H AND K EMISSION IN LATE-TYPE STARS: DEPENDENCE OF LINE WIDTH ON LUMINOSITY AND RELATED TOPICS

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ABSTRACT

The H and K emission lines of Ca II have been studied on 10-A/mm spectrograms of 185 stars of types G, K, and M. Nearly all stars of type G0 or later in the list of MK standards (Johnson and Morgan 1953) have been included. Emission-line widths have been measured, as well as displacements of the emission and absorption components. The displacements are determined with respect to nearby low-excitation reversing-layer absorption lines.

When the logarithms of the emission-line widths (corrected for instrumental width) are plotted against the Yerkes absolute spectroscopic magnitudes, the points define a straight line which extends over a 15-mag. range of M_v and which indicates that the line width varies as the one-sixth power of the luminosity. Stars with weak or strong lines and of all spectral types later than G0 seem to fit the linear relationship equally well. The widths therefore cannot be dependent upon line intensity or stellar surface temperature.

Evidence from the solar spectrum, from ζ Aurigae, from Hyades stars, and from four visual binaries point to the conclusion that the relationship described here is not of a statistical nature. Therefore, it is probable that the Ca II emission-line widths can be used as luminosity indicators. Internal consistency considerations indicate that one good spectrogram should fix the absolute magnitude of any late-type star with suitable lines to within ±0 5 mag.

It is found that, for displacements within ±6 km/sec, negative values are more frequent than positive for the emission components of H and K. On the other hand, between +4 and −4 km/sec, positive values are more common for the absorption components. The naïve interpretation is that the emitting layer is rising and that the absorbing material is falling slowly inward. Statistics of the larger displacements common among the intrinsically luminous stars are discussed briefly. In particular, it is found that among the M-type giants and supergiants the negative displacements of the absorption components are not correlated with absolute magnitude.

I. INTRODUCTION

It has long been known that central emission components are present in the H and K absorption lines of Ca II in the spectra of many stars of later spectral type. The initial discovery was made by Schwarzschild and Eberhard (1913), and since then an extensive literature has been published. Much of this material is in the form of brief notes and announcements and need not be referred to here. Comprehensive catalogues by Joy and Wilson (1949) and by Bidelman (1954) are available, which include not only lists of stars known to have bright H and K lines but also complete references to the papers containing the original observations. In addition, Joy and Wilson give intensity estimates of the emission and absorption components wherever possible.

Despite the large volume of published material on the H and K emission lines, no systematic study of them, based on a homogeneous set of spectrograms, has yet been made, and the present work is intended partially to fill this gap. The investigation originated in 1938–1939 when one of the authors obtained spectrograms of 10-A/mm dispersion, with the 32-inch camera of the Mount Wilson coudé spectrograph, of about two dozen late-type stars. These objects were selected from the Mount Wilson catalogue of spectroscopic absolute magnitudes (Adams, Joy, Humason, and Brayton 1935) with the intention of covering as wide a range of luminosity as possible, and the list extended from 61 Cygni at one extreme to α Scorpii and α Orionis at the other. The primary goal, at first, was to look for variations in emission intensity between these first plates and

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others which it was planned to obtain at some later time. Second-epoch spectrograms were not secured until 1951–1952, at which time some additional stars were also observed. In 1954 some preliminary measures of this material (Wilson 1954) showed that a rather close correlation existed between the widths of the emission lines and the Mount Wilson spectroscopic absolute magnitudes, and this result led to the decision to make a major effort to extend and explore the relationship further.

II. OBSERVATIONS AND MEASUREMENTS

The principal aim of the investigation is to see how the emission-line widths behave as a function of (*a*) the emission intensity, (*b*) the stellar luminosity, and (*c*) the spectral type. It is known from previous work that there is likely to be a considerable range in Ca II emission strength in any reasonably sized group of late-type stars of otherwise similar characteristics. No attempt, therefore, was made to consider item *a* in forming an observing list. Both spectral types and absolute magnitudes must be known in advance, however, and the observations should include stars of all intrinsic luminosities. Moreover, the luminosities should be known with maximum precision. Consideration of these requirements suggested that the list of MK standard stars published by Johnson and Morgan (1953) would provide the most useful and homogeneous source of material. Accordingly, all the stars in this list of spectral type G0 or later were placed on the observing program, and spectrograms of nearly all have been obtained. Other stars for which we have observations have been included for a variety of reasons: for some there were existing suitable spectrograms taken by other observers; some were suggested to us by P. C. Keenan; some were merely on hand when appropriate amounts of telescope time, not otherwise needed, happened to be available; a few were taken from the list of Joy and Wilson (1949) because of high emission intensity; and a number of them are stars of high space velocity.

All the spectrograms were made either with the 32-inch camera at Mount Wilson or with the 36-inch at Palomar. The range of dispersion does not exceed about 10 per cent, which is small enough to insure against systematic differences between groups of spectrograms made with the two cameras. Most of the exposures were made on baked II*a*-O plates, except that baking was omitted for some of the brighter stars. Eastman 33 emulsion had been used for the majority of the 1938–1939 spectrograms, and, as a result, a number of them were underexposed, as judged by later standards, and could not be included in the measures. The brighter stars were trailed on the slit so as to yield spectra 0.3–0.5 mm in width; but many of them were too faint for this procedure and were held fixed.

In dealing with weak Ca II emission, there is an effective deficit of the order of 2 mag. to allow for in judging exposure times, since the weak emissions occur at the bottoms of the strong, broad H and K absorption lines. Occasional failures, usually in the direction of underexposure, are thus encountered, but experience soon reduces these to a fairly small percentage.

Measurements of the Ca II emission features were made differentially with respect to nearby metallic absorption lines in the star spectrum and without reference to the comparison spectrum of iron. All plates were measured in both directions and then reduced in the usual manner, using the stellar absorption lines as standards to derive wave lengths for the Ca II emissions. The star lines chosen as standards and their wave lengths taken from Miss Moore's multiplet table (1945) are shown in Table 1. Settings were made on the outer edges of the Ca II emission lines and also on any absorption features associated with them, either centrally located or displaced. Wherever possible, both K and H were measured, although on many spectrograms H was either too weak or was so obviously being interfered with by other lines as to be unusable.

Let the wave lengths derived in this fashion for the violet edge, red edge, and sharp absorption component in a Ca II emission line be λ_v, λ_r, and λ_a, respectively. If λ_0 is the laboratory wave length of the line in question, $\lambda_v - \lambda_0 = \delta_v$, $\lambda_r - \lambda_0 = \delta_r$, and $\lambda_a - \lambda_0 = \delta_a$ are the displacements in angstroms, and the displacement of the emission as a whole is $(\delta_v + \delta_r)/2$, while its measured width is $W = \delta_r - \delta_v$.

All the measuring was done by one individual (O. C. W.), so there is no question of adjustment for the personal equation. Also, the measuring was completed before any of the data were plotted, in order to avoid the possibility of subconscious bias. The only exceptions to this statement were a few plates taken later to confirm the locations of some of the plotted points.

TABLE 1

WAVE LENGTHS OF STELLAR ABSORPTION LINES USED AS STANDARDS IN REDUCTION OF H AND K MEASURES

Element	λ	I P (v)	Element	λ	I P (v)
Fe I	3920 260	0 12	Al I	3944 009	0 00
	3922 914	0 05		3961 523	0 01
	3927 922	0 11	Fe I	4005 246	1 55
	3930 299	0 09			

TABLE 2

WEIGHTING PROCEDURE FOR WIDTHS OF K LINES

Plate Quality	H	K	Weight	Plate Quality	H	K	Weight
Fair	×	×	2	Good	×	×	3
Fair	—	×	1	Good	—	×	2

When the plates were set up on the measuring engine, two things were done besides the measuring itself. First, an estimate was made of the emission intensity. As far as possible, under the circumstances, these estimates referred to the peak emission strengths and ignored the widths. A scale of 0–5 was adopted for the intensities with these meanings: intensity 0 indicates that no emission could be seen. Where a small but unmeasurable emission was suspected, it was assigned the symbol "Tr?" Intensity 5 means that the peak emission was equal to, or in excess of, that in the neighboring continuous background outside the absorption wings of H or K. Intensity 1 signifies the weakest lines on which it was judged that reliable measures could be made. The other values—2, 3, 4—were assigned as best they could be in the judgment of the observer. This procedure, admittedly crude, certainly separates out the weak, medium, and strong lines and is sufficient for present needs.

Second, the plates were assigned a quality rating of good, fair, or poor at the time of measurement. A plate rated as poor was forthwith rejected without being measured. H and K were both measured except on plates where H was clearly unacceptable. After the reductions were made, H was included or not on the basis of the following arbitrary, but not unreasonable, criterion: If the width of H agreed with that of K to within 10 per cent, it was assumed that the difference was not more than could be accounted for by observational error. In this case the straight mean of the widths of H and K was used. If, however, the difference between H and K exceeded 10 per cent, H was rejected entirely on the grounds that the discrepancy was most probably due to real interference of other lines with H.

Finally, a weight was assigned to each spectrogram on the basis of the following simple scheme shown in Table 2. The weights given in Table 3 for each star are simply the sum of the weights of the plates measured, and the widths tabulated are the correspondingly weighted means. Although this procedure is again somewhat rough and arbitrary, it is hoped that it gives an approximate idea of the relative reliability of the line widths shown in Table 3.

III. TABULATED RESULTS

Table 3 contains the results of the measurements, together with other data, and requires explanation. Columns 1, 2, and 3 are self-explanatory. In column 4 the spectral types are Yerkes estimates when accompanied by a Roman numeral indicating luminosity class. If the star is one of the MK standards, this fact is indicated by the symbol "MK" in the column headed "Notes." Some stars which are not MK standards also have Yerkes spectral types and luminosity classes in column 4. These were supplied by Dr. P. C. Keenan, except for the high-velocity stars, for which the data are from the paper by Miss Roman (1955). When no Yerkes data were available, the spectral types from the Mount Wilson catalogue (Adams *et al.* 1935) were entered in column 4. The visual magnitudes in column 5 were taken from the list of MK standards (Johnson and Morgan 1953) or from the Mount Wilson 1935 catalogue.

Under the heading "K" in Table 3 are six columns containing data relating to the Ca II emission lines. The significance of the intensity ("Int.") and of the weight ("Wt.") have already been described. Under "W" is entered the measured K-line width in kilometers per second. The columns headed "ΔA" and "ΔE" contain, respectively, the displacements in kilometers per second of the absorption component (K_3, H_3) or components and of the emission as a whole. Because of the method of measurement, all displacements are determined with respect to low-excitation lines of the stellar reversing layers. Lastly, under "log W_0" is entered the common logarithm of the quantity ($W - 15$ km/sec). The reason for this correction will be established in the next section.

Four columns are found under the heading "M_v" (absolute visual magnitude). The first ("Mt. W.") contains the values from the Mount Wilson catalogue (Adams *et al.* 1935). Yerkes ("Y") determinations are from the luminosity classes given by Johnson and Morgan (1953) or by Miss Roman (1955), transformed to M_v by means of Table 1.5 of the well-known paper by Keenan and Morgan (1951). Yerkes M_v's are entered also for some stars which are not MK standards or high-velocity stars. These were supplied by P. C. Keenan. Under "Trig." are given the M_v's calculated from trigonometric parallaxes taken from the Yale *Catalogue* (Jenkins 1952). Only those values have been included for which the parallax exceeds its probable error by a factor of at least 4. Lastly, under "K" are the absolute visual magnitudes derived from the measures of the Ca II lines by means of the straight solid line of Figure 1.

The column headed "$B - V$" contains values of this quantity taken from Johnson and Morgan (1953), Eggen (1955), or Stebbins and Kron (1956). Eggen's colors were transformed to $B - V$ by means of equations given in his paper, and the $V - G$ values of Stebbins and Kron were similarly treated by means of a relationship established by Morgan, Harris, and Johnson (1953).

In the last column ("Notes") an "MK" indicates that the star is one of the MK standards; "HV" that it occurs in the catalogue of stars of high space velocity of Miss Roman (1955); and "R" that there is a remark pertaining to the object in the footnotes to the table.

