

ABUNDANCES OF METALS, CN, AND CH IN GIANT STARS

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ABSTRACT

On coude spectrograms of four standard stars and twelve peculiar stars, of types G4–K0, total absorptions of 35 atomic lines, 10 band lines of CN, and 18 band lines of CH have been measured. The relative numbers of atoms or molecules are derived by the use of a mean curve of growth combining the curves found for γ Leo A and γ Leo B by Wehlau. The differences between the mean values of $\log N$ for each peculiar star and for the four standard stars are given separately for the metal atoms, CN, and CH, respectively. These values of $D = \log N (*) - \log N$ (standards) are corrected for the relatively small differences in type and luminosity of each star. Eight high-velocity stars are included, and only one of them, 32 v^2 Cnc, is found to be a population I star. The seven population II stars show abundance deficiencies (D) of -0.18 for metals and -1.00 for CN, but all except one of them have normal amounts of CH, as compared to the standard stars. Two stars which had been included as stars with a possible excess of CN are found to be nearly normal population I stars. The star BS 885 = HD 18474, described by Bidelman as a carbon-poor star, is found to have logarithmic deficiencies of -1.55 in CN and -1.97 in CH. The object BS 6791 = HD 166208 is a similar star with a less extreme deficiency of carbon; HD 191046 may be an example of a carbon-poor star with a high space velocity.

INTRODUCTION

Among the giant stars of types G and K, a considerable number have been observed to have CN and/or CH bands of abnormal intensity in their spectra. The observations of these effects are based on a wide variety of spectrograms. They extend from visual estimates of intensity on small-scale plates (taken usually for programs of spectral classification) to spectrophotometric measurements on coude plates with dispersions of a few angstroms per millimeter.

Quantitatively, there is considerable divergence between the results of these various observations. There is fairly good agreement, however, on the existence of one group of peculiar stars—those having the CN bands definitely *weaker* than the average for stars of the same temperature and luminosity. Several of these stars with CN deficiency (particularly δ Lep and Boss 2527 = HD 81192) were reported originally by Morgan, Keenan, and Kellman (1943), and many cases have since been established. One of the best-known examples is γ Leo A, which was studied on coude spectrograms by Gratton (1953), by Schwarzschild and Schwarzschild (1950), and by Schwarzschild, Schwarzschild, Searle, and Meltzer (1957).

It has been well established also that the weakening of CN is most common among stars having high space velocities, and thus it seems to be a physical characteristic of at least some population II stars. It is not so clear, however, how closely the amount of the CN deficiency is correlated with other suggested traits of population II, such as a general weakening of the metallic lines. This is partly because the measurements on spectrograms of fairly high dispersion have been limited to a few bright stars and have necessarily omitted most of the stars with the strongest population II characteristics.

The other peculiarities that have been reported have been studied much less completely than the CN deficiency. The peculiar groups include (1) stars with a CN excess, (2) carbon-poor stars, and (3) stars with unusually strong hydrogen lines.

1. Stars with a CN Excess

Members of the group with a CN excess are distinguished from normal G or K giants by having the blue bands of CN appreciably stronger than normal, and from carbon stars in not having so much carbon in their atmospheres that the Swan bands of C₂ are noticeable on small-scale spectrograms. They were discovered by Morgan and Nassau in the course of a search for late-type supergiants. Several of them are included among the stars listed by Roman (1952) as having unusually strong absorption in the vicinity of λ 4150.

2. Carbon-poor Stars

In a footnote to Bidelman's paper (1950) on carbon stars that are deficient in hydrogen, he called attention to a contrasting star, HD 18474 = BS 885, which "exhibits the unique peculiarity of a spectrum resembling that of a giant G-type star but with no observable CH or CN absorption." An extensive investigation of this star on Mount Wilson coudé plates is being made by J. Humblet. It is the fact that *both* CN and CH are weak in the spectrum of this star that distinguishes it from the high-velocity (h-v) stars with weak CN, for the G band has at least normal strength in the spectra of the latter and in some cases has been reported as stronger than normal (see the descriptions in Table 1). A similar star, HD 30297, was discovered by Morgan and Nassau.

3. Stars with Unusually Strong Hydrogen Lines

The possession of unusually strong hydrogen lines is a peculiarity which is difficult to establish with certainty, for the hydrogen lines are so sensitive to both temperature and luminosity that they are used as criteria for classification throughout types F, G, and K. This means that an apparent anomaly in the intensities of the Balmer lines can usually be made to vanish by changing slightly the type or luminosity class assigned to the star. It is only when several criteria involving other features agree in defining a type which cannot be reconciled with the appearance of the Balmer lines that we can say that a peculiarity in the hydrogen-line strength has been established. In spite of this difficulty, Hossack (1954) was able to conclude, from his oscillographic estimates of intensity, that a number of K-type stars do have hydrogen lines stronger than normal. Most of his examples (such as ζ And and η And) are spectroscopic binaries, but four of them (56 Peg, HD 49500, HD 166208, and α^2 Cap) show no evidence of variations in their radial velocities.

The purpose of the present program was to make an objective examination of the spectra of several typical stars to which one or more of these peculiarities has been ascribed. The method was to compare total absorptions of *individual* rotational lines in the bands of CN and CH (and of some relatively unblended atomic lines) in the spectra of the peculiar stars and of a group of "normal" stars. The spectrograms were mostly taken by one of us (J. L. G.) with dispersions of 2.8 or 4.5 Å/mm with the coudé spectrograph of the 100-inch reflector at Mount Wilson. Only two of the program stars—HD 5544 and HD 191046—were observed with the lower dispersion (10 Å/mm) of the 32-inch camera on the same spectrograph.

The stars observed are listed in Table 1. Except for HD 18474 (G4p), all stars included in the program have types between G7 and K0. They all lie close to the giant branch. The faintest luminosity class represented is III-IV (ϕ^2 Ori), and the brightest is II-III (HD 180262). These limitations were set in order to minimize the corrections for the effect of temperature and luminosity on the absorption by bands and lines. The stars were all classified on the MK system from small-scale spectrograms taken with the 69-inch Perkins reflector. The differences from other published types for the same stars are not large and in several cases undoubtedly reflect the difficulty of classifying stars which show peculiarities in their spectra. The seventh column of the table lists the space motion, w . For HD 191046 the radial velocity is given as a minimum value of w , since an

accurate determination of the proper motion is lacking. The last column gives the main peculiarities as estimated visually on small-scale plates.

The first four stars listed are the "normal" giants included as standards of comparison. Of the eight high-velocity stars which follow, all except $32 \nu^2$ Cnc have been described as exhibiting the weakening of CN typical of this group, and the brighter ones have been included by Miss Roman among the stars with relatively weak metallic lines. Star $32 \nu^2$ Cnc was described by Keenan and Keller (1953) as having CN slightly stronger than normal. It might be thought of as one of the few population I stars which are moving just fast enough to fall barely within the range of the h-v stars.

Although HD 18474 has a relatively early type, it was included as a specimen of a presumably carbon-poor star. HD 166208 was taken from Hossack's list of stars having hydrogen lines too strong for the type. The stars apparently having CN stronger than normal are represented by HD 180262 and HD 5544.

TECHNIQUE OF MEASUREMENT

The spectrophotometric calibration was twofold. Both the step-slit intensity scale of the 100-inch coude spectrograms and the linear wedges taken with a separate calibrating

TABLE 1
LIST OF OBSERVED STARS

No	STAR		m_v	1950		TYPE	w (km/sec)	DESCRIPTION
	Name	HD		α	δ			
1	ι Gem	58207	3 89	7 ^h 22 ^m 6	+27° 54'	G9 III ⁺	18	Normal standard
2	κ Gem	62345	3 68	7 41 4	+24 31	G8 III	11	Normal standard
3	ϵ Vir	113226	2 95	12 59 7	+11 14	G8 III	39	Normal standard
4	η Her	150997	3 61	16 41 2	+39 01	G8 III ⁺	26	Normal standard
High-Velocity Stars								
5	ϕ^2 Ori	37160	4 39	5 34 2	+ 9 16	G8 III-IV	94	CN and atomic lines weak
6	δ Lep	39364	3 90	5 49 2	-20 53	G8 ⁺ III	185	CN weak
7	ν^2 Cnc	72324	6 41	8 30 0	+24 15	G9 III	90	CN slightly strong
8	γ Leo A	89484	2 61	10 17 2	+20 06	K0 III	113	CN slightly weak
9	γ Leo B	89485	3 80	10 17 2	+20 06	G7 III ⁺	113	CN slightly weak
10	BS 6853	168322	6 10	18 15 5	+40 54	G9 III ⁻	125	CN weak, CH slightly strong?
11		191046	7 17	20 03 6	+36 05	G9 III	>100	CN weak
12	γ Psc	219615	3 85	23 14 6	+ 3 01	G7 III	140	CN weak
Peculiar Stars								
13	BS 885	18474	5 61	2 56 4	+47 01	G4p	31:	CN and CH weak
14	BS 6791	166208	5 11	18 06 0	+43 27	G8 III ⁻	41	Hydrogen lines strong; CN, CH weak*
15		5544	7 71	0 54 7	+ 0 04	K0 III	34	CN, CH strong
16	BS 7300	180262	5 69	19 13 2	+15 00	G8 II-III	11	CN, CH strong

* BS 6791 = HD 166208 Previous to Hossack's (1954) inclusion of this star in his group of those having anomalously strong hydrogen lines, Miss Roman (1952) had classified it as K0p, with the description: "CN and Sr II are weak. The hydrogen lines are strong enough to indicate a fairly high luminosity." The weakness of CH appears to have been first noticed on the Perkins spectrograms taken for classification of the star.

