THE RADIAL VELOCITY OF DELTA CEPHEI*

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ABSTRACT

Twenty-seven observations of δ Cephei made in 1950 with the Mills spectrograph are combined with observations obtained in 1907 and 1923 in order to determine a definitive velocity-curve. The following conclusions are drawn: (1) there is no evidence of any long-period change in the mean velocity; (2) there is no evidence of change in the form of the velocity-curve; (3) the mean velocity is -16.1 ± 0.2 km/sec; the velocity range is 39.4 km/sec. An attempt is made to ascribe the velocities measured to some identifiable level in the stellar atmosphere.

The variable radial velocity of δ Cephei was discovered by Belopolsky (1894) and has since been observed extensively by the discoverer (Belopolsky 1909) and by Moore (1913), Jacobsen (1926), Petrie (1934), and others. Many photometric observations are also available, including a photoelectric series in six colors by Stebbins (1945).

The Mills three-prism spectrograph, which gives a dispersion of 11 A/mm at λ 4500, was used both by Moore in 1907 and by Jacobsen in 1923 for observations of δ Cephei. The same instrument was used in the present program in 1950. Since these three series of plates provide a particularly homogeneous group, it was decided to confine the current discussion to these observations. The objects of the investigation were to obtain a definitive velocity-curve, to study the possibility of changes in the form of the curve over a long period, and to test the reality of the small changes in the mean velocity reported by some observers.

During the summer of 1950, twenty-seven spectrograms of δ Cephei were obtained. The plates were distributed evenly over the cycle, except for additional observations obtained on the part of the curve where the velocity changes most rapidly. A slit 0.001 inch wide was used, to give a projected width of 19 μ. The exposures on Kodak II*a*-O emulsion averaged about 30 minutes. The plates were all well exposed and of good quality with the exception of an underexposed one, which was consequently assigned half-weight.

All the plates of the 1950 series were measured on a Hartmann spectrocomparator with a standard plate of γ Cygni that afforded a good match in the region measured ($\lambda\lambda$ 4391–4634). The velocity adopted for the standard spectrum was that published by Moore (1932), corrected for annual and diurnal motion of the earth and for flexure of the telescope tube. Since the constancy of the radial velocity of γ Cygni has been questioned by Adams and MacCormack (1935), it was not surprising to find from measurements of constant-velocity stars that it was necessary to apply an appreciable correction to the velocities obtained by comparison with this plate. The measurement of the plates of the two series by Moore and by Jacobsen is described in their original publications.

In order to reduce all velocities to the system defined by the *General Catalogue of Stellar Radial Velocities* (Wilson 1953), the following procedure was used. A group of eight stars of constant velocity was selected for the similarity of their spectra to that of δ Cephei, which varies from F5 I*b* to G2 I*b* according to Code (1947). From among plates of these stars in the observatory file, sixteen Mills spectrograms were chosen, eight of which had been taken at about the same time as each of the two earlier δ Cephei series. In addition, eight representative spectrograms were selected from each of these two series of δ Cephei plates. All these plates were measured by the writer in the same manner as

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those of the 1950 series. Comparison of the measures of the constant-velocity stars with the velocities published by Wilson provided corrections by which the writer's measures for each epoch could be reduced to the system of Wilson's *Catalogue.* Comparison of the writer's measures of the plates of δ Cephei with the velocities published by Moore and by Jacobsen for the same plates afforded corrections by which their measures could be reduced to the writer's system for the corresponding epoch and thence to the system of Wilson's *Catalogue.*

In 1950 and 1951 eight more plates of the constant-velocity stars were obtained, and these were measured by the writer in 1951 in order to establish a correction to the velocities derived from the latest series of plates of δ Cephei. At the same time, six plates from the 1950 series were remeasured so as to detect any change in the writer's personal equation between 1950 and 1951, but no such change was found.

All the data used in reducing the measures to the velocity system of Wilson's *Catalogue* are given in Table 1. The dates of observation are Greenwich Mean Time prior to 1925.0 and Greenwich Civil Time thereafter. Spectral types of the constant-velocity stars are based on the original Yerkes *Atlas* (MKK) system. Slit-widths are given in units of 0.001 inch. The symbols V_M and V_J refer, respectively, to Moore's and Jacobsen's published velocities of δ Cephei: V_{50} and V_{51} are the writer's measures in 1950 and 1951; V_W are the velocities published in Wilson's *Catalogue.* The probable errors of the corrections are based on the internal consistency within each group. The quality of the plates of constant-velocity stars is more variable than that of the plates of δ Cephei, and weights were assigned accordingly. Here, as elsewhere in this paper, all computations were carried out to an accuracy of 0.01 km/sec, but published values are rounded off to the nearest 0.1 km/sec.

