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CURRENT PROBLEMS IN THE EXTRAGALACTIC DISTANCE SCALE*

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ABSTRACT

In principle, a decision between the simplest cosmological models (exploding cases with $\Lambda = 0$, $k = +1, 0, -1$, or the steady-state case) is possible from the observed velocity-distance relation. Two numbers are needed. These are H and the deceleration parameter, $\ddot{R}_0/R_0H^2$. This paper discusses the determination of H.

Problems connected with the use of cepheid variables as distance indicators are discussed. Because of a finite width of the instability region for cepheids in the color-magnitude (C-M) diagram, intrinsic scatter in the period-color and period-luminosity (P-L) relation is expected. The observed period-color relation at median light for field cepheids in our Galaxy shows that Eggen's type C variables are oscillating in a higher mode than cepheids of Eggen type A, B. The data show $Q_{A,B}/Q_C \approx 1.9$. If the region of instability for cepheids in the C-M diagram has a width of $\Delta(B-V) = 0.2$ mag, then the P-L relation is expected to have a scatter of 1.2 mag. The intrinsically bluest cepheids should be the brightest. Arp's two-color data for cepheids in the SMC confirm these predictions.

The brightest stars are discussed as distance indicators for galaxies beyond the local group. It is probable that knots identified by Hubble as brightest stars in more distant resolved galaxies are really H II regions. From data in M100, the stars appear to be 1.8 mag. fainter than the knots. This correction, together with a correction of 2.3 mag. to Hubble's moduli for galaxies in the local group, suggests a total correction of about 4.1 mag. to the 1936 scale of distances. This gives $H \approx 75$ km/sec 10^6 pc or $H^{-1} \approx 13 \times 10^9$ years, with a possible uncertainty of a factor of 2. The connection of this value of H with the time scale of exploding cosmologies is briefly discussed.

I. INTRODUCTION

The velocity-distance relation is perhaps the most important single instance where cosmological theory meets the test of observational astronomy. In principle, if both the shape and the absolute calibration of this relation are known, a decision between various world models is possible (Robertson 1955; Hoyle and Sandage 1956). Observation must provide two numbers. These are (1) the present numerical value of the expansion rate and (2) the change of this rate with time. Curiously, the second datum, often called the "deceleration parameter," may be the easiest to determine because only the *ratio* of distances to galaxies rather than their absolute values are required. Deviations from linearity of the *relative* velocity-distance relation alone is sufficient.

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The velocity-distance relation appears to be strictly linear at any given cosmic time. This is predicted by most cosmological models and is confirmed by observation (Hubble 1929, 1936, 1953; Humason, Mayall, and Sandage 1956). The relation is expressed as

$$\frac{c\Delta\lambda}{\lambda}=H r , \tag{1}$$

where H is often called the "Hubble constant." H must be independent of spatial coordinates if the cosmological principle is obeyed but will, in general, be a function of time. The time dependence is the cause of the non-linearity in the *observed* velocity-distance relation because, when great distances are surveyed, the universe is sampled at increasingly earlier cosmic times. The more distant sources are observed to have the expansion rate valid a time τ ago, where τ is the travel time for light. And because, in general, $H(t) \neq H(t-\tau)$, these distant galaxies will deviate from the linear relation valid for the nearby sources unless corrections are made to reduce all observations to the same cosmic time. Consequently, the non-linearity arises from two circumstances: (1) we look back in time as we look out in space, and (2) the Hubble constant is time-dependent. Although discussion of the deceleration parameter $q_0 = \ddot{R}_0/R_0H_0^2$ is beyond the scope of this paper, the sign and numerical value of the parameter are important for the conclusions of the last section. The present observational data for q_0 are given by Humason *et al.* (1955) and more recently by Baum (1957).

This paper is concerned with the absolute value of H at the present cosmic time rather than its time derivative. The determination of H is very difficult. The problem is equivalent to finding the distances to galaxies sufficiently remote to have systematic red shifts larger than the mean random velocity of 300 km/sec (Hubble 1936*b*), yet not so remote that the very brightest distance indicators have faded below the plate limit and are undetected. Precision indicators can be isolated and measured in galaxies with distance moduli smaller than about 31.0. Systematic red shifts larger than 1000 km/sec probably do not exist for galaxies with moduli smaller than $\sim$30.0. Consequently, the overlap distance is very small where the two circumstances of resolvability into stars and large red shifts are simultaneously met; and, further, in this twilight zone, probable errors of measurement begin to mount even with the 200-inch telescope.

There are three principal steps to find H: (1) Distances to the nearby galaxies can be found by the use of cepheid variable stars and normal novae if the details of the cepheid and novae criteria are understood. (2) In the same galaxies where cepheids are isolated, brighter distance indicators must be calibrated with respect to the cepheids. The only reliable indicators seem to be the brightest stars in the Sb and Sc galaxies and the globular clusters in elliptical systems. (3) The brightest stars and globular clusters must be isolated and photometrically measured in galaxies with $c\Delta\lambda/\lambda \geq 1000$ km/sec. The photometric data, together with the calibration from step 2, solve the problem.

II. THE CEPHEID PROBLEM

From 1925 to the present day, the period-luminosity (P-L) relation for cepheid variables has been almost exclusively used to find distances to galaxies in the local group (Hubble 1925, 1926, 1929*b*). Justification for its use has always come from the empirical data rather than from theoretical considerations, even though the extensive investigations in the Magellanic Clouds by Shapley, Nail, and collaborators have shown that a large scatter exists in the relation. Most investigators, following Shapley, have usually assumed that the true P-L relation is narrow and that the observed scatter is the result of photometric uncertainties together with the presence of internal absorption in the Clouds. But the possibility exists that the P-L relation is not unique and the scatter real. The first observational data bearing on this point were the precise photometry of Arp (1955) on the cepheids in globular clusters. These data showed that the P-L relation was *not* unique but rather has a scatter of more than 1 mag. at a given period. Observational errors can be excluded here because the precision of Arp's data was very high. The existence of intrinsic scatter in $P = f(L)$ suggests that a third parameter is involved.

In what follows, a semitheoretical relation is found which indeed does involve three parameters. These are P, M_v, and the color $B-V$. The intrinsic scatter predicted by this treatment makes the use of cepheids for distance determinations more difficult than was previously believed.

a) The Semitheoretical P, M_v, $B-V$ Relation

Synopsis.—In what follows, $P(\bar{\rho}/\bar{\rho}\odot)^{1/2} = Q$ is assumed. Appropriate approximations to the mass and radius factors in $\bar{\rho}$ are made in terms of the observed parameters M_v and $B-V$. The resulting equation $P = f(M_v, B-V, Q)$ can then be separated into the period-color and P-L relations if the region in the color-magnitude (C-M) diagram occupied by the cepheids is known. A finite width in color index to this region of instability produces intrinsic scatter in the period-color and P-L relation.

Details.—Assume that

$$P\left(\frac{\bar{\rho}}{\bar{\rho}\odot}\right)^{1/2} = Q\,. \tag{2}$$

This transforms to

$$\log P + \tfrac{1}{2}\log\frac{\mathfrak{M}}{\mathfrak{M}\odot} + 0.3M_{\text{bol}} - 0.3M\odot_{\text{bol}} + 3\log\frac{T_e}{T_e\odot} = \log Q\,. \tag{3}$$

We wish to eliminate the mass by the mass-luminosity relation, change M_{bol} to M_v by the $\Delta M_{\text{bol}} = f(B-V)$ relation, and express $\log T_e$ in terms of $B-V$.

Consider, first, the mass-luminosity relation. Kuiper's data (1938*b*) are well fitted by

$$M_{\text{bol}} = 3.96 - 8.22\log\frac{\mathfrak{M}}{\mathfrak{M}\odot} \qquad (-8 < M_{\text{bol}} < +1)\,. \tag{4}$$

If the evolutionary tracks of a cepheid are similar to those of other stars in the giant region, then the customary assumption of a 1-mag. rise from the age-zero main sequence gives

$$M_{\text{bol}} = 2.96 - 8.22\log\frac{\mathfrak{M}}{\mathfrak{M}\odot} \qquad (-8 < M_{\text{bol}} < +1) \tag{5}$$

as an approximation to the mass-luminosity law for cepheids.

Consider, next, the bolometric correction. Some of the stars used by Kuiper (1938*a*) to define the correction for stars later than A0 have been measured photoelectrically by Eggen (1956). The resulting $\Delta M_{\text{bol}} = f(B-V)$ relation can be represented adequately in the range $1.0 > B-V > 0.3$ by

$$M_{\text{bol}} = M_v + 0.202 - 0.406\,(B-V) \qquad (1.0 > B-V > 0.3)\,. \tag{6}$$

Finally, the $\log T_e = f(B-V)$ relation is represented in the range $0.9 > B-V > 0.3$ by

$$\log T_e = 3.969 - 0.318\,(B-V) \qquad (0.9 > B-V > 0.3)\,, \tag{7}$$

which is an approximation to the full non-linear relation obtained from the temperature–spectral class relation quoted by Morgan and Keenan in Hynek's *Astrophysics*, and the spectral class–color relation of Johnson and Morgan (1953).

Substitution of equations (5), (6), and (7) in equation (3) gives

$$\log P - 1.051\,(B-V) + 0.239M_v = \log Q + 0.588\,, \tag{8}$$

where $M_{bol} \odot = +4.84$ and $T_e \odot = 5713°$ K have been adopted. Here $M_{\bar{v}}$ is strictly the visual absolute magnitude of the cepheid taken as a time average over the light-curve expressed in intensity units and converted back to a magnitude scale. This is because $\bar{\rho}$ in equation (2) refers to the static equilibrium structure of the star about which the pulsation perturbations are applied.