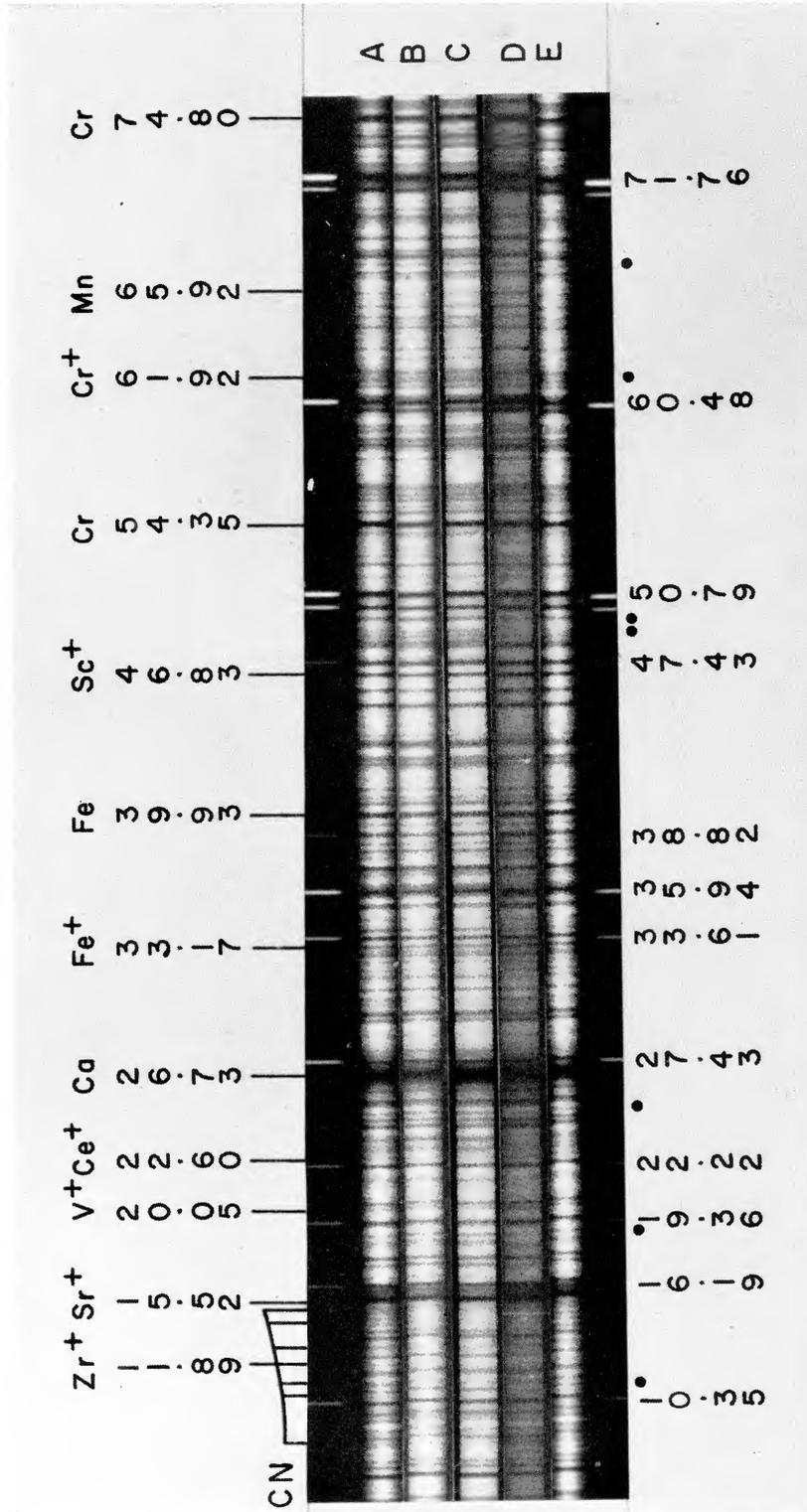


FIG. 1.—Spectra of late G giants, λ 4205–4276. Stars are A, ι Gem, G9 III⁺, normal standard; B, ϕ^2 Ori, G8 III–IV, high velocity; C, δ Lep, G8⁺ III, high velocity; D, HD 166208, G8 III⁻, strong H, weak CH, CN; E, κ Gem, G8 III, normal standard. Star wave lengths above and some CN features marked by lines. The apparent emission line near λ 4209.1 is actually a high spot of relatively undisturbed continuum. Iron-arc wave lengths below, and dots for some stellar CH features.

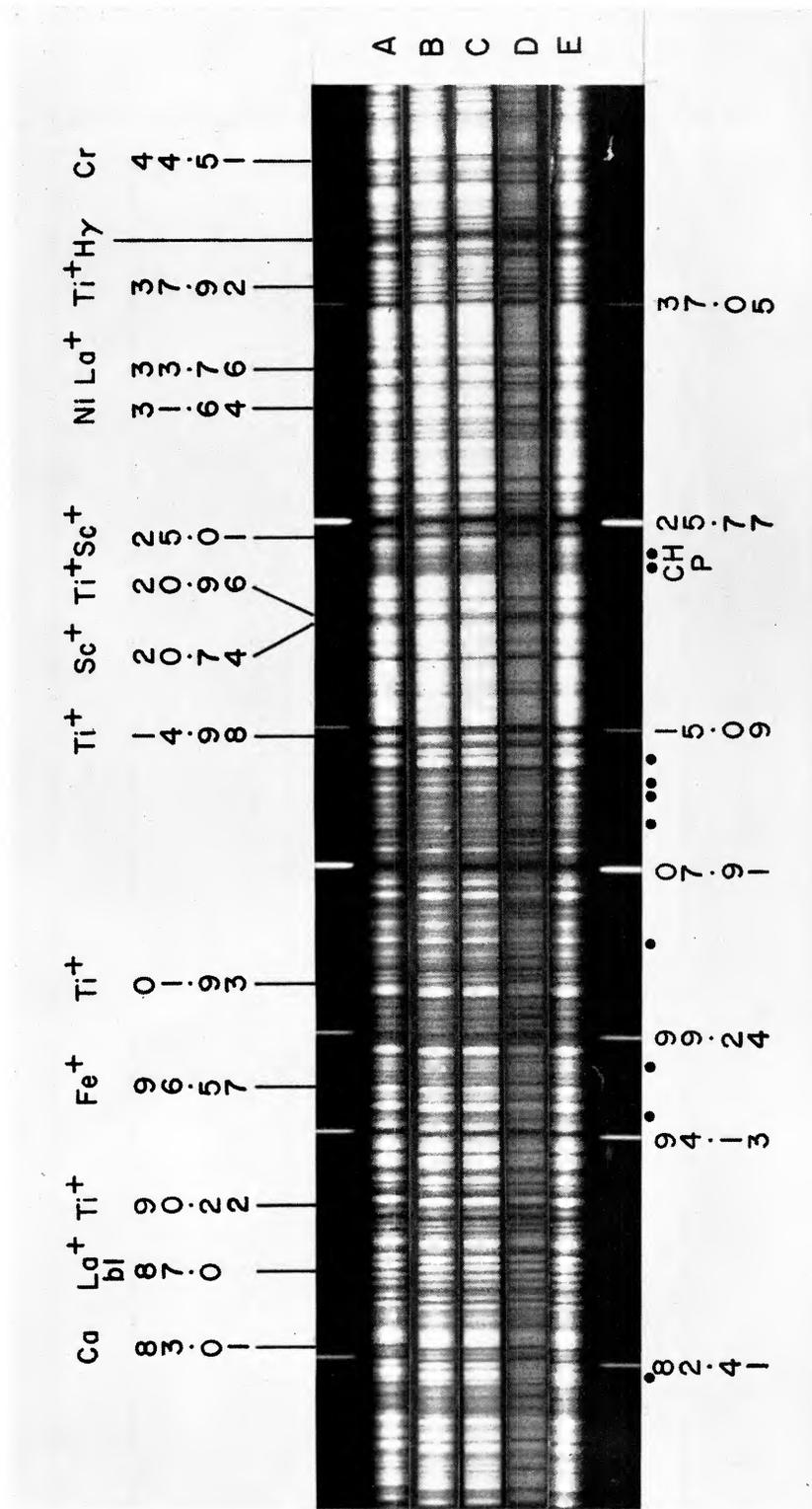


FIG. 2.—Spectra of late G giants, $\lambda\lambda$ 4275–4348. Stellar identifications as in Fig. 1. Region of the G band; some CH features marked by dots below, together with iron-arc wave lengths. Star wave lengths above.