The corrections presented in Table 1 were applied to the velocities of δ Cephei published by Moore and by Jacobsen and to the measures made by the writer, and the resulting velocities are given in Table 2, column 5. In the case of the six plates of δ Cephei measured in both 1950 and 1951, both measures were reduced to the system of Wilson's *Catalogue,* and the mean was used. These plates were assigned only unit weight, since general experience shows that the most important errors are usually intrinsic in the plates and are repeated on remeasurement, particularly by the same individual; however, they are identified by a dagger (†) in column 6. The weights in this column are those assigned by the original observer. The Julian Day given in column 2 has been reduced to the sun.

By means of an approximate period derived in a preliminary investigation, all the observations listed in Table 2 were plotted on a single-cycle diagram similar to Figure 1. After combining all the observations into twenty-five normal points, a mean velocity-curve was drawn freehand, and the residual of each observation from this curve was measured. The three series of observations were then considered separately, and for each series a first least-squares solution was made, to determine how much ($\Delta\Gamma$) the curve would have to be shifted along the ordinate axis and ($\Delta\Phi$) along the abscissa axis to obtain the best fit with the observations. The results of this computation are presented in columns 2 and 3 of Table 3. Column 4 lists the estimated probable error of an observation of unit weight. We have used the customary formula, $\text{p.e.}(1) = \pm 0.6745\sqrt{[pvv/(n - m)]}$, where n is the total number of observations and m is the number of variables. The problem of determining m was complicated by the fact that a freehand curve was drawn through the observed points. Had a suitable interpolation formula been used, we could have proceeded by combining all three series of observations, using appropriate weights, and making a least-squares solution for the six variables considered above and for corrections to the parameters in the interpolation formula. Since this rigorous but cumbersome procedure was not used, and the solution was performed in three parts, it was necessary to compensate for the fact that some of the information contained in the observations was used up in determining the form of the velocity-curve. This was done by estimating

TABLE 1

CORRECTIONS TO MEASURED VELOCITIES

PART 1. MEASURES OF CONSTANT-VELOCITY STARS

Plate No. (1)	Date (2)	Star (3)	Spectrum (4)	Slit-Width (5)	V_{51} (6)	V_W (7)	$V_W - V_{51}$ (8)	Wt. (9)
3642	1905 Jan. 2 GMT	α Per	F5 I*b*	1 3	− 5 3	− 2 4	+2 9	1 0
3880	1905 July 4 GMT	β Aqr	G0 I*b*	1 3	+ 4 1	+ 6 5	+2 4	1 0
4175	1906 Jan. 28 GMT	β Cam	G0 I*b*	1 3	− 4 7	− 1 7	+3 0	0 25
4278	1906 July 3 GMT	α Aqr	G2 I*b*	1.3	+ 4 5	+ 7 5	+3 0	1 0
4317	1906 July 16 GMT	9 Peg	G5 I*b*	1 4	−25 2	−22 3	+2 9	1 0
4863 ..	1907 Aug. 10 GMT	α¹ Cap	G3 I*b*	1 5	−26 3	−25 9	+0 4	0.5
5172	1908 Mar. 12 GMT	HR 3459	G2 I*b*	1 6	+29 2	+31 4	+2 2	0.5
5250 .	1908 Apr. 27 GMT	ε Leo	G0 II	1 4	+ 0 7	+ 5 0	+4 3	0.25
						Mean	+2 6±0 2	
12753.	1923 June 26 GMT	α¹ Cap	G3 I*b*	2 0	−27 0	−25 9	+1.1	0 5
12799.	1923 July 20 GMT	α Aqr	G2 I*b*	1 5	+ 6 7	+ 7 5	+0 8	1 0
12800.	1923 July 20 GMT	α Aqr	G2 I*b*	1 5	+ 5 6	+ 7 5	+1 9	0 5
12848.	1923 Aug. 8 GMT	α¹ Cap	G3 I*b*	1 5	−24 8	−25 9	−1.1	0 25
12962. ..	1923 Oct. 10 GMT	β Cam	G0 I*b*	1 5	− 3 1	− 1 7	+1 4	0 25
12973.	1923 Oct. 18 GMT	9 Peg	G5 I*b*	1 8	−23 9	−22 3	+1 6	0 25
13260.	1924 Mar. 6 GMT	HR 3459	G2 I*b*	2 0	+27 8	+31 4	+3 6	0 25
14134.	1925 July 8 GCT	β Aqr	G0 I*b*	1 0	+ 5 9	+ 6 5	+0.6	0.5
						Mean	+1 1±0 2	
33459.	1950 July 2 GCT	β Aqr	G0 I*b*	1 0	+ 3.5	+ 6 5	+3 0	2 0
33692.	1950 Aug. 20 GCT	α Per	F5 I*b*	1 0	− 4 6	− 2 4	+2 2	2 0
33819.	1950 Oct. 5 GCT	β Aqr	G0 I*b*	1 0	+ 4 5	+ 6 5	+2 0	1 0
34252.	1951 Apr. 15 GCT	β Cam	G0 I*b*	1 0	− 4 4	− 1 7	+2 7	1 0
34272.	1951 Apr. 21 GCT	ε Leo	G0 II	1 0	+ 3 1	+ 5 0	+1 9	1 0
34432.	1951 June 18 GCT	α¹ Cap	G3 I*b*	1 0	−29 7	−25 9	+3 8	0 5
34433.	1951 June 18 GCT	9 Peg	G5 I*b*	1 0	−26 5	−22 3	+4 2	0 5
34434.	1951 June 18 GCT	α Aqr	G2 I*b*	1 0	+ 3 9	+ 7 5	+3 6	1 0
						Mean	+2 8±0 2	

