PROPERTIES OF TWO INTERGALACTIC GLOBULAR CLUSTERS*

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ABSTRACT

Data for two very distant globular clusters are given. The 11^h cluster at R.A. (1950) = $11^h26^m.6$, Dec. (1950) = +29°15′, has an apparent distance of 1.2×10^5 pc if $\overline{M}_v$ is 0.00 for the horizontal branch, has a diameter of about 90 pc, an integrated absolute magnitude $M_v = -6.3$, an estimated mass of 1.8×10^4 $\mathfrak{M}_\odot$, and an estimated total number of stars of 4.3×10^4. The 10^h cluster at R.A. (1950) = $10^h3^m.0$, Dec. (1950) = +0°18′ has an apparent distance of 1.3×10^5 pc, a diameter of about 80 pc, an integrated M_v of -5.9, an estimated mass of 1.1×10^4 $\mathfrak{M}_\odot$, and an estimated total number of stars of 2.7×10^4.

The diameters of both clusters are unusually large compared with systems like M3 and M13, which are more populous by factors of at least 10. Large diameters are expected if the tidal force of the Galaxy, which limits the radii of most clusters, has always been small. The observed diameters can be explained if the two clusters have never passed closer than 9000 pc to the galactic center. This result requires that the clusters were formed at least 9000 pc from the galactic center.

The 10^h and 11^h clusters could reach their present distances in about 10^9 years if they are moving in parabolic orbits around the galactic center. If the clusters are escaped members of M31, they could reach their present position in about 10^{10} years if the clusters have the escape velocity and if we neglect the influence of the Galaxy. The true travel time from M31 will be less than 10^{10} years because of the attraction of the galactic system. Computation of the correct travel time from M31 is given by the restricted three-body problem, which is not discussed.

INTRODUCTION

Intergalactic globular clusters have been discovered with the 48-inch Schmidt telescope. Hubble found the first cluster (R.A. = $11^h26^m.6$, Dec. = +29°15′) on a 48-inch Schmidt plate taken in 1949 before the Sky Survey began. Later, this cluster was rediscovered on regular Sky Survey plates, along with many other new globulars. A finding list for 13 of the new clusters has been published by Abell (1955).

Beginning in 1951, plates of the most distant of Abell's clusters (Nos. 1, 3, 4, 12, and 13) have been taken sporadically with the 200-inch Hale telescope. Data for the two most distant clusters, Nos. 3 and 4, are given in this paper. These are called here the 10^h and 11^h clusters. Eleven blue plates (103*a*-0 + GG13 filter), and eleven yellow plates (103*a*-D + GG11) are available for the 11^h cluster. Three plates in each color are available for the 10^h cluster.

I. THE 11^h CLUSTER

a) *The C-M Diagram and Distance Modulus*

Figure 1 shows a reproduction of a limiting 200-inch exposure of the 11^h cluster on a 103*a*-D plate behind a GG11 filter. The faintest stars clearly visible on the original plate are at an apparent visual magnitude of about $V \approx 22.0$. The brightest cluster

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stars have $V = 17.5$. The apparent diameter of the cluster, traced to the faintest stars visible on the best plates, is about 150 seconds of arc. The small degree of central concentration places this and the 10^h cluster in the group with the well-known globular clusters NGC 5053 and NGC 5897. The concentration class for our clusters is either XI or XII.

The form of the C-M diagrams of the 10^h and 11^h clusters was not known at the start of this investigation. The first plates revealed immediately, however, that the brightest stars were red. Before photometric scales were determined in either cluster, we used a trick to find the general character of the C-M diagrams. Because calibration-curves for modern iris photometers are nearly linear in the plot of magnitude against astrophotometer reading, the general shape of the C-M diagram for any cluster can be found by plotting the astrophotometer readings on the 103*a*-D plates (called "D") as ordinate, and the difference between the readings on the 103*a*-D and the 103*a*-O

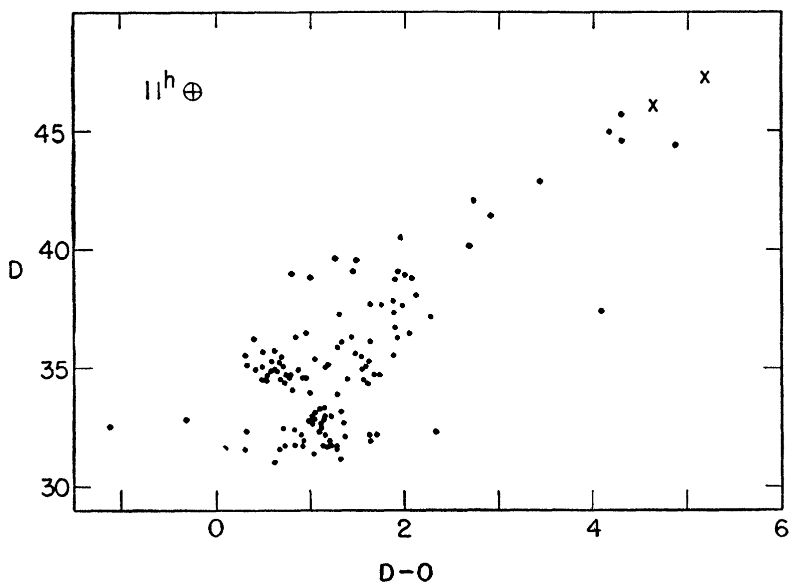

FIG. 2.—The pseudo C-M diagram for the 11^h cluster. The photometer readings on the 103*a*-D plate are plotted against the difference of the readings on the 103*a*-D and 103*a*-O plates. Variables 25 and 44 are plotted as crosses on the giant branch.

plates (called "D − O") as abscissa. If there are single-valued functions $D = f(V)$ and $O = h(B)$ and if these functions are nearly linear, then the D versus D−O plot will be the C-M diagram on an uncalibrated scale which may be stretched from a Pogson scale but not greatly distorted.

Figure 2 shows this pseudo C-M diagram for the 11^h cluster. The similarity with normal C-M diagrams for globular clusters is evident. Two variable stars, found during the photometry, are plotted as crosses in Figure 2 at their median readings. They will be discussed later.

