# LINE BROADENING IN HIGH-LUMINOSITY STARS. I. BRIGHT GIANTS* 

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#### Abstract

Line profiles have been obtained from high-dispersion spectra of a random sample of ten A-and F-type bright giants. These profiles match very well ones computed with the Doppler velocity from curves of growth combined with various amounts of rotation. The mean projected rotational velocity of $26 \mathrm{~km} / \mathrm{sec}$ is in good agreement with that predicted from B stars and the Sandage-Schwarzschild evolutionary tracks, provided that the bright giants do not rotate as rigid bodies.


## I. INTRODUCTION

It has long been known that the line profiles in supergiant spectra are not characteristic of rotation alone and that the lines are broader than is compatible with reasonable rotational velocities for these large stars. It has been felt that turbulence contributes appreciably to the line widths, but just how much is due to this cause and how much to rotation has not yet been determined. We are familiar with the contribution of turbulence (or some other type of atmospheric mass motion) to the intensities of lines as shown by curves of growth. Is the same mean turbulent or Doppler velocity from curves of growth also applicable to line profiles, and does it, together with rotation, fully account for the observed profiles?

In this series of papers we shall discuss and interpret a homogeneous set of high-dispersion spectra of high-luminosity stars. The set includes nearly all the known bright giants and supergiants of types A and F and brighter than $m_{v}=8.0$. It is believed to be a random sample of such stars. The general procedure involves a comparison of observed and computed profiles. From this we wish to determine whether all the profiles can be explained satisfactorily by the Doppler velocity from curves of growth plus rotation or whether it is necessary to invoke some additional broadening mechanism. A second test is whether the rotational velocities derived are reasonable.

A preliminary survey of the line widths on these spectra is shown in Figure 1. The widths plotted still include an instrumental half-half-width of 0.113 A . We see that the bright giants (luminosity class II) exhibit a large range of line widths (from 0.22 to 0.60 A ), whereas for supergiants the range is shorter and the minimum width increases with luminosity. In this paper we shall discuss only the bright giants.

## II. OBSERVATIONS

The spectrograms were obtained with the B camera (dispersion $8 \frac{1}{2} \mathrm{~A} / \mathrm{mm}$ ) of the McDonald 82 -inch coudé spectrograph during September and October, 1956. The instrumental broadening was constant within 10 per cent and can be described by the Voigt parameters (van de Hulst and Reesinck 1947) $h=0.226$ A and $\beta_{1} / \beta_{2}=0.59$. Spectra were obtained for ten of the twelve A- and F-type bright giants known at the time. Table 1 gives, for each star observed, the spectral classification and its source.

To compute a line profile, it is necessary to know the temperature, $T$; electron pressure, $P_{e} ;$ Doppler velocity, $v_{T}$; and damping constant, $a$. These quantities were obtained from curves of growth for both sharp- and broad-lined stars, with the results tabulated

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Fig. 1.-Observed half-half-widths of $\lambda 4508 \mathrm{Fe}$ II in stars of various absolute magnitudes, $M_{v}$. The solid dots represent stars for which individual luminosities were available from membership in associations or from hydrogen-line intensities. The circles represent stars for which only luminosity classes are available; all these were plotted at the mean $M_{v}$ for the class.

TABLE 1
List of Stars Observed

| Star | Spectral Class | Source* | Log $W / \lambda$ | $\eta_{0}$ | $V \sin i(\mathrm{kM} / \mathrm{SEC})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Present | Herbig and Spalding | Initial <br> (A) | Initial <br> (B) |
| 19 Aur. | A5 II | 1 | -4 23 | 40 | 15 |  | 29 | 62 |
| HR 1242 | F0 II | 1 | -4 18 | 65 | 10 | <20 | 24 | 50 |
| 22 And. | F2 II | 1 | -4 20 | 54 | 58 | 40 | 157 | 324 |
| 1 Mon | F2 II | 2 | -4 28 | 21 | 26 | 25: | 70 | 145 |
| $\nu \mathrm{Her}$ | F2 II | 1 | -4 27 | 25 | 28 | 30 | 76 | 156 |
| $\pi \mathrm{Sgr}$ | F2 II | 2 | -4 18 | 63 | 28 | 30 | 76 | 156 |
| $\nu$ Per. | F5 II | 1 | -4 16 | 130 | 46 | 45 | 146 | 322 |
| 41 Cyg | F5 II | 1 | -4 11 | 320 | 12 | $<15$ | 38 | 84 |
| $\rho$ Pup | F6 II | 2 | -4.10 | 360 | 15 | 15 | 54 | 123 |
| 11 Pup | F8 II | 2 | -407 | 600 | 20 | 25 | 94 | 199 |
| Mean |  |  |  | $\ldots$ | 26 | $\ldots$ | 76 | 162 |

[^0]in Table 2. Values of the ionization temperature, $T_{\text {ion }}$, and $P_{e}$ were obtained through a comparison of continuous opacities and ionization of Fe I with Greenstein's (1948) results for a Persei. Only lines of $\mathrm{Fe}_{\mathrm{I}}$ and Fe II were used. From Table 2 we deduce that the Doppler velocity is nearly the same for all bright giants and probably independent of rotational velocity.