spectrograph were available. Unbaked IIa-O, with D19 development, at 4.5 A/mm form the majority of the plates; some plates at 2.8 A/mm and two at 10 A/mm were also used. The slit-width at the plate averaged 20 μ ; spectra were 0.35 mm high, except at the higher dispersion, where they were 0.50 mm. The grating has negligible scattered light. Tracings were made on the Babcock direct-intensity microphotometer, using the wedge calibration, at a scale of 28 mm/A, i.e., about 60 per cent of that of the Utrecht *Atlas*, although of much lower resolution (Greenstein 1957). The linearity of wedge and microphotometer was checked by running the step-slit calibration at three wave lengths. The wave-length range used is so limited ($\lambda\lambda$ 4170–4410) that no change in the characteristic curve was detectable. Typical spectra of normal and peculiar stars are shown in Figures 1 and 2.

Serious blending arises from the late type of the stars and makes the location of the true continuum difficult. The best adopted continuum was drawn between apparent high points in the same way on all tracings. Second-order effects must persist, between weak- and strong-line stars, since even the apparent peaks are depressed; an error in the continuum appears roughly doubled in the equivalent width, and there is reason to believe that the weakening of lines in a weak-lined star must be *underestimated*. The observed tracing profile is roughly triangular, and extensive wings are not seen except in a few strong lines. Thus, dependent on strength and degree of crowding, equivalent widths were derived either by (1) planimetry, giving W ; (2) measurement of an equivalent tri-

TABLE 2
AVERAGE DEVIATION, $\langle |\Delta \log \eta_0| \rangle$

Strength	Metals	CH, CN	Strength	Metals	CH, CN	Strength	Metals	CH, CN
Weak	± 0.06	± 0.11	Moderate	± 0.12	± 0.14	Strong	± 0.06	± 0.17

angle, giving W_t ; or (3) from central depths A_c for blended lines. Empirical relations for sample lines were obtained, permitting conversion from W_t or A_c to W . One feature, λ 4323, the detached branch of CH, was measured as a complex profile, as nearly as possible the same in all stars, with obvious atomic features omitted. Inside the G band the stellar continuum is depressed, and some extrapolation of observed profiles is necessary. The measurements were largely carried out by Mildred S. Matthews.

The accuracy of W , from internal agreement, can be obtained for the few stars for which there are several plates. Since one curve of growth was used for all stars, the more useful measure of accuracy is the interagreement of the final quantities, $\log \eta_0$, for each line. The average deviation $\langle |\Delta \log \eta_0| \rangle$ is approximately $4 \times \langle |\Delta \log W| \rangle$ on the flat part of the curve of growth and a smaller multiple elsewhere. The metallic lines, for which there existed a relatively great freedom of selection, were less blended than the CN and CH features, as can be seen from Table 2, which lists the average deviations of a single $\log \eta_0$. The metallic lines occur over a wide range of strength, as do the CH lines; the CN lines are individually weak. The greater errors for CH and CN cannot, apparently, be avoided in spite of the high dispersion used. The measuring error, deduced from Table 2, is about 7 per cent for metallic lines and 20 per cent for CH and CN. Thus, although the visual impressions of differences in CN strength are an excellent luminosity criterion, their quantitative calibration is difficult. We shall see that our measures do not completely confirm visual estimates of CH strengthening.

From external consistency, a different type of estimate of errors can be made. When the neutral metallic lines in two stars are intercompared, the differences in $\log \eta_0$ should be essentially constant. Per difference, this $\langle |\Delta \log \eta_0| \rangle$ is found to be ± 0.22 ; each line in each star should have $\langle |\Delta \log \eta_0| \rangle = \pm 0.15$. This exceeds the observed errors of

Table 2, presumably because the blending and the location of the continuum differ more from star to star than from plate to plate. Nevertheless, with about 33 metallic lines, the expected average deviation of the mean $\log \eta_0$ for a star should be about ± 0.03 .

An indirect comparison is possible with Schwarzschild *et al.* (1957), who give A_c for 20 lines in ϕ^2 Ori (their star H). Figure 3 compares these measures with our W . The scatter from a smooth curve is very small, even for weak lines. The dashed curve in Figure 3 is obtained from their approximate calibration formula for expected equivalent widths, W_p , as follows:

$$W_p = 0.283 \left(1 - \frac{I_l}{I_c} \right), \quad (1)$$

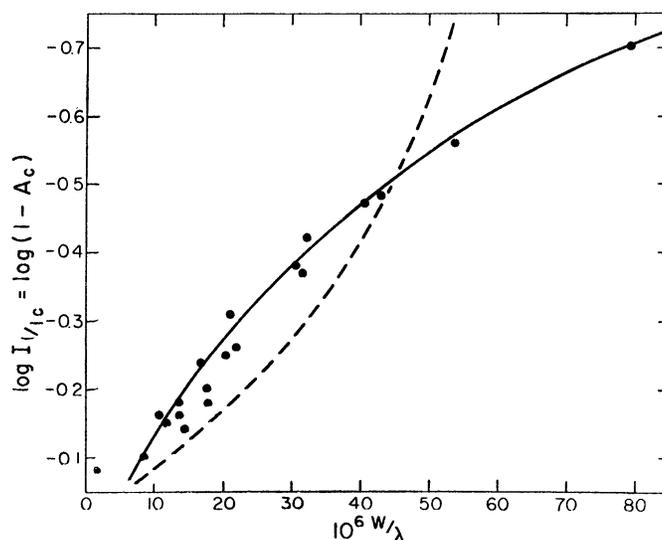


FIG. 3.—Comparison of measured residual central intensities (*ordinates*) in ϕ^2 Ori (Schwarzschild *et al.*) and the equivalent widths of the present investigation (*abscissae*). The dashed line is a theoretical expression (eq. [1]) for weak lines. The measuring errors (deviations from the empirical solid line) are about 7 per cent.

which is valid only for weak lines. For lines with $40 < W < 150$ mÅ, our $W < W_p$; the sign reverses for strong lines, where equation (1) fails. The use of depths alone apparently gives satisfactory internal agreement.

Wehlau (1956) has measured two Mount Wilson coude spectrograms each of γ Leo A and γ Leo B, at a dispersion of 2.8 Å/mm. These were analyzed with different microphotometer and reduction techniques and therefore give an interesting comparison of accidental and systematic errors. Figure 4 shows the results for 15 lines common to the two investigations. A systematic difference, notably in the strength of weak lines, reaches 0.06 in $\log 10^6 W/\lambda$. The maximum accidental error is 0.14, and the average deviation between the two sets of observations is ± 0.05 . Thus there seems to have been some difference in the method of drawing the lines and the continuum; the total deduced photometric plus measuring accidental error is ± 9 per cent (if shared equally between the two observers).

The dependence of the strength of H γ and of Ca I, λ 4227, is shown as a function of our spectral types in Figure 5, separately for the two population types. In this comparison ν^2 Cnc has been included in population I. There is a slight tendency for H γ to be weaker in population II; this may also be true for λ 4226.

The absorption by the blue CN bands extending for more than a hundred angstroms

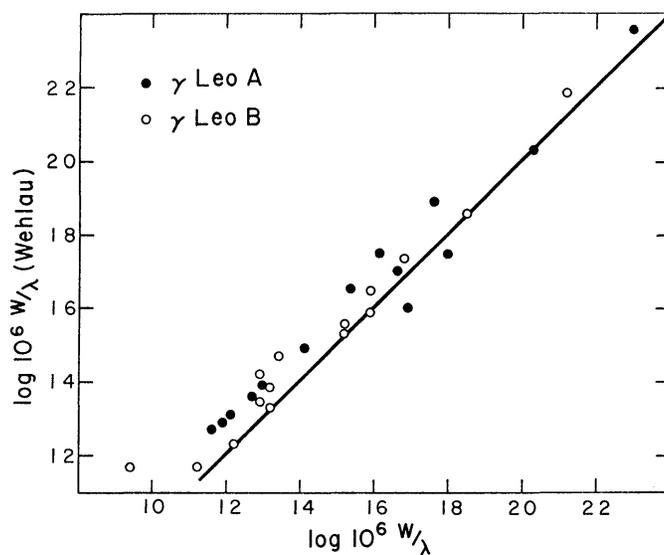


FIG. 4.—Equivalent widths on Mount Wilson spectrograms measured by Wehlau (*ordinates*) and in this investigation (*abscissae*) in γ Leo A and γ Leo B. The 45° line demonstrates a small systematic error; the accidental errors are about 9 per cent.