Equation (8) is the final result. It is an equation containing not only the period and luminosity of a cepheid but a color term as well. It is this color term which introduces intrinsic dispersion in the P-L relation.

We can now isolate both the mean P-L relation and the mean period-color relation by elimination of either $B-V$ or $M_{\bar{v}}$ from equation (8) with the equation for the region of instability of cepheid variables in the C-M diagram. Figure 1 shows the composite C-M diagram for selected galactic clusters. The region of the cepheids is drawn in this diagram from (1) Arp's precise two-color data for cepheids in the Small Magellanic

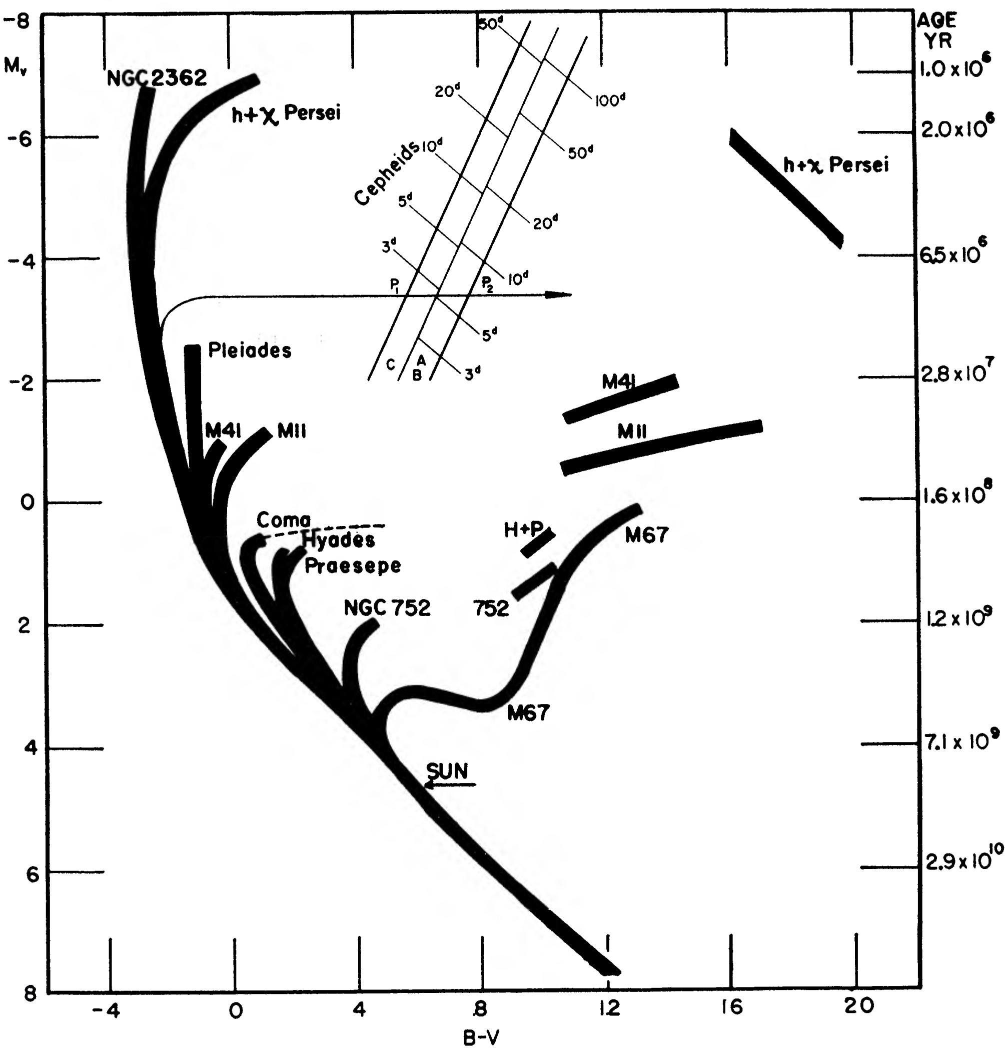

FIG. 1.—A composite C-M diagram for a number of galactic clusters. The region of the cepheids is shown. The Eggen type C variables are separated from the type A and B variables in analogy to the RR Lyrae domain. Lines of constant period are drawn from equation (8) with log $Q_{AB} = -1.39$ and log $Q_C = -1.66$.

Cloud (Arp 1958, unpublished) and (2) data compiled by Kraft (1957) from the work of Irwin (1955), Arp, Johnson, Kraft, and Sandage in the four galactic clusters M25, NGC 6087, NGC 6664, and NGC 7790, which contain the cepheids U Sgr, S Nor, EV Sct, and CF Cas. I am indebted to H. C. Arp for permission to quote his results before publication.

By analogy with the RR Lyrae domain in globular clusters, it is likely that the classical cepheid domain is of finite width in $B-V$, and, indeed, Arp's data show this to be the case. We have drawn the domain in Figure 1 with an intrinsic width of $\Delta(B-V) = 0.2$ mag. The mean line through the center of the domain has an equation

$$M_{\dot{v}} = 4.0 - 11.1\,(B - V) \tag{9}$$

or

$$B - V = 0.36 - 0.090 M_{\dot{v}}\,. \tag{9a}$$

If we neglect for the moment the width of the domain, then substitution of equation (9) or (9*a*) in equation (8) gives

$$M_{\dot{v}} = -3.0 \log P + 3.0 \log Q + \text{const.} \tag{10}$$

for the predicted P-L relation and

$$(B - V)_{\text{mean}} = 0.27 \log P - 0.27 \log Q + \text{const.} \tag{11}$$

for the predicted period-color relation. These are to be compared with the observed relations of

$$M_{\dot{v}} = -2.5 \log P - 1.77 \tag{12}$$

and

$$(B - V)_{\text{med}} = 0.30 \log P + 0.37 \tag{13}$$

for combined Eggen types A, B, and C. The evidence for the observed equations (12) and (13) is given in the appendix. The predicted equations (10) and (11) agree remarkably well with the observed relations (12) and (13) when the errors in the approximations of equations (5), (6), and (7) are considered.

b) Discussion of Equations (8), (10), and (11)

Equations (8), (10), and (11) contain the information for the cepheid problem important to the extragalactic astronomer. We first look at the period-color relation given by equation (11).

Figure 2 is a plot of normal colors of galactic cepheids at median light on the assumption that $(B-V)_{\max} = 0.47$ with no period dependence. This maximum color relation is given by Kraft (1957) from a compilation of all data on cepheids in galactic clusters available to January 1, 1958. When the color amplitude observed by Eggen, Gascoigne, and Burr (1957) is transformed to the $B-V$ system and half its value is applied to $(B-V)_{\max}$, the $(B-V)_{\text{med}}$ are obtained. The procedure, described more fully in the appendix, provides the data for Figure 2. The cepheids appear to divide into two groups. The blue group is composed mostly of Eggen's type C variables, while the red group is predominantly Eggen's type A and B stars. However, the separation is not unique. Separation into two color groups at median and minimum light was first noted by Eggen (1951, Fig. 37) and later by Stibbs (1955). At constant period, Figure 2 shows a separation of 0.07 mag. in $B-V$. If equation (11) correctly describes the situation, we are forced to conclude that the Q-value for the C variables differs from Q for the A and B variables. Differentiation of equation (11) at constant log P and substitution of $\Delta(B-V) = 0.07$ give $\Delta \log Q \equiv \log Q_{A,B}/Q_C = 0.27$ or

$$\frac{Q_{A,B}}{Q_C} = 1.86\,. \tag{14}$$

A similar situation exists for the RR Lyrae stars (Schwarzschild 1940; Roberts and Sandage 1955), where the Q ratio between Bailey types a, b and type c is $\sim$1.5. This suggests that classical cepheids of type C are vibrating in the second mode!

We can now estimate the expected intrinsic scatter in the period-color and P-L relation. Scatter in these relations is produced both by $Q_{A,B} \neq Q_C$ and by the finite width of the variable-star domain in the C-M diagram. As previously mentioned, an intrinsic

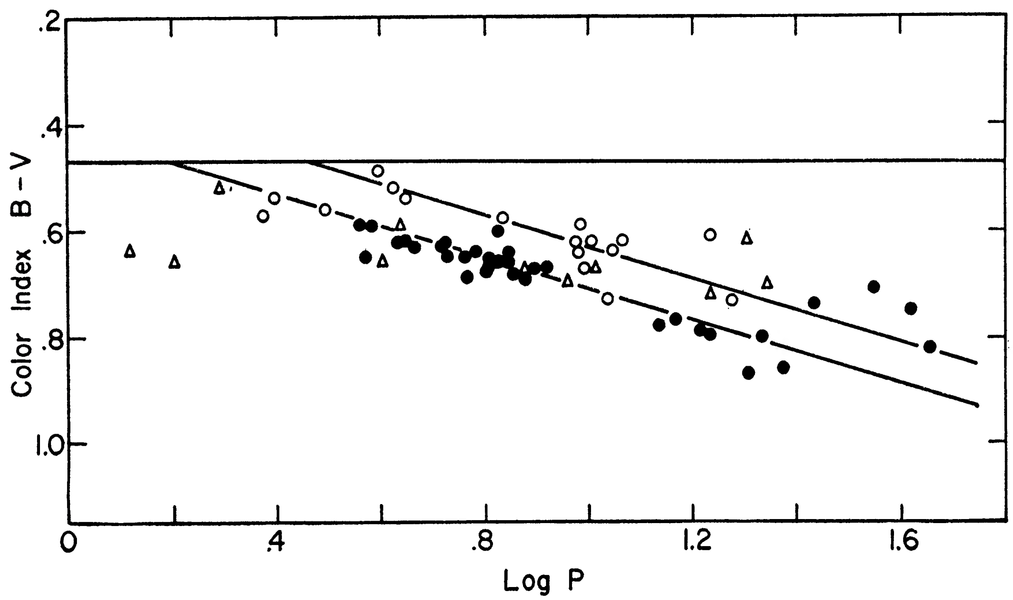

FIG. 2.—Normal median colors for 63 galactic cepheids observed by Eggen and by Eggen, Gascoigne, and Burr. Upper solid line is the assumed relation at maximum light. Open circles are Eggen type C variables; closed circles are Eggen type A, B variables; and triangles are either unknown type or called II by Eggen.

width of $\Delta(B-V) = 0.2$ mag. is assigned to the domain. We further assume that the domain is uniformly filled with cepheids and is 0.1 mag. wide for A and B stars and 0.1 mag. wide for C stars, with the C stars bluer. Consider, first, the period-color relation.