TABLE 3

H D	1950 α	1950 δ	Sp	mv	K: Int	K: W (km/sec)	K: Wt	K: Δ A (km/sec)	K: Δ E (km/sec)	Log Wo	Mv: Mt. W	Mv: Y	Mv: Trig	Mv: K	B−V	NOTES	
1013	0-12- 1	+19 56	M2 III	4 9	3	94	3	−16	−3	1 90	+0 2	−0 4		−0 5	+1 58	χ Peg	MK
1522	0-16-53	− 9 06	K3	3 8	1	103	2	+ 2	+1	1 94	+0 2			−1 2		ι Cet	
3627	0-36-39	+30 35	K3 III	3 5	2	88	4	− 1	+1	1 86	+0 2	+0 5	+0 4	+0 1	+1 31	δ And	MK
4128	0-41-05	−18 16	G6	2 2	2	80	9	0	−1	1 81	+0 5		+1 0	+0 9	+1 05	β Cet	
4614	0-46-03	+57 33	G0 V	3 6	0						+4 8	+4 4	+4 9		+0 58	η Cas	MK
5286	0-52-17	+23 21	K 1	6 1	1	56	2	0	−1	1 61	+2 3			+4 0	+1 03	36 And	
6805	1-06-04	−10 27	K 1	3 6	2	77	2	0	+1	1 79	+0 5		+1 1	+1 2		η Cet	
6860	1-06-55	+35 21	M0 III	2 4	4	98	4	−22	−6	1 92	+0 2	−0 4	+0 6	−0 9	+1 60	β And	MK
9270	1-28-48	+15 05	G8 III	3 7	1	90	3	0	0	1 88	+0 4	+0 4		−0 2	+0 98	η Psc	MK
9927	1-34-55	+48 23	K3 III	3 8	2	85	3	+ 1	+1	1 84	+0 2	−0. 1		+0 4	+1 28	51 And	MK
10307	1-38-44	+42 22	G2 V	5 1	0						+4 4	+4 7	+4 8		+0 63		MK
10761	1-42-45	+ 8 54	G6	4 5	Tr?						+0 6					ο Psc	
12533	2- 0-49	+42 05	K3	2 3	2	108	3	− 1	−2	1 97	−0 2			−1 6	+1 15	γ' And	
12929	2-04-21	+23 14	K2 III	2 2	2	72	4	+ 2	+2	1 76	+0 4	0 0	+0 6	+1 7	+1 15	α Ari	MK
16901	2-40-49	+44 05	cG0	5 6	1	163	1	+ 2	−6	2 17	−2 1			−4 7		14 Per	
17506	2-47-02	+55 41	K3 Ib	3 9	3	164	1	−44,−1	−12	2 17	−2 5	−4 5		−4 7		η Per	MK
18322	2-53-59	− 9 06	K2	4 0	1	73	3	+1	0	1 76	+0 6		+1 2	+1 7		η Eri	
18884	2-59-40	+ 3 54	M2 III	2 8	3	106	3	−24	−5	1 96	−0 1	−0 4		−1 5	+1 64	α Cet	MK
20630	3-16-44	+ 3 11	G5 V	5 0	3	48	2		0	1 52	+4 7	+5 1	+5 1	+5 4	+0 68	κ Cet	MK
20797	3-20-19	+64 25	M0 II	5 6	4	141	1	−16,+19	−4	2 10	−1 7	−2 4		−3 6			
21120	3-22-07	+ 8 51	G8 III	3 8	1	72	1	−1	−4	1 76	+0 3	+0 4		+1 7	+0 89	ο Tau	MK
22049	3-30-34	− 9 38	K2 V	3 8	4	42	6		0	1 43	+6 0	+6 4	+6 2	+6 8	+0 86	ε Eri	MK
23249	3-40-51	− 9 56	K0 IV	3 7	1	50	4	+3	−2	1 54	+4 6	+3 4	+3 9	+5 1	+0 95	δ Eri	MK
26630	4-11-13	+48 17	G0 Ib	4 3	2	158	2	+1	+12	2 16	−2 0	−4 5		−4 6		μ Per	MK
26965	4-12-58	− 7 44	K0	4 5	2	47	4		0	1 51	+6 0		+6 0	+5 6	+0 82	ο^2 Eri	
27022	4-15-57	+65 01	G5 III	5. 4	2	73	3	+1	+1	1 76	+0 9	+0 2		+1 7	+0 82		MK
27371	4-16-57	+15 31	K0 III	3 9	1	88	6	+1	+2	1 86	+0 6	+0 2		+0 1	+0 99	γ Tau	MK
27697	4-20-03	+17 26	K0 III	3 9	1	83	4	+2	+2	1 83	+0 6	+0 2		+0 6	+0 98	δ Tau	MK
28305	4-25-42	+19 04	K0 III	3 6	1	82	5	0	+1	1 83	+0 7	+0 2		+0 6	+1 03	ε Tau	MK
28307	4-25-43	+15 51	K0 III	4 0	2	80	3	−2	−2	1 81	+1 0	+0 2		+0 9		θ Tau	MK
29139	4-33-03	+16 25	K5 III	1 1	4	86	6	+2	+6	1. 85	0 0	−0 3	−0 4	+0 2	+1 51	α Tau	MK
31398	4-53-44	+33 05	K3 II	2 9	3	113	1	−4	−1	1 99	−0 5	−2 3		−2 0	+1 53	ι Aur	MK
31767	4-55-57	+ 1 38	K2 II	4 7	2	113	1	−1	−1	1 99	−0 7	−2 2		−2 0		π^6 Ori	
31910	4-58-58	+60 22	G0 Ib	4 2	2	175	2	+7	+3	2 20	−2 5	−4 5		−5 2		β Cam	MK
32068	4-58-59	+41 00	K4 II		2	116	4			2 00				−2 1		ζ Aur	
36389	5-29-17	+18 34	M2 Ib	4 7	2	169	2	−10	+5	2 19	−3 2	−4. 5		−5 1	+1 95	119 Tau	MK
39400	5-49-51	+ 1 51	K2 II	5 0	2	112	2	−2	−3	1 99	−1 5	−2 2		−2 0		56 Ori	MK
39801	5-52-28	+ 7 24	M2 Iab	0 9	3	186	6	0	+4	2 23	−4 0	−6:		−5 7		α Ori	MK
40035	5-55-25	+54 17	K0 III	3 9	1	76	2	−1	0	1 79	+0 8	+0 2	+0 4	+1 2	+0. 99	δ Aur	MK
40239	5-56-13	+45 56	M3 II	4 6	4	130	2	−18	−4	2. 06	−1 3	−2 4		−3 0		π Aur	
44478	6-19-56	+22 32	M3 III	3 2	4	104	3	−14	−3	1 95	−0 6	−0 5	−0 2	−1 3	+1 67	μ Gem	
44537	6-21-03	+49 19	M0 Iab	5 1	2	183	2	+2	+1	2 23	−2 1			−5 7		ψ^5 Aur	MK
45416	6-24-40	+ 0 20	K1 II	5 3	1	110	2	−1	−1	1 98	−1 4	−2 2		−1 8		77 Ori	MK
47731	6-38-12	+28 15	G5 Ib	6 5	2	148	2	+4	−4	2 12	−1 6	−4 5		−4 0		25 Gem	MK
48329	6-40-51	+25 11	G8 Ib	3 2	3	154	2	−13	−3	2 19	−2 1	−4 5		−5 1	+1 41	ε Gem	MK
50877	6-52-03	−24 07	K3 Iab	4 1	2	173	3	−2	+2	2 20	−2 4			−5 2		o' CMa	MK
52497	6-59-22	+24 17	G5 II	5 2	2	131	3	+7	−8	2 06	−1 2	−2 0		−3 0		ω Gem	MK
60522	7-32-51	+27 01	M0	4 2	3	93	4	0	0	1 89	0 0			−0 4		υ Gem	
62058	7-38-56	−31 33	G0 Ia	6 6	0												MK
62044	7-40-11	+29 00	K1p	4 6	5	108	3		+4	1 97	+0 5			−1 6		σ Gem	
62345	7-41-26	+24 31	G8 III	3 7	Tr						+0 6	+0 4	+0 7		+0 68	κ Gem	MK
62509	7-42-16	+28 09	K0 III	1 2	1	74	2	+1	+1	1 77	+0 9	+0 2	+1 0	+1 5	+1 00	β Gem	MK
62721	7-43-14	+18 38	K5 III	5 3	2	81	3	−2	−1	1 82	+0. 2	−0 3		+0 7	+1 45	81 Gem	HV
67594	8-06-05	− 2 50	G2 Ib	4 4	2	167	2	+1	+8	2 18	−2 2	−4 5		−4. 9		ζ Mon	MK
69267	8-13-48	+ 9 20	K4 III	3 8	2	82	2	−3	−1	1 83	+0 2	−0 2		+0 6	+1 48	β Cnc	MK
74395	8-41-13	− 7 03	G2 Ib	4 7	1	149	2	+1	+7	2 13	−1. 4	−4 5		−4 1		31 Mon	
80493	9-18-01	+34 36	M0	3 3	3	93	5	−5	−3	1 89	−0 2			−0 4		α Lyn	
81192	9-21-57	+20 00	G8 III	6 7	1	55	1	−3	−4	1 60	+3 7	+0 4		+4 1	+0 94		HV
81797	9-25-08	− 8 26	K3 II-III	2 2	3	100	3	0	0	1 93	−0 8	−1 2	−1 6	−1 0	+1 44	α Hya	
83425	9-35-51	+ 4 53	K3 III	4 8	2	80	2	−1	−2	1 81	+0 2	−0 1		+0 9	+1 30	2 Sex	HV
84441	9-43-01	+24 00	Go II	3 1	2	117	7	0	−3	2 01	−1 3	−2		−2 2	+0 85	ε Leo	MK
86663	9-57-34	+ 8 17	M2 III	4 9	3	99	3	−14	−4	1 92	−0 1	−0 4		−0 9		π Leo	MK
88230	10- 8-19	+49 42	M0	6 8	5	42	3		0	1 43	+8 3		+8 5	+6 8	+1 38		
89484	10-17-13	+20 06	Ko III−	2 6	2	83	7	−1	−1	1 83	+0 3	−1 2		+0 6	+1 14	γ' Leo	
89485	10-17-13	+20 06	G7 III+	3 8	1	72	3	+2	0	1 76	+1 1	0		+1 7		γ^2 Leo	
91612	10-32-12	+ 7 13	G8 II-III	5. 2	Tr						+0 8	−1 0			+0 93	48 Leo	HV
92125	10-35-55	+32 14	G3 II:	4 8	2	143	3	+2	−2	2 11	−1 6	−2 0:		−3 8		37 LMi	
92095	10-36-00	+53 56	K3 III	5 7	2	82	2	−3	−3	1 83	+0 4	−0 1		+0 6	+1 27		HV
94264	10-50-31	+34 29	K0 III-IV	3 9	1	69	3	+1	−2	1 73	+2 2	+1 8		+2 1	+1 03	46 LMi	
94705	10-53-26	+ 6 27	M5 III	6 0	2	102	3	−10	−2	1 94	−0 9	−0 5:		−1 2		56 Leo	

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TABLE 3 (Cont'd)