For purposes of computing line profiles, the stars in Table 1 were divided into two groups. Table 3 gives the parameters assumed for each group. The ratio of local to mean opacity was computed at $\lambda 4508$ for $\mathrm{H}^{2}$ and $\mathrm{H}^{-}$.

One necessary quantity is not determined by the curves of growth, namely, the ratio $\epsilon$ of line absorption to line absorption plus scattering. All computations were made for the line $\lambda 4508 \mathrm{Fe}$ II, a moderately strong one originating at 2.8 ev . It was found that,

TABLE 2
Results from Curves of Growth

| Star | Spectral Class | $\log a$ | $\begin{gathered} v_{T} \\ (\mathrm{~km} / \mathrm{sec}) \end{gathered}$ | $T_{\text {ion }}$ | $\log P_{e}$ | Source |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HR 1242 | F0 II | -20 | 55 | 6680 | 0.47 |  |
| $\nu \mathrm{Her}$ | F2 II | -26 | 58 |  |  |  |
| 41 Cyg | F5 II | -30 | 63 | 6220 | 18 |  |
| $\rho$ Pup | F6 II | -27 | 50 | 6070 | 04 | Greenstein (1948) |

TABLE 3
AsSumed Physical Parameters

| Group | $\log a$ | $\begin{gathered} \binom{v_{T}}{(\mathrm{~km} / \mathrm{sec})} \end{gathered}$ | $T_{\text {ion }}$ | $\log P_{e}$ | $\kappa_{\nu} / \bar{\kappa}$ | $\epsilon$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A5-F2 | $-20$ | 55 | 6680 | 047 | 034 | 0.0 |
| F5-F8. | -27 | 55 | 6150 | 030 | 054 | 00 |

for stars in rapid rotation ( $V \sin i>15 \mathrm{~km} / \mathrm{sec}$ ), various assumptions for $\epsilon$ over the complete range from 1.0 to 0.0 produced no change in the rotational velocity derived but did change the abundance by factors as large as 2 . For smaller rotations it was possible to find pairs of values of $\epsilon$ and $V \sin i$ each of which would produce agreement with the observed profiles. The extreme case is that of HR 1242, which fits either $\epsilon=1.0$ and $V \sin i=0 \mathrm{~km} / \mathrm{sec}$ or $\epsilon=0.0$ and $V \sin i=10 \mathrm{~km} / \mathrm{sec}$. However, for two stars it was impossible to explain the large depths of the line with large values of $\epsilon$. From 41 Cygni we learned that $\epsilon \leq 0.06$ and from $\rho$ Puppis, $\epsilon<0.02$. For consistency we assumed $\epsilon=0.0$ for all the stars.

## III. COMPUTED PROFILES

It is easy to show that the line depth, $R(\lambda)$, integrated over the disk of a star rotating with an equatorial linear velocity $V$ is given by

$$
\begin{equation*}
R(\lambda)=\frac{\int_{0}^{1} \frac{I_{\lambda}(0, \mu)}{I_{\lambda}(0,1)} \mu d \mu \int_{0}^{2 \pi} R_{\mu}\left(\lambda-\lambda_{0}-\frac{V \sin i \sin \phi \sin \theta}{c} \lambda_{0}\right) d \phi}{2 \pi \int_{0}^{1} \frac{I_{\lambda}(0, \mu)}{I_{\lambda}(0,1)} \mu d \mu} \tag{1}
\end{equation*}
$$

where $I_{\lambda}(0, \mu) /\left(I_{\lambda}(0,1)\right.$ is the monochromatic limb darkening at $\mu=\cos \theta ; \lambda_{0}$ is the wave length of the center of the line; $i$ is the angle of inclination of the axis of rotation with the line of sight; $\theta$ is the angle at the center of the star between any point on the surface and the subsolar point; and $\phi$ is an angle around the line of sight measured from a projection of the rotation axis onto a plane perpendicular to the line of sight. The line depth, $R_{\mu}$, was computed from Chandrasekhar's (1950) exact solution to the equation of transfer as a function of angular distance from the center of the disk. Thus we take into account not only the limb darkening for the wave length of the line but also the center-to-limb variation in the strength and shape of the line.