Figure 1 shows that, at constant $B-V$, the spread in ΔM_v read vertically is $\Delta M_v \approx$ 1.0 mag. across the domain for both the A, B stars and the C stars. At constant $B-V$, equation (8) shows

$$\Delta \log P = -0.239 \Delta M_v \tag{15}$$

for contant Q. Consequently, the difference in log P for variables of the same color will be $\Delta \log P = 0.239(1.0) = 0.239$. This is the expected intrinsic scatter about both the A, B line and the C line, read at constant $B-V$ in Figure 2. This scatter, expressed in $\Delta(B-V)$ at constant P, is found from equation (11) to be $\Delta(B-V) = 0.27\ \Delta \log P =$ 0.06 mag. about each line. This is close to what the data of Figure 2 suggest.

Consider, now, the more important question of intrinsic scatter in the P-L relation. That scatter will exist is evident from Figure 1. Lines of constant period are shown there.

They were computed from equation (8) with log $Q_{A,B} = -1.39$ and log $Q_C = -1.66$. Consider cepheids at the same M_v along, for example, the suggested evolution track. Let the periods at the edges of the cepheid domain be P_1 and P_2. Because the lines of constant period slope downward, $P_2 > P_1$ at constant M_v. The ratio P_2/P_1 at constant M_v follows from equations (8) and (9). Differentiation of equation (8) gives

$$\Delta \log P = 1.051 \Delta (B - V) + \Delta \log Q \,. \qquad (16)$$

If $\Delta \log Q = 0$ across the gap (i.e., $Q_{A,B} = Q_C$), then $\Delta \log P = 0.21$ or $P_2/P_1 = 1.62$ for a gap width of $\Delta (B-V) = 0.2$ mag. Because the slope of the observed P-L relation is 2.5 (eq. [12]), this $\Delta \log P$ corresponds to a spread of 0.52 mag. if the P-L relation is read at constant P. The more realistic case is for $Q_{A,B}/Q_C = 1.86$. Then $\Delta \log P = 0.21 + 0.27 = 0.48$ or $P_2/P_1 = 3.0$ for $\Delta (B-V) = 0.2$ mag. at constant M_v. This cor-

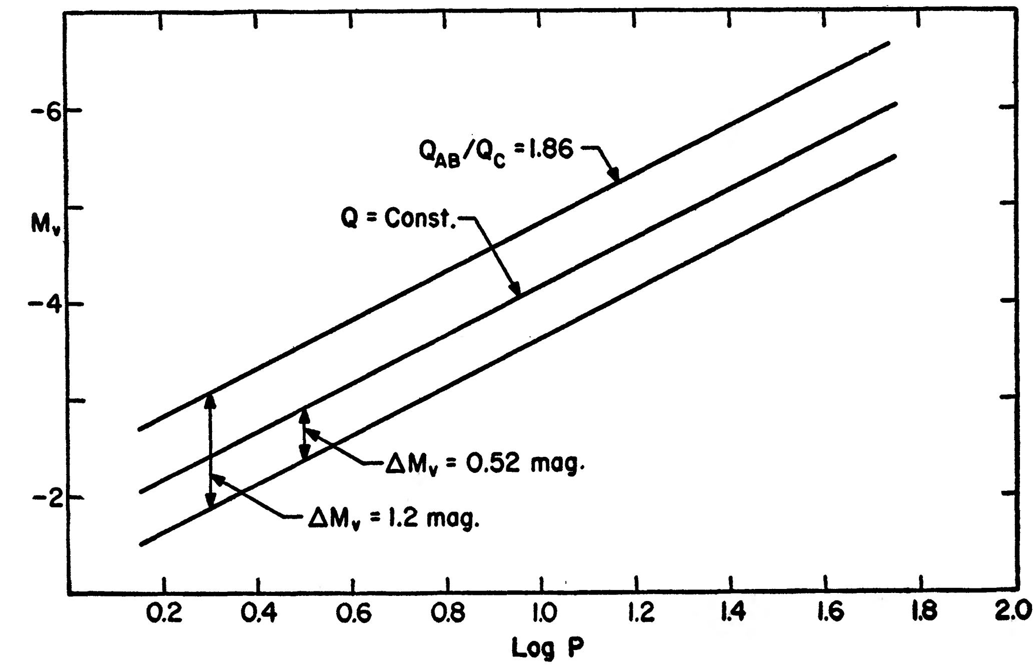

FIG. 3.—Schematic P-L relation for cepheids. If $Q_{A,B} = Q_C$, then variables should scatter throughout the region bounded by the two lowest lines. If $Q_{A,B} = 1.86\, Q_C$, then variables should scatter between the lower and upper lines.

responds to $\Delta M_v = 2.5(0.48) = 1.2$ mag. intrinsic spread in the P-L relation. The type C variables are predicted to be brighter than the A, B variables. Figure 3 shows the P-L relation of equation (12) with the foregoing predicted scatter for the two cases of Q. Comparison of Figure 3 with Arp's unpublished data for SMC variables shows good agreement.

This scatter is so large that problems of zero-point calibration of the P-L relation become serious. For example, Blaauw and H. R. Morgan (1954) used only eighteen cepheids in their proper-motion determination of 1954, and this group contains a mixture of type A, B, and C stars. With so few stars we cannot be certain to what point in the scatter their mean zero point refers. Similarly, it is not surprising that the individual cepheids in galactic clusters (U Sgr, S Nor—Irwin; EV Sct, DL Cas—Arp and Kraft; CF Cas—Sandage and Kraft) give different zero-point calibrations. It is evident that the three-parameter equation (8) must be used if precision is to be gained.

The foregoing analysis shows that the intrinsically bluer cepheids will lie on the bright edge of the P-L relation. Again Arp's preliminary two-color data for the SMC variables confirm the result. Unfortunately, a similar correlation exists for reddened stars. The ratio of absorption to reddening for interstellar extinction is $\Delta M_v/\Delta(B-V) = 3.0$. The ratio due to the gap characteristics for the cepheids is found from equation (8) to be $\Delta M_v/\Delta(B-V) = 4.4$ for P and Q constant. The very similar ratios make observational separation of temperature reddening and interstellar reddening difficult.

Use of the cepheids for precision extragalactic studies may still be possible if such parameters as the shapes of light-curves and the amplitudes of variation can be correlated within the scatter of Figure 3. For example, it is expected that the amplitude of light-variation will decrease nearly to zero at the edges of the domain (as in the RR Lyrae case). If this is so or if the normal color of each cepheid is known, then the scatter in the P-L relation can be allowed for.

Clearly, detailed studies of the properties of cepheids in the Magellanic Clouds must be made before the step to M31, M33, NGC 6822, and other members of the local group can be accomplished. But a situation exists which is even more serious than the scatter in the P-L relation. The evidence is good that cepheids in globular clusters obey a mean P-L relation which is at least 1.5 mag. fainter than the P-L law for classical cepheids (Arp 1955). This zero-point difference is probably caused by a mass difference. The classical cepheids perhaps obey a mass-luminosity law like equation (5), while the globular-cluster cepheids have a constant mass of 1.3 $\mathfrak{M}_\odot$ independent of period (Wallerstein 1957; Cox and Whitney 1958). Unless evolutionary arguments can exclude intermediate masses, we cannot a priori neglect the possibility that cepheids can exist whose P-L relations are intermediate between Figure 3 and the globular-cluster case.

It would appear that, because of (1) the intrinsic scatter, (2) the problem of zero-point calibration, (3) the lack of knowledge of the color-ΔM_v correlation at constant period, and (4) the possible non-uniqueness of the P-L relation, we do not yet know the modulus of M31 to better than probably $\pm$ 0.5 mag. from the cepheid criterion alone.

c) Summary of the Cepheid Problem

1. Equation (8) follows from $P(\bar{\rho})^{1/2} = Q$ when $\bar{\rho}$ is expressed in the observed quantities $B-V$ and M_v.

2. If the region of cepheid instability is localized in the M_v, $B-V$ plane with a mean position given by $M_v = f(B-V)$, then equation (8) can be separated to give the P-L and period-color relations. But these separate relations will have intrinsic scatter if the region of cepheid instability has a finite width. In particular, if the intrinsic width is $\Delta(B-V) = 0.2$ mag., then the spread in the P-L relation will be $\sim$1.2 mag. The intrinsically bluest cepheids are expected to have the brightest absolute magnitudes for a given period. Arp's two-color data for cepheids in the SMC follows these predictions.

3. Evidence from the observed period-color relation shows that Eggen's type C variables have a Q-value $\sim$1.9 times smaller than the A, B variables. The same situation holds for type c RR Lyrae variables and suggests that classical cepheids of Eggen type C are pulsating in the first overtone.