	1950				K						Mv						
H D	α	δ	Sp	mv	Int	W (km/sec)	Wt	ΔA (km/sec)	ΔE (km/sec)	Log Wo	Mt. W	Y	Trig.	K	B-V	NOTES	
95272	10-57-20	-18 02	K0 III	4 2	1	80	1	+2	0	1.81	+0 7	+0 2		+0 9	+1 10	α Crt	HV
95735	11-00-37	+36 18	M2 V	7 6	5	27	3		-1	1 08	+10 7	+10 1	+10 5	+12 2	+1 51		MK
95689	11-00-40	+62 01	K0 III	2 0	2	88	3	-1	-1	1 86	0 0	+0 2	-0 5	+0 1	+1 06	α UMa	MK
98118	11-14-43	+ 2 17	M0 III	5 4	3	84	3	-2	0	1.84	+0 6	-0 4		+0 4	+1.52	75 Leo	HV
98230	11-15-31	+31 48	G0	5 6	0						+4 6		+6 1			ξ UMa	
98839	11-20-05	+43 45	G8 II+	5 1	2	91	6	+1	+1	1 88	+0 1	-1 6		-0 2		56 UMa	MK
99196	11-22-23	+11 42	K4 III	6 0	2	83	2	-1	0	1 83	+0 2	-0 2		+0.6	+1 38		HV
100029	11-28-28	+69 36	M0 III	4 1	4	89	3	-15	-3	1 87	-0 3	-0 4	+1 0	-0.1		λ Dra	
101501	11-38-25	+34 29	G8 V	5 5	3	50	3		0	1 54	+5 2	+5 6	+5 7	+5 1	+0 69	61 UMa	MK
107274	12-17-21	+49 16	M 1	5 6	3	95	2	-23	-3	1 90	-0 8			-0 5		3 CVn	
107328	12-17-49	+ 3 35	KI III	5 1	2	83	3	-3	-3	1 83	+0.4	+0.1		+0 6	+1 13	16 Vir	HV
109358	12-31-22	+41 38	G0 V	4 3	0						+4.2	+4 4	+4 5		+0 59	β CVn	MK
109379	12-31-45	-23 07	G4	2 8	1	89	3	0	0	1 87	0 0		0 0	-0 1		β Crv	
111028	12-43-50	+ 9 49	K1 IV	5 9	2	58	5	0	-1	1 63	+2 4	+3 4	+2 5	+3.7	+1 00	33 Vir	HV
111812	12-49-16	+27 49	G0 III	5 1	1						+2 3	+0 7				31 Com	R
112300	12-53-05	+ 3 40	M3 III-	3 7	3	90	3	-16	-3	1 88	-0 1	-1 1		-0 2		δ Vir	
113226	12-59-41	+11 14	G6	3 0	1	84	5	+1	+1	1 84	+0 4	+0 4	+0 8	+0 4	+0 93	ε Vir	
114710	13-09-32	+28 08	G0 V	4 3	1	51	3	-1	-1	1 56	+4 5	+4 4	+4 7	+4 8	+0 56	β Com	MK
115043	13-11-34	+56 58	G1 V	6 7	4	58	3		0	1 63		+4 6		+3 7	+0 58		
115617	13-15-47	-18 02	G6	4 8	0						+5 2		+5 1		+0 70	61 Vir	
119228	13-38-51	+54 56	M2 III	4 8	3	104	2	-17	-2	1 95		-0 4		-1.3		83 UMa	MK
119425	13-40-33	+ 3 47	K2 III	5 7	1	75	1	0	-1	1 78	0 7	0 0		+1 3	+1 10	84 Vir	HV
121370	13-52-18	+18 39	G0 IV	2 8	0						+3 2	+3 2	+2 8		+0 59	η Boo	MK
124897	14-13-23	+19 27	K 1 III	0.2	2	80	7	0	+1	1 81	+0 2	0 1	0 0	+0 9	+1 23	α Boo	
127665	14-29-40	+30 35	K3 III	3 8	2	92	2	0	+1	1 89	+0 5	-0 1	+0 8	-0 4	+1 29	ρ Boo	MK
128902	14-36-20	+43 51	K4 III	5 9	3	82	3	-11	-3	1 83	+0 1	-0 2		+0 6	+1 48		HV
129336	14-39-19	+11 52	G8 III	5 6	1	86	2	+2	+3	1 85	+0 6	+0 4		+0.2	+0 94	32 Boo	HV
129989	14-42-48	+27 17	K0 II-III	2 8	1	98	5	0	-2	1 92	-0.1	-1.0		-0 9	+0 96	ε Boo	
131156br	14-49-05	+19 18	G8 V	4 6	5	46	9		0	1 49	+5 3	+5 6	+5 6	+5 9	+0 75	ξ Boo	MK
131156ft	14-49-05	+19 18	K4 V	6 8	5	38	6			1 36	+6 9	+7 5	+7 8	+7.9		"	
131507	14-50-10	+59 30	K4 III	5 7	2	83	2	+1	0	1.83	+0 4	-0 2		+0 6	+1 36		HV
131873	14-50-50	+74 22	K4 III	2 2	3	88	3	0	0	1 86	-0 5	-0 2	-0 3	+0 1	+1 47	β UMi	MK
131511	14-51-07	+19 21	K 1	6 0	4	49	3		-1	1 53	+5 5		+5 6	+5 2			
131977	14-54-32	-21 11	K 5	5 8	5	44	3		-1	1 46	+6 9		+7 0	+6 3			
135722	15-13-29	+33 30	G8 III+	3 5	1	73	4	0	-2	1 76	+0 8	+0 8	+0 7	+1 7	+0 95	δ Boo	MK
137759	15-23-49	+59 08	K2 III	3 5	2	75	3	0	0	1 78	+0 2	0 0	+1 0	+1 3	+1 17	ι Dra	MK
137704	15-24-20	+34 31	K4 III	5 9	2	71	2	-1	0	1 75	+0 1	-0 2		+1 8	+1.40		HV
138716	15-31-26	- 9 54	K1 IV-	4 8	2	62	4	0	0	1 67	+2 1	+2.6	+1 7	+3 0	+1 01	37 Lib	HV, R
139446	15-36-01	-19 08	G2	5 5	Tr?						+2 3					41 Lib	
140573	15-41-48	+ 6 35	K2 III	2 8	2	76	6	0	0	1 79	+0 5	0 0	+1 1	+1.2	+1 16	α Ser	
142574	15-52-22	+20 27	K4 III	5.8	2	89	2	-12	-5	1 87	-0 5	-0 2		-0 1	+1.59		HV
142980	15-54-56	+14 33	K1 IV	5 7	1	70	1	+1	-2	1 74	-0 1	+3 4		+2.0	+1 14	φ Ser	HV
143761	15-59-07	+33 27	G0	5 4	0						+3 9		+3 5		+0 61	ρ CrB	
145148	16-06-43	+ 6 31	K0 IV	6 0	1	58	1	0	0	1 63	+5 6	+3 4	+3 1	+3.7	+1.00		HV
145328	16-07-08	+36 37	K0 III-IV	4 9	1	64	4	+2	+2	1 69	+1 3	+1 8		+2 7	+1 03	τ CrB	MK HV
146051	16-11-43	- 3 34	M0	3 0	2	92	3	-8	-4	1 89	-0 1	-0 4	+0 3	-0 4	+1 57	δ Oph	
148387	16-23-19	+61 38	G8 III	2 9	2	65	2	+3	+3	1 70	+0 7	+0 4	+1 1	+2.6	+0 92	η Dra	MK
148349	16-25-02	- 7 29	M2	5 4	4	85	2	-12	-1	1 84	-0 4			+0 4			
148478	16-26-20	-26 19	MI	1 5	3	168	6	-8	-4	2 18	-3 8	-5 2		-4 9	+1 84	α Sco	
148856	16-28-04	+21 36	G8 III	2 8	1	89	2	+4	+4	1 87	+0 1	+0 4		-0 1	+0 94	β Her	MK
148897	16-28-23	+20 35	G8 II	5 3	2	86	1	-3	0	1 85	+0 6	-2 1		+0 2			
149161	16-30-16	+11 36	K4 III	4 9	3	85	2	-3	0	1.84	0 0	-0 2		+0 4	+1 50	29 Her	HV
150275	16-32-46	+77 33	K III	6 4	1	70	2	+1	0	1 74	+2 2	+0 1		+2.0	+1 00		HV
149661	16-33-44	- 2 13	K0	5 9	4	51	3		-1	1 56	+5 4		+5 6	+4 8	+0 82	12 Oph	
150680	16-39-24	+31 42	G0 IV	3 0	0						+3.7	+3 2	+3 3		+0 64	ζ Her	MK
153210	16-55-18	+ 9 27	K2 III	3 4	1	82	3	+2	+3	1 83	+0 5	0 0	+0 5	+0 6	+1 17	κ Oph	MK
154733	17-04-11	+22 09	K4 III	5 7	2	78	2	+1	+1	1 80	+0 9	-0 2		+1 0	+1.30		HV
156014	17-12-22	+14 27	M5	3 6	3	114	6	-6	-4	2 00	-1 9	-2 5		-2 1	+1 36	α' Her	
156283	17-13-18	+36 52	K3 II	3 4	2	101	8	-1	-2	1 93	-0 1	-2 3		-1.0	+1 44	π Her	MK
159181	17-29-18	+52 20	G2 II	3 0	3	186	5	+7	+2	2 23	-1.7	-2		-5 7	+0 95	β Dra	MK, R
161096	17-41-00	+ 4 35	K2 III	2 9	1	74	2	-1	-4	1 77	+0 5	0 0	-0 3	+1 5	+1 16	β Oph	MK
161797	17-44-30	+27 45	G5 IV	3 5	0						+4 2	+3 4	+3 7		+0 75	μ Her	MK
162076	17-46-16	+20 35	G5	5 8	3	69	3	-1	0	1 73	+2 3			+2 1			
163588	17-52-40	+56 53	K2 III	3.9	2	75	3	+1	+1	1 78	+0 6	0 0	+1 4	+1 3		ξ Dra	MK
163770	17-54-32	+37 15	K1 II	4 0	3	121	6	-70,+1	+2	2 03	-1 3	-2 2		-2.5		θ Her	MK
164058	17-55-27	+51 30	K5 III	2.4	3	101	2	-1	-1	1 93	-0 1	-0 3		-1 0	+1 52	γ Dra	MK
165341br	18-02-56	+ 2 30	K0 V	4 1	3	48	6		0	1 52	+5 9	+6 0	+5 7	+5 4	+0 86	70 Oph	MK
165341ft	18-02-56	+ 2 30	K6	6 0	5	42	6		+1	1 43	+7 3		+7 6	+6 8		70 Oph	
165438	18-03-35	- 4 45	K1	5 9	2	59	2	-4	-3	1 64	+2 1			+3.6			
167042	18-09-30	+54 16	K1 III	5 9	1	55	2	+1	+1	1 60	+2 2	+0 1		+4 1	+0 94		HV

TABLE 3 (Cont'd)