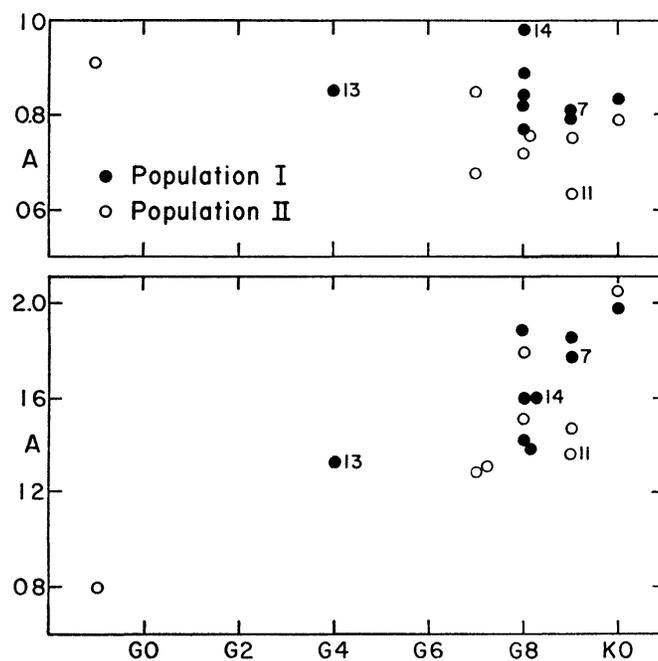


FIG. 5.—Equivalent widths of $H\gamma$ (at top) and $\lambda 4227$ of Ca I (at bottom) as a function of spectral type. Dots are low-velocity stars, including v^2 Cnc (No. 7); HD 166208 (No. 14) has abnormally strong hydrogen lines. Open circles are h-v stars. The star HD 225239, F9, has been added to show more clearly the trend with spectral type.

below their 4216 Å head is so strong in giants of types G and K that intensities of atomic lines in this region are unreliable. Consequently, the 35 atomic lines measured as relatively free from blending all lie in the range from 4222 to 4410 Å.

Since the number of good lines is not sufficient to allow the construction of accurate curves of growth, it was decided to use one mean curve for all the stars and then to correct the derived numbers of atoms for effects of differences in temperature and luminosity. It will be seen that, because these differences are quite small for our program stars, the corrections are less than the usual uncertainties in curves of growth. Wehlau (1956) has published curves of growth for two of these stars— γ Leo A and γ Leo B. Since the differences in temperature and luminosity between these two stars span nearly the whole range of these variables in all our stars, Wehlau's curves of growth were adopted as representative. The procedure was to reduce the values of W/λ to $\log N$ for each star by applying the curves of growth for γ Leo A and γ Leo B separately and then to average the logarithms of the numbers of atoms to obtain $N_0(*)$.

For the molecular bands it is still more difficult to set up reliable curves of growth. Much higher dispersion would have been required to resolve adequately the individual band lines of CN and CH. Wehlau had set up curves of growth for CH by displacing the curves of iron by an amount corresponding to the difference between the thermal motions of iron and those of CH. Our total absorptions of both CH and CN were reduced by Wehlau's CH-curves. The zero points of $\log N$ for the two molecules are, of course, displaced by an unknown amount. Consequently, the ratio CH/CN in any one star cannot be found, but the relative numbers of each molecule in any two stars can be obtained, after correction for differences in temperature and luminosity, just as for the atomic lines.

RESULTS: ATOMIC LINES

The stars selected covered only a small range of spectral type and luminosity but a considerable range of spectral peculiarity. It was impossible to select lines completely free of blends, or even with the same blends in different stars—especially so for the CN lines in h-v stars and the CH lines in objects with weak CH. Of the 35 atomic lines measured, the very strong lines, H γ and λ 4227, were omitted from the statistical analysis. Of the metallic lines, 14 belonged to iron, 7 to ionized metals, and the balance to calcium, chromium, cobalt, manganese, nickel, and vanadium. The excitation potentials ranged from 0 to 3.6 eV. Nine "lines" of CN in the λ 4215 band were measured, mostly blends of two rotational lines. The CH band provided sixteen useful features and the *P* branch at λ 4323. Since all lines were within a 240 Å wave-length interval, opacity variation was negligible. Table 3 gives a résumé of our results for the differences of $\log \eta_0(*)$ from star to star. We define D as follows:

$$D = \langle \log \eta_0(*) - \log \eta_0(0) \rangle, \quad (2)$$

where the group of "normal standard" stars, Nos. 1–4, provide a mean $\log \eta_0(0)$ for each line. [In forming $\log \eta_0(0)$, the two good plates of η Her were each given equal weight with the single plates of the other standard stars.] The quantity D represents the logarithmic difference between a given star and the normal standards, averaged over all lines; D represents a useful statistical average, which we must correct for differences of excitation temperature, pressure, degree of ionization, opacity, etc. Means of $\log \eta_0$ are taken because of the nature of a curve of growth and of the Boltzmann distribution over excited states. The standard stars are of mean type G8 III; the temperature differences to the program stars are small, so that the averaging over $\log \eta_0$ should not increase the errors, and provides a rapid method of analysis. In Table 3 we also give an estimate of the internal probable error of the mean D . A measure of quality is included, based on the dispersion, number, and character of the plates used. The internal probable error does

not include systematic errors in photometry. From Table 2, the photometric errors between plates, we conclude that systematic errors larger than 0.10 are not common, because the total $\langle |\Delta \log \eta_0| \rangle$ includes both measuring and plate errors.

The quantity D depends on abundance differences, $z(*)/z(0)$, opacity and temperature differences, as well as in differences in level of ionization, $(x/1-x)$ for which corrections can be applied:

$$D = \log \frac{z(*)}{z(0)} + \log \frac{\phi(T_0)P_e(0)}{\phi(T^*)P_e(*)} + \log \frac{V(0)}{V(*)} - \overline{EP}(\theta_* - \theta_0) \quad (3)$$

$$+ \left[\log \frac{(1-x)_*}{(1-x)_0} \right], \text{ neutral ; or } + \left(\log \frac{x_*}{x_0} \right), \text{ ions .}$$

TABLE 3
OBSERVED MEAN LOGARITHMIC DIFFERENCES, D ,
WITH PROBABLE ERRORS

No	Star	Quality	Atomic Lines ($n=33$)	CN ($n=10$)	CH ($n=17$)
1	ι Gem	B	+0 19±0 03	+0 32±0 07	+0 27±0 02
2	κ Gem	C	- 17 02	-0 02 08	-0 28 04
3	ϵ Vir	A	+ 06 02	+0 02 06	-0 16 02
4	η Her	A	- 05 02	-0 16 04	+0 12 02
5	ϕ^2 Ori	B	- 29 03	-1 18 07	+0 09 02
6	δ Lep	A	- 27 03	-1 13 08	+0 04 02
7	ν^2 Cnc	B	+ 40 04	+0 13 09	+0 03 05
8	γ Leo A	B	+ 25 04	-0 22 08	+0 11 03
9	γ Leo B	B	- 35 03	-0 94 07	-0 21 06
10	168322	B	- 08 04	-0 75 10	+0 15 06
11	191046	C	- 31 05	-1 28 10	-0 68 08
12	γ Psc	B	- 15 03	-0 62 04	-0 01 94
13	18474	A	- 03 03	-2 17 13	-1 97 10
14	166208	B	+ 46 04	-0 51 12	-0 76 06
15	5544	C	+ 44 05	+0 12 12	-0 02 04
16	180262	B	+0 29±0 03	+0 12±0 09	-0 16±0 04

The second term arises from the opacity and the third from the effect of turbulent velocity on the Doppler width, $\Delta\lambda_D$. The fourth involves the excitation potentials and temperature; the last term differs for neutral and ionized lines. Since we have about four times as many neutral as ionized lines, we can replace the last term by a linear combination,

$$0.8 \log \frac{(1-x)_*}{1-x_0} + 0.2 \log \frac{x_*}{x_0}.$$

Equation (3) should be written separately for each line and then averaged; for convenience, we have replaced this by the weighted average of the metals, for which $\langle \text{I.P.} \rangle = 7.50$ ev, $\langle \text{E.P.} \rangle = 1.73$ ev, $\log 2u_{r+1}/u_r = +0.34$. We neglect the possible variation in turbulent velocity from star to star; it has little influence on equation (3) but would introduce larger errors into $\log \eta_0$, because of the possible vertical shifts of the curve of growth. We use equation (3) to obtain small differential corrections dependent on differences of T_e from star to star.