4. An intrinsic scatter of 1.2 mag. in the P-L relation makes use of cepheids for the extragalactic distance scale difficult. The extragalactic astronomer must decide where in the scatter any particular cepheid lies. This may perhaps be done by differences in the light-amplitude, by differences in the shape of the light-curve, or by intrinsic color measurements if and when such relations can be established by study of cepheids in the Magellanic Clouds.

III. THE BRIGHTEST-STAR CRITERION

a) The Problem

Cepheids cannot be located in galaxies with distance moduli greater than $m-M \approx 28$. The only galaxies which are closer than this are members of the local group, the

M81 group, and the M101 group (Holmberg 1950). But we have seen in the introduction that systematic red shifts emerge from the random velocities only for galaxies more distant than $m-M \approx 30$. Hence distance indicators brighter than cepheids must be used to bridge the 2–3-mag. gap beyond the nearby groups.

Hubble used the brightest resolved stars as the most luminous individual indicators. But at least three criticisms can be raised against the procedure. These have been mentioned by Hubble (1936*a*): (1) The possibility exists that bright, compact H II regions will appear as small, starlike images on blue-sensitive plates of distant galaxies. Mistaken identification will give systematically incorrect answers because the calibration of correctly identified brightest stars in the local, M81, and M101 groups will refer to a different class of objects. (2) It is probable that the M_v for the brightest stars depends on nebular type and on the total absolute magnitude of the systems in question. For example, it is well known that resolution of spiral arms into stars occurs along the sequence of nebular classification somewhere between the Sa and Sb types. Hubble's early data showed that stars in Sb galaxies average 0.7 mag. fainter than those in Sc galaxies. But the data were few, and the separation of real stars from H II regions was not certain. (3) The brightest stars do not occur singly but rather in associations and clusters. Isolation and photometry of single stars in galaxies at $m-M \approx 30$ is difficult. These objections can be largely overcome today.

1. Fast red-sensitive plates have become available in the past decade, so that it is possible to separate the H II regions from the stars by comparing photographs taken in Hα light with those taken in an emission-free comparison wave length.

2. The 200-inch telescope has increased the realm of cepheid detection beyond the local group. There are now nearly thirty calibration systems instead of the 7 available to Hubble with the 100-inch (Galaxy, SMC, LMC, IC 1613, M31, M33, and NGC 6822). These are in the local group, the M81 group, and the M101 group. The nebular types of these thirty galaxies range from intermediate Sb through late Sc to Irr. The absolute magnitudes range from $M_{pg} \approx -20$ for M31 and M81, through $M_{pg} \approx -18$ for M33 to $M_{pg} \approx -14$ for the Wolf-Lundmark-Melotte system and the Sextans system as outriding members of the local group and Ho I, Ho II, and NGC 2366 as dwarf Irr in the M81 group. This range of type and absolute magnitude will allow calibration of the $M_s = f(M_g$, galaxy type) relation. (The subscripts s and g refer to star and galaxy.)

3. We now consider in more detail the identification and isolation of single stars and H II regions. Location of H II regions in the nearby galaxies is easy. Ordinary blue plates are often sufficient for detection because of the strong double line λ 3727 [O II]. Figure 4 is a reproduction of a photograph of M33 taken with the 100-inch telescope diaphragmed to 58 inches on an 8 × 10-inch Eastman 103*a*-O plate without filter. The band pass of the atmosphere, telescope, and emulsion combination is from $\lambda \approx 3200$ to $\lambda \approx 5000$ A. The three marked areas are shown in more detail in Figures 5, 6, and 7, which are from plates taken with the 200-inch telescope on either Eastman 103*a*-D plates behind a Schott GG14 filter or Eastman 103*a*-E plates behind a Schott RG2 filter. The D plate is shown on the left, and the E plate is on the right in each figure. The *a*-D+GG14 combination transmits $\lambda = 5200$ to $\lambda = 6300$ A, where few emission features of appreciable strength occur. In particular, the strong N_1 and N_2 lines of [O III] at $\lambda = 4959$ A and $\lambda = 5007$ A are cut out. The *a*-E+RG2 combination transmits $\lambda = 6400$ A to $\lambda = 6600$ A, which contains Hα plus the two conspicuous lines of [N II] at $\lambda = 6548$ A and $\lambda = 6583$ A.

The large emission patches in Figures 5 and 6 are so conspicuous that visual observers found them prior to 1900. The large patch in Figure 5 is NGC 604. The patches on the extreme left and lower right of Figure 6 are NGC 595 and NGC 592, respectively. Figures 5, 6, and 7 illustrate the difficulty that many of the brightest stars in galaxies occur in clusters and small groups. This phenomenon, so evident in M33, has also been noted and discussed by Sharpless (1954) for H II regions in our own Galaxy. On the best 200-inch plates, NGC 604 in M33 contains at least twenty individual bright stars (M_{pg} brighter than -8.5 if $m-M = 24.5$). Similar clustering into associations is even more evident in Figure 7, which is an enlarged segment of the conspicuous arm 20′ south-preceding the nucleus. The six prominent associations shown here are about the same intrinsic size as h or χ Persei. If M33 were placed at a distance corresponding to $m-M = 30$, the angular size of the features in Figures 4, 5, and 6 would shrink to less than 0.1 of their present diameters, and nearly all detail would be lost. In particular, the bright associations of Figure 7, which average about 50 parsecs across (15 seconds of arc at the adopted distance of M33 of $m-M = 24.5$), would be reduced to images of about the stellar seeing disk on 200-inch plates. Only in good to excellent seeing could the clustering be detected. Therefore, even if separation of stars from H II regions is possible for galaxies with $m-M \approx 30$, the identification of *associations* of stars as distinct from individual stars will require the very best observational conditions. This means good seeing and a telescope of large aperture and long focal length. At present, only the 200-inch Hale telescope and possibly the 120-inch at Lick are candidates.

b) Correction to Hubble's 1936 Distances

It is certain that a number of knots which Hubble identified as brightest stars in his calibration of the distance scale in 1936 are, in reality, H II regions. Figures 8 and 9 are reproduced from Humason, Mayall, and Sandage's discussion of the red shifts (1956). Figure 8 is a reproduction of M100 from a plate taken with the 200-inch telescope on Eastman 103*a*-D emulsion behind a GG14 filter. M100 is the brightest Sc galaxy in the Virgo cluster. There are a number of knots along the spiral arms which look, without further evidence, like images of bright stars. Figure 9 is an enlarged section of the main spiral arm south-following the nucleus, where three of the most prominent knots appear. Again the left section of Figure 9 is reproduced from a 103*a*-D plate behind a Schott GG14 filter, while the right section is from a 103*a*-E plate behind a Schott RG2 filter. Figure 9 shows that the knots are all H II regions. The stars do begin to appear, but at $m_{pg} \approx 20.8$, which is 1.8 mag. fainter than Hubble's value of 19.0. This circumstance makes it certain that the H II regions were misidentified as stars in Hubble's study of 1936. All other distant galaxies where the necessary *a*-D and *a*-E plates have been assembled show the same effect.

Hubble's 1936 distance scale depends on only two data. These are (1) the mean absolute magnitude of the brightest stars obtained from Table 2 of *Ap. J.*, **84,** 285, 1936, for galaxies in the local group (these values are tied directly to the cepheid variables) and (2) the red-shift–apparent magnitude relation for resolved "brightest stars" in more distant galaxies. The current program for recalibration of H calls for new observations and interpretation of the cepheid criterion within the local group and the isolation and photometry of stars from H II regions in all galaxies of type Sb, Sc, SBb, SBc, and Irr north of $\delta = -15°$ with red shifts between 500 and 2000 km/sec. Calibration of $M_s = f(M_g,$ type) from the thirty nearby galaxies within the cepheid zone will solve the problem. This project has only just begun, and the results are not yet available. It is, however, of interest to ask what the possible final value of H is likely to be. In what follows we correct (1) Hubble's calibration of the brightest stars by means of modern distances to members of the local group and (2) Hubble's data for the more distant resolved galaxies by the magnitude difference, H II$-$star, found from M100.

Present values for M_{pg} for stars in our Galaxy and in other members of the local group are summarized in Table 1. The data for the Galaxy, LMC, and SMC are values for individual stars measured by the people quoted. Data for NGC 6822, M31, and M33 depend upon Hubble's statistical counts, which give the apparent magnitude where an appreciable excess of stars occurs in the luminosity function for the Galaxy compared with the neighboring field. These statistical upper limits will not pick out the very