Three photographic transfers in each color were made to the 11^h cluster from SA 51 or SA 57 on nights of good seeing. The unpublished photoelectric sequences of Baum were used for both selected areas. The internal consistency of the transfers suggests that our photometric scales are accurate to at least ±0.2 mag. in both B and V. This accuracy is clearly high enough to obtain preliminary values of the distances, true diameters, and integrated absolute magnitudes for each of the clusters.

Magnitudes in two colors were found for 109 stars in the 11^h cluster by the usual

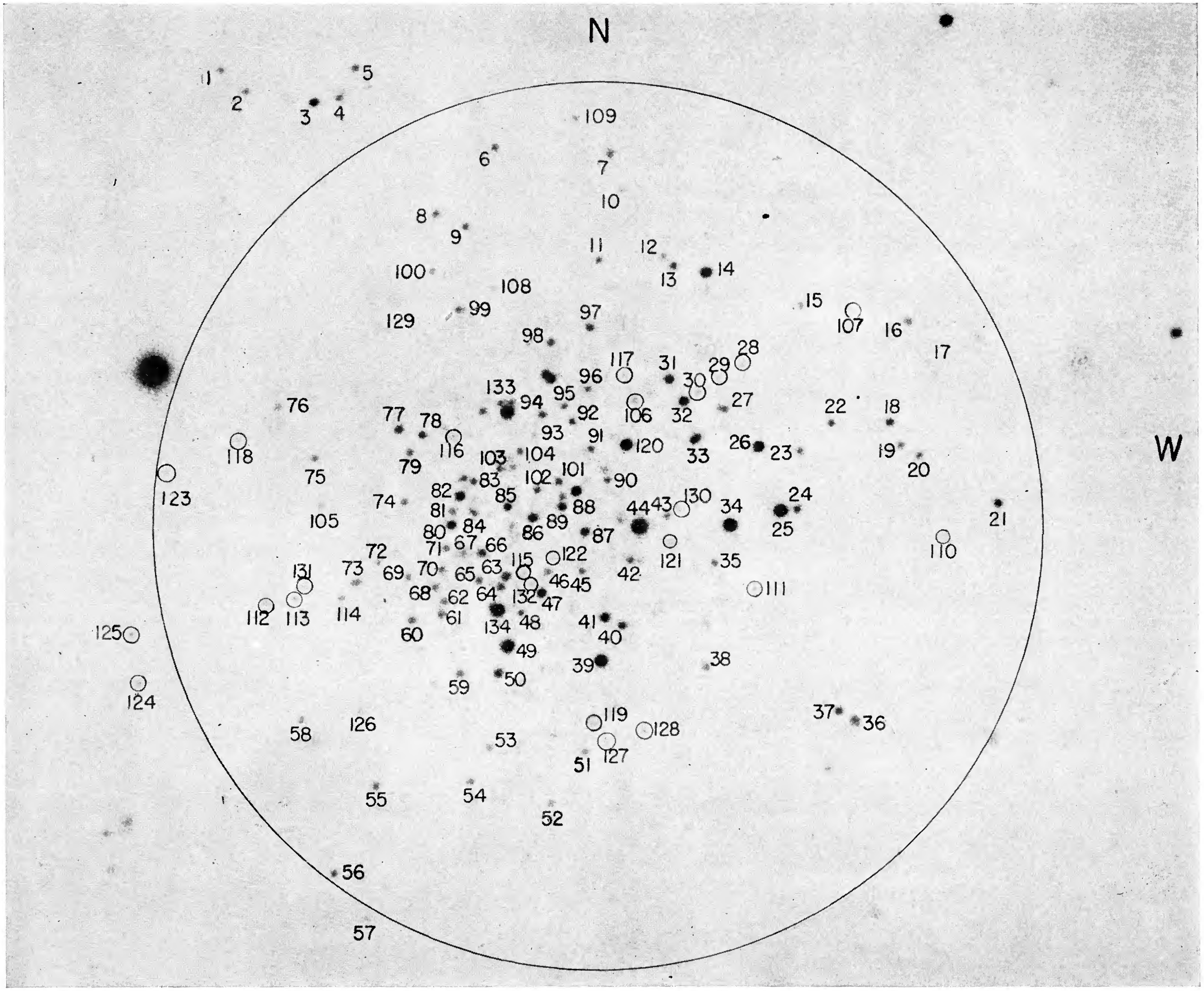

FIG. 3.—The identification chart for stars measured for color and magnitude. The V and $B - V$ for these stars are given in Table 1. The diameter of the circle is 190 seconds of arc.

methods of photographic photometry. Four plates in each color were measured with the Eichner iris-diaphragm photometer at the California Institute of Technology. Table 1 gives the resulting V and $B - V$ values for the program stars. The internal probable error for a single measure of V and B is ± 0.032 mag.; for a single measure of $B - V$ it is ± 0.045 mag.; for a tabulated V it is ± 0.016 mag.; and for a tabulated $B - V$ it is ± 0.023 mag. The true error is, of course, larger because of the larger errors for the transfer from the selected areas. Figure 3 is the identification chart for the stars listed in Table 1. The circle in Figure 3 has a radius of 95 seconds of arc.