PART 2. REMEASURES OF δ CEPHEI PLATES

Plate No. (1)	V_M (2)	V_J (3)	V_{50} (4)	V_{51} (5)	$V_{51} - V_{M,J,50}$ (6)	Wt. (7)	$V_W - V_{M,J,50}$ (8)
4945	− 5 0	...	...	− 6 6	−1.6	1 0	...
4953	−18.5	...	...	−20.1	−1.6	1.0	...
4965	−12 0		...	−13 8	−1.8	1 0	...
4969	+ 3 2			+ 1 5	−1 7	0 5	...
4990	−16.3			−17 7	−1.4	1 0	...
5004	−35 3	...	...	−37 5	−2 2	1 0	...
5009	−26.8		...	−29 0	−2 2	1 0	...
5011	−24 3		...	−25 7	−1.4	0 5	...
				Mean	−1 8±0.1	...	+0.9±0 2
12779	...	−23 3	...	−25 5	−2.2	1.0	...
12827	...	−18 0	...	−20 0	−2 0	1.0	...
12836	...	−24 3		−26 4	−2.1	1 0	...
12877	...	−30 2	...	−31 2	−1.0	1.0	...
12894		− 9 1	...	−10 5	−1 4	1 0	...
12961	...	−14 9	...	−15 9	−1 0	1.0	...
12970	...	− 1 4	...	− 2 5	−1 2	1.0	...
13005	...	+ 2 8	...	+ 0 3	−2 4	1 0	...
				Mean	−1 7±0 1	...	−0.5±0.3
33447	...	...	−17 8	−17.6	+0 3	1 0	...
33492	...	...	−21.9	−21 2	+0.7	1 0	...
33554	...	...	+ 1.1	+ 0 8	−0 3	1.0	...
33721	...	...	−32.2	−31.9	+0 2	1.0	...
33733	...	...	− 9.8	−10.4	−0 6	1.0	...
33792	...	...	−35 9	−36 1	−0 2	1 0	...
				Mean	0.0±0.1	...	+2.8±0.2