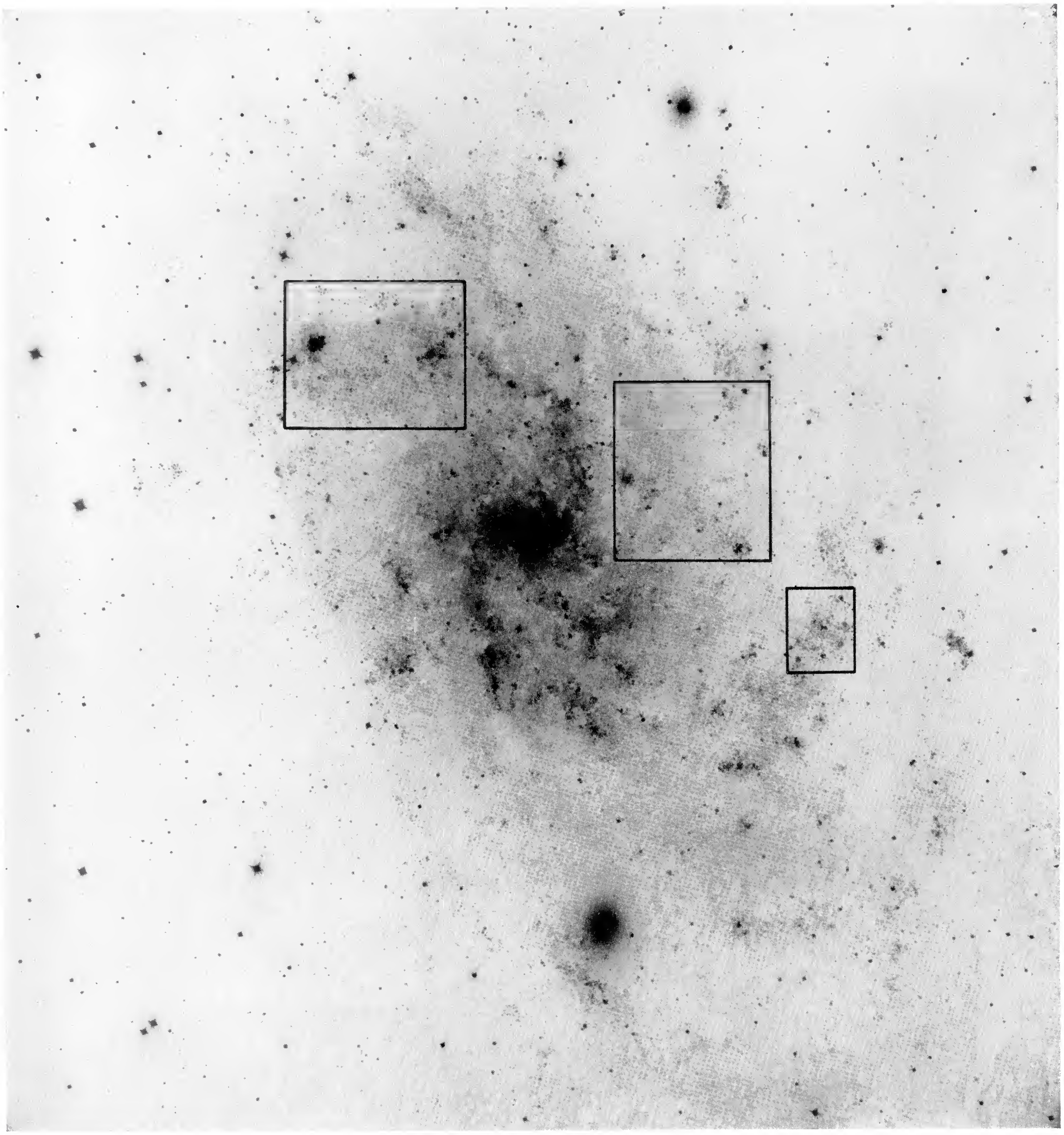

Fig. 4.—M33 taken with the 100-inch telescope on Eastman 103a-O without filter. Areas marked are shown in Figs. 5, 6, and 7.

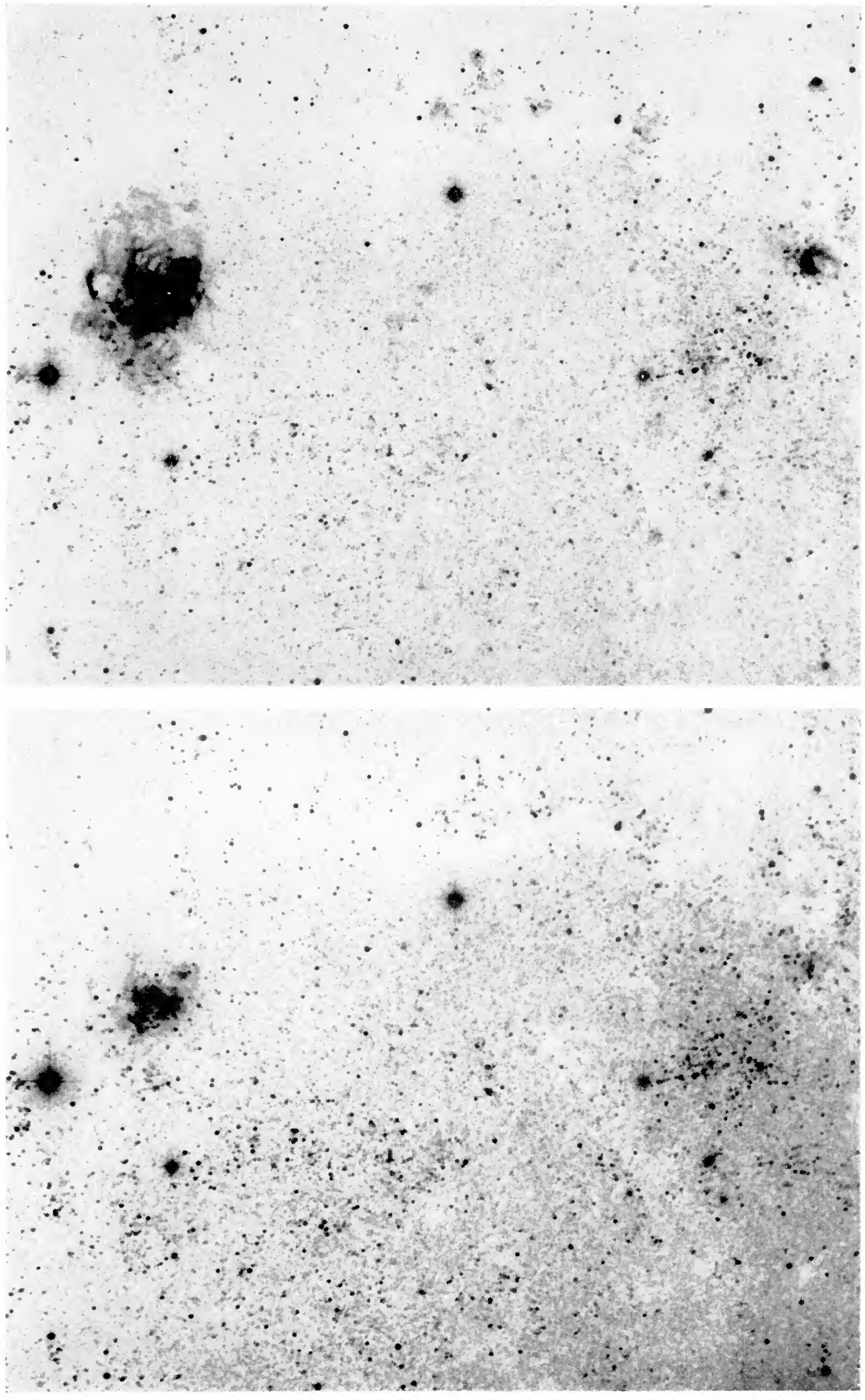

FIG. 5.—Region of NGC 604 in M33. This area is at the upper left in Fig. 4. The reproduction at the top is from a 200-inch plate taken on 103*a*-E behind a Schott RG2 filter. The reproduction at the bottom is from a 200-inch plate taken on 103*a*-D behind a Schott GG14 filter.

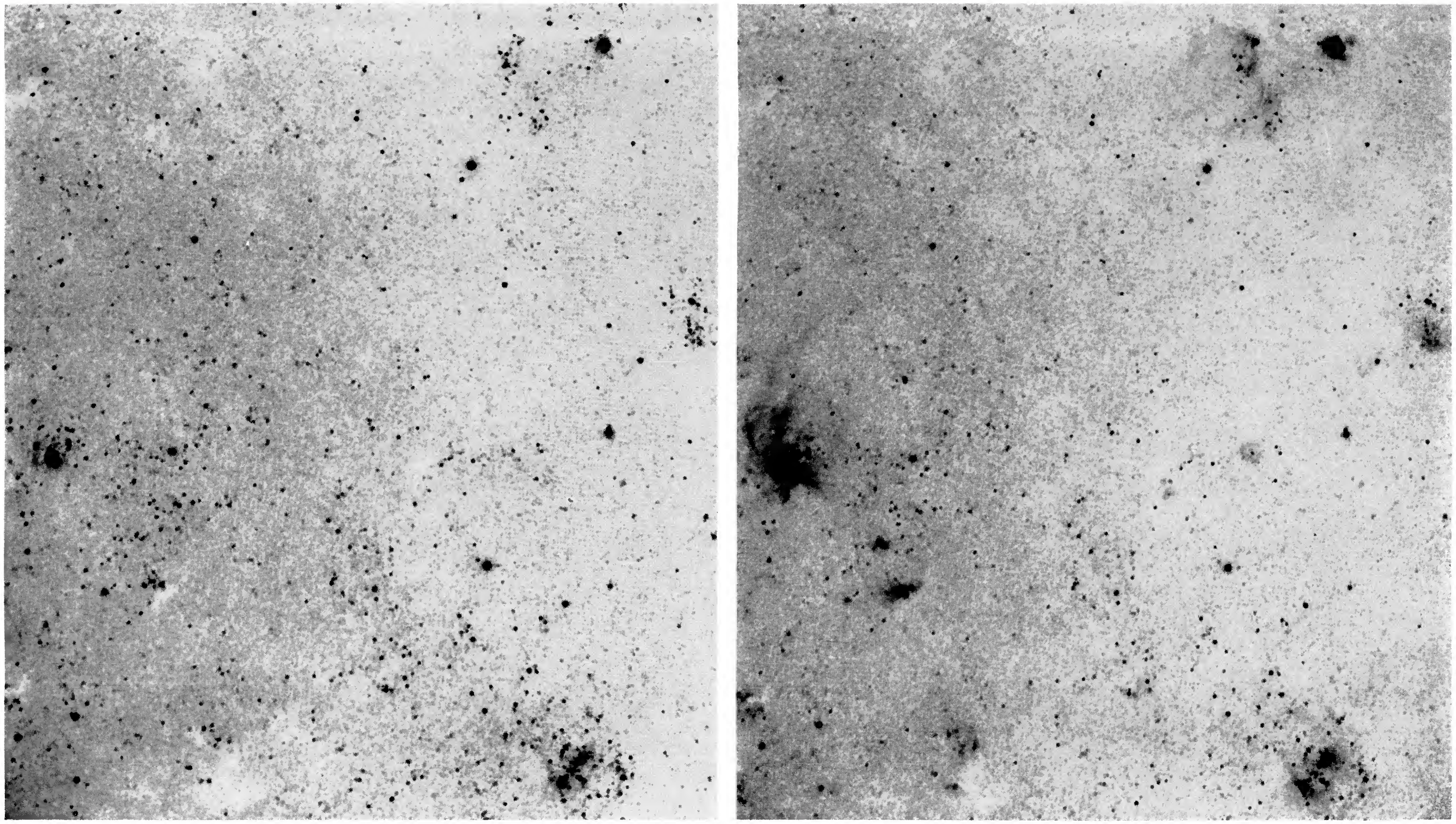

FIG. 6.—Region of NGC 595 and NGC 592 outlined in Fig. 4 in the large area on the right. The a-D plate is on the left; the a-E plate is on the right