If we take the mean observed values of D separately for ions and neutral atoms, we find considerable scatter from star to star. The average of the differences of D for ions and neutral atoms proves to be zero for the h-v stars, within the internal error of about ± 0.08 . In other words, the ionization differs only slightly from star to star, for our average metal. Of course, second-order effects in lines used in luminosity criteria (like Sr II, with abnormally low I.P.) must exist; unfortunately, no such lines were included in our program. For ϕ^2 Ori the differential ionization is -0.12 ± 0.07 ; for γ Leo B, $+0.06 \pm 0.06$; and for HD 180262, $+0.09 \pm 0.07$. These residuals are not significant but do correlate with luminosity. Since we cannot determine the ionization level and do not know P_e , we shall make various assumptions transforming equation (3) to usable form.

EFFECTS OF TEMPERATURE

The low-dispersion spectral classification is supposed to be largely a temperature classification, on which are superposed second-order ionization differences. We list in Table 4

TABLE 4
METAL ABUNDANCES, $\text{LOG } z(*)/z(0)$

No	STAR	OBSERVED		COMPUTED ABUNDANCES		
		θ	(D)	(A)	(B)	(C)
1	ι Gem	1 135	+0 19	+0 14	+0 18	+0 17
2	κ Gem	1 135	- .17	- 22	- 19	- 19
3	ϵ Vir	1 135	+ 06	+ 01	+ 05	+ 05
4	η Her	1 105	- 05	+ 06	- 05	- 02
	Mean standard	1 125	+ 01	00	00	00
5	ϕ^2 Ori	1 08	- 29	- 08	- 20	- .25
6	δ Lep	1 15	- 27	- 39	- 29	- 39
7	ν^2 Cnc	1 165	+ 40	+ 20	+ 39	+ 38
8	γ Leo A	1 20	+ 25	- 12	+ 29	+ 13
9	γ Leo B	1 09	- 35	- 16	- 27	- 35
10	168322	1 17	- 08	- 31	- 09	- 22
11	191046	1 165	- 31	- 51	- 32	- 43
12	γ Psc	1 115	- 15	- 10	- 16	- 22
13	18474	1 06	- 03	+ 30	+ 07	+ 09
14	166208	1 15	+ 46	+ 34	+ 45	+ 43
15	5544	1 20	+ 44	+ 06	+ 47	+ 46
16	180262	1 165	+0 29	+0 09	+0 28	+0 27

the values of $\theta = 5040/T_e$, where the effective temperatures, T_e , are taken from the table by Keenan and Morgan (1951) giving T_e as a function of type and luminosity. The total range, dT/T , is only 15 per cent; the zero point of the T_e scale is not important, since we compare the program stars with a group of standards at nearly the same T_e . We shall not distinguish between T_e , T_{ion} , T_{exc} ; we assume essentially that they preserve a constant ratio for the small changes in T_e .

The electron pressure, P_e , for a K giant is not known; mass and radius are both uncertain. In addition, the masses may differ between h-v and low-velocity stars, as well as A , the abundance ratio of hydrogen to the metals. The behavior of P_e with T_e and g in cool giants is shown by Aller (1953), who finds only a small change in P_e in this range. Rough interpolation formulae can be derived from the opacity theory for H^- , which gives

$P_e \propto (P_g/A)^{0.8}$. Integrating through an isothermal atmosphere to a "representative point," τ_0 , we obtain

$$dP_g = \frac{g}{\kappa} d\tau \propto \frac{g A^{0.8} d\tau}{\phi(T) P_g^{0.8}}, \quad (4)$$

$$P_g(\tau_0) \propto \left[\frac{g\tau_0}{\phi(T)} \right]^{0.55} A^{0.44}, \quad (5)$$

$$P_e(\tau_0) \propto \frac{g^{0.44}}{A^{0.36} \phi(T)^{0.44}}. \quad (6)$$

We shall show that A is larger by a factor of 2 in the h-v stars. The latter may be evolving stars, excessively luminous for their mass, but it is improbable that their $\log g$ can be less than in population I giants by a factor of more than -0.3 . The resultant maximum decrease in $\log P_e$, at a given luminosity and T_e , is by -0.24 . Between populations I and II red giants of the same T_e we might therefore expect an increased ionization at τ_0 . However, the small decrease in P_e could easily be balanced in the ionization equilibrium by a systematic shift in T_e (at a given low-dispersion spectral class) of about 5 per cent. We can make three assumptions as to the meaning of spectral classification in this temperature range and modify equations (3) and (6) to obtain the abundances.

A. Type is a measure of level of ionization. Stars of different T_e have a compensating change in P_e , to keep the level of ionization constant.

B. P_e is constant, and the ionization is variable

C. P_e is determined from equation (6) as a function of T_e , g , and A ; the ionization is variable.

The raw data of Table 3 are repeated in Table 4, together with the mean abundances of the metals derived under these three assumptions. We must make a definite choice of P_e for the normal standard stars. We have $\theta_0 = 1.125$; at $\log g = +2.8$ we find $\log P_e = +0.2$ in the standard treatment. For the average metal, $\log K(\theta_0)$ is $+0.2$, $\log x_0/(1-x_0) = +0.34$, where

$$\log K(\theta) = \log \frac{x}{1-x} P_e - \log \frac{2u_{r+1}}{u_r} \quad (7)$$

is the ionization equation. We neglect the variation in the partition functions with T . The fact that $K(\theta)$ is near unity presents some difficulties; the metals are only partially ionized, and certain simplifying approximations valid for hotter stars like the sun cannot be used.

Assumption A

If $x/(1-x)$ remains constant from star to star, equation (7) gives $\log P_e = \log K(\theta) + \text{constant}$. In a comparison with the standard stars, at θ_0 , we obtain the abundance of the average metals as

$$\frac{z(*)}{z(0)} = D + \log \frac{\phi(T*)}{\phi(T_0)} + \log \frac{K(\theta_*)}{K(\theta_0)} + 1.73(\theta_* - \theta_0). \quad (8)$$

The P_e in the concentration of neutral atoms cancels the P_e in the opacity. The terms involving T_0 , on the right-hand side, give a constant for the normal stars equal to $+6.31$. The corrections to D are the same for neutral and ionized atoms. The resulting abundances are given under the heading A in Table 4.

Assumption B

Equation (7) shows that, for a fixed P_e , the relative concentration of neutral atoms to ions is

$$\log \frac{1-x}{x} = -\log K(\theta) + \log P_e - \log \frac{2u_{r+1}}{u_r}. \quad (9)$$

If the metals were largely ionized ($x \rightarrow 1$), the P_e term would again cancel, and assumptions A and B give the same result. In fact, equation (9) must be used carefully, separating neutral atoms and ions, when combined with equation (3). Since x and $1-x$ vary in opposite senses, the corrections to D become smaller in general. The result is

$$\log \frac{z(*)}{z(0)} = D + \log \phi(T_*) + 1.73 \theta_* + 0.8 \log \frac{(1-x)_0}{(1-x)_*} + 0.2 \log \frac{x_*}{x_0} + 6.50. \quad (10)$$

The results in Table 4, column B, show a somewhat larger scatter, in particular giving positive deviations for stars at low T_e , e.g., γ Leo A, and negative residuals for hot stars. It is probable that the ionization decreases with T_e less rapidly than predicted for constant P_e ; from opacity theory it is, in fact, probable that P_e decreases with T_e . Thus the low-dispersion classification seems not to be purely a temperature classification but may take partially into account the level of ionization.

Assumption C

Note that in both cases A and B, the star v^2 Cnc is outstandingly peculiar among the h-v objects in showing a large positive residual. Since it also has strong CN, we shall consider it to be a population I star. We find that a mean deficiency of metals of at least -0.20 exists in the other h-v stars. Assume (rather uncertainly) that $\log g$ is also -0.20 smaller. From equation (6) we can obtain the value of P_e , given T_* , g , and A , as follows:

$$\log P_e = +0.20 - 0.44 \log \frac{\phi(T_*)}{\phi(T_0)}, \quad \text{Population I,} \quad (11a)$$

$$\log P_e = +0.04 - 0.44 \log \frac{\phi(T_*)}{\phi(T_0)}, \quad \text{Population II.} \quad (11b)$$

We shall assume that all stars except Nos. 5, 6, 8, 9, 10, 11, and 12 obey equation (11a) and belong to population I. The abundances are derived from

$$\log \frac{z(*)}{z(0)} = D + \log \phi(T_*) + \log P_e(*) + 1.73 \theta_* + 0.8 \log \frac{1-x_0}{1-x_*} + 0.2 \log \frac{x_0}{x_*} + 6.30. \quad (12)$$

ABUNDANCES OF THE METALS

Let us assume that the metallic lines here studied are representative of all the metals. The abundance deficiencies or excesses measured by $\log z(*)/z(0)$ will be used as a measure of A . Table 5 gives a brief résumé of the results when various groups of stars are intercompared. The last column (D) is based directly on the line strengths not corrected for temperature or pressure differences. It shows that the essential features of assumptions A, B, and C cannot be so far wrong as to conceal the real effect, which is apparent in

the line strengths themselves. The h-v G8–K0 giants are, in fact, weak-line stars and have logarithmic abundance deficiencies by at least a factor of -0.20 and probably not more than -0.35 . The intrinsic and observational scatter combine to give the probable error of the ratio $\log_{10} A_i/A_j$ of about ± 0.08 for the average groups. This scatter is slightly less on assumption A than in the others; it is definitely larger, about ± 0.10 , in column D, in Table 5. Therefore, we conclude that the temperature and pressure corrections reduce the scatter within a group. Inspection of Tables 4 and 5 shows that the different models do not completely remove the effects of temperature on the derived abundances within a group of stars but that the uncertainties of the corrections are smaller than the abundance deficiencies themselves. Some individual stars require comment.