	1950				K						Mv						
H D	α	δ	Sp	mv	Int	W (km/sec)	Wt	ΔA (km/sec)	ΔE (km/sec)	Log Wo	Mt W	Y	Trig	K	B-V	NOTES	
168723	18-18-43	- 2 55	K0 III-IV	3 4	0						+1 4	+1 8			+0 94	η Ser	HV
171443	18-32-29	- 8 17	K3 III	4 1	2	84	2	0	-1	1 84	+0 3	-0 1		+0 4		α Sct	
173764	18-44-31	- 4 48	G5 II	4 5	2	154	5	-2	+1	2 14	-1 9	-2 0		-4 3		β Sct	MK
180809	19-14-38	+38 03	K0 II	4 5	1	119	2	+4	+4	2 02	-0 6	-2 1		-2 4	+1 26	θ Lyr	MK
181276	19-15-57	+53 17	K0 III	4 0	1	83	2	+2	-6	1.83	+0 5	+0 2		+0 5		κ Cyg	MK
181391	19-17-53	- 5 31	K0	5 4	3	76	3		-1	1 78	+2 3		+2 3	+1 3		26 Aql	
183439	19-26-37	+24 34	M0 III	4 6	4	94	2	-9	-4	1 90	+0 1	-0 4		-0 5	+1 51	α Vul	HV
184406	19-31-39	+ 7 16	K3 III	4 6	2	69	2	0	+1	1 73	+0 9	-0 1	+2 5	+2 1	+1 17	μ Aql	HV
185144	19-32-28	+69 35	K0 V	4 8	1	50	2		+1	1 54	+5 6	+6 0	+6 1	+5 1	+0 79	σ Dra	MK
185351	19-35-05	+44 35	K0 III-IV	5 2	1	67	1	0	-1	1 72	+2 4	+1 8		+2 3			
185758	19-37-52	+17 54	G0 II	4 4	1	113	4	0	-2	1 99	-1 4	-2		-2 0		α Sge	MK
186408	19-40-29	+50 24	G2 V	6 3	0						+4 8	+4 7	+4 0		+0 64	16 Cyg prec	MK
186427	19-40-32	+50 24	G5 V	6 4	0						+4 4	+5.1	+4 1		+0 66	16 Cyg foll	MK
186791	19-43-53	+10 29	K3 II	2 8	3	115	7	-75,-3	-1	2 00	-0.8	-2 3		-2 1	+1 51	γ Aql	MK
188119	19-48-21	+70 08	G8 III	4 0	1	80	2	+2	-1	1 81	+0 9	+0 4		+0 9		ε Dra	MK
188512	19-52-51	+ 6 17	G8 IV	3 9	0						+4 0	+3 4	+3.1		+0 86	β Aql	MK
189319	19-56-32	+19 21	M0	3 7	1	99	2	+1	0	1 92	-0 1	-0 4		-0 9		γ Sge	
192713	20-13-21	+23 21	G2 Ib	5 4	2	182	6	+12	+7	2 22	-2 3	-4 5		-5 5		22 Vul	MK
197989	20-44-11	+33 47	K0 III	2 6	2	75	4	+1	0	1 78	+0 7	+0 2	+0 9	+1 3	+1 03	ε Cyg	MK
198149	20-44-16	+61 39	K0 IV	3 6	1	63	2	0	+2	1 68	+2.6	+3.4	+2 7	+2 9	+0 92	η Cep	MK HV
200905	21-03-07	+43 44	K5 Ib	3 9	5		3	-85,-54,-6			-2.0	-4 5			+1 64	ξ Cyg	MK, R
201091br	21-04-40	+38 30	K5 V	5 6	5	41	2		+1	1 41	+7 7	+7 8	+8 0	+7 1	+1 19	61 Cyg	MK
201091ft	21-04-40	+38 30	K7 V	6 3	5	37	3		0	1 34	+8 6	+8.5	+8.7	+8 2	+1 38	61 Cyg	MK
202109	21-10-48	+30 01	G8 II	3 4	1	77	8	0	+1	1 79	-0 6	-2 1		+1 2	+1 02	ζ Cyg	MK
204867	21-28-56	- 5 48	G0 Ib	3 1	1	169	3	-93,-2	+2	2 19	-2 5	-4 5		-5 1	+0 83	β Aqr	MK
206778	21-41-44	+ 9 39	K2 Ib	2 5	4	158	7	-34	-6	2 16	-2 3	-4 5		-4 6	+1 58	ε Peg	MK
206936	21-41-59	+58 33	M2 Ia	4 4	0						-3 0	-6 5			+2 41	μ Cep	
206859	21-42-08	+17 07	G5 Ib	4 5	2	148	3	+2	+3	2 12	-1 3	-4 5		-4 0	+1 18	9 Peg	MK
207089	21-43-46	+22 43	K1 Ib?	5 4	2	194	3	-124,-80,0	-4	2 25	-2 8	-4 5		-6 0		12 Peg	
208606	21-53-51	+61 18	G8 Ib	6 2	2	170	4	-24,+14	-2	2 19		-4 5		-5 1		HR 8374	MK
209750	22-03-13	- 0 34	G2 Ib	3 2	2	182	4	-2	+2	2 22	-2 2	-4 5		-5 5	+1 00	α Aqr	MK
210745	22-09-07	+57 57	K1 Ib	3 6	3	168	4	-28,+2	-3	2 18	-2 3	-4 5		-4 9	+1 55	ζ Cep	MK
212943	22-25-20	+ 4 27	K0	4 9	2	61	3	-1	-3	1 66	+2 3		+1 5	+3 2	+1 07	35 Peg	
216131	22-47-35	+24 20	G8 III-III+	3 7	1	79	2	+2	-2	1.81	+0 9	+0 7	+1 3	+0 9	+0 94	μ Peg	
216386	22-50-00	- 7 51	M2	3 8	4	96	8	-10	+2	1 91	-0 2	-0 4		-0 7		λ Aqr	
216640	22-52-07	-16 32	K4	5 7	1	71	1	+2	+2	1 75	+1 9			+1 8		77 Aqr	
216946	22-54-14	+49 28	K5 Ib	5 1	4	169	2	-39	-14	2 19	-2 4	-4 5		-5 1			
216953	22-54-42	- 5 05	G6	6 4	1	88	2	-1	-2	1 86	+0 6			+0 1			
217476	22-57-58	+56 41	cG3	5 5	1?						-3 2				+1 29		
217906	23-01-21	+27 49	M2 II-III	2 6	4	109	6	-8	-2	1 97	-0 6	-1 4		-1 6	+1 67	β Peg	
218356	23-04-40	+25 12	cK0	5 0	4	116	6	+6	-2	2 00	-1 6	-2.2		-2 1		56 Peg	
219134	23-10-54	+56 54	K3 V	5 6	3	43	3		0	1.45	+6 6	+6 9	+6 5	+6 5	+1 01		MK
222107	23-35-06	+46 11	G7	4 3	5	80	5		-1	1 81	+2 3	+1 9	+2 2	+0 9	+1.02	λ And	
222404	23-37-17	+77 21	K1 IV	3 4	1	65	2	+1	-2	1 70	+2 1	+3 4	+2 1	+2 6	+103	γ Cep	MK
223719	23-49-24	+ 2 39	K4 II	5 8	3	84	3	-5	+1	1 84	+0 1	-2.4		+0 4		22 Psc	

REMARKS

HD 111812, 31 Com H and K emission and all absorption lines are broadened, presumably by rotation

HD 138716, 37 Lib Spectral type and Mv(Y) by Keenan Miss Roman gives K1 III, Mv = +0 1.

HD 159181, β Dra Mv (Y) from MK list However Keenan and Morgan (1951) orginally classified this star as G2 Ib.

HD 200905, ξ Cyg Violet edge of emission obscured by absorption component Assuming line width to be equal to twice the distance from center to red edge, corresponding Mv (K) is -3 7

IV. RELATIONSHIP BETWEEN EMISSION-LINE WIDTH AND LUMINOSITY

In the earlier plot (Wilson 1954) of Ca II emission-line widths against Mount Wilson spectroscopic absolute magnitudes, the relationship turned out to be a curve. At least some of the curvature may originate in three probable sources: (*a*) no correction was applied for instrumental line width, a factor of great importance for intrinsically narrow lines; (*b*) the Mount Wilson spectroscopic absolute magnitudes are known to be systematically too faint for stars brighter than about $M_v = 0$; and (*c*) the plot is logarithmic in one co-ordinate only. In the present version these three items have been modified as follows:

First, 15 km/sec have been subtracted from all measured line widths to allow for the instrumental contribution. Primarily, this figure was arrived at because it represents approximately the mean projected slit-width, which was generally in the range of 15–20 μ for the spectrograms used, plus a small additional allowance for optical imperfections, photographic spreading in the emulsion, etc. Measures of some of the weaker iron lines in the comparison spectrum yielded widths of about this value, which, it seems, cannot be far from correct. It must be borne in mind, however, that spectrograms do differ somewhat in sharpness of definition, sometimes for reasons which are quite obscure, and, while the use of individual corrections for each plate is quite impractical, the use of a constant correction for all may contribute appreciably to the scatter of the measured widths for stars with very narrow lines, i.e., for objects in the lower portion of the main sequence. Second, the Yerkes absolute-magnitude system has been adopted, since it is undoubtedly more nearly correct for the brighter stars; and, third, the logarithms of the corrected widths have been plotted against absolute magnitude.

The results of the foregoing procedure are shown in Figure 1, which includes all stars, except the high-velocity objects, for which we have both K-line widths and Yerkes M_v's. The solid straight line has been drawn through the points by eye in an attempt to strike a reasonable mean as closely as possible. It is important to note that this line has no other significance at present, either theoretical or empirical. The two dashed lines represent the spread corresponding to ± 10 per cent error in the measurement of the line widths. As we shall see later, the internal consistency of the measures indicate that, as far as measuring errors are concerned, all the points should fall within, or nearly within, the dashed lines.

In Figure 1 there is no distinction as to spectral type, but there is a separation into three intensity groups representing, roughly, weak, medium, and strong lines. No systematic deviations according to emission intensity are apparent, and weak and strong lines seem to be pretty thoroughly mixed throughout the diagram. One must conclude that the widths are at least very nearly independent of the intensities.

Figure 2 is a repetition of Figure 1, except that the stars are now separated into the spectral-type groups G, K, and M. As far as the material allows, the conclusion is that all three groups fit the straight line equally well; hence the line widths are not, to any appreciable degree, a function of spectral type, i.e., of surface temperature.

Figure 3 is a plot of log W_0 against the trigonometric absolute magnitudes for those stars whose trigonometric parallaxes are at least four times larger than the probable errors. The straight line is the same as that in Figures 1 and 2, and the fact that it fits the points well is not surprising, since the spectroscopic absolute magnitudes are largely determined by trigonometric parallaxes below $M_v = 0$.

The evidence presented here strongly suggests that the Ca II emission-line width is a function only of the absolute magnitude of the star. If so and if the accuracy attainable is sufficient for the purpose, the bright H and K lines may provide one of the most powerful methods of measuring the intrinsic luminosity of the giants and supergiants of later spectral types. These points are discussed in the following section.

V. VALIDITY OF CA II EMISSION-LINE METHOD OF DETERMINING M_v AND ESTIMATE OF ACCURACY ATTAINABLE

At present there is no theory which relates Ca II emission-line width to intrinsic luminosity. Therefore, any attempt to make use of the line widths as luminosity indicators must rest entirely on the empirical calibration of the method. Moreover, the question arises: Is the relationship only of a statistical nature, or is there actually a one-to-one correspondence between line width and stellar luminosity? There are four items which contribute significantly to supplying an answer to this question. They relate to (*a*) the sun, (*b*) ζ Aurigae, (*c*) the Hyades, and (*d*) visual double stars.

The sun is classified on the Yerkes system as G2 V, and the corresponding M_v is +4.7 (Keenan and Morgan 1951). Sky plates of 10-A/mm dispersion, even those taken on days when the plage area on the solar disk is large, fail to show a trace of H_2 and K_2. On the other hand, spectrograms of high dispersion taken with solar equipment show these emission components virtually everywhere on the disk at all times. Indeed, the better the quality of the spectrograms, the more complicated is the appearance of K_2, and in the most recently published photographs of this sort (McMath, Mohler, Pierce, and

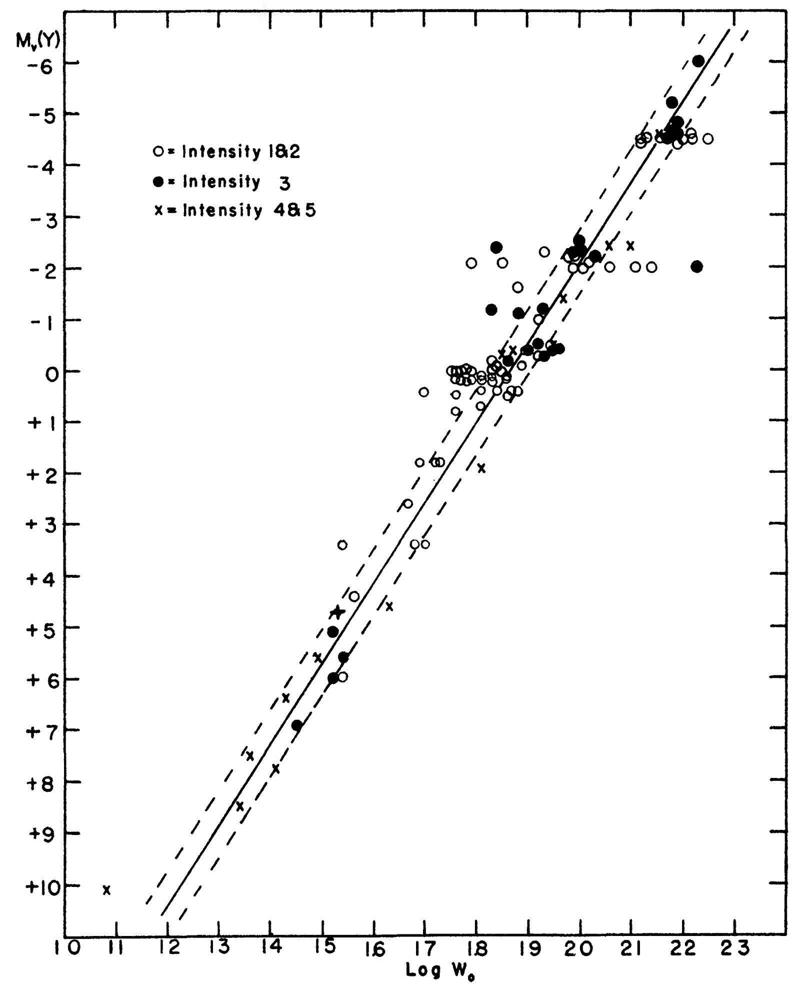

FIG. 1.—Logarithm of corrected Ca II emission-line widths plotted against Yerkes spectroscopic absolute magnitude. Stars divided into three intensity groups

Goldberg 1956) it is evident that even a precise definition of line width would be difficult. Nevertheless, if one attempts to measure the average emission-line widths on these published prints, the resulting values are such as to place the sun very close to the straight line of Figure 1.

Similar measures have been made on a Mount Wilson solar spectrogram. On this plate far less detail is shown than by the Michigan equipment, but the H_2 and K_2 components vary over a considerable range of intensity along the line covered by the slit. The strongest regions (plages) were not measured; those of moderate to weak intensity yielded widths in the range of 39–33 km/sec, and the corresponding log W_0's, 1.59 and 1.52, straddle the value 1.56, which is read from the straight line of Figure 1 for an M_v of +4.7.