TABLE 2

RADIAL-VELOCITY OBSERVATIONS OF δ CEPHEI

Plate No (1)	JD (Heliocentric) 2400000+ (2)	Slit-Width (3)	Phase (Period) (4)	Observed Velocity* (km/sec) (5)	Wt (6)	Residual O−C (km/sec) (7)
4938	17837 701	1 4	0 5428	−13 3	1 0	+0 5
4939	17837 764	1 4	5546	−14 6	1 0	−1 3
4940	17837 839	1 4	5686	−11 4	0 5	+1 2
4943	17838 695	1 4	7281	− 5 9	1 0	−0 7
4945	17838 852	1 4	.7573	− 4 1	1 0	−0 4
4953	17842 746	1 4	4830	−17 6	1 0	−0 5
4954	17842 828	1 4	.4982	−16 2	1 0	0 0
4957	17844 740	1 4	8545	+ 3 1	1 0	+0 6
4958	17844 830	1 4	8713	+ 4 4	1 0	+1 0
4962	17845 727	1 6	0385	−28 1	1.0	−0 7
4963	17845 812	1 4	0543	−29 7	1 0	+0 8
4965	17848 719	1.4	.5960	−11 1	1 0	+0 1
4966	17848 814	1 4	6137	− 9 4	1 0	+1 0
4969	17855 758	1 4	9077	+ 4 1	0.5	+0 1
4970	17855 813	1 4	9179	+ 3 5	0 5	−0.4
4976	17856 718	1 4	0866	−35 1	1.0	−0 3
4977	17856 781	1 4	.0983	−35 0	1 0	+0 2
4978	17856 839	1 4	.1091	−34 7	1 0	+0 5
4984	17858 799	1 4	4744	−18.7	0 5	−1.0
4985	17859.673	1 4	6372	− 9 0	1.0	+0 4
4990	17861 608	1 4	.9978	−15 4	1.0	−0 3
4992	17862 730	1 4	2069	−31.4	1 0	+0 2
4993	17862 772	1 4	2147	−30 8	1 0	+0 4
5001	17866 669	1 4	.9409	+ 3 6	1 0	+1 0
5002	17866.763	1 4	9584	+ 0 1	0 5	+0 9
5004	17867 739	1 4	.1403	−34 4	1 0	−0 1
5005	17867 785	1 4	1489	−33 7	1.0	+0 3
5009	17868 618	1 4	.3041	−25 9	1 0	+0 8
5011	17879 724	1.4	3737	−23 4	0 5	−0.1
5031	17890.679	1 4	.4151	−21 2	0.5	−0 1
5037	17893 647	1 4	9682	− 3 0	1 0	+0 6
5038	17893 690	1 4	9762	− 5 5	1 0	+0 6
5039	17893 739	1 4	9853	− 9 2	1 0	+0 4
5042	17894 713	1 4	1668	−33 8	1 0	−0 5
5054	17904 661	1 4	0206	−23 8	1 0	−0 5
5068	17909 665	1 4	9530	− 1 1	1 0	−1 7
5077	17922 625	1 4	3681	−24 3	1 0	−0 7
5093	17946 611	1 4	8378	+ 0 6	0 5	−0 8
12776	23609 957	1 5	1819	−32 8	1 0	−0 2
12779	23610 996	1 5	.3756	−23 8	1 0	−0 6
12782	23611 983	1 5	5595	−13 0	1 0	0 0
12784	23612 993	1.5	7477	− 4 2	1 0	0 0
12788	23613.990	1 5	9335	+ 2 5	1 0	−0 8
12789	23615 005	1 5	1226	−35 3	1 0	−0 3
12797	23620 972	1 5	2346	−30 7	1 0	−0 5
12798	23621 874	1 5	4026	−21.0	1 0	+0 8
12801	23622 000	1 5	.4261	−20 4	1 0	+0 1
12803	23622.992	1.5	0.6110	−11 2	1 0	−0.7

* Including corrections derived from data given in Table 1 In addition, several small computational errors affecting previously published velocities have been corrected.

TABLE 2—*Continued*

Plate No. (1)	JD (Heliocentric) 2400000+ (2)	Slit-Width (3)	Phase (Period) (4)	Observed Velocity* (km/sec) (5)	Wt (6)	Residual O−C (km/sec) (7)
12806	23623 991	1 5	0.7971	− 0 6	0 5	+0 6
12827	23632 963	1 5	.4690	−18 5	1 0	−0 5
12833	23634 977	1 5	8443	+ 0 5	0 5	−1 3
12836	23635 954	1 5	0264	−24 8	1 0	−0 1
12854	23645 991	1 5	.8968	+ 4 2	1 0	+0 2
12868	23652 876	1 5	.1798	−33 0	1 0	−0 3
12877	23663 889	1 5	2320	−30 7	1 0	−0 4
12894	23676 826	1 5	6428	− 9 7	1 0	−0 5
12922	23686 875	1 5	.5154	−15 0	1 0	+0 3
12948	23694 683	1 5	9704	− 4 8	1 0	−0 5
12961	23702 901	1 5	5018	−15 4	1 0	+0 6
12970	23709 800	1 5	.7874	− 1 9	1.0	0 0
12986	23717.735	1 5	2661	−28 6	1 0	−0 1
13005	23726 767	1 5	9491	+ 2 2	1 0	+0 8
13013	23730 620	1 5	.6671	− 7 1	1 0	+0 9
13491	23969 908	1 5	.2577	−28 7	1 0	+0 3
33436	33452 913	1 0	.3926	−22 6	1.0	−0 2
33440	33453 951	1 0	.5861	−11 5	1 0	+0 2
33447	33458 994	1 0	.5258	−15 0	1 0†	−0 2
33452	33459 927	1 0	6997	− 6 6	1 0	0 0
33457	33464 967	1 0	.6389	− 9 3	1 0	0 0
33464	33467.945	1 0	.1938	−32 4	1 0	−0.2
33485	33470 928	1 0	.7497	− 4 0	1 0	+0.1
33489	33473 938	1 0	.3106	−26 4	1.0	0 0
33492	33474 734	1 0	.4589	−18 8	1 0†	−0 2
33501	33476 828	1 0	.8491	+ 1 7	0 5	−0 4
33522	33483.752	1 0	.1394	−33 9	1 0	+0 5
33530	33484 932	1 0	3593	−23 0	1.0	+1 0
33554	33487 796	1 0	8930	+ 3 7	1 0†	−0 3
33571	33488 862	1 0	0917	−35 1	1 0	0 0
33577	33489 917	1 0	2883	−28 0	1.0	−0 6
33603	33498 830	1 0	.9492	+ 2 0	1 0	+0 6
33684	33509 951	1 0	0215	−22 8	1 0	+0 8
33721	33521 910	1.0	.2501	−29 3	1 0†	+0 1
33728	33524 827	1 0	.7937	− 1 7	1 0	−0 2
33732	33525 777	1 0	9707	− 4 4	1 0	0 0
33733	33525 820	1 0	9787	− 7 3	1 0†	−0 3
33735	33525 907	1 0	9949	−14 4	1 0	−0 5
33738	33525 994	1 0	0111	−19 9	1 0	+0 6
33740	33527 847	1 0	3564	−24 2	1 0	0 0
33752	33530 840	1 0	9142	+ 4 1	1 0	+0 1
33789	33536 906	1 0	0446	−29 0	1 0	−0 3
33792	33537 044	1 0	0 0703	−33 2	1 0†	−0 1

* Including corrections derived from data given in Table 1 In addition, several small computational errors affecting previously published velocities have been corrected

† Mean of two measurements

the number of parameters that would be required in an interpolation formula in order to represent the variation of velocity to the same degree of detail as does the freehand curve. This number was taken as thirteen, and it was apportioned between the three solutions in accordance with the number of observations in each. Then m was estimated by adding 2, the number of variables in each solution, to this quantity. It should be noted that the resultant probable errors will not be very sensitive to this estimate; if 26 is substituted for 13, we find that the probable errors are increased by only about 9 per cent. The fact that corrections to the form of the curve could not be included in the solutions was not a serious drawback, since it subsequently appeared that the form of the curve did not require correction. Henceforth, we shall call quantities computed as described here "probable errors," with the understanding that they are not computed in a mathematically rigorous manner but in the belief that they are realistic estimates of the probable errors of the quantities considered.

The progressive reduction in probable error from one series to the next may be attributed to the use of shorter exposures and a narrower slit in the 1950 series, permitted by faster photographic plates, and to the correction of the 1923 and 1950 series for flexure

TABLE 3

CORRECTIONS AND PROBABLE ERRORS FROM LEAST-SQUARES SOLUTIONS

Series	Solutions for $\Delta\Gamma$ and $\Delta\Phi$			Solutions for $\Delta\Phi$ Alone	
	$\Delta\Gamma$ (km/sec)	$\Delta\Phi$ (Period)	p e (1) (km/sec)	$\Delta\Phi$ (Period)	p.e. (1) (km/sec)
(1)	(2)	(3)	(4)	(5)	(6)
1907.	-0 01$\pm$0 09	$+0$ 0004$\pm$0 0005	$\pm$0 48	$+0$ 0005$\pm$0.0005	$\pm$0 47
1923 . .	$-$ .12$\pm$ .08	$+$ .0008$\pm$ .0008	$\pm$ 38	$+$ .0008$\pm$ 0008	$\pm$ 38
1950 . .	-0 03$\pm$0 06	-0.0005 ± 0.0004	$\pm$0 29	-0 0004$\pm$0 0003	$\pm$0 28

effects in the telescope tube, as discussed by Jacobsen (1926). The importance of this effect was not appreciated in 1907, and it is not now possible to apply the same corrections to the earlier observations.

The values of $\Delta\Gamma$ given in column 2 of Table 3 depend upon the manner in which the mean velocity-curve was drawn. If it had been drawn systematically 0.05 km/sec lower, then the mean of the three values of $\Delta\Gamma$ would have been zero. Thus the mean value of $\Delta\Gamma$ represents an error in drawing the mean velocity-curve, while the differences between the individual values of $\Delta\Gamma$ would, if significant, represent either changes in the mean velocity of the variable between epochs or differences in the velocity systems. If we assume that there is no compensating error in the velocity systems, then the agreement between the three values of $\Delta\Gamma$ implies that there has been no detectable change in the mean velocity of the variable.

After a correction of -0.05 km/sec was applied to all velocities read from the mean curve so as to reduce the mean value of $\Delta\Gamma$ to zero, a second set of least-squares solutions was made, this time for $\Delta\Phi$ alone, since there was no longer any reason to allow for a change in the mean velocity. The results are given in column 5 of Table 3. There is no significant difference between the solutions, as might have been expected from the small differences between the values of $\Delta\Gamma$. The slight reduction of some of the values of p.e. (1) in column 6 results from the fact that there is now one less variable in each solution. Since there now appeared no justification for making any further correction to the observed velocities, it was decided to adopt the second set of solutions as final.

The residuals from the first set of least-squares solutions were plotted against phase, and for none of the series of observations was there any significant deviation from the zero axis. This result implies, first, that there was no significant change in the form of the velocity-curve between epochs and, second, that no correction to the form of the preliminary velocity-curve was needed.

The mean velocity from the preliminary curve was computed by numerical integration and was found to be -16.08 km/sec. After correcting for the systematic error of $+0.05$ km/sec made in drawing the freehand curve, we obtained a final mean velocity of -16.1 ± 0.2 km/sec. Here the probable error is estimated on the basis of the uncertainty of the velocity system as determined from the data in Table 1.

The phases given in column 4 of Table 2 were computed by applying, to the preliminary phases used in the least-squares solutions, the corrections from column 5 of Table 3, with the opposite sign, plus a constant shift to bring zero phase to the intersection of the steep (descending) branch of the velocity-curve with the mean velocity axis. After application of these corrections, the formula for the time of zero phase becomes $\mathrm{JD}_{\odot}$ $2433450.806 + 5.366296E - 0.0114 \times 10^{-6} E^2$. This expression is in satisfactory agreement with those derived by Hertzsprung (1919), Danjon (1927), and Stebbins (1945). A period determined from velocity observations, however, suffers from the disadvantage, when compared with a photometric period, that a single observation requires a longer exposure and hence the time resolution is inferior. In addition, this period is less certain than it might otherwise be because there are few good observations on the descending branch of the velocity-curve in the 1923 series. On the other hand, all observations used here were made with the same instrument and the same observing technique; thus they may be combined with less difficulty than photometric observations made with different instruments and possibly on different color systems.

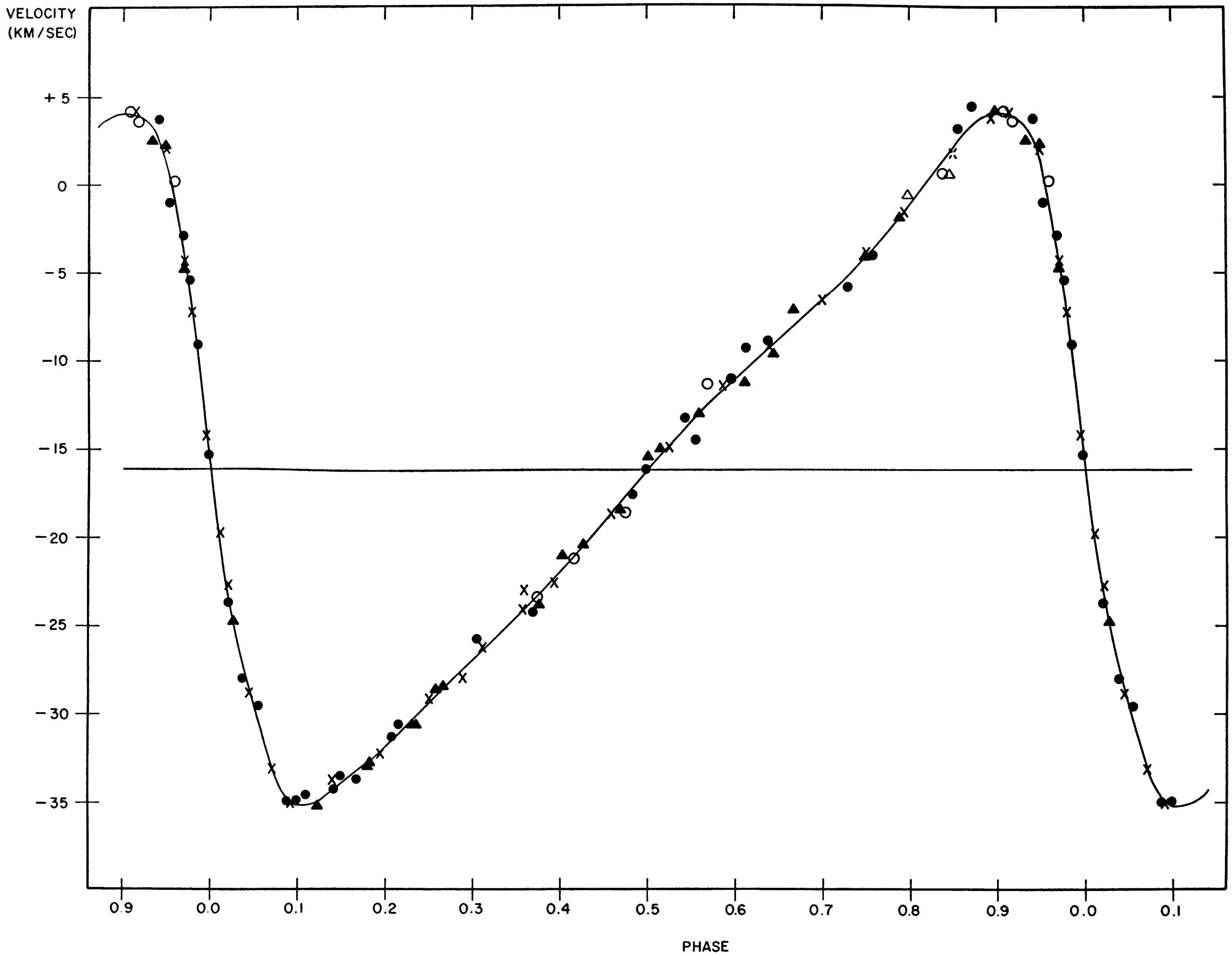
After the preliminary mean velocity-curve was shifted vertically by -0.05 km/sec and horizontally so that its steep branch crossed the mean velocity axis at zero phase, it was adopted as the final velocity-curve, which is reproduced in Figure 1. Ordinates were read off the curve at intervals in phase of $0\overset{\mathrm{P}}{.}01$, and these are presented in Table 4. The velocities in this table are given to an accuracy of 0.01 km/sec, not because it is thought that this accuracy is attained but rather to facilitate the drawing of a smooth curve through the points. Figure 1 also includes all the observations, with phases and velocities from Table 2. Full- and half-weight observations from each of the three series are distinguished by symbols as explained in the caption.

Finally, the residuals listed in column 7 of Table 2 were computed by subtracting from the observed velocities, given in column 5, the computed velocities obtained by interpolation in Table 4 with phases from column 4 of Table 2.

In most cases where velocities have been determined from individual lines in cepheid variables, it has been found that the velocity varies from line to line, and it is thought to depend, in general, on the level in the atmosphere at which the line originates. Among the most thorough high-dispersion studies of this type is one made recently by Grandjean (1956), who used two plates each of η Aquilae and ζ Geminorum. In both stars he found differential velocities of the order of 1.5–2.5 km/sec, depending on phase. Efforts to detect effects of this type in δ Cephei have been made by several investigators, notably by Jacobsen (1926), Petrie (1934), and Brück and Green (1942), but the only investigation in which they have been clearly detected in δ Cephei is a more recent study by Jacobsen (1949). He found differential velocities of approximately the same magnitude as those he measured in a similar study of η Aquilae (Jacobsen 1950). In 1926, however, he had been able to measure differential velocities in η Aquilae but not in δ Cephei. These studies suggest that the effect in δ Cephei is probably, at most, of no greater magnitude than that studied by Grandjean in η Aquilae and ζ Geminorum. Nevertheless, it seems desirable to attempt to assign the velocities measured in δ Cephei to some level in the atmosphere.

Grandjean has discussed correlations between differential velocity and (1) the chromo-

FIG. 1.—The radial velocity-curve of δ Cephei. Observations of the 1907 series are represented by circles; those of the 1923 series by triangles; and those of the 1950 series by crosses. Open symbols represent observations of half-weight. The phase is in units of the period.



spheric height, as determined by Mitchell (1913) from solar measurements; (2) the residual central intensity of the line; and (3) the excitation potential for lines of Fe I. Since the lines measured in the present program are due to a variety of elements, the possibility of using criterion 3 was eliminated. In choosing between the first and second criteria, it was decided to use the chromospheric height because, although residual central intensity is probably a more direct measure of the level in the atmosphere, it would be difficult to relate, in a meaningful way, the values that could be measured on the present plates with those derived by Grandjean from plates of η Aquilae and ζ Geminorum. Furthermore, Grandjean noted considerable dependence of central intensity on phase, and, as similar variations no doubt exist in δ Cephei, the problem of correlation would be further complicated.

After all the plates had been measured, the standard plate of γ Cygni was re-examined in order to decide which lines should be considered as having entered into the velocity determination, and 116 such lines were picked out, the selection including all the strongest lines in the region $\lambda\lambda$ 4391–4634. In those parts of the spectrum where strong lines were not present, several of the strongest lines available were listed, although these may

TABLE 4

ORDINATES OF FINAL VELOCITY-CURVE OF δ CEPHEI (KM/SEC)

Phase	$+0^{P}.00$	$+0^{P}.01$	$+0^{P}.02$	$+0^{P}.03$	$+0^{P}.04$	$+0^{P}.05$	$+0^{P}.06$	$+0^{P}.07$	$+0^{P}.08$	$+0^{P}.09$
$0^{P}.00$	−16 13	−20 11	−23 16	−25 62	−27 76	−29 68	−31 46	−33 07	−34 32	−34 97
.10	−35 23	−35 22	−35 05	−34 72	−34 35	−33 97	−33 56	−33 16	−32 75	−32 33
.20	−31 90	−31 43	−30 94	−30 43	−29 88	−29 35	−28 83	−28 32	−27 82	−27 34
.30	−26 87	−26 41	−25 94	−25 48	−25 01	−24 53	−24 05	−23 56	−23 05	−22 53
40	−22 00	−21 43	−20 87	−20 29	−19 70	−19 11	−18.50	−17 90	−17 31	−16 71
.50	−16 14	−15 57	−15 04	−14 50	−13 97	−13 47	−12 97	−12 47	−11 99	−11 51
.60	−11 04	−10 59	−10 14	− 9 71	− 9 27	− 8 82	− 8 38	− 7 94	− 7 50	− 7 03
.70	− 6 57	− 6 10	− 5 62	− 5 12	− 4 61	− 4 07	− 3 52	− 2 93	− 2 32	− 1 70
.80	− 1 06	− 0 41	+ 0 23	+ 0 88	+ 1 54	+ 2 19	+ 2 78	+ 3 27	+ 3 69	+ 3 95
0 90	+ 4 05	+ 4 03	+ 3 85	+ 3 47	+ 2 72	+ 1 22	− 1 23	− 4 18	− 7 50	−11 67

have been weaker than lines not considered in other regions. These 116 lines were identified with the aid of a table of the lines observed in α Persei by Dunham (1929), and the identifications were verified by comparison with the wave-length table of lines in δ Cephei published by Walraven (1948). Of the 116 lines considered, only 27 were found to be seriously blended on plates taken with the Mills spectrograph. As all plates were measured on a spectrocomparator where the line profile is matched against the profile of the same line on a plate of a similar star, the velocity derived from a blended line should be less sensitive to variations in the relative strengths of the components than when the center of gravity of the blend is measured with a micrometer wire. Hence it is felt that blending is not a serious source of error. Of the lines considered, 113 could be tentatively identified with the chromospheric lines listed by Mitchell, with about 90 per cent of these identifications being reasonably definite. Of these 113 lines, 94 (83 per cent) are assigned to chromospheric heights between 350 and 600 km. Of the remaining 19 lines, 4 are assigned to chromospheric height 300 km, and the other 15 to heights greater than 600 km, the highest being 2500 km. Thus it appears that the velocities measured in the present program are heavily weighted to correspond to a relatively restricted atmospheric level. An examination of Figure 2 of Grandjean's paper reveals that the range in velocity corresponding to chromospheric heights between 350 and 600 km is, at most, half the total differential. Since the effect of differential velocity in δ Cephei is probably no greater than in η Aquilae or ζ Geminorum, it appears that the differences in velocity between the majority of the lines measured was, at most, in the neighborhood of 1 km/sec.

The radial velocity-curve of δ Cephei has been observed at three different epochs over a total span of 43 years, and no evidence is found of change either in the form of the velocity-curve or in the mean velocity, which is determined as -16.1 ± 0.2 km/sec; the velocity amplitude is 39.4 km/sec. The velocities measured are ascribed to an effective atmospheric level where lines originate that are identified with chromospheric heights of 350–600 km in the sun.

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REFERENCES

Adams, W. S., and MacCormack, E. 1935, *Ap. J.*, **81**, 129; *Mt. W. Contr.*, No. 279.
Belopolsky, A. 1894, *A N.*, **136**, 281.
———. 1895, *Ap. J.*, **1**, 160.
———. 1909, *Mitt. Pulk. Obs.*, **3**, 63.
Brück, H. A., and Green, H. E. 1942, *M.N.*, **101**, 376.
Code, A. D. 1947, *Ap. J.*, **106**, 310.
Danjon, M. A 1927, *J. d'obs* , **10**, 1.
Dunham, T. 1929, *Princeton Contr.*, No. 9.
Grandjean, J. 1956, *Acadêmie Royal de Belgique, Classe des Sciences, Mêmoires, Collection in -8°*, Ser. 2, Vol **29**, No. 7; *Liêge, Institut d'Astrophysique, Collection de Mêmoires -8°*, No. 383.
Hertzsprung, E. 1919, *A.N.*, **210**, 17.
Jacobsen, T. S. 1926, *Lick Obs. Bull.*, **12**, 138 (No. 379).
———. 1949, *J R.A.S. Canada*, **43**, 142.
———. 1950, *ibid* , **44**, 104.
Mitchell, S A. 1913, *Ap J.*, **38**, 407.
Moore, J. H 1913, *Lick Obs Bull.*, **7**, 153 (No. 234).
———. 1932, *Pub. Lick Obs.*, **18**, 168.
Petrie, R. M. 1934, *Pub Obs. Michigan*, **6**, 37 (No 3).
Stebbins, J. 1945, *Ap J.*, **101**, 47; *Mt. W. Contr.*, No. 704.
Walraven, Th. 1948, *Pub Astr. Inst. U. Amsterdam*, No. 8.
Wilson, R. E. 1953, *General Catalogue of Stellar Radial Velocities* (Carnegie Inst. Washington Pub. No. 601).