The integration over $\mu$ was performed numerically for ten equally spaced values of $\mu$. The integration over $\phi$ was performed numerically at variable increments $\Delta \phi$ such that

$$
\begin{equation*}
\Delta\left(\frac{V \sin i \sin \phi \sin \theta}{c} \lambda_{0}\right)=0.05 \mathrm{~A} . \tag{2}
\end{equation*}
$$

This enabled us to make use of the same set of profiles, $R_{\mu}$, for each different $\phi$. The increments $\Delta \phi$ were then determined from equation (2), namely

$$
\begin{equation*}
\Delta \phi=\Delta \operatorname{arc} \sin \left(\frac{0.05 c}{V \lambda_{0} \sin } \frac{i \sin \theta}{\sin }\right) \tag{3}
\end{equation*}
$$

Finally, instrumental broadening was applied to obtain the computed profile $R_{\text {comp }}(\lambda)$,

$$
\begin{equation*}
R_{\mathrm{comp}}(\lambda)=\int_{-\infty}^{\infty} I\left(\lambda-\lambda^{\prime}\right) R\left(\lambda^{\prime}\right) d \lambda^{\prime}, \tag{4}
\end{equation*}
$$

where $I\left(\lambda-\lambda^{\prime}\right)$ is the instrumental profile described in the previous section and $R\left(\lambda^{\prime}\right)$ is the result of equation (1).

Profiles were computed for each of the two groups in Table 3 for values of $V \sin i$ differing in steps of $10 \mathrm{~km} / \mathrm{sec}$ and for several line strengths.

## IV. RESULTS AND DISCUSSION

The computed and observed profiles for a sample of the stars are given in Figure 2. The agreement is within the accuracy of the photometry for all ten stars. In fitting the observed profiles it was possible to distinguish a difference in $V \sin i$ of $1 \mathrm{~km} / \mathrm{sec}$. However, because of possible systematic errors, due especially to the representation of stellar atmospheres by mean parameters, we estimate the probable error in $V \sin i$ for individual stars to be $\pm 3 \mathrm{~km} / \mathrm{sec}$. Table 1 gives the equivalent widths $(\log W / \lambda)$ of $\lambda 4508$; the central line strengths, $\eta_{0}$; and $V \sin i$. Primarily because of the statistically small sample involved, a probable error of $\pm 5 \mathrm{~km} / \mathrm{sec}$ in the mean $V \sin i$ would be realistic.

It is of interest to compare these rotational velocities with visual estimates by Herbig and Spalding (1955). Their values (Table 1) are in very good agreement with the present ones, except for 22 Andromedae. For this star, Slettebak (1953) derived a rotational velocity of $65 \mathrm{~km} / \mathrm{sec}$.

We have succeeded in explaining the observed profiles of $\lambda 4508$ by using as broadening mechanisms only the Doppler velocity from curves of growth and rotation. However, the solution may not be a unique one-we may have underestimated the contribution by turbulence and overestimated the rotational contribution-so we shall inquire as to the reasonableness of the rotational velocities.

According to the evolutionary tracks by Sandage and Schwarzschild (1952), the bright giants originated from about B7.5 on the main sequence. Assuming (1) $M_{v}=-2.0$ for all these ten bright giants; (2) the temperature calibration by Keenan and Morgan (1951);
and (3) Kuiper's (1938) bolometric corrections, we obtain evolutionary changes in radii by factors of $4.2-10.0$. What should be the resulting changes in rotational velocities?

Following the technique of Oke and Greenstein (1954), we shall try two extreme cases in the application of conservation of angular momentum. These correspond to (A) a free radial transfer of angular momentum within the star and (B) no such transfer. Case A is that of rigid-body rotation and case B of differential rotation in which each thin shell retains its own angular momentum. We have computed the initial rotational velocities for these two cases (Table 1), employing, in case A, Oke and Greenstein's calculations


Fig. 2 -Observed (circles) and computed (solid lines) profiles of $\lambda 4508 \mathrm{Fe}$ II in the four stars identified by name, spectral class, and value of $V \sin i$ in $\mathrm{km} / \mathrm{sec}$.
of changes in moment of inertia. The mean projected rotational velocities for the two cases A and B are $76 \pm 15$ and $162 \pm 31 \mathrm{~km} / \mathrm{sec}$, respectively. These should be compared with Slettebak's (1954) value of $152 \mathrm{~km} / \mathrm{sec}$ for B8 V stars.

We see that there is excellent agreement with the model of differential rotation in bright giants. However, if the evolutionary track is correct, our rotational velocities are too small by a factor of 2 to be compatible with rigid-body rotation in bright giants. It could be that we have overestimated the contribution of rotation to the line profiles for the following reason: One of these stars, $\rho$ Puppis, is known to be a light-variable (Eggen 1957) and probably the others are also slightly variable (Abt 1957). Pulsation will contribute to the line width but will produce less broadening in this case than a rotation of
about $3 \mathrm{~km} / \mathrm{sec}$. However, it does not seem possible that the rotational contribution has been underestimated, since the Doppler motion which contributes to line intensities must also be present in the profiles.

We conclude that the profiles of lines in bright giant spectra are entirely due to the Doppler velocities from curves of growth and rotation. Furthermore, the rotational velocities indicate that if the Sandage-Schwarzschild evolutionary tracks for bright giants are correct, these stars do not rotate as rigid bodies.

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[^0]:    * The sources are: (1) Johnson and Morgan (1953); and (2) Bidelman (1951).