TABLE 1

MAGNITUDES AND COLORS IN THE 11^h CLUSTER

Star	V	$B-V$	Star	V	$B-V$	Star	V	$B-V$
1	21 02	0 80	49	18 00	1 39	93	21 32	−0 08
2	20 97	0 81	50	19 67	0 90	94	20 18	0 75
3	19 76	1 58	51	21 15	0 85	95	20 46	0 50
4	20 59	0 53	52	21 25	0 77	96	20 56	0 82
5	20 51	0 61	53	21 00	0 91	97	20 00	0 65
6	20 55	0.62	54	20 96	0 81	98	20 12	0 86
7	20 55	0 68	55	20 59	0 59	99	20 51	0 55
8	20 57	0 60	56	20 60	0 62	100	21 08	0 85
9	20 44	0 86	58	21.12	0 78	101	20 05	0 80
10	21 60	0 79	59	20.46	0 56	102	20 28	0 46
11	20.62	0 90	60	20 48	0 56	103	20 11	0 76
12	21.25	0 81	61	20.36	0 88	104	20 32	0 68
13	20 30	0 90	62	20 73	0 73	105	21 44	0 77
14	19 06	1 05	63	19 40	0 88	106	21 19	0 76
15	21 10	0 89	64	19 68	0 85	111	21 57	1 09
16	20 59	0 55	65	20 25	0 82	113	21 56	0 64
18	20 06	0 94	66	19 65	0 87	114	21 29	0 80
19	20 54	0 67	67	20 69	0 66	115	21 09	0 79
20	20 50	0 93	68	20 46	0 57	116	21 25	0 95
21	19 84	1 02	69	20 76	0 84	119	21 59	1 13
22	20 40	0 74	70	20 41	0 59	120	18 59	1 04
23	20 53	0.63	71	20 44	0 51	121	21 35	0 80
24	19 94	0 91	72	21 42	0.64	122	21 44	0 41
26	18 74	1 10	74	20 52	0 92	124	21 70	1 11
30	21 74	0 62	76	21 15	0 25	125	21 73	0 79
31	19 58	0 94	77	19 78	0 88	129	21 67	0 90
32	19 34	0 88	78	20 07	0 62	132	21 36	0 51
34	17 97	1 61	79	20 42	0 87	133	17 92	1 36
35	20.48	0 65	80	19 33	0 75	134	17 74	1 39
37	20 03	0 97	81	20 22	0 56			
38	20 61	0 91	82	19 19	0 70			
39	18 39	1 19	84	20 09	0 48			
40	19 80	0 73	85	19 67	0 81			
41	19 36	0 91	86	19 21	0 75			
42	20 25	0 52	87	19.39	0 93			
43	20 49	0 58	88	18 96	0 88			
45	20 38	0 48	89	19 39	0 63			
46	20 29	0 84	90	20 42	0 52			
47	19 35	0 58	91	20 36	0 59			
48	20 29	0 58	92	20 35	0 56			

Blinking of the plates showed that stars Nos. 25 and 44 are long-period variables. Table 2 gives the B and V magnitudes of these stars on each of the 22 available plates. The material is inadequate to obtain unique light-curves and periods, but all data, with the exception of that on JD 2433412.75 for variable 25, fit a period of 166.7 days for variable 25 and a period of 131.6 days for variable 44. These must be checked by additional data, but they are normal for similar variables known in other clusters (Arp 1954). The amplitude of the light-curve is at least 2.1 mag. for variable 25 and 1.3 mag. for No. 44. The median V and B values are $V = 17.67$, $B = 19.37$ for variable 25; $V = 17.37$, $B = 18.97$ for variable 44. The light-curves are not sufficiently well observed to obtain precise values. When this investigation was nearly completed, we learned that L. Rosino had also discovered these two variables independently with the 120-cm telescope at Asiago and the 60-cm telescope at Loiano, Italy. From observations

TABLE 2

MAGNITUDES FOR VARIABLE STARS 25 AND 44

JD 2433+	STAR 25		STAR 44	
	V	B	V	B
061 70		18.69		18 54
062 70		18 71		18 65
062 72	16 73		16 93	
412 75		19 71		18 74
1370 04	17 51		16 99	
1397 87		18 60		19.23
1502.74		20 12		18 48
1502.77	18.52		16.94	
1504 74		20.20		18.54
1504.76	18 42		16.91	
1515.70	18.41		17.16	
1532.72		19.54		19 74
1532.74	18.04		17.90	
1533 70	18 04		17.99	
1533.72		19.53		19.84
1544.70	17.23		18.54	
1778.99		19 73		18.96
1779.02	18.08		17.27	
1888.73	17.09		16.89	
2874 90	17 48		18.73	
2874 93		19 04		20 37
2874 99		18 99		20.42

made during 1956 and 1957 he derived periods of about 100 days for both these variables (Rosino 1957). There are no RR Lyrae stars in this cluster. Charles Whitney blinked five pairs of plates and found no variables.

The C-M diagram plotted from the data of Table 1 is shown in Figure 4. Certain characteristic features of a normal globular cluster are evident: (1) the top of the giant branch occurs at $B - V = 1.6$; (2) the giant sequence slopes blueward for fainter magnitudes until the bifurcation point is reached at $B - V = 0.8$, where the horizontal branch leaves the nearly vertical subgiant branch; (3) the magnitude difference between the horizontal branch and the top of the giant branch is $\Delta V = 3.0$. These features are all normal. The serious abnormality of Figure 4 is the lack of stars on the horizontal branch bluer than $B - V = 0.4$. While it is true that the greatest variation in the C-M diagrams for the globular clusters studied so far (Arp 1955) occurs along this branch, there is no known cluster in our Galaxy that looks like Figure 4. In the seven clusters studied by Arp, the density of stars along the horizontal branch varies in a systematic way. The known clusters can be grouped in the order M13, M10, M2, M92, M15, M5, M3, where clusters near the M13 end have a heavy population at the extreme blue end of the horizontal branch, while clusters near the M3 end are about equally populated on the blue and red ends. This ordering also places the clusters approximately in a sequence of increasing redness of the giant branches and decreasing mean period of the RR Lyrae stars. An interpretation of these differences (Sandage 1957) assigns different absolute magnitudes to the horizontal branches of the various globular clusters.

What does this have to do with the 11^h cluster? Simply this: The stars along the horizontal branch are all pushed toward the red end. In this respect the 11^h cluster may be placed in the sequence M13, M10, M2, M92, M15, M5, and M3 at the extreme right end beyond M3. The correlation of the density distribution along the horizontal branch