Probably the best solar data we have at present are contained in a print kindly sent to us by Dr. O. C. Mohler, of the University of Michigan Observatory. The original plate was taken by moving the solar image across the slit during a 20-minute exposure and avoiding plage regions. On the print the K_2 line is thus structureless along its length, and the dispersion is so great, 1 A = 11.6 mm, that the line width can easily be measured with a millimeter scale. The result is W_0 = 34 km/sec, log W_0 = 1.53, and this last value is plotted as + in Figure 1.

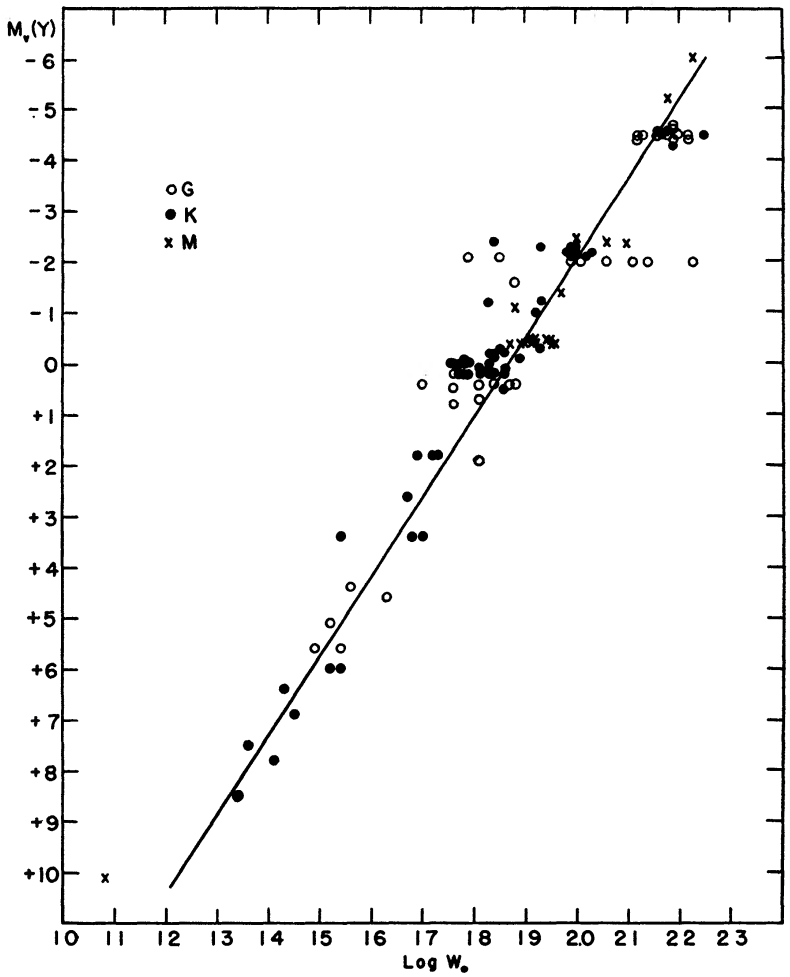

FIG. 2.—Same plot as Fig. 1 except that stars are grouped by spectral type G, K, or M. Line intensities not indicated.

The conclusion is inescapable that, to within the accuracy with which the relationship of Figure 1 has been established, the sun conforms to it, and this despite the fact that the solar H_2 and K_2 lines are too weak even to be visible in integrated light at the dispersion used in making the stellar observations! .

Some years ago Wellmann (1951) made a careful study of the binary star ζ Aurigae. His purpose was to derive the best values of the physical constants of the system by considering the spectra of both components and adjusting the parameters for best consistency with observation. One of the results was that he found the late-type component to be K4 II, $M_v = -2.5$. This value of M_v does not depend upon the Yerkes calibration of class II stars but chiefly upon Wellmann's estimate that the B-type star is B7 V. Actually, according to the Keenan and Morgan calibration (1951), M_v for a K4 II star should be between -2.3 and -2.4. Apparently, therefore, M_v for the K-type component of ζ Aurigae is pretty reliably located in the range -2.3 to -2.5 and cannot in any case differ greatly from these values without doing violence to the data relating to the other component. Hence this star is the best test object we have in the upper part of Figure 1. Reference to Table 3 shows the results of the measurement of the K line on four plates of fair quality (for this purpose) taken during totality: $M_v = -2.1$.

Table 3 also contains data on the stars γ, δ, ε, and ϑ′ Tauri, which are the bright-

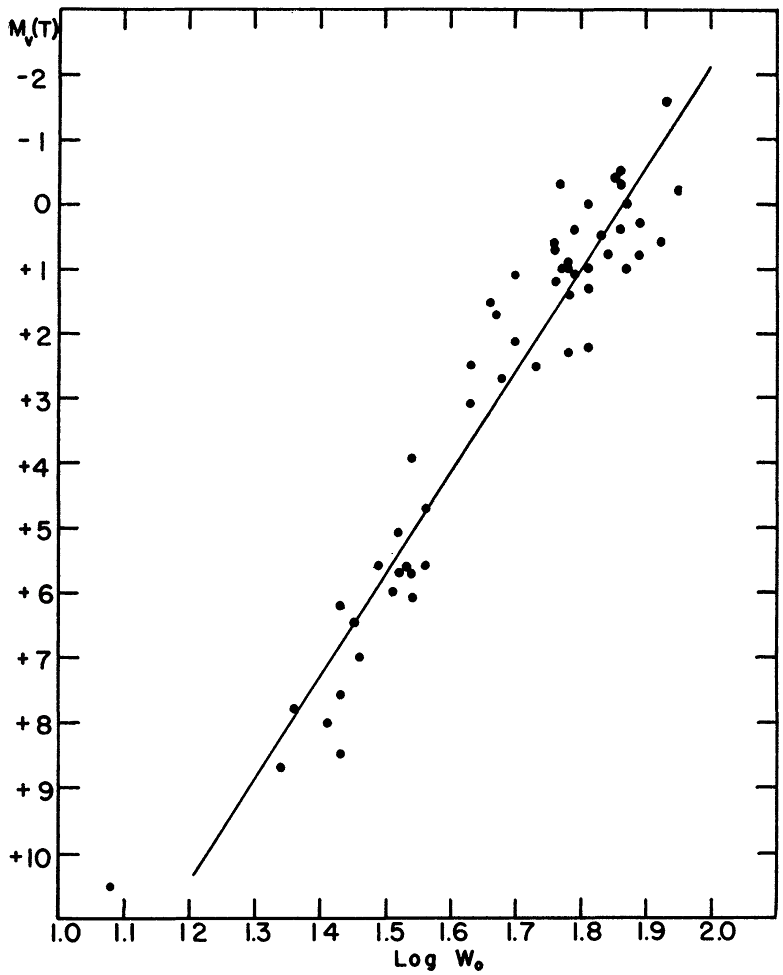

FIG. 3.—Logarithm of corrected Ca II emission-line widths plotted against absolute magnitudes derived from trigonometric parallaxes Straight line same as in Fig 1.

yellow giants in the Hyades[1] and hence have a very well-determined distance. In Table 4 are the results of a comparison between the observed magnitudes of these stars and the magnitudes computed from the M_v's read from Figure 1, together with van Bueren's (1952) modulus of 3.03 mag. The mean deviation, taking account of the signs, is only 0.15 mag. for these four stars.

There are four visual pairs in Table 3, for both components of which M_v(K) has been derived. Three of these doubles consist of main-sequence stars, for which the percentage accuracy is reduced because the emission lines are narrow and only one (γ Leo) is composed of giants. In Table 5 we compare the differences in magnitude derived from the K lines with the observed differences. The mean difference between the two values of Δm, with regard to sign, is only -0.1 mag. This indicates that the slope of the straight line in Figure 1, based upon fairly large groups of unrelated stars, is correct also for stars which are members of binary systems.

TABLE 4

COMPARISON OF APPARENT MAGNITUDES DERIVED FROM K-LINE WIDTHS WITH OBSERVED VALUES FOR HYADES STARS

Star	M_v(K)	Calculated Mag	Observed Mag	O−C	Star	M_v(K)	Calculated Mag.	Observed Mag	O−C
γ Tau	+0 1	3 1	3 7	+0 6	ϵ Tau	+0 6	3 6	3 5	−0 1
δ Tau	+0 6	3 6	3 8	+0 2	ϑ' Tau	+0 9	3 9	3 8	−0 1

TABLE 5

OBSERVED AND DERIVED MAGNITUDE DIFFERENCES OF COMPONENTS OF VISUAL BINARIES

Star	Δm(K Line)	Δm(Obs)	Star	Δm(K Line)	Δm(Obs)
γ Leo .	1 1	1 2	70 Oph .	1 4	1 9
ξ Boo	2 0	2 2	61 Cyg	1 1	0 7

The evidence presented in this section thus points with high probability to the conclusions that (*a*) the K emission-line width is determined uniquely by the absolute magnitude; (*b*) the relationship between width and luminosity is therefore not statistical in nature; and (*c*) the preliminary calibration represented by the straight line of Figure 1 is already close enough to the truth to be of value in deriving absolute magnitudes with considerable accuracy. Of the evidence discussed, that relating to the sun and to the Hyades stars is probably the most impressive.

If these conclusions are correct, why is there such a large scatter, particularly in certain regions, in Figure 1? In principle, this scatter could be attributed to errors in either co-ordinate, but note that most of it is such as to spread the plotted points out into horizontal bands. This suggests either (*a*) that there are large errors in the measurement of the line widths or (*b*) that the Yerkes system of assignment to luminosity classes is rather coarse, so that groups of stars actually covering a considerable range in luminosity are all placed in the same class. The fact that the horizontal scatter is less in the

[1] This fact was not realized until after the observations and measurements had been completed. One of the penalties of coudé work is that the observer is often unaware of the exact part of the sky in which he is operating.

lower part of the diagram, where, although the percentage accuracy of measurement is reduced because the lines are narrow, the M_v's are more reliable, suggests that alternative *b* is probably correct. This interpretation can be strengthened greatly by considering the errors of measurement.

There are fifty-four stars in our list for which two or more plates have been measured. For these objects the differences of individual plates from the mean have been averaged, without regard to sign, and the results are summarized in Table 6 in terms of the per cent of the mean width. This relatively high accuracy was altogether unexpected and is rather surprising. It must mean, however, that as far as measuring error is concerned, nearly all the points in Figure 1 should lie between the ± 10 per cent lines shown on the diagram and that the scatter of the plotted points must be due largely to erroneous values of M_v. Actually, in computing the data of Table 6, only a few deviations were found which exceeded 10 per cent, and then only by small amounts. Hence it seems reasonable to estimate that one good plate of a star with suitable emission lines should, in general, yield an absolute luminosity accurate to the order of 0.3–0.5 mag. (apart, of course, from residual zero-point or scale errors in the calibration).

The question of the precision of the K-line method can be attacked also from the standpoint of stars with well-determined trigonometric parallaxes. In Table 3 there are

TABLE 6

AVERAGE DEVIATIONS OF WIDTH MEASUREMENTS IN PER CENT

Lum Class	No Stars	Av Per Cent Dev	Lum Class	No Stars	Av Per Cent Dev
I*ab*	3	0 6	III	21	3 4
I*b*	9	2 5	IV	3	4 8
II	12	4 4	V	6	1 4

nineteen stars (including both members of three visual pairs) whose trigonometric parallaxes exceed 0".080 and for which M_v(K) values have been derived. In the Yale *Catalogue* (Jenkins 1952) two or more independent parallax determinations are listed for all but one of these objects. For these stars M_v was calculated from each of the separate parallax values, which were taken to be of equal weight for this purpose, and the residual of each determination from the mean was obtained. In all, sixty-nine such residuals, treated as independent and of equal weight, led to a mean error for the trigonometric method of 0.25 mag. Similarly, the nineteen M_v(K) values gave residuals from the means of the trigonometric determinations which resulted in an observed mean error for the K-line method of 0.62 mag. Correcting the latter for the errors in the trigonometric method, one finds 0.6 mag. for the true mean error of the K-line method, with a corresponding probable error of 0.4 mag. These figures agree well with the estimate given in the preceding paragraph.

This discussion based on trigonometric parallaxes puts the K-line method in the worst possible light, since most of the stars involved are on the main sequence, where the emission lines are narrow. For intrinsically brighter stars, the accuracy should be better, since the measurement of the line widths, percentagewise, should be more precise.