The group of four normal standards includes κ Gem, which has a large negative residual compared to all other low-velocity stars. In fact, omission of κ Gem raises the observed mean D (stars 1, 3, 4) to $+0.07$. The somewhat peculiar stars—Nos. 13, 14, 15, 16—have a large positive residual D (Table 5, second ratio) which is somewhat reduced if κ Gem is omitted. It seems improbable that κ Gem is a h-v star; its proper motion is smaller than that of ι Gem, and its radial velocity is moderate and due largely to solar

TABLE 5
LOG₁₀ A_i/A_j , METALLIC ABUNDANCE RATIOS

Ratio	A	B	C	Uncorrected D
Stars 5, 6, 8–12/stars 1–4 (population II/normal standards)	–0 25	–0 15	–0 25	–0 18
Stars 7, 13–16/stars 1–4 (peculiar population I/normal standards)	+ 20	+ 33	+ 33	+ 14
Stars 5, 6, 8–12/stars 1–4, 7, 13–16 (population II/all population I)	–0 35	–0 34	–0 43	–0 35

motion. The spectrum used is overexposed, so that the deviation may arise from photometric errors; the quality rating was C (Table 3).

Individual differences may not be significant, given expected total photometric and analysis errors of about ± 0.15 in $\log A$ per star. One of the h-v stars, γ Leo A, tends to show positive deviations in all models, as does HD 166208 among the low-velocity group. We cannot ascribe the positive deviation for γ Leo A to real abundance excess, since its physical companion, γ Leo B, shows a typical negative deviation; γ Leo A is of somewhat higher luminosity, and consequently our T_e correction may be insufficient. It is interesting to note how clearly ν^2 Cnc, in spite of its large space motion, qualifies as a population I star. The carbon-poor star HD 18474 does not show a significant weakening of the metals. HD 166208, with smaller deficiencies of CN and CH, has, if anything, a positive deviation in metal abundance.

If we omit κ Gem, the residual for the peculiar low-velocity stars and ν^2 Cnc is reduced to $+0.07$ in D and about $+0.11$ – $+0.27$ (models A, B, and C) in abundance. It is not at all certain that the visual impression of overstrong CN is confirmed by the data in Table 3 (and the discussion below). Consequently, there is no a priori reason to exclude these from a larger group of standard “population I” stars, though they might be younger, metal-rich giants. In comparison with this larger group, in the last line of Table 5, the average h-v star shows metal deficiencies between -0.33 and -0.43 .

It would be very desirable to have an independent temperature calibration of the h-v giants from color indices rather than types. An attempt was made to use the published photoelectric colors of stars classified accurately on the MK system. At types of G8 III and of K0 III, the observed spread of $U - V$ colors (very wide base line) was 0.30 mag.,

while the change in color with type was 0.26 mag. A probable error of half a spectral subdivision in classification results in a probable error of the deviation of one star, from the ($U - V$ type) relation of 0.07 mag., consistent with the observed spread of color at each type. An error of 0.07 mag. on a $U - V$ base line corresponds to an error, for a black body, of dT/T near 0.03; the error in our temperature calibration corresponding to half a subdivision is about 0.015 in dT/T . Not only are the errors comparable using types or colors, but the colors may be affected by the spectral lines. The B and especially the U effective wave lengths must be seriously depressed by the lines, and a decrease in line strength will therefore affect the color scale. There is a slight indication that the colors of the h-v giants by Roman (1954) are bluer than those of normal giants; unfortunately, the accuracy of classification seems lower, presumably because of the fainter apparent magnitudes of the former group.

Figure 5 shows the measured equivalent widths of $H\gamma$ and $\lambda 4227$ as a function of spectral type. (An additional star, of earlier type and rather large proper motion, HD 225239, has been added to check the trend over a larger temperature range.) Note that there is a slight tendency for the h-v stars to have weaker lines of both $H\gamma$ and Ca I. The star noted by Hossack (1954) is No. 14 = HD 166208. It has the strongest hydrogen line in the group; No. 13 = HD 18474 may be slightly high. In the diagram for $\lambda 4227$, it is clear that the line strengthens with decreasing temperature and that the h-v stars definitely have the weaker lines at a given type. In spite of its apparently positive residuals in the abundance table, the h-v star No. 9 = γ Leo A shows a weakened $\lambda 4227$.

RESULTS: MOLECULAR BANDS

The correction of the values of D for temperature and pressure effects is more difficult for the band lines of CN and CH than for atomic lines. Although ionization of the constituents of these molecules can be neglected, the dissociative equilibrium of the compounds is sensitive to the relative abundances of hydrogen, carbon, nitrogen, and oxygen in the atmospheres and to any differences in g_{eff} .

Calculations of the partial pressures for several assumed chemical compositions, temperatures, and total pressures have been carried out by de Jager and Neven (1957) and Stanger (1957). From these partial pressures the values of $\log N$ can be computed for stellar atmospheres constructed according to some assumed model, if the variation in the effective gravitational constant, g_{eff} , is known as a function of temperature and pressure. At the present time there is enough uncertainty in our knowledge of some of the atmospheric parameters, particularly the chemical composition and the values of g_{eff} for the more luminous stars, to prevent the computations from leading directly to a good representation of the observed behavior of the more abundant molecules. Perhaps the best immediate use of the equilibrium computations is to give additional relations leading to more consistent values for the physical properties of the atmospheres.

For this reason it is preferable to find observational corrections to our values of D as a function of type and luminosity. This can be done for CN. The magnitude of the break in the spectrum at the $\lambda 4216$ head of the blue bands has been measured for a number of G- and K-type stars on small-scale prismatic spectrograms (104 Å/mm at $H\gamma$) taken with the Perkins 69-inch reflector. The measurements within the band cover a range of about 2 Å centered near 4212 Å, i.e., just far enough from the head to avoid the strongest part of the Sr II line at 4215.5 Å. The most nearly comparable data from the Mount Wilson coude spectrograms are the central depths of the four CN band lines between 4212 and 4214.6 Å. In Figure 6 the averages of these central depths are plotted as abscissae and the percentages of absorption at the break, measured on the Perkins plates, as ordinates for the 10 stars measured at both observatories. The correlation is good enough to indicate that essentially the same physical quantity is being measured on both the high- and the low-dispersion spectrograms.

Consequently, the following procedure was adopted. A diagram was prepared showing the mean low-dispersion CN break as a function of type and luminosity in 60 stars measured at Perkins. The correlation between the break and the actual strength of CN lines was available from stars measured at Mount Wilson and shown in Figure 6. We adopt the same curve of growth for all stars and obtain the observed variation in $\log N$ of the CN molecule as a function of MK spectral type and luminosity. Thus differential corrections could be made between the normal standard stars and the program stars, given the MK type. These corrections have been applied, and the results are shown as D' in the fourth column of Table 6. The corrections are large only for HD 18474, which has an earlier type, G4, than any of the others, and for HD 180262, for which the correction of -0.43 corresponds to the luminosity class II-III, and reflects the rapid strengthening of CN with increasing luminosity. The method includes an empirical allowance for change in opacity as a function of MK type but not for population type. Similarly, pos-

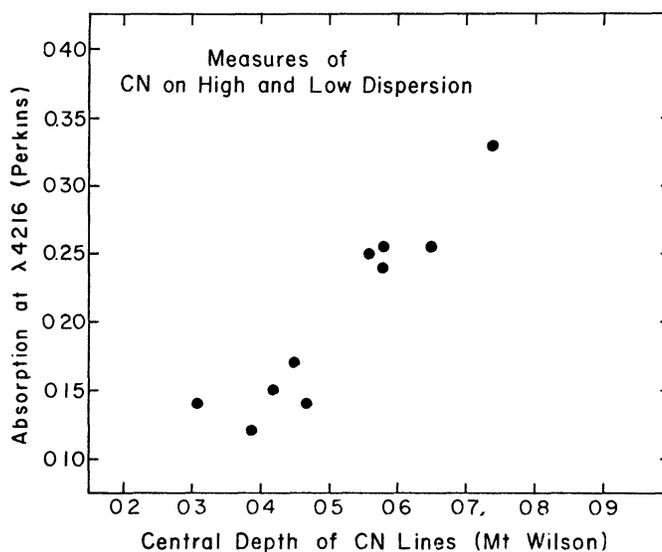


FIG 6—Correlation of fraction of light absorbed at the CN break at $\lambda 4216$ from Perkins prismatic spectrograms (*ordinates*), with the mean central depth of the CN band lines at $\lambda\lambda 4212\ 4$, $4212\ 8$, $4213\ 8$, and $4314\ 6$ from Mount Wilson coude spectrograms (*abscissae*)

sible changes in curve of growth as a function of luminosity are implicitly assumed to be the same in both populations.