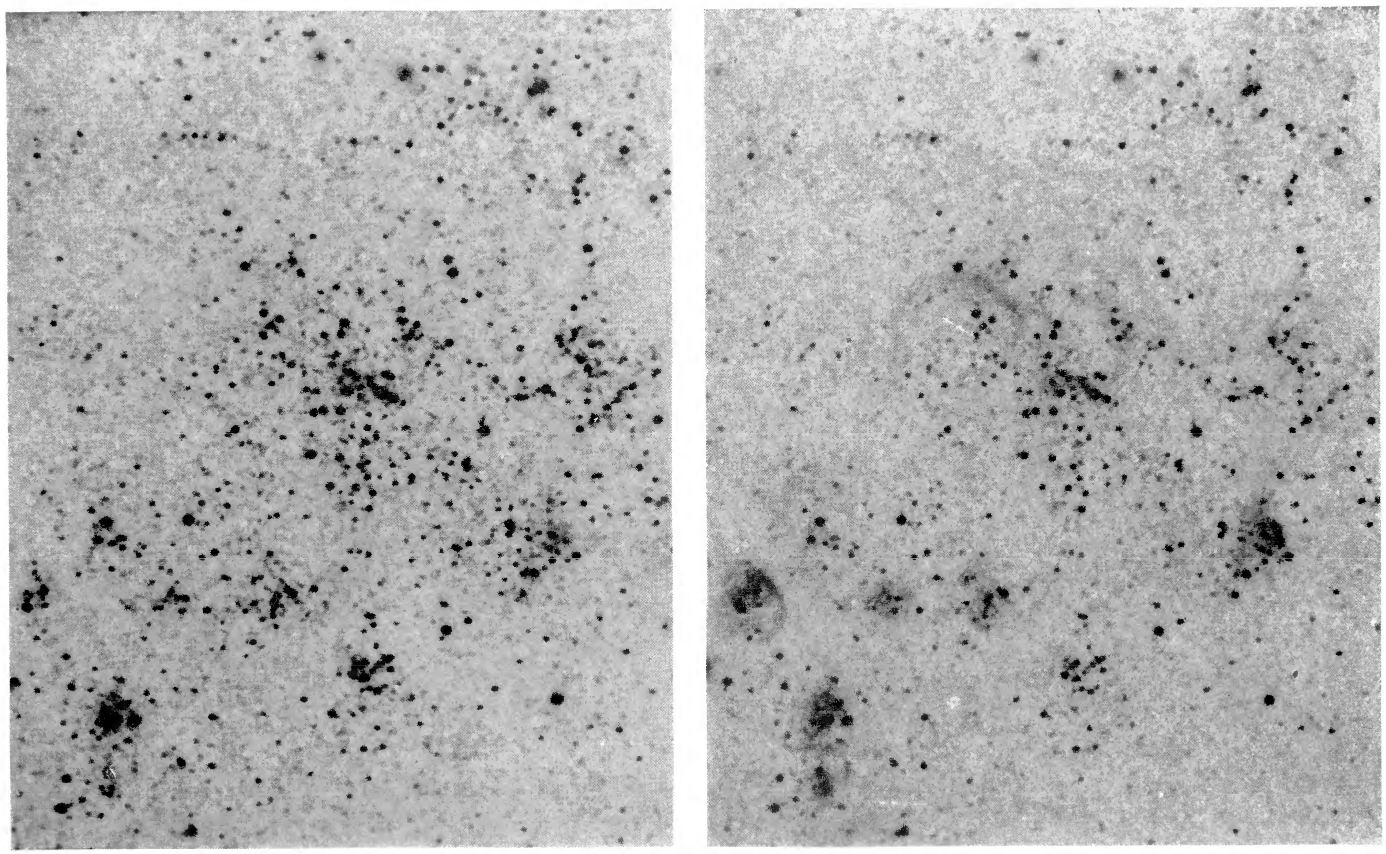

FIG. 7.—The small outlined region in the south-preceding arm of M33 shown in Fig. 4. The *a*-D plate is on the left; the *a*-E plate is on the right. The large association in the middle of the *a*-D plate is of the same order of size as h or χ Persei. Two compact clusters of stars are visible near the extreme upper edge near the middle of both prints.

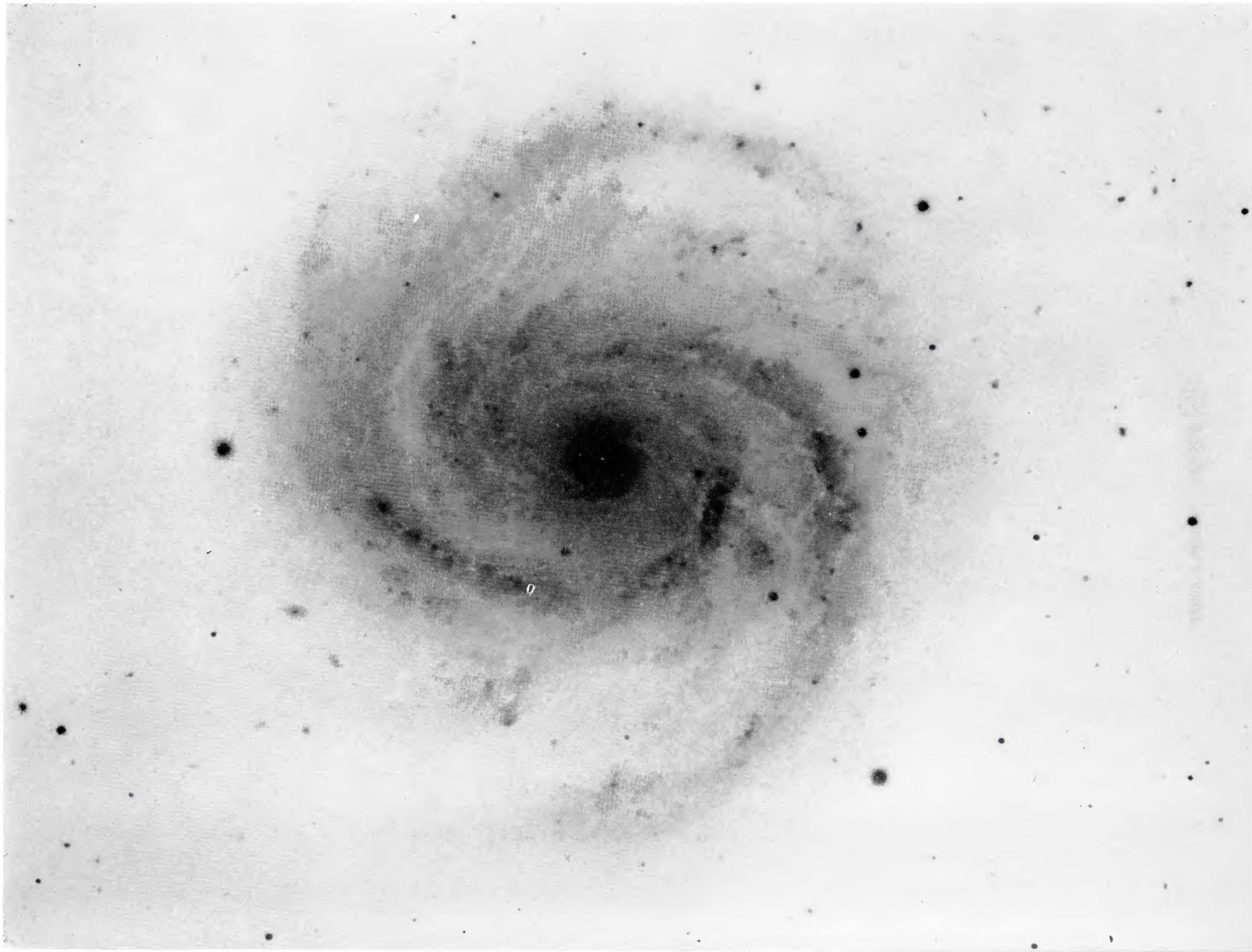

FIG. 8.—NGC 4321 (M100) taken with the 200-inch telescope with a 103a-D plate behind a Schott GG14 filter. The band pass of this combination is from $\lambda = 5200$ A to $\lambda = 6300$ A.