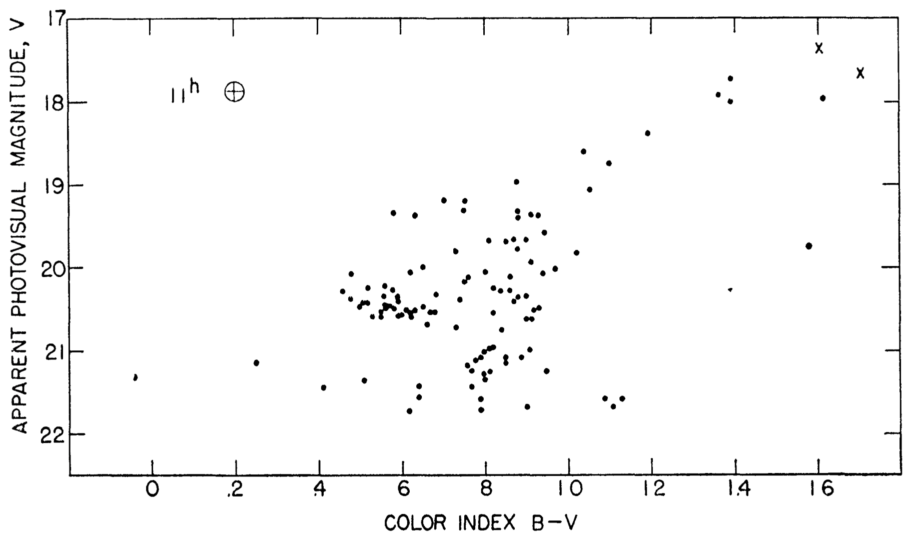

FIG. 4.—The C-M diagram for the 11^h cluster. The data are from Table 1

with the mean period of the RR Lyrae stars in these clusters gives warning that we cannot be certain of the modulus of the 11^h cluster. The $\Delta \log P = f(\Delta M_v)$ relation (Sandage 1957) gives $\Delta M_v = 0.2$ mag. as the expected difference in absolute magnitude between variables in the M3 group and the M92 group. Variables in the M3-type cluster are expected to be *fainter*. If we use the horizontal-branch density distribution as an indication of M_v, we expect 0.2 mag. $< M_v <$ 0.5 mag. for the 11^h cluster, but more data are obviously needed on this point.

The horizontal branch of the 11^h cluster occurs at $V = 20.5$. If $M_v = 0.00$ for these stars, then $m - M = 20.5$. Little or no reddening is expected because the galactic latitude is $+73°$. (No correction for the conventional "half-thickness" absorption of 0.25 mag. has been applied.) A modulus of 20.5 corresponds to a distance of 125000 parsecs. This is more distant than the Magellanic Clouds, whose modulus is presently adopted to be about 19.2. The apparent modulus of M31 is presently adopted to be 24.6 (Th. Schmidt 1957). The 11^h cluster is therefore about twice as distant as the Clouds and about one-fifth as distant as the Andromeda Nebula. The apparent diameter of 150 seconds of arc at a distance of 125000 parsecs gives a true diameter of 90 parsecs. If the correct absolute magnitude of the horizontal branch is $+0.5$, then the true modulus is 20.0, which corresponds to a distance of 10^5 parsecs and a diameter of about 70 parsecs. These abnormally large diameters are discussed in Section III.

b) *The Luminosity Function*

Counts in appropriate magnitude intervals in Table 1, together with counts of stars unmeasured due to crowding, give the luminosity function tabulated in Table 3. Here $\phi(M_v)$ is defined as the number of stars at M_v in $\Delta M_v = \pm 0.1$ within the circle of 95-sec. radius. There are only 123 stars in the 11^h cluster brighter than $M_v = +1.1$. We have assumed in this tabulation that $m - M = 20.50$. Figure 5 shows a histogram for $\phi(M_v)$. The luminosity function for M3 (Sandage 1954) multiplied by a reduction factor of 0.072 is shown as a solid line. The fit is quite satisfactory. Integration of $\phi(M_v)$ weighted by $10^{-0.4V}$ gives an integrated apparent magnitude of $V = 14.57$ for stars brighter than $M_v = +1.1$. In M3, stars brighter than $M_v = 1.1$ contribute only 0.684 of the total

TABLE 3

THE LUMINOSITY FUNCTION FOR THE 11^h CLUSTER

V	M_v*	$\phi(M_v)$	V	M_v*	$\phi(M_v)$
17 30	−3 2	1	19.50	−1 0	4
17 50	−3 0	0	19 70	−0 8	5
17 70	−2 8	2	19 90	−0 6	3
17 90	−2 6	2	20 10	−0 4	11
18 10	−2 4	1	20 30	−0.2	14
18 30	−2 2	1	20 50	0 0	27
18 50	−2 0	1	20 70.	+0 2	7
18 70	−1 8	1	20 90	0 4	5
18 90	−1 6	1	21 10	0 6	11
19 10 .	−1 4	2	21 30	0 8	9
19 30 .	−1 2	7	21 50	1 0	8

* Computed with the apparent modulus $m - M = 20\ 50$,

$$\sum_{17\ 30}^{21\ 50} \phi(M_v) = 123\,.$$

light. If we assume a similar distribution for the 11^h cluster, the *total* integrated apparent magnitude becomes $V = 14.15$. This corresponds to an absolute visual magnitude of $M_v = -6.35$ for $m - M = 20.5$. Normal globular clusters have an international color index of about $CI \approx 0.50$, which requires $M_{pg} = -5.85$. Christie's study (1940) of the integrated absolute magnitudes of 68 globular clusters in the Galaxy shows that only 5 clusters are fainter than $M_{pg} = -5.5$. Therefore, the 11^h cluster is one of the intrinsic faintest globulars known. W. Lohmann's more recent study (1953) gives similar results.

An estimate of the mass follows if we assume that the mass-to-light ratio is the same as for normal globular clusters ($\mathfrak{M}/L \approx 1$ in solar units). This estimate is $\mathfrak{M}_T = 1.8 \times 10^4\ \mathfrak{M}_\odot$ for the 11^h cluster. The total number of stars is estimated to be 4.3×10^4. This was found by scaling the $\phi(M)$ for M3 by 0.072.

Sidney van den Bergh has studied the 11^h cluster on prints of the Palomar Sky Survey (1956). He obtained a magnitude scale by measuring image diameters directly on the Sky Survey prints with calibration-curves found from known stars in M67 and M3. The inherent errors in this method are large because of the improbability that the Schmidt plates which contain M3, M67, and 11^h cluster were exposed and developed in identical fashion. Furthermore the Survey plates for M3, M67, and the 11^h cluster were taken several years apart. Nevertheless, van den Bergh obtained values for the integrated properties of the 11^h cluster which are moderately close to our data. Furthermore, he was the first to call attention to the globular-cluster characteristics of this object.

