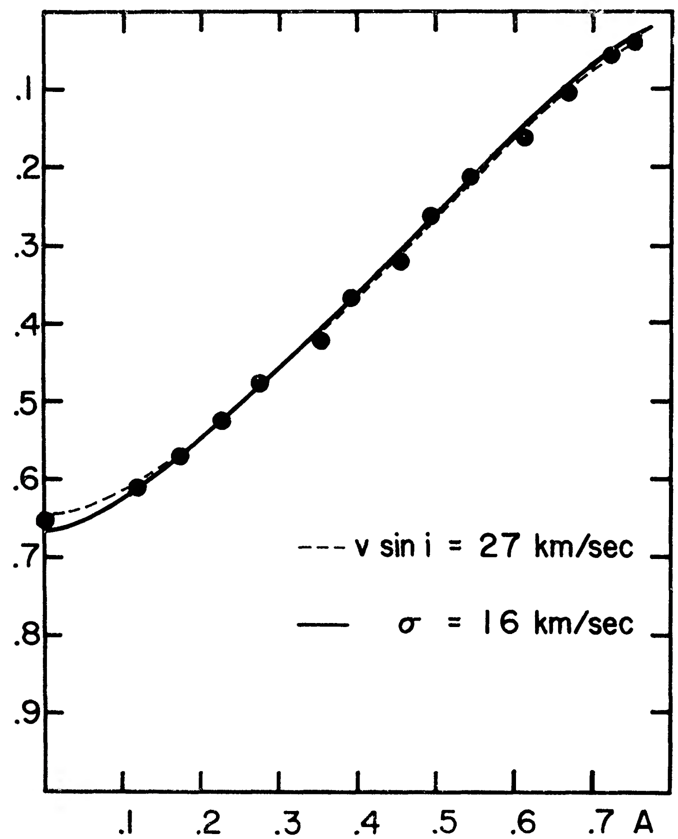

FIG. 2.—Macroturbulence versus rotation at phase 0.95P in SV Vul. The dashed line represents the computed profile of λ 4508 broadened by rotation with $v \sin i = 27$ km/sec (same as in Fig. 1). The solid curve represents the same profile broadened by macroturbulence with a characteristic turbulent velocity of 16 km/sec. The profiles contain an instrumental broadening describable by Voigt parameters.

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[1] We are indebted to Dr. M. P. Savedoff for calling our attention to this possibility.

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