The percentage errors of the corrections are undoubtedly large but are difficult to determine, since they must be due largely to uncertainties in the spectral classification of the stars. We are not prepared to estimate this uncertainty for the stars on this program, most of which were included because of suspected peculiarities in their spectra. It is only for the last two stars in Table 6 that the corrections would appreciably change the interpretation of the measurements. These two stars, HD 5544 (No. 15) and HD 180262 (No. 16), had been measured because the bands of both CN and CH had appeared on small-scale spectrograms to be stronger than normal. The uncorrected abundances for CN and CH differ relatively little from the values for the four standard stars, and the corrected figures for CN imply that this molecule may actually be *less* abundant in stars Nos. 15 and 16. We conclude that there is no real evidence of peculiarities in molecular abundances in these two stars.

Among the four standard stars, only ι Gem, with a corrected excess of 0.35 in D , differs appreciably from the mean. This star shows positive residuals in the number of

atoms and of CH molecules also; the differences are of doubtful significance and may be explained if the continuum had been drawn slightly too high on the spectrograms of ι Gem.

The behavior of CN in the h-v stars Nos. 5-12 parallels that of the atomic lines. Again No. 7, ν^2 Cnc, shows no significant peculiarity and appears as a typical population I star. The remaining 7 h-v stars all show CN deficiencies ranging from -0.49 to -1.47 in the corrected value of $\log N$. Application of the corrections has brought the values for γ Leo A and γ Leo B into good agreement. The average value of the uncorrected D is -0.91 and of the corrected D is -0.98 . Thus our sample of h-v stars has a ninefold deficiency of CN in comparison to the population I standard stars. γ Psc and the two components

TABLE 6
ABUNDANCES OF CN AND CH

No	STAR	CN		CH			
		Observed D	Empirically Corrected D'	Observed D	Corrected for Opacity Only D'		
					A	B	C
1	ι Gem	+0 32	+0 35	+0 27	+0 22	+0 28	+0 27
2	κ Gem	-0 02	-0 10	-0 28	-0 33	-0 27	-0 28
3	ϵ Vir	+0 02	-0 06	-0 16	-0 21	-0 15	-0 15
4	η Her	-0 16	+0 03	+0 12	+0 28	+0 08	+0 10
	Mean standard	+0 04	+0 05	-0 01	-0 01	-0 01	-0 01
5	ϕ^2 Ori	-1 18	-0 91	+0 09	+0 49	-0 01	-0 13
6	δ Lep	-1 13	-1 27	+0 04	-0 11	+0 07	-0 10
7	ν^2 Cnc	+0 13	-0 06	+0 03	-0 23	+0 08	+0 06
8	γ Leo A	-0 22	-0 58	+0 11	-0 38	+0 22	+0 01
9	γ Leo B	-0 94	-0 69	-0 21	+0 05	-0 28	-0 41
10	168322	-0 75	-1 11	+0 15	-0 14	+0 21	+0 02
11	191046	-1 28	-1 47	-0 68	-0 94	-0 63	-0 81
12	γ Psc	-0 62	-0 49	-0 01	+0 07	-0 04	-0 19
13	18474	-2 17	-1 55	-1 97	-1 51	-2 09	-2 04
14	166208	-0 51	-0 76	-0 76	-0 91	-0 73	-0 74
15	5544	+0 12	-0 20	-0 02	-0 52	+0 08	+0 04
16	180262	+0 12	-0 31	+0 03	-0 23	+0 08	+0 06

of γ Leo have the smallest deficiency, averaging 0.38 of the normal number of CN molecules. The most extreme case is HD 191046 (No. 11), in which the abundance of CN is low by a factor of 20 or 30. Much of this dispersion in the values of the CN deficiency is evidently real; for example, γ Leo A and B look more nearly like population I giants on low-dispersion plates than do stars such as HD 191046, for which the usual CN depression in the blue is conspicuously weak. Significant deficiencies in CN are shown also by stars Nos. 13 and 14, mentioned previously as possible carbon-poor stars.

For the bands of CH it is not easy to derive empirical corrections to D . The G band has a relatively open structure, and the band lines are generally more affected by blending than are the strong heads of the blue CN bands. Consequently, it has proved difficult to obtain reliable intensities of CH on spectrograms with a scale smaller than about 10 A/mm. The measured intensities of Setterberg (1947) and Tchong-Kien (1951) are based upon relatively small-scale plates and do not agree very closely with each other or

with our unpublished estimates from Yerkes plates in defining the variation of CH with luminosity and temperature. In general, it appears that CH shows a *negative* luminosity effect near the giant branch for late G and early K stars, although the magnitude of the effect is less than the gradient of the positive effect for CN. The maxima of the curves of intensity against spectral type are also flatter for CH than for CN. For giants the CH maximum is not far from type G8. Thus there is reason to believe that the corrections to the measured D would be correspondingly less than those derived from CN. We have chosen to leave the observed values of D_{CH} in the fifth column of Table 6 uncorrected for changes in dissociation equilibrium. The empirical corrections can be applied at a later time when more accurate measurements from coude spectrograms of standard stars become available.

The only stars for which D is large enough to indicate significant anomalies in CH abundance are Nos. 11, 13, and 14, for all of which the bands are weaker than in the standard stars. All the h-v stars excepting No. 11 have nearly normal CH absorption. For Nos. 5, 6, 8, 9, 10, and 12 the mean $D = +0.03 \pm 0.04$ m.e.

It may be argued that at least some theoretical correction to the observed D for CH should be applied. The most obvious known factor is the opacity, which is a function of T and P_e . In our discussion of the metallic lines, various assumptions were made as to the meaning of spectral classification. In assumption A it was assumed that the level of ionization defines a type. In that case we can derive

$$D'(\text{CH}) = D(\text{CH}) + \log \frac{\phi(T_*)}{\phi(T_0)} + \log \frac{\kappa(\theta_*)}{\kappa(\theta_0)}, \quad (13)$$

where $D(\text{CH})$ is the uncorrected value. For a fixed P_e , i.e., assumption B, only the $\phi(T)$ term appears in the opacity; for assumption C, the variation in P_e with T and population type must be included, as well as the $\phi(T)$ term. The last three columns in Table 6 give $D'(\text{CH})$ computed on the basis of these assumptions. No preference for systematic effects is apparent; the scatter in CH abundances persists; if we exclude v^3 Cnc and HD 191046, the mean differences in D between h-v and normal stars are $+0.01$, $+0.03$, and -0.10 , depending on the corrections adopted. It should be remembered that the molecular dissociation equilibria for CH have been omitted in both the D and the D' values.

DISCUSSION

In principle, we have in Table 6 data for the abundance determination of C and N in h-v stars relative to normal stars. Table 7 contains the results of the analysis based on various assumptions. We employ the mean results derived from assumptions A, B, and C for the metals, $\log(z_*/z_0)$, and for $D'(\text{CH})$ and use the empirically corrected $D'(\text{CN})$. If we assume that our method of analysis has sufficiently corrected for opacity variations, we determine separately the changes in $\log A$, $\log z_{\text{C}}$ (abundance of C/H), and $\log z_{\text{N}}$ (abundance of C/N). The method of Schwarzschild, Spitzer, and Wildt (1951) gives abundances compared to the source of opacity, H^- , i.e., the hydrogen-to-metal abundance ratio appears also in the CN and CH strengths; we use their case 2 as probably relevant for G8 giants (metals neutral, carbon and nitrogen mostly free); for this purpose we must use our uncorrected, observed D . Their method gives a mean B for the ratio of abundance of hydrogen to the carbon, nitrogen, and oxygen group as a unit, while our observations and analysis apparently require a differential effect between carbon and nitrogen. We must emphasize that our group of h-v stars was selected to include those objects which show the h-v characteristics most strikingly in their spectra. It is a representative *spectroscopic* sample, and the mean abundance anomalies for a *dynamical* sample may well be smaller. The small effects found by Schwarzschild *et al.* (1957) arose from the use of a dynamical sample, and, of their stars, only ϕ^2 Ori is spectroscopically an extreme case. Another discordance between our results is in our small or zero value of

$D(\text{CH})$ and $D'(\text{CH})$; in addition, on visual, low-dispersion classification, CH was stated to be strengthened in h-v stars by Keenan and Keller (1953), e.g., in HD 168322 and HD 134063. The most likely explanation is that the confirmed weakening of the atomic lines reduces the amount of blending and the depression of the continuum and allows the bands to stand out more conspicuously to the eye and in relatively low-dispersion spectrophotometry.