Table 4 summarizes the available data for the 11^h cluster. We have not applied the absorption correction of 0.25 mag. to van den Bergh's basic material.

II. THE 10^h CLUSTER

The 10^h cluster (Abell No. 3) is at R.A. (1950) = $10^h3^m.0$, Dec. (1950) = 0°18′ (l = 209°, b = +43°). The data for this cluster are not nearly so accurate as for the 11^h

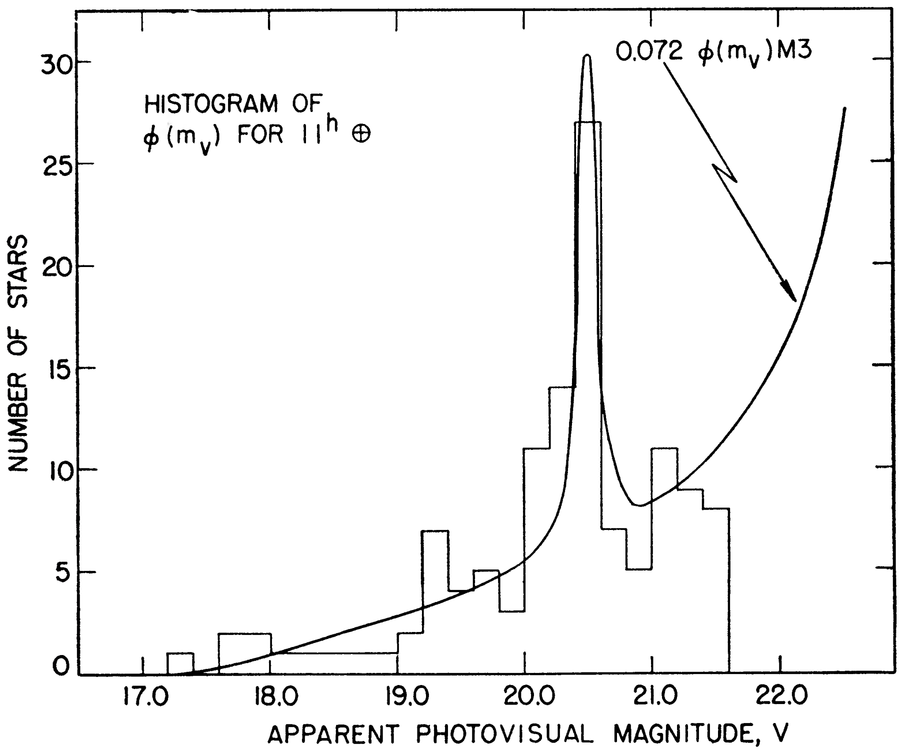

FIG. 5.—The luminosity function for the 11^h cluster. The solid line is the $\phi(M_v)$ for M3 but reduced by a factor of 0.072.

TABLE 4

PROPERTIES OF THE 11^h CLUSTER

	Present Investigation	Van den Bergh
Apparent diameter	150 sec. of arc	85 sec. of arc
Distance modulus	20.5, 20 0	21.05
Distance	125000 pc, 100000 pc	162000 pc
True diameter	90 pc, 73 pc	67 pc
No. of stars brighter than $M_v = +1$ 1	123	. . . .
Integrated V	14 15	14 4
Integrated M_v	−6 35, −5 85	−6 65
Estimated mass	1 8×10⁴ 𝔐⊙	
Estimated total no. of stars	4 3×10⁴	

cluster because the photometric scale depends on only one photographic transfer in each color from SA 57. Nevertheless, the data do give preliminary values which are probably not in error by more than ± 0.3 mag.

A reproduction from a limiting 200-inch exposure on a 103*a*-D plate behind a Schott GG11 filter is shown in Figure 6. The diameter of the circle is 168 seconds of arc and is somewhat larger than the cluster can be traced. The estimated cluster diameter is about 130 seconds of arc.

The pseudo C-M diagram is shown in Figure 7. The D and D − O values are the means of two plates in each color; the means were taken after the plates had been reduced to a common system of astrophotometer readings. Preliminary calibration of the D and D − O values has been made from the one photographic transfer available in each color and is given in Table 5. Note that the end of the giant branch occurs at $V \approx 17.8$, $B - V \approx 1.6$; that the splitting of the subgiant and horizontal branches occurs at $V \approx 20.6$, $B \approx 0.9$; and that the mean apparent magnitude of the horizontal branch is $V \approx 20.6$. The difference between the top of the giant sequence and the horizontal branch is $\Delta V = 2.8$ mag. These data, together with Figure 7, show that the 10^h cluster has a typical globular-cluster C-M diagram. It is more normal than the 11^h cluster because the horizontal branch extends to bluer colors. We shall adopt a distance modulus of $m - M = 20.6$ and will not correct for unknown galactic obscuration, if any.

One variable has been found in this cluster. It is identified on Figure 6 and is shown as a cross in Figure 7. The variable is near maximum light in Figures 6 and 7. It is almost certainly an RR Lyrae star, but its period and light-curve are unknown. No variable stars are present on the giant branch.

Star counts on the original plates give 76 stars brighter than $V = 21.7$ ($M_v = +1.1$) within the circle shown in Figure 7. This is to be compared with 123 stars in the same absolute-magnitude interval for the 11^h cluster. The integrated V for the 10^h cluster is then 14.68 if the two clusters have a similar $\phi(M_v)$. This corresponds to $M_v = -5.92$ for $m - M = 20.60$. If $CI = 0.50$, then $M_{pg} = -5.42$, which places the 10^h cluster among the faintest globular clusters known. A summary of the data is given in Table 6.

III. DISCUSSION

a) The Linear Diameters

Tables 4 and 6 show that both clusters have abnormally large diameters compared with normal halo-type globular clusters (M3, M92, M15, M13, etc.). The diameter of a globular cluster is not well defined. For differential purposes, we may compare the diameter to which cluster stars can be traced on photographic plates exposed to reach say 1.5 mag. fainter than the horizontal branch. This corresponds to limiting exposures on the 10^h and 11^h clusters with the 200-inch telescope (see Figs. 4 and 7). A series of plates on M3 exposed for this limit was examined. The diameter of M3 estimated from these plates was 760 seconds of arc, which corresponds to a linear diameter of 50 parsecs at a distance modulus $m - M = 15.7$. This diameter is appreciably smaller than the values for the 10^h and 11^h clusters when the extreme sparseness of these clusters is considered.