Among the stars in Table 3 with well-determined trigonometric parallaxes, there are five for which the difference $M_v(T) - M_v(K)$ equals or exceeds ± 0.9 mag. Two of these —HD 88230 and HD 95735—were not included in the computation of the mean error and do not cause any concern. They are both M-type dwarfs in whose spectra the Ca II emission lines are comparable in width with the lines in the comparison spectrum. Obviously, for such narrow lines a dispersion of 10 A/mm is simply incapable of yielding useful information concerning the line widths, and it is fair to ignore the corresponding discordant values of M_v(K). The three other stars with discrepant M_v(K)'s are HD 23249 δ Eridani, α Bootis, and HD 185144 σ Draconis. In the spectrum of δ Eridani the Ca II emissions are exceedingly weak—so weak, in fact, that it is questionable whether they should have been measured. It is quite possible that this weakness is the cause of the erroneous absolute magnitude. In the other two stars, however, there is no obvious explanation of the discrepancy, particularly for α Bootis, where the results depend upon three spectrograms. It may be significant that both α Bootis and σ Draconis are high-velocity stars (Roman 1955). It is conceivable that the relationship between line width

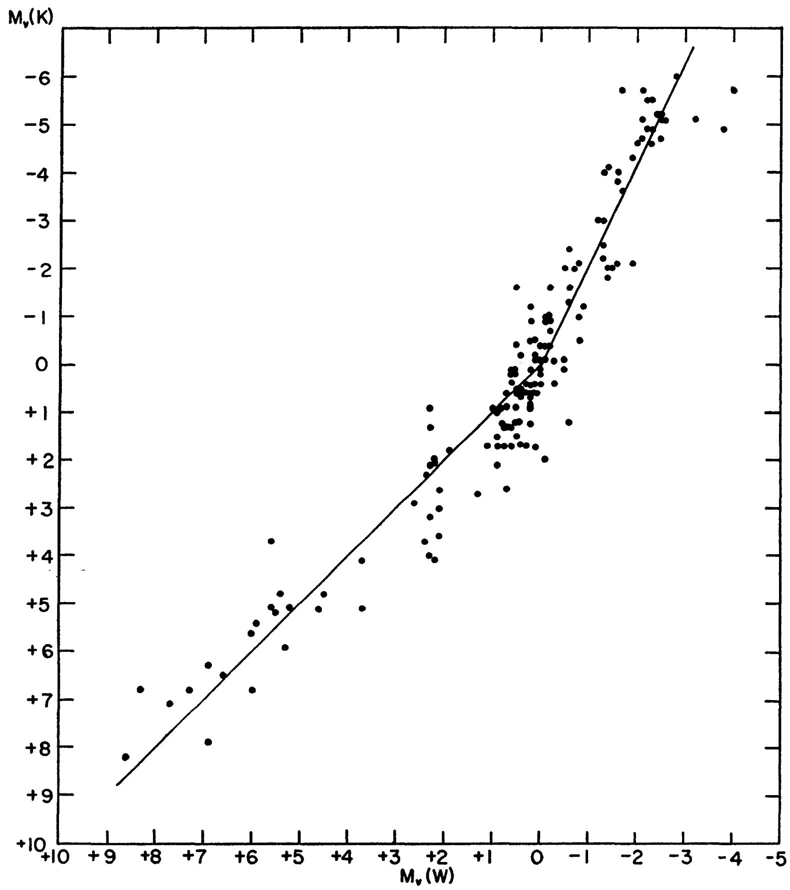

FIG. 4.—Absolute magnitudes on Yerkes system derived from Ca II emission lines versus Mount Wilson spectroscopic absolute magnitudes.

and M_v is somewhat different for these objects than for population I stars, and this possibility must be kept in mind.

One further diagram of some interest in this connection is shown in Figure 4. Here the values of M_v(K) are plotted against the Mount Wilson absolute magnitudes. The M_v(K)'s have been read from the straight line of Figure 1 and are thus on the Yerkes system, but the grouping of stars into luminosity classes has been removed by this procedure. The 45° straight line in the lower part of the diagram serves to show that below $M_v \sim 0$ the Yerkes and Mount Wilson systems are in agreement; the upper line has merely been drawn in by eye. Note, however, the large vertical scatter in the points near M_v(W) $= +2$ and again for those between M_v(W) $= +1$ and M_v(W) $= -1$. These must be regions where the Mount Wilson methods of spectroscopic absolute-magnitude determination have not had sufficient discrimination and groups of stars have been lumped together which actually are spread over a luminosity range of 3 mag. or so. They are analogous to the horizontal groupings of points in Figure 1 which are due to similar deficiencies in the Yerkes data. The upper part of the diagram (except for the extreme end) indicates that, on the whole, the Mount Wilson methods were successful in placing the bright stars in the right order of luminosity with a scatter of not over about 2 mag. but that all these stars were assigned luminosities considerably too faint.

To conclude this section, it is necessary to point out that, although the K-line method appears very promising and may indeed yield accurate results, it does have some serious limitations, one of which is that it is restricted to late-type stars. Among the main-sequence stars, Ca II emission is rare in those with spectral types near G0; but this is not too important, since the absolute magnitudes of these objects may be readily ascertained by other methods. For the intrinsically luminous stars of type G0, Table 3 shows that Ca II emission, though somewhat weak, is usually present; this fact suggests that it will be interesting to see how far the method can be pushed toward earlier spectral types for these objects.

The method is limited in scope also by the fact that it requires a large telescope, at least moderate dispersion, and a modern spectrograph capable of reaching H and K without excessive exposure time. Even with the 200-inch telescope it is impractical to observe any large number of objects fainter than the seventh magnitude. However, at least for stars brighter than $M_v \sim 0$, it would seem that useful results could be expected for dispersions considerably less than the 10 A/mm employed thus far, and this approach will be tested in the near future. To what limiting magnitude the usefulness of the K-line technique can be extended by these means remains to be seen.

VI. AN APPLICATION OF THE METHOD

An interesting application of the K-line method of measuring absolute magnitudes is shown in Figure 5. This is a color-magnitude diagram constructed from the K-line data of Table 3 for those objects whose weights are 3 or more and for which $B - V$ is known. Hitherto such plots have been largely restricted to clusters, since for field stars, except for the main sequence, uncertainties in M_v have been altogether too large to yield useful results.

In the diagram the main sequence for "age zero" (Johnson and Hiltner 1956) is shown by the solid line, and the observed main-sequence stars in general agree well with it. Deviations near the main sequence occur in the sense that some points are found above and/or to the right of it, as would be expected according to current evolutionary ideas. There are too few of these points as yet to draw any certain conclusions. However, it seems reasonable to expect that, by extending the observations to enough stars, it should be possible to make a comparison between the ages and limiting chemical compositions of the stars in the solar neighborhood and those of cluster members. It should also be possible to select with certainty from such a diagram pairs of stars of identical color but widely different absolute magnitude and perhaps, from detailed study of high-dispersion spectra, to find out whether such stars differ in age or in chemical composition or both.

VII. DISPLACEMENTS OF EMISSION AND ABSORPTION COMPONENTS OF H AND K

We turn now to a study of some of the other information contained in Table 3, namely, the displacements of the Ca II emission lines and associated absorption components. The accuracy of the emission displacements (ΔE in Table 3) is, in percentage terms, very much less than that of the widths, since most of the displacements are quite small. For those stars with multiple plates, we have seen that deviations from the mean of the order of 10 per cent or more are rare in the case of width measurements. The individual displacements, on the other hand, frequently deviate by several hundred per cent, i.e., by several kilometers per second, and not much can be learned about the reliability of the measured displacements. In Table 3 the ΔE's have been rounded off to the nearest kilometer per second, and the simplest procedure—and the only one which appears justified—is to construct a histogram of the tabulated values of ΔE. Figure 6 shows the result.

Relatively large displacements of, say, 6 km/sec or more, either plus or minus, are rare and, in the small sample available, appear to be more or less equally probable. In

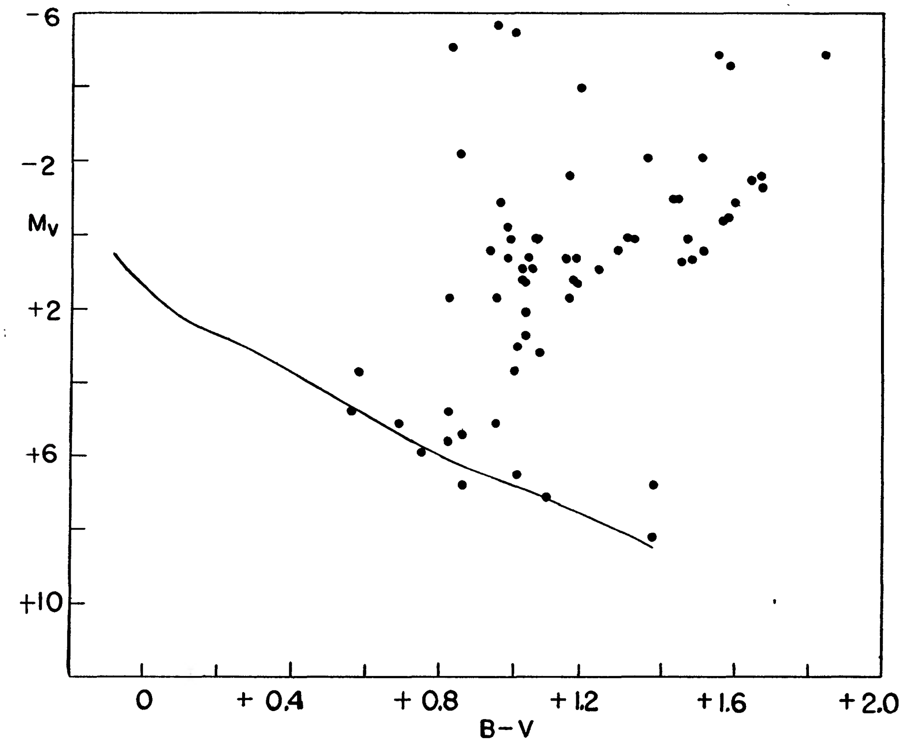

FIG. 5.—Color-magnitude diagram for field stars "Age zero" main sequence shown by solid line

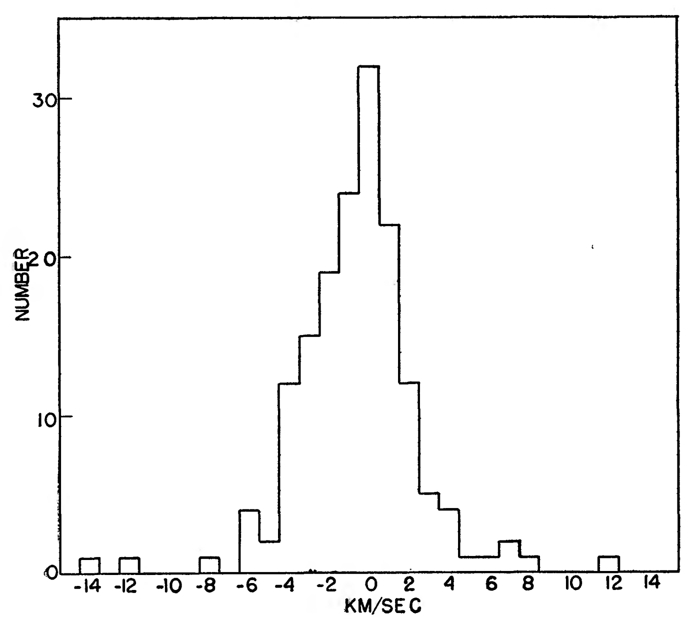

FIG. 6 —Frequency distribution of displacements of Ca II emission lines

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Table 3 there are twelve stars for which $|\Delta E| \geq 6$ km/sec. These stars are listed in Table 7, where values of ΔE from individual plates are given. Unfortunately, there is not enough such material to enable any statement to be made other than this: It appears that ΔE values of the order of 6 km/sec or more have a good chance of being real. There is apparent also a strong tendency for large ΔE values to belong to stars of high luminosity. The table also indicates that among the I*b* stars the earlier spectral types tend to be associated with positive displacements and the later types with negative, but here the numbers involved are becoming too small for reliability. In any event, it is clear that the questions of the reality and significance of the larger emission-line displacements require further study before sound conclusions may be drawn.

The histogram of Figure 6 shows clearly, however, that the smaller emission-line displacements are distributed unsymmetrically about zero. There is a very noticeable bias in favor of small negative, rather than small positive, values, and careful examination of the original measures shows that this cannot be due to encroachment of other lines onto the red wing of H. If we take this result at its face value, the interpretation is that, in general, the material responsible for the Ca II emission lines is moving outward from the stellar reversing layers with velocities of the order of 1–2 km/sec. Unfortunately, the observations are not adequate for more detailed analysis.