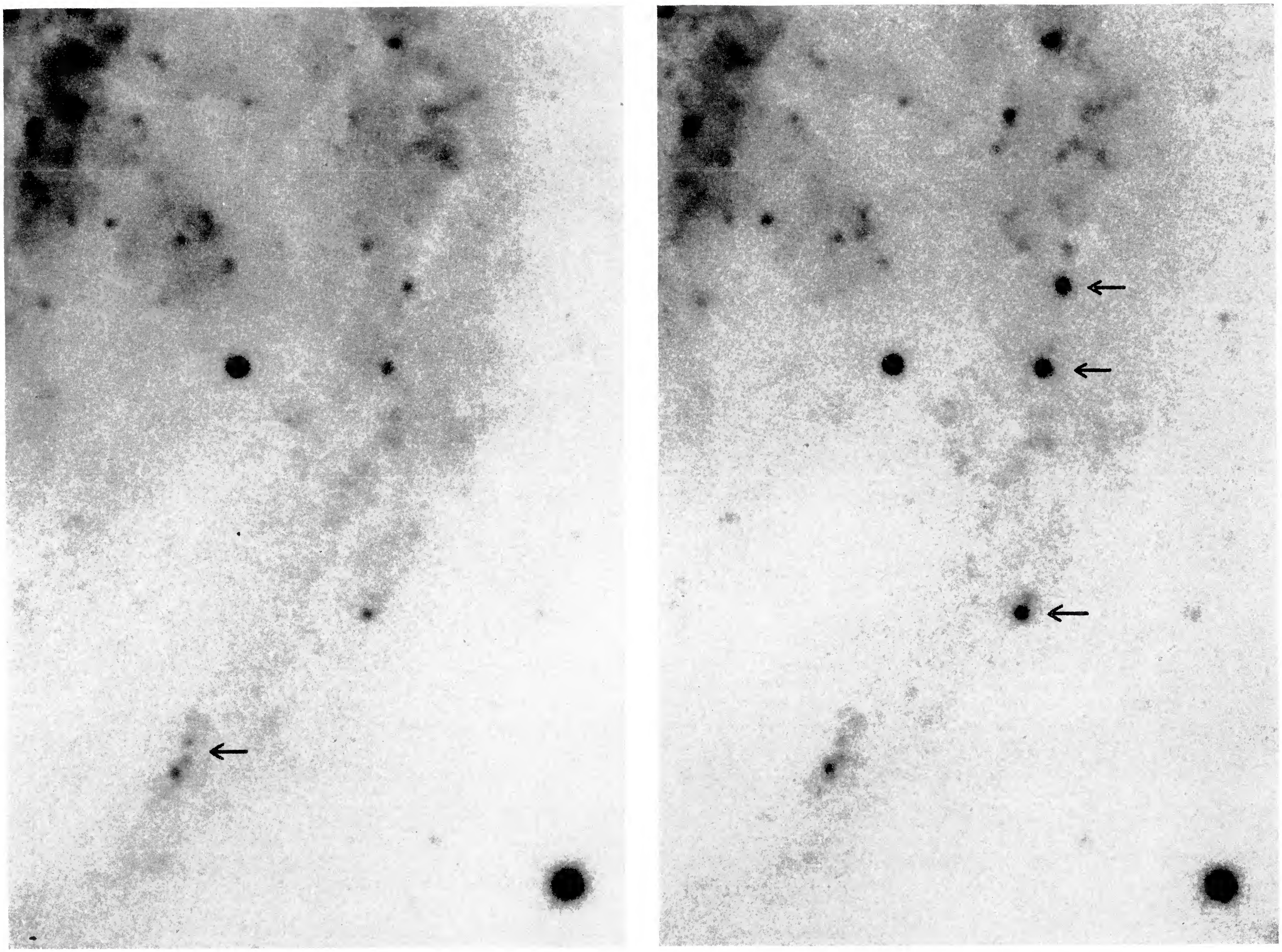

FIG. 9.—Part of the south-following spiral arm of M100. The left exposure is from the a-D plate; the right exposure is from an a-E plate behind an RG2 filter. H II regions are conspicuous on the a-E plate and are marked with arrows. A few objects identified as stars are marked on the left.

brightest stars but will rather sample to some fainter limit. It is not surprising, therefore, that the values for NGC 6822, M33, and M31 are fainter than for the Galaxy and LMC.

The zero point of the distance moduli for Table 1 are based on *normal novae alone.* Because of the difficulties discussed in Section II, cepheids are considered to be unusable at this time. Schmidt's recalibration (1957) of the life-luminosity relation for normal novae in our Galaxy, together with the photometry of novae in the Clouds and in M31 (Arp 1956), gives $m-M = 19.2$ for LMC and SMC and $m-M = 24.6$ for M31. Schmidt's value of 24.6 for M31 is preferred over Baade and Swope's (1955) value of $m-M = 24.2$ based on cepheids because these authors did not take into account the intrinsic scatter in the P-L relation but rather they fitted their cepheid data with an upper envelope line. According to Figure 3, this envelope is about 0.5 mag. brighter than the mean line to which the zero-point calibration of Blaauw and Morgan probably refers.

TABLE 1

BRIGHTEST STARS IN GALAXIES

Star Name	Galaxy	Spectra	M_{pg}	Reference*
No. 12 VI Cyg	Own	B5	− 9 8	1
ζ^1 Sco	Own	B1 5 Ia^+	− 9 4	2, 3
β Ori	Own	B8 I*a*	− 8 8	4
HDE 26970	LMC†	B2 Ia^+	− 9 8	5
HDE 269781 .	LMC†	B9 I*a*	− 9 5	6
HD 33579	LMC†	A2 I*a*	−10 1	6
HD 7583 . .	SMC†	A0 I*a*	− 8 8	5, 7
HD 6884 ...	SMC†	B9 I*ap*	− 8 5	5
.	M33†	.	− 8 9	Hubble (statistical)
. .	M31†		− 8 6	Hubble (statistical)
	NGC 6822†	.	− 8 3	Hubble (statistical)

* The references are as follows:

1 Sharpless, S. 1957, *Pub A S P*, **69,** 239.
2 Houck, T 1955, thesis, University of Wisconsin
3 Code, A , and Houck, T 1958, *Pub. A S P* (in press).
4 Morgan, W. W., Strömgren, B., and Johnson, H M. 1954, *A J*, **59,** 188
5. Code, A., and Houck, T. 1956, unpublished material.
6 Feast, M W , Thackeray, A. D , and Wesselink, A. J. 1955, *Observatory,* **75,** 216.
7 Gascoigne, S C. B 1955, *Australian J. Sci*, Vol **17,** No 3

† Assumed moduli are LMC = SMC = 19 2; M33 = 24 5; M31 = 24 6; NGC 6822 = 24 1

Hubble's relative moduli to M31, M33, and NGC 6822 are then used to find the ratio of distances within the local group. The final adopted values for local group members are: $m-M = 19.2$ for LMC and SMC; 24.6 for M31; 24.5 for M33; and 24.1 for NGC 6822. These should be compared with Hubble's old moduli of $m-M = 17.1$ for LMC and SMC; 22.1 for M31; 22.0 for M33; and 21.6 for NGC 6822. The new values represent an average change of 2.3 mag. from Hubble's old data.

Corrections to Hubble's 1936 distances beyond the local group can be approached by at least three different routes.

Case A.—Hubble obtained (1936*c*)

$$\log \frac{c\Delta\lambda}{\lambda} = 0.2\, m_s - 1.00 \tag{17}$$

for the velocity-magnitude relation for brightest resolved knots. Suppose (1) that Hubble actually did observe the brightest stars and not H II regions; (2) that his photometric scales from $m_{pg} = 18$ to $m_{pg} = 21$ were correct; and (3) that his value of $M_{pg} =$

-6.11 was incorrect. Replace r in equation (1) by the equivalent expression involving magnitudes. This gives

$$\log \frac{c\Delta\lambda}{\lambda} = 0.2\, m_s + (\log H - 0.2M + 1)\,. \tag{18}$$

Consequently,

$$\log H - 0.2M + 1 = -1.00\,. \tag{19}$$

If the correct value of M_{pg} is -9.5 instead of -6.11, then $H = 125$ km/sec 10^6 pc. This value is almost certain to be too high because suppositions 1 and 2 are probably incorrect.

Case B.—Here we assume that all knots identified by Hubble as brightest stars are actually H II regions. We further assume that the photographic magnitude difference between H II patches and stars is 1.8 mag., as given by the M100 case. Because the value of $m_{pg} = 20.8$ for stars in M100 is based on a true Pogson scale transferred from Baum's photoelectric sequence in SA 57, the 1.8-mag. difference is compounded of the true difference between stars and H II regions, together with any error in Hubble's magnitude scale. If the value of 1.8 mag. holds for all galaxies used to find equation (17), then the correct red-shift–magnitude equation for actual stars on a true Pogson scale is

$$\log \frac{c\Delta\lambda}{\lambda} = 0.2\, m_s - 1.36\,. \tag{20}$$

Again, if M_{pg} (stars) $= -9.5$, then $H = 55$ km/sec 10^6 pc. This value represents an increase by a factor of almost 10 in the distance scale of 1936. But 55 km/sec 10^6 pc may be too small because the picking of the brightest stars on a photograph is a statistical operation somewhat like counting over the face of a galaxy to find the bright end of the luminosity function. The data for M31, M33, and NGC 6822 in Table 1 show that the counting procedure gives a value close to $M_{pg} = -8.5$ instead of -9.5. If $M_{pg} = -8.5$, equation (20) requires $H = 87$ km/sec 10^6 pc. But clearly these arguments on the choice of $M_{pg} = -8.5$ or $M_{pg} = -9.5$ are very weak.