Von Hoerner (1957) has shown that the diameter of globular clusters is probably determined by the disruptive action of the galactic tidal force. The acceleration on a star in a globular cluster is composed of the acceleration of the galactic mass plus the acceleration of the cluster as a whole. For a star at some distance r_s from the cluster center, in the direction of the galactic center, these accelerations will be equal. Stars will be evaporated from the cluster when $r > r_s$. Von Hoerner shows that the stability radius of the cluster is given by

$$r_s = R_g \left(\frac{m}{2 \mathfrak{M}_g} \right)^{1/3}, \tag{1}$$

1958ApJ...127..527B

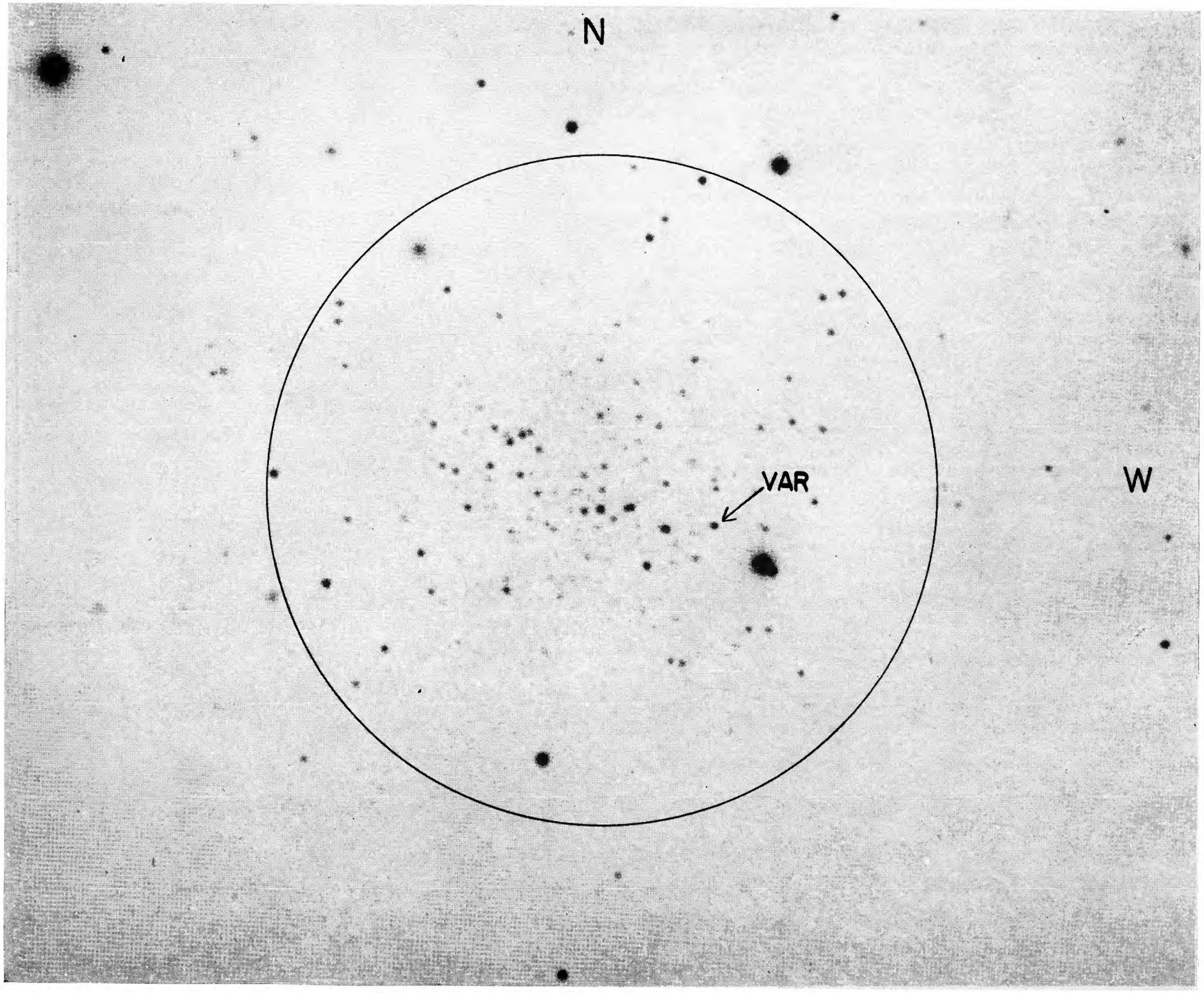

FIG. 6.—A 200-inch photograph of the 10^{h} cluster. The plate is a 103a-O behind a Schott WG2 filter. The diameter of the circle is 168 seconds of arc.

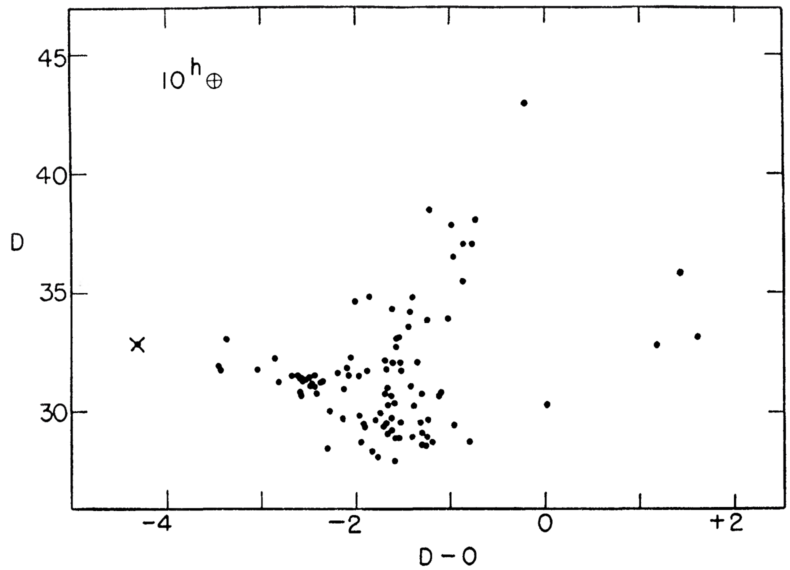

FIG. 7.—The pseudo C-M diagram for the 10^h cluster. The one known variable is plotted as a cross