TABLE 7

STARS WITH MEASURED $|\Delta E| \geq 6$ KM/SEC

Star	Type	M_v(K)	ΔE	Star	Type	M_v(K)	ΔE
β And	M0 III	−0 9	−4, −7	HD 216946	K5 I*b*	−5 1	−14
14 Per	cG0	−4 7	−6	μ Per	G0 I*b*	−4 6	+12
η Per	K3 I*b*	−4 7	−12	α Tau	K5 III	+0 2	+5, +6
ω Gem	G5 II	−3 0	−8	ζ Mon	G2 I*b*	−4 9	+8
κ Cyg	K0 III	+0 5	−6	31 Mon	G2 I*b*	−4 1	+7
ϵ Peg	K2 I*b*	−4 6	−6, −8, −6, −6	22 Vul	G2 I*b*	−5 5	+7, +7

The frequency distribution of the absorption components of H and K lying between +20 and −20 km/sec is plotted as a histogram in Figure 7. Here one notes for the smaller values—say between +4 and −4 km/sec—that there is a distinct asymmetry in favor of the positive side, which is the reverse of the situation in Figure 6. In all probability, therefore, where all displacements concerned are not over 3 or 4 km/sec, the Ca II responsible for the absorption components is drifting inward at velocities of the order of 1–2 km/sec. Since the absorbing atoms must lie above those which produce the emission, the general kinematic picture is one of slow outward motion in the lower chromospheric layers relevant to our problem, followed by a slow downward return from somewhere above.[2] It is reasonable to suppose that this behavior is that to which we might attach the word "normal" and that the larger displacements, either of emission or of absorption components, represent anomalies of some kind in chromospheric motions.

It would appear from Figure 7 that the absorption-component anomalies set in at values of ΔA in the vicinity of ± 5 or ± 6 km/sec, since from here on there is no doubt that the relative frequency of + and − displacements is reversed from what it is for the range +4 to −4 km/sec. Indeed, the reversal is much more marked than shows on the diagram, since there are fourteen negative displacements too large to plot in the figure, although all the positive values are included. That fairly large positive displacements do really occur, however, cannot be doubted; the observations are amply good to insure the

[2] That this interpretation may, however, be incorrect is indicated by the statement on p 9 of the paper by McMath *et al.* (1956).

reality of those shown. From Deutsch's (1956) work on α Herculis, the occurrence of negatively displaced Ca II absorption components is probably to be attributed in all cases to the presence of a circumstellar envelope containing matter ejected from the star. What the significance of the positive displacements may be is entirely obscure at present.

Let us adopt the working definition that an absorption displacement of $|\Delta A| \geq 6$ km/sec is "large," i.e., that it is large enough to be considered as a real anomaly in the sense of the preceding paragraph. In Table 3 there are thirty-eight stars with at least one absorption component which falls in this range, and one regularity appears at once. For thirty-three of these stars there is a negative ΔA, and, of these thirty-three, twenty-eight also have a negative ΔE. Likewise, five stars have a positive ΔA, of which three also have a positive ΔE. Thus, for thirty-one out of thirty-eight objects, the signs of ΔA and of ΔE are the same, a preponderance which is unlikely to be due to chance. This observation seems also to dispose of one worrisome point. If—and this is particularly true of the M stars—the absorption component is displaced shortward, one would expect that any absorption wings belonging to this component would have their greatest effect

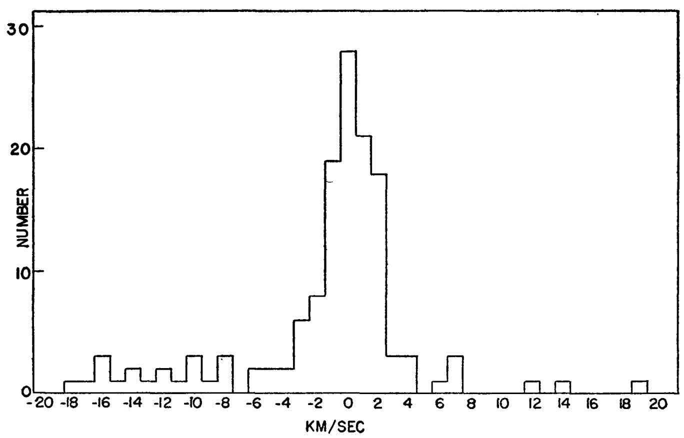

FIG. 7.—Frequency distribution of displacements of Ca II absorption components

on the shortward edge of the emission component and tend to give it an apparent bodily displacement toward the red. That this does not happen is shown by the foregoing figures, and, except in a very few instances where the absorption happens to coincide almost exactly with the shortward edge of the emission, the displacement and presumably also the width of the latter are unaffected. The only star in which an absorption component is located so as obviously to spoil the shortward edge of the emission is ξ Cygni.

We now give some brief statistics concerning the displaced absorption components for the stars in Table 3. In doing so, only those displacements in excess of 6 km/sec in either direction are counted. It is to be noted that, among the G stars, only those brighter than $M_v = -2$ have displaced absorptions; among the K's such phenomena are found in stars of about $M_v = 0$ and brighter, and the same is true of the M stars—hence the brightness limits in Table 8. In so far as the numbers are significant statistically, one sees that positive displacements are virtually restricted to the intrinsically bright G-type stars; that among the K's the prevalence of negative displacements increases rapidly with luminosity; and that a large majority of M-type stars of all luminosities above $+1.5$ have negatively displaced absorption components.

When the negative absorption displacements are plotted against absolute magnitude, M_v(K), as in Figure 8, there is no correlation. Perhaps the most striking feature of the plot is the relatively small horizontal scatter of the points representing the giant and supergiant M-type stars. For these objects, regardless of the large range in M_v, the displacement has a median value of about -14 km/sec and a spread of the order of 10 km/sec on either side of the median. Whatever is the mechanism responsible for the ejection of matter from these stars, it is not sensitive to the intrinsic luminosity. Another plot, not reproduced, shows that within the group of M-type stars there is also no correlation between spectral type and negative absorption velocity.

VIII. INTERPRETATION

The observations described here raise a number of interesting questions relating to the physical processes which underlie the various phenomena. Unquestionably, the most important problem of all is how to explain the relationship between Ca II emission-line width and absolute magnitude shown in Figures 1 and 2. Such a simple correlation, extending over 15 or more mag., must imply that the mechanism which produces the line widths is the same for all stars and that it is of a very fundamental nature. We have seen that the line widths are independent of spectral type, i.e., of surface temperature. It is difficult to see how surface gravity can play any simple role either. Thus, going up

TABLE 8

STATISTICS OF DISPLACED ABSORPTION COMPONENTS IN LATE-TYPE STARS

Type	Range of M_v	All Stars	Negative Disp	Positive Disp
G	Brighter than -2	19	3	5
K	$+1$ 5 to -1 5	47	2	0
K	-1 5 to -3 5	8	2	1
K	Brighter than -3 5	7	6	0
M	$+1$ 5 to -1 5	19	15	0
M	-1 5 to -3 5	3	3	0
M	Brighter than -3 5	4	2	0

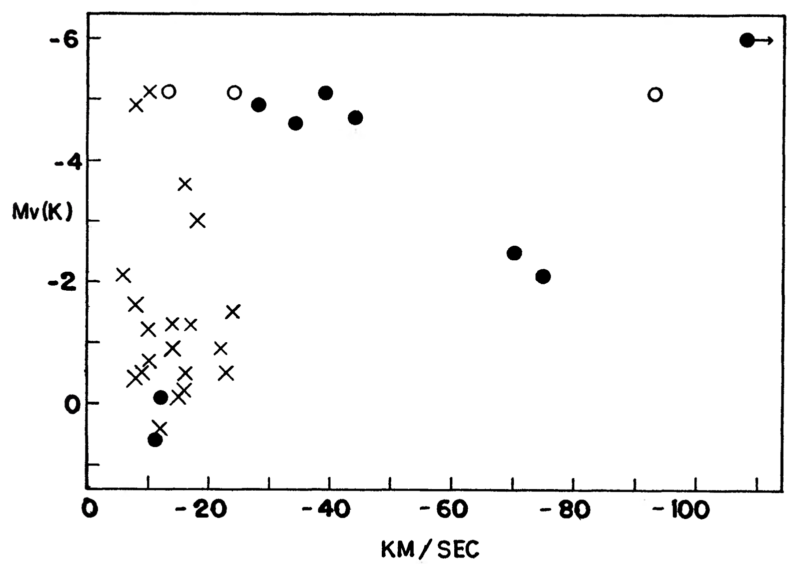

FIG. 8.—Large negative displacements of Ca II absorption components plotted against absolute magnitude. Circles, filled circles, and crosses are G, K, and M stars, respectively.

the main sequence from M5 to G0, g decreases by less than a factor of 10, whereas from the main sequence to normal giants the decrease in g is between 10^2 and 10^4, depending upon spectral type, with a further decrease of 10 to 10^2 between the normal giants and supergiants (Allen 1955). Yet all these stars obey the same relationship between line width and luminosity. One must then ask: How can it be that a phenomenon taking place in the outer fringe region of a star is governed not by the local conditions but by the total rate of energy generation deep inside? This is indeed a puzzling question, and we can offer only a few vague and uncertain speculations which may or may not contribute to a solution.

It is clear that the first point to be considered concerns the means by which the Ca II emission lines are widened. There are only two possibilities: either the widths are a manifestation of Doppler effect, i.e., motions, presumably of a turbulent nature, or they are due to abundance broadening as a result of large optical thickness. So far as we are aware, this question has never been settled, but, in looking through the literature, one gains the impression, at least in the case of the sun, that turbulent motion is generally considered to be the source of the line widths. If the point can be decided for any one object—the sun, for example—the same explanation must almost certainly apply to all, because of the correlation mentioned previously.

If the Ca II emission components originate in a region which is optically thin and are widened by motion, the H_2 and K_2 lines should have the same widths (at the same fraction of the peak intensity), and their intensities should be in the ratio of 1:2. The relative strengths of the lines are affected by the H_3 and K_3 absorption components, but their widths are probably not much influenced. In Section V some measures of the solar K_2 width were mentioned. Widths of H_2 were measured at the same time, and no significant difference was found. This is in accord with the Doppler effect explanation and in disagreement with the idea of abundance broadening, according to which the ratio of the width of K to the width of H should be about $\sqrt{2}$. Although this method of measuring is crude and should be supplemented by a photometric study, the fact remains that the solar H_2 and K_2 lines have rather well-defined edges (on the Mount Wilson spectrograms); it is unlikely that the line widths can really be very unequal. This evidence, for what it is worth, favors turbulent motion as the cause of the Ca II emission-line widths in the sun.

There are indications from the stellar observations that point in the same direction, but they are not conclusive. First consider the relative widths in the stars of the H and K emissions. There are a great many instances in which the measured widths of these lines are the same to within 10 per cent, and among these H is measured wider about as often as K is, which presumably indicates that the differences are due chiefly to observational error. When the disagreement exceeds 10 per cent, K is nearly always measured wider than H; but this occurs among the lines of greater width, where interference from Fe I λ 3969.26 or even from Hϵ may be expected. Indeed, in many cases it is apparent to the eye that the longward side of H is being cut into, and such lines have not been measured. These facts suggest, but do not prove, that equality of width of H and K is the normal situation, deviations from which result from extraneous circumstances having nothing to do with the Ca II lines themselves.

Second, on plates of suitable density it is often possible to see that the maximum intensity of K_2 is great than that of H_2. The interpretation here is complicated by the presence of the H_3 and K_3 absorption components. In fact, it seems possible to make the final emission ratio of K to H either greater or less than unity, depending upon the circumstances assumed in the upper absorbing layer. However, in some of the stars in which this intensity difference is observed, the absorption components appear sufficiently sharp and narrow to give the impression that they probably do not have wings of sufficient extent to obscure the true intensity ratio of the underlying emission. If this is so, the observed intensity differences again point to optically thin emitting layers. It must be conceded that, without further study, not much weight can be given to this evidence.

It appears, therefore, that the most probable source of Ca II emission-line widening is turbulence in the emitting chromospheric layer. If so, Figures 1 and 2 show that the mean turbulent velocity varies very nearly as the one-sixth power of the luminosity. However, recent work by L. Goldberg (communicated privately) on the solar H and K lines indicates strongly that it may be possible to account for them largely by abundance effects in an optically thick layer. Whether the major features of the stellar observations may be explained in this fashion remains to be seen. Until this matter is clarified, we are still somewhat inclined to favor turbulence as the source of emission-line width.

We have no theory to account for the proportionality of log W_0 to log L. The observed fact seems to suggest something like the following picture: In all late-type stars there is an outflow of matter from the interior to the surface, perhaps in the form of jets or streams and perhaps related to the hydrogen convective zone. The flow velocity is a function only of luminosity. At some height, corresponding to that of the lower chromosphere in the sun, the laminar flow in the jets is transformed into turbulence, the mean velocity spread of which bears some simple relationship to the original outward-flow velocity. In this region, perhaps owing partly to the transformation of kinetic energy of flow into heat and thermal excitation, line emission takes place. Probably the observation of this emission has been restricted to the H and K lines simply because they are resonance lines of large f-value and because they are seen against a favorably reduced background of photospheric radiation. In this emitting region, although most of the original outward-flow velocity is lost, a fraction of it remains, and there is a slow residual upward drift. Somewhere above the emitting region the material is cooler, and absorption components of H and K are produced by matter which is falling slowly inward.

Miyamoto (1953) has given an approximate theory for the general shape of the Ca II emissions in the solar chromosphere based on the idea of trapping of resonance radiation in the material and its ultimate escape in the emission-line wings due to thermal modification of the frequencies of the quanta. However, the systematic differences in displacement (Figs. 6 and 7) between the emission and absorption components seem to us to be more suggestive of a simple two-layer explanation.

The foregoing discussion may be completely at variance with the real circumstances and, in any case, is mere suggestion. A thoroughgoing theoretical explanation of the major result of this paper may be difficult, but it would seem to be a matter of some importance to understand the nature of the physical processes involved.

We wish to express our thanks to Dr. Philip Keenan for supplying us with estimates of spectral type and luminosity for a number of stars not in the MK list and to Dr. Orren Mohler for the print of the solar K line described in Section V. We are grateful also to W. Baade, L. Goldberg, D. H. Menzel, and A. Unsöld for stimulating discussion or correspondence concerning various problems discussed in this paper.

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NOTES TO FIGURES 9 AND 10

Spectra

The spectra, including stars of nearly all absolute magnitudes from the main sequence to the supergiants, have been selected to illustrate most of the points discussed in the paper (see Figs. 9 and 10). Data on all but one of these objects will be found in Table 3. The sole exception is No. 7, HD 191026, which is represented by a spectrogram taken after the paper was written. It is included here to fill out the main sequence.

Where a choice was available, there was a strong tendency to select the stars with stronger emission lines, since these will naturally best survive the process of reproduction. Thus the weaker lines are not represented to the same extent that they occur in the observations, although they are not completely lacking. It is equally obvious that the best spectrograms have been chosen for reproduction and hence that the over-all quality of those shown is somewhat superior to the general average. The spectra are reproduced with the red to the right.

All the stars have been ordered in what is believed to be a luminosity sequence. Main-sequence objects are in order of spectral type; above the main sequence they are arranged according to the M_v(K) values of Table 3. The objects illustrated are listed in Table 9. Following the table are some remarks calling attention to items of interest.

TABLE 9

LIST OF SPECTRA

No	Star	Sp	M_v	No	Star	Sp	M_v
1	HD 95735	M2 V	+10 1	19	75 Leo	M0 III	+0 4
2	61 Cyg B	K7 V	+ 8 5:	20	β UMi	K4 III	+0 1
3	61 Cyg A	K5 V	+ 7 8	21	η Psc	G8 III	−0 2
4	HD 219134	K3 V	+ 6 9	22	α Lyn	M0	−0 4
5	70 Oph br	K0 V	+ 6 0	23	β And	M0 III	−0 9
6	ξ Boo br.	G8 V	+ 5 6	24	π Leo	M2 III	−0 9
7	HD 191026	dG3	+ 4 8	25	μ Gem	M3 III	−1 3
8	HD 115043	G1 V	+ 4 5	26	γ And	K3	−1 6
9	33 Vir	K1 IV	+ 3 7	27	β Peg	M2 II–III	−1 6
10	35 Peg	K0	+ 3 2	28	γ Aql	K3 II	−2 1
11	37 Lib	K1 IV	+ 3 0	29	ε Leo	G0 II	−2 2
12	τ CrB	K0 III–IV	+ 2 7	30	ω Gem	G5 II	−3 0
13	HD 162076	G5	+ 2 1	31	π Aur	M3 II	−3 0
14	ξ Dra	K2 III	+ 1 3	32	ε Peg	K2 I*b*	−4 6
15	ε Cyg	K0 III	+ 1 3	33	ζ Cep	K1 I*b*	−4 9
16	β Cet	G6	+ 0 9	34	α Sco	M1	−4 9
17	λ And	G7	+ 0 9	35	α Aqr	G2 I*b*	−5 5
18	51 And	K3 III	+ 0 4	36	β Dra	G2 II	−5 7

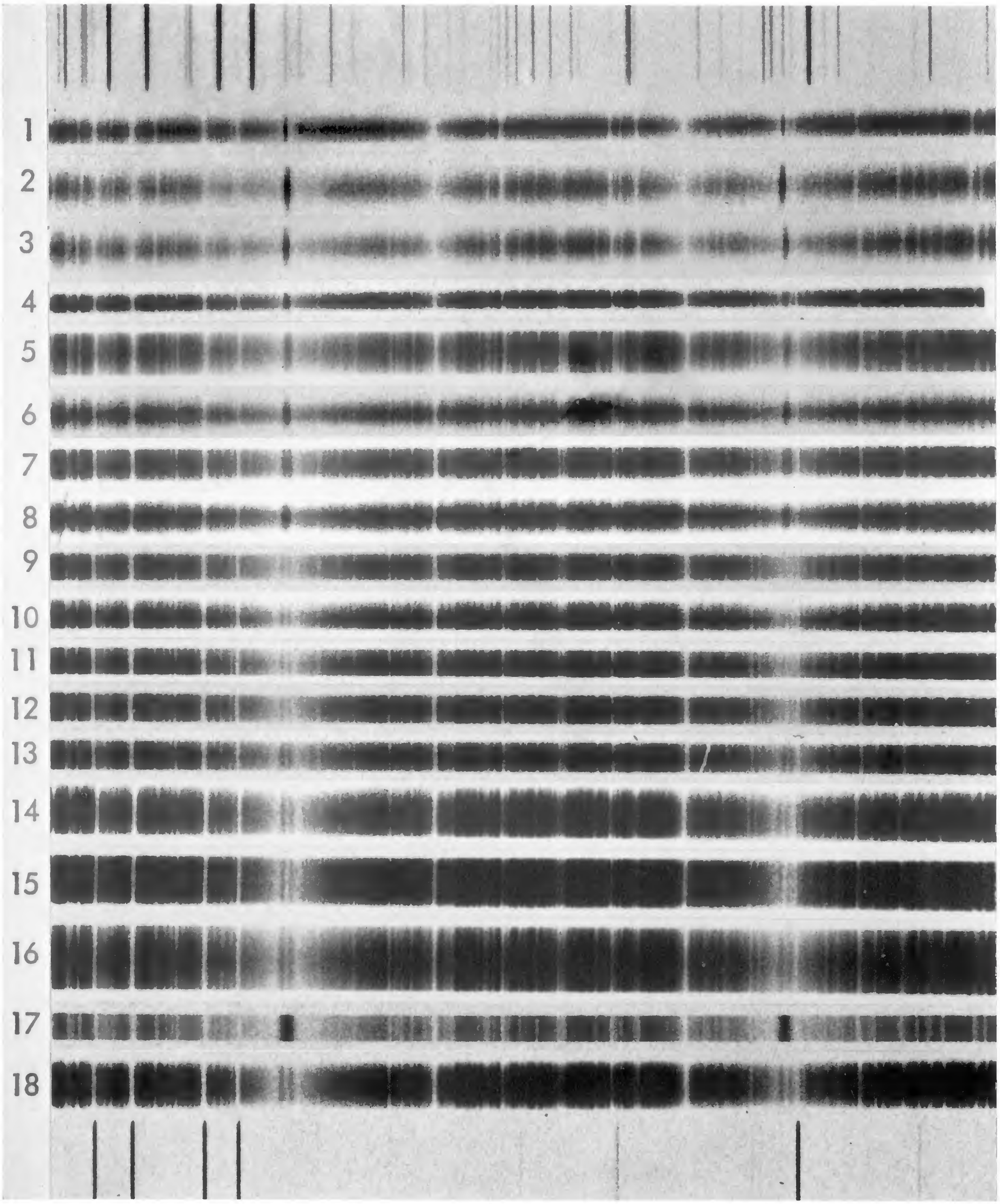

Fig. 9.—Spectra

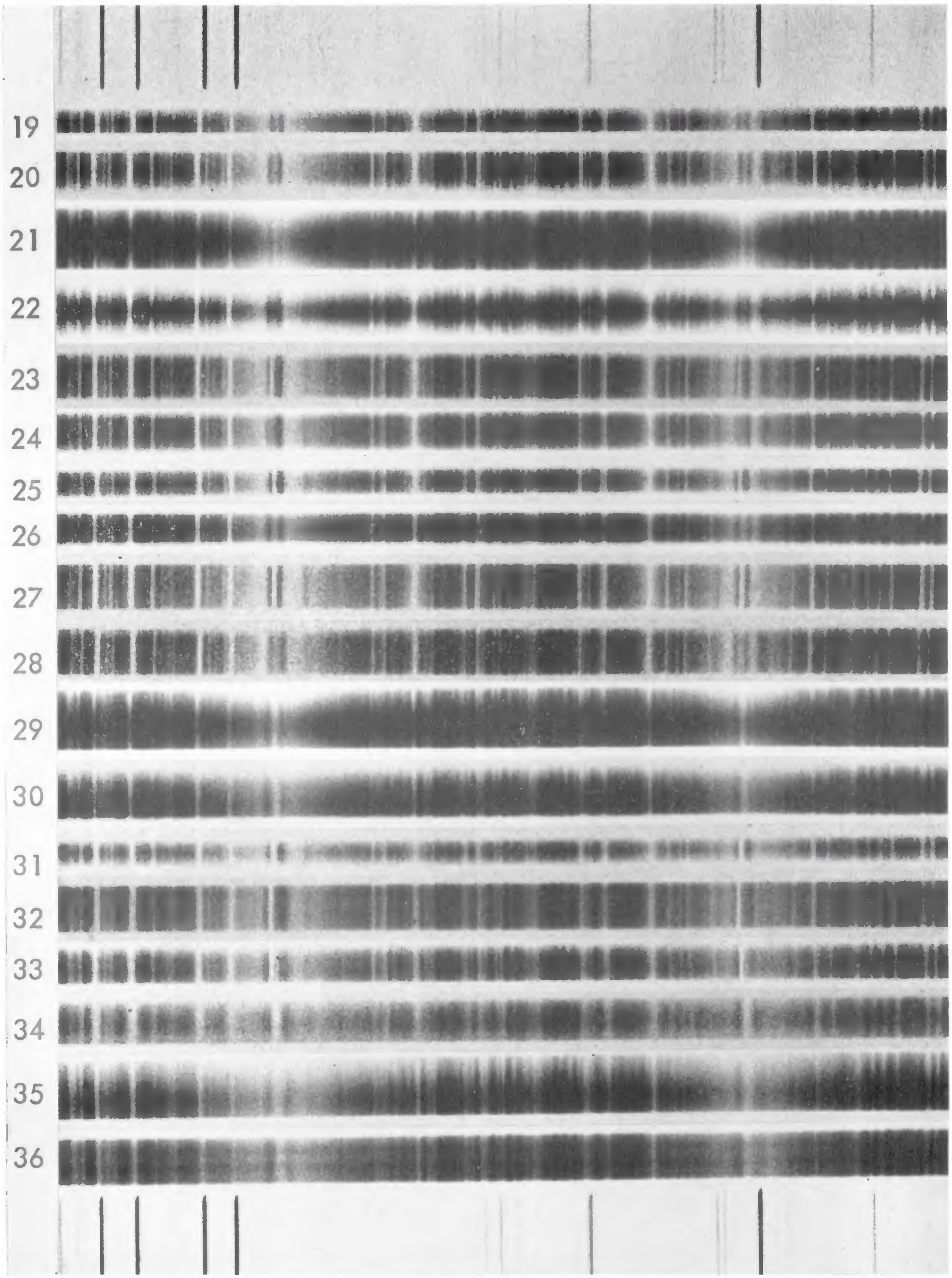

FIG. 10.—Spectra

Remarks

No