Case C.—A modulus to the Virgo cluster will give a value of H because the systematic red shift of this cluster may be large enough to be statistically significant. Hubble gives $m-M = 26.7$ for this cluster. The value is based on "the most frequent apparent magnitude for the brightest [resolved knots] in the Virgo Cluster" together with an assumed absolute magnitude of $M_{os} = -6.11$ for the stars (Table 2 of *Ap. J.*, **84**, 285, 1936). The average difference between the present moduli to local group galaxies and Hubble's old values is, as previously mentioned, 2.3 mag. Therefore, $M_{pg,s} = -6.11 - 2.3 = -8.4$ mag. Again with M100 as the basis, Δm(H II − stars) = 1.8 mag. This gives a total correction of $2.3 + 1.8 = 4.1$ mag., which results in a corrected modulus of the Virgo cluster of $26.7 + 4.1 = 30.8$. If anything, this value is on the low side. $M_{pg,s}$ may be brighter than -8.5.

The value 30.8 is quite uncertain and depends upon the consistency of Hubble's measurements at apparent magnitude levels of about 16 in the local group and 20 in the Virgo cluster. But a modern independent estimate serves as a check. Baum (1955) has measured the apparent magnitude of certain globular clusters in M87 and has compared the data with clusters in M31. The result gives a difference in moduli between these galaxies of 6.0 mag. If $m-M = 24.6$ for M31, then $m-M = 30.6$ for the Virgo cluster. If $m-M = 30.7$ and if $c\Delta\lambda/\lambda = 1136$ km/sec, then $H = 83$ km/sec 10^6 pc.

These arguments do not constitute a rediscussion of the distance scale. Precise new measurements are required for this. But they show that if we adopt Hubble's data as they stand and apply corrections, then H can range between 50 and 100 km/sec 10^6 pc with a very large uncertainty. These values represent corrections to the old distance scale by factors of about 10 and 5, respectively.

IV. THE TIME SCALE

The numerical value of H is important because it tests the time-scale criteria in various world models. By itself, H does not give the time since the "beginning" of the expansion. Knowledge of the world model must also be available. As mentioned in Section I, this can be found, in principle, from the deceleration parameter together with a physical theory. The simplest theory is the general theory of relativity together with Friedman's exploding models with $\Lambda = 0$. Within this context, the observational evidence of Humason *et al.* (1956) suggested $k = +1$. This means that the intrinsic geometry of the universe is closed with positive space curvature and the models are of the oscillating type. More recent evidence by Baum (1957) also requires a deceleration parameter such that $k = +1$, but the spatial curvature value is somewhat closer to the Euclidean case than the data of Humason *et al.* For the Euclidean case ($k = 0$), the time since the "beginning" of the expansion is $T = \frac{2}{3} H^{-1}$ for pressure-free models. For a closed universe ($k = +1$), T will be smaller than $\frac{2}{3} H^{-1}$, depending on the value of the radius of curvature.

These remarks show why the numerical value of H is so important. Because, if the time computed from H and $\ddot{R}_0/R_0 H^2$ is incompatible with the age of the solar system or the age of the heavy elements ($\sim 7 \times 10^9$ years according to Burbidge, Burbidge, Fowler, and Hoyle 1957), then the simple exploding models lose much of their significance. But if the correct value of H lies between 50 and 100, then $\frac{2}{3} H^{-1}$ lies between 13.3×10^9 years and 6.6×10^9 years. These values are not inconsistent with exploding models with $k = 0$ or even with $k = +1$ with large radii of curvature, as current inconclusive evidence suggests.

The major conclusion is that there is no reason to discard exploding world models on the evidence of inadequate time scale alone, because the possible values of H are within the necessary range.

It is a pleasure to thank Dr. H. C. Arp not only for permission to quote his unpublished results for the cepheids in the SMC but also for the many day-to-day discussions on the general problems of variable stars, which has, in large measure, led to the approach of Section II.

APPENDIX

The Observed P-L and Period-Color Relations

Equations (12) and (13) of the text come from the following assumptions. Arp's preliminary data for variables in the SMC give a period–apparent B magnitude relation,

$$\dot{B} = 17.8 - 2.2 \log P \; . \tag{A1}$$

If the modulus of the Cloud is taken from Schmidt's discussion of the normal novae (1957) to be $m - M = 19.2$, then

$$M_{\dot{B}} = -1.4 - 2.2 \log P \; . \tag{A2}$$

Kraft's discussion (1957) of the four galactic clusters which contain cepheids, together with Gascoigne and Eggen's (1957) estimated color excesses for α UMi and δ Cep, give normal colors at maximum light

$$(B - V)_{\text{max}} = 0.47 \pm 0.02 \; , \tag{A3}$$

which is independent of period. Eggen, Gascoigne, and Burr (1957) give color amplitudes for 63 galactic cepheids on the $(P-V)_E$ color system. Half of these color amplitudes, transformed to the $B-V$ system by $(P-V)_E = -0.125 + 1.038\,(B-V)$ (Eggen 1955), added to equation

(A3) give the $(B-V)$ at median light. The resulting median colors are plotted against log P in Figure 2 of the text. The relations fitted to Figure 2 at median light are

$$(B-V)_{\rm med}=0.33+0.30 \log P\,, \quad \text{Type C variables}\,, \tag{A4}$$

$$(B-V)_{\rm med}=0.40+0.30 \log P\,, \quad \text{Type A, B variables}\,, \tag{A5}$$

$$(B-V)_{\rm med}=0.37+0.30 \log P\,, \quad \text{Average of A, B, C variables}\,. \tag{A6}$$

Equation (A6) is equation (13) of the text. Equation (A6) substituted in equation (A2) gives

$$M_v=-2.5 \log P-1.77\,, \tag{A7}$$

which is equation (12) of the text.

We must state explicitly that equations (A4), (A5), and (A6) depend upon an assumed normal color for cepheids at maximum light, given by equation (A3). If this is incorrect, then the absolute calibration of Figure 2 and the coefficients of equation (A6) will be incorrect in detail. But, regardless of slight uncertainties in the coefficients, the arguments of Section II, *a*, go through unchanged in principle.

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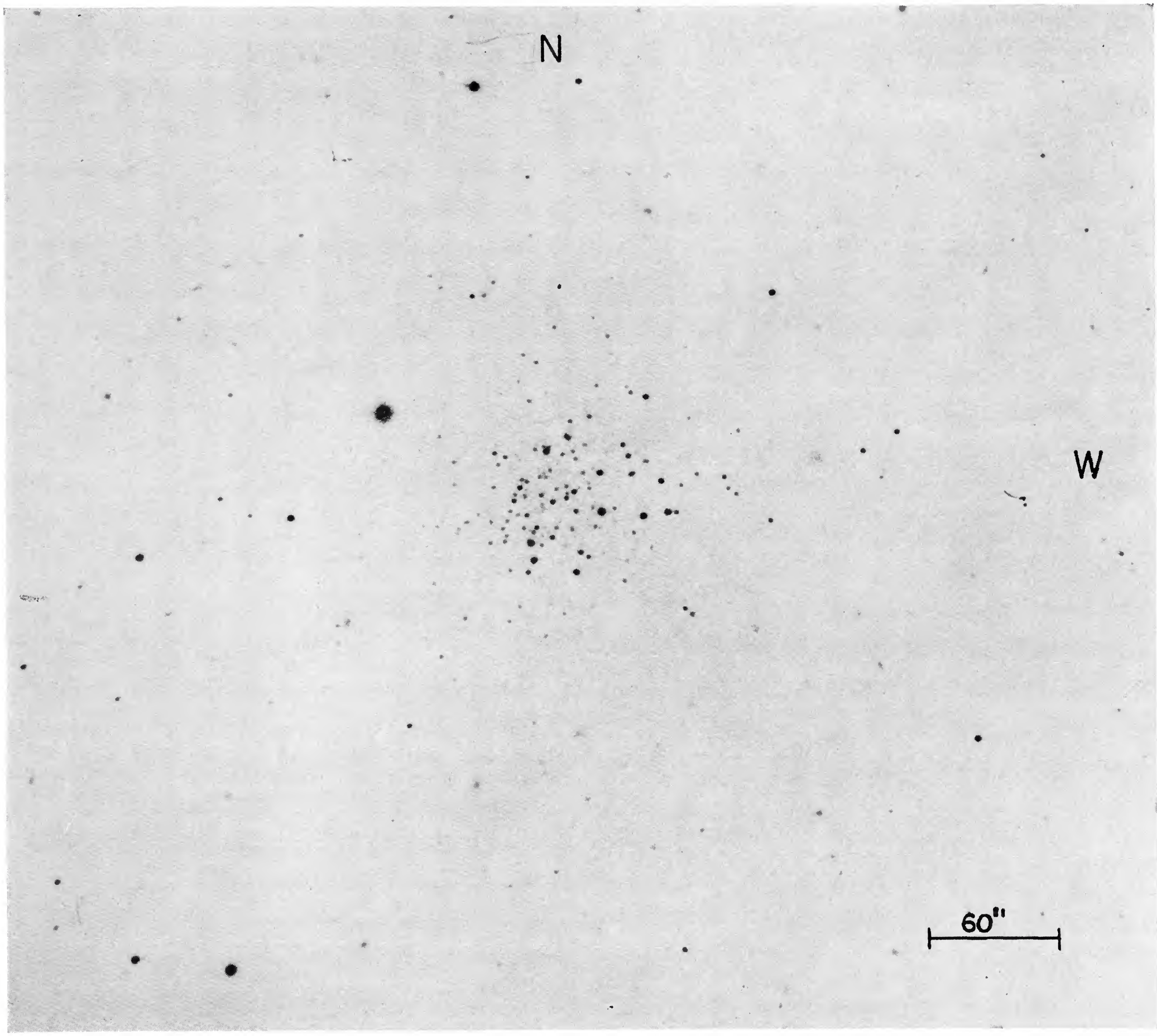

FIG. 1.—A 200-inch photograph of the 11^h cluster. The plate is a 103*a*-D behind a Schott GG11 filter. The faintest visible stars are about $V = 22.0$.