TABLE 5

PRELIMINARY CALIBRATION OF FIGURE 7 FOR THE 10^h CLUSTER

D	V	D−O	$B-V$
	Giant and Subgiant Branch		
38 0	17 8	−0 65	1 6
37 0	18 1	−0 80	1 5
36 0	18 4	−0 95	1 4
35 0	18 8	−1 10	1 3
34 0	19 2	−1 22	1 2
33 0	19 7	−1 35	1 1
32 0	20 3	−1 45	1 0
31 0	20 8	−1 55	0 9
30 0	21 5	−1 64	
	Horizontal Branch		
31 5	20 6	−2 0	0 7
31 3	20 7	−2 5	5
31 5	20 6	−3 0	2
32 0	20 3	−3 5	0 1

where R_g is the distance of the cluster from the galactic center, m is the mass of the cluster, and $\mathfrak{M}_g$ is the mass of the Galaxy.[1] Equation (1) assumes that all the mass of the Galaxy is concentrated at the center and neglects the time-dependence of the evaporation process. Nevertheless, the equation should be correct as to order of magnitude.

The introduction of the tidal effect explains several facts in globular-cluster astronomy. (1) Kurth, Lohmann (1953), and van den Bergh (1956) have shown that there is a systematic increase in the diameter of globular clusters with increasing distance from the galactic nucleus. Van den Bergh suggests that the tidal force is at work stripping cluster members as the clusters approach regions of high galactic accelerations. (2) The clusters near the center of the Galaxy, such as NGC 6522 and the seven clusters described by Morgan (1956), have very small linear diameters. The clusters in Morgan's Table 2 have average angular diameters of only 3 minutes of arc, which corresponds to about 7 parsecs at the distance of the nucleus. Of course, these small diameters may result from inability to trace the outer regions of clusters against the dense background of nuclear stars. But this is not a likely explanation, because (*a*) for NGC 6522, Baade states that the RR Lyrae stars associated with this cluster indicate a radius of less than 10 parsecs and (*b*) 200-inch plates of NGC 6440, NGC 6637, and NGC 6356, all of which are near the nucleus, clearly show that the angular diameters are much smaller than would be the case if M3 were superposed on an equal background field at the same distance. Equation (1) gives an adequate answer to the problem.

TABLE 6

PROPERTIES OF THE 10^h CLUSTER

Apparent diameter	130 sec. of arc
Distance modulus	20.6
Distance	132000 pc.
True diameter	83 pc
No. of stars brighter than $M_v = +1$ 1	76
Integrated V	14 68
Integrated M_v	−5 92
Estimated mass	1 $1\times10^4\,\mathfrak{M}\odot$
Estimated total no. of stars	2 7×10^4

It is now of interest to compute R_g from equation (1) with the m and r_s values for the 10^h and 11^h clusters. If $\mathfrak{M}_g = 7 \times 10^{10}\,\mathfrak{M}\odot$ (M. Schmidt 1956), then equation (1) gives $R_g \approx 9000$ parsecs. This means that both the 10^h and 11^h clusters must never have passed closer than $\sim 10^4$ parsecs from a mass of order $10^{11}\,\mathfrak{M}\odot$. Otherwise their radii would be smaller than that observed. This result shows that both clusters were created at large distances from the galactic center and emphasizes their extragalactic character.

It might be argued that this result does not necessarily follow. It is conceivable that the clusters at some time could have been closer than 9000 parsecs from the galactic nucleus. At the distance of closest approach, R_1, the limiting radius of the clusters would have been r_1, given by equation (1). If $R_1 < 10000$ parsecs, then $r_1 < 50$ parsecs for the same mass. At a later time, when the clusters were far from the galactic nucleus (as they are now), it might be argued that the stars remaining in the cluster have moved outward to the presently observed radius. Such an event would invalidate the conclusions of the last paragraph. But this possibility appears to be unlikely, for, if it had happened, energy must have been supplied to the expanding stars equal to the work they did against

[1] There is an error in the position of the factor 2 in Von Hoerner's equation (33). It has been corrected here.

the gravitational field of the cluster. This energy must have come either from an outside agent or from a rearrangement of the cluster itself. Outside energy sources are quite unlikely. This leaves only the internal source. If the central regions of the cluster were to contract, a release of part of the gravitational energy would occur, and this energy would be available to the outer stars for their expansion. This is an example of the result that, for two configurations of the same potential energy, that configuration with the greatest central concentration has the largest radius.

The physical reason for the foregoing sequence of events is the exchange of energy between the cluster members by interaction. In one relaxation time, an equilibrium velocity distribution is set up in which stars in the Maxwell tail will always populate the outer regions. These stars have robbed the central stars of part of their kinetic energy thus causing the initial contraction. But if this mechanism has operated in the 10^h and 11^h clusters, we must expect some degree of central concentration. However, the observations show an almost uniform density distribution. The conclusion, therefore, is that the radii of the 10^h and 11^h clusters have never been smaller than their present values, and this requires that R_g has never been smaller than about 10000 parsecs.

b) Travel Times

It is of interest to mention some characteristic times for various assumptions concerning the orbits of the two clusters. If we neglect the gravitational influence of M31 and other members of the local group, then the time for the 11^h cluster to reach R = 1.25×10^5 parsecs from the galactic center can be computed if it has the escape velocity. If we approximate the gravitational effect of the Galaxy by a Newtonian central force and assume rectilinear orbits, this time is given by

$$t = \frac{2}{3}\left(\frac{R}{2G\mathfrak{M}}\right)^{1/2} R . \qquad (2)$$

We have seen in the last subsection that rectilinear orbits are not permissible because the distance of closest approach must be greater than 9000 parsecs. The time for the 11^h cluster to reach $R = 1.25 \times 10^5$ parsecs on a parabolic orbit with R (min.) = 9000 parsecs is obtained exactly from Euler's theorem, but this time is given to close approximation by equation (2). If $R = 3.8 \times 10^{23}$ cm, $\mathfrak{M} = 1.4 \times 10^{44}$ gm, then $t = 1.1 \times 10^9$ years. The present parabolic orbital velocity would be about 70 km/sec. Consideration of the geometry of the situation (earth, galactic center, $11^h \oplus$) shows that the observed radial velocity for the 11^h cluster will be approximately 70 cos 10° = 69 km/sec if it is indeed traveling in a parabolic orbit around the galactic center with R (min.) = 9000 parsecs. The orbit, of course, need not be parabolic. If we do not specify R (min.), then the problem is indeterminate, and radial-velocity measurements will tell us little concerning a choice between hyperbolic, parabolic, or elliptical orbits unless the measured radial velocity is >70 km/sec, in which case the present orbit is hyperbolic.