Table 7 includes our quantitative results, together with mean abundance changes in $\log z_*/z_0$, which may be taken as a measure of $\Delta \log A$ for the metals, and $\Delta \log z_C$ and $\Delta \log z_N$, on the assumption that the partial pressures of carbon and nitrogen are unaffected by the association of carbon monoxide and that there are no systematic differences in the associative equilibria of CN and CH between the h-v and the normal stars. Our lack of knowledge of the equilibria makes these results only tentative first approximations.

TABLE 7
TENTATIVE ABUNDANCES OF METALS, CARBON, AND NITROGEN
BASED ON MEAN OF ASSUMPTIONS A, B, C

GROUP OF STARS	METHOD OF THIS PAPER			METHOD OF SCHWARZSCHILD	
	$\Delta \log A$	$-\Delta \log z_C$	$-\Delta \log z_N$	$\Delta \log A$	$\Delta \log B$
Stars 5, 6, 8-10, 12/1-4 (population II/standards)	+0 21	+0 03	+0 86	+0 28	{ +0 15* +0 53
Stars 5, 6, 8-10, 12/1-4, 15, 16 (population II/population I)	+ 37	0 00	+ 79		
Stars 11/1-4 (C-poor population II/standards)	+ 4	+0 8	+ 6		
Stars 13-14/1-4 (C-poor population I/standards)	-0 3	+1 3	0 0		

* Note that the system of equations and observational data is overdeterminate; the two values given are derived, respectively, from the ratio of CH/metals and CN/metals

The quantities of z_C or z_N , as abundances, are reciprocal to the usual quantities A (H/metals) and B (H/C + N + O). In Table 7 we see that the h-v stars have abundance deficiencies in the metals—a larger deficiency in nitrogen and a smaller deficiency in carbon. The peculiar h-v star HD 191046, No. 11 (called “C-poor”), seems to have greater deficiencies of carbon and nitrogen than of the metals. The peculiar carbon-poor stars Nos. 13 and 14 seem to have normal nitrogen and a possible excess of metals. Detailed interpretation of these data on theories of nucleogenesis is premature; the burning of C^{12} in an equilibrium carbon cycle can yield high N^{14} abundance, with a reduction in the $\text{C}^{12}/\text{N}^{14}$ ratio. On an evolutionary theory of nucleogenesis, the abundance of $\text{N}^{14}/\text{C}^{12}$ should increase with time, as does the metal content. Therefore, old stars could be nitrogen-poor, as well as metal-poor; C^{12} abundance variations depend on unknown details of the late helium-burning stages of nuclear evolution. The agreement between our results and those of Schwarzschild *et al.* (1957) is satisfactory when we consider how different are the methods of analysis. We have adopted temperatures, where they have used the colors; we used 4.5 A/mm against their 10 A/mm; we had usually one plate per star as against their four plates; and our choice of lines is quite independent. We also have used level of ionization and opacity considerations based on theory rather than their interpolation methods.

Bidelman's carbon-poor star, HD 18474, No. 13, is outstanding as showing the greatest observed deficiencies of both CN and CH. The metallic lines have nearly normal

strength for its type of about G4. Smaller but similar deficiencies of both molecules obtain for HD 166208, No. 14. This object was classified by Hossack (1954) among his group of stars showing strong hydrogen lines and is one of the few stars in his list without marked variations in radial velocity; the excess strength of hydrogen is only moderate. Since visual examination of several other stars in Hossack's group revealed no indication of weakness in CN or CH, it appears that HD 166208 does not belong with them but has more in common with the two other known carbon-poor stars, HD 18474 and HD 30297. It is, however, a less extreme example. HD 5544 and 180262, included as examples of stars with an apparent CN excess, are found to have rather strong metallic lines but nearly normal strengths of CN and CH. In view of the great sensitivity of CN to luminosity near type K0, we can say only that in these two there is no indication of a cyanogen excess greater than the uncertainty of our measurements and classification. There should be careful measurements made on any stars which are reported as showing unusually strong CN bands, for it is of obvious importance to define the characteristics and membership of this group.

The results for the metals (Table 4) and the two molecules (Table 6) and the derived abundances (Table 7) confirm these known characteristics of h-v population II stars of the giant branch near G8: a moderate reduction in the abundance of metals and a large deficiency in the abundance of CN. In all except HD 191046, the concentration of CH, while difficult to determine, appears to be about the same as in population I stars.

All data agree also in indicating that 32 ν^2 Cnc is a normal population I star which happens to have a space motion (about 90 km/sec) well down on the large-velocity side of the distribution curve for speeds of population I stars, i.e., that it is an evolved "runaway" population I star.

RESULTS OF VISUAL INTENSITY ESTIMATES

Spectra such as are reproduced in Figures 1 and 2 provide an excellent opportunity to search for possible small line-intensity changes from star to star. The region $\lambda\lambda$ 4205–4345 contains at least four hundred distinguishable "lines" (which even on this dispersion are often blends) for visual comparison. The correspondence of the line spectra is remarkably close in a comparator; the eye quickly accommodates to small systematic strengthening or weakening and notes only differential effects. The two h-v stars ϕ^2 Ori and δ Lep were compared with the standards ι Gem and κ Gem; there is hardly any difference in type or luminosity. After a careful survey, 53 identifiable lines were suspected of intensity differences. Identifications were based on the *Revised Rowland* table of solar lines, after the relative-intensity changes had been noted. The results were that in the h-v stars 14 lines of CN weakened 1.9 steps; 10 lines of CH strengthened 0.6 steps; 16 lines of iron, chromium, and titanium with excitation potentials > 2 ev weakened 0.8 steps; 13 lines of Fe⁺, Ti⁺, and rare-earth ions weakened 0.7 steps. It should be emphasized that these changes are relative to the behavior of the average lines in the spectrum. The step values are arbitrary; a change of two steps is a very large difference. The most significant effect, confirmed by the quantitative results given in Table 3, is the weakening of CN. The strengthening of CH is small and may be due to the contrast effect discussed. However, the weakening of the ions and high-excitation lines of the neutral metals is a new fact of some interest. The h-v stars seem to have a slightly lower excitation temperature and level of ionization than indicated by their low-dispersion spectral classification. It is particularly interesting to note that the same conclusion is suggested by the results of Schwarzschild *et al.* (1957) for ϕ^2 Ori, their h-v star H. In their Figure 1, H deviates from other stars of similar $B - V$ color in having Fe I (low E.P.)/Fe (medium E.P.) and Ti I/Fe II greater than normal, i.e., in showing a lower level of excitation and ionization. A similar effect is that Ti II/Ti I is less than normal. Therefore, the star has a lower temperature than is implied by its color; we find in addition that its line spectrum has a lower temperature than is implied by its low-dispersion spectral type. It therefore seems

safe to conclude that T_e may be lower than we have used in the theoretical analysis. Therefore, the conclusion as to the reduced abundances of the metals in the h-v stars is strengthened.

The star HD 166208, which has strong hydrogen lines and weak CN and CH, according to our measures, was compared with ι Gem. The plates were less well matched in density, and, in addition, from Table 3 it can be seen that the average metallic lines were strongest in this star. Thus a considerable readjustment of the visual intensity scale was required. However, 91 identifiable lines were noted as different in intensity (presumably relative to the average intensity difference). The mean change between HD 166208 and ι Gem is as follows: 9 lines of CN weakened 1.7 steps; 32 lines of CH weakened 1.2 steps; 13 lines of metals with E.P. > 2 ev strengthened 0.8 steps; 12 lines of metals with E.P. < 2 ev strengthened 0.7 steps; 16 lines of Fe^+ , Ti^+ , V^+ , Cr^+ , and Sc^+ strengthened 0.8 steps; 11 lines of ionized rare earths, Zr^+ , and Y^+ strengthened 0.6 steps. Note that there is no differential behavior between lines of high and low excitation or between the ionized normal metals and rare earths. However, both CN and CH are weakened (in strong contrast to the h-v stars) and indicate a different type of apparent abundance anomaly.

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