For the case of a circular orbit with semimajor axis of 1.25×10^5 parsecs, the period of complete revolution around the galactic center is

$$t_{\text{orbit}} = 2\pi R \left(\frac{R}{G\mathfrak{M}}\right)^{1/2} = 1.5 \times 10^{10} \text{ years} .$$

These times should be compared with the lifetime of the cluster of about 6×10^9 years.

It is interesting to note that the travel time for the clusters from the Andromeda Nebula is about 10^{10} years if we assume that the clusters have the escape velocity and if we neglect the influence of the Galaxy. The added acceleration due to the Galaxy will decrease this quoted time. The exact solution to the problem is found in the restricted three-body problem, which will not be discussed.

c) Mean Density

Clusters of the 10^h and 11^h type are very difficult to discover because of their faint absolute magnitude. They may populate space in great numbers and remain undetected. It is therefore of interest to estimate their contribution to the mean density of space. Suppose such clusters are spread uniformly with a density of 20 clusters in a sphere of radius 1.25×10^5 parsecs. This number is a generous upper limit compared with the known clusters in Abell's list. (The assumption of uniform space distribution, of course, presupposes that such clusters have no concentration around galaxies.) If each cluster has an average mass of 1.5×10^4 $\mathfrak{M}_\odot$ (3×10^{37} gm), then the mean density due to these objects is 3×10^{-33} gm/cm^3—a number negligible compared with the mean density of visible matter in the universe.

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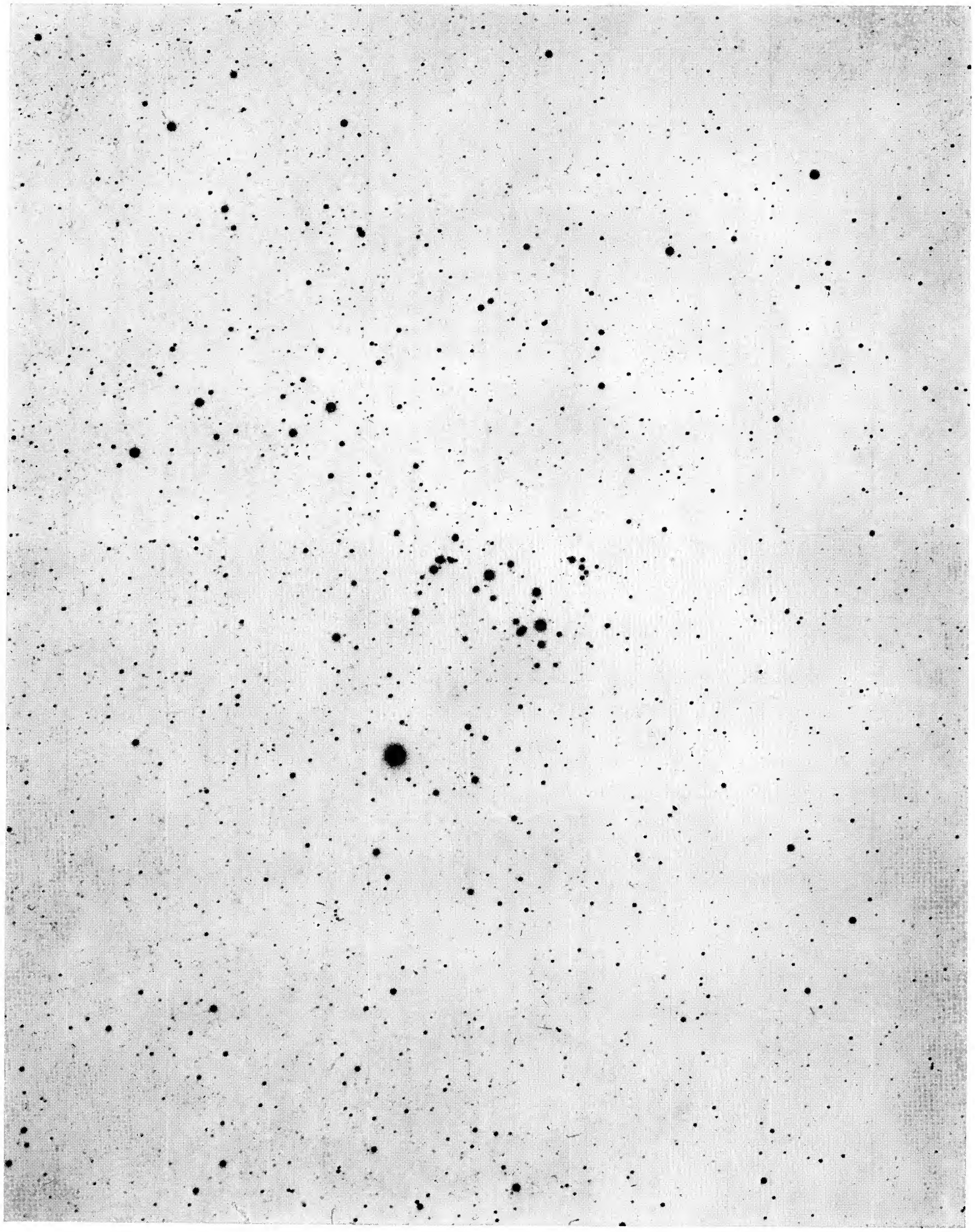

FIG. 1.—Photograph of NGC 6633 taken with the 10-inch Cooke at the McDonald Observatory. North is at the top and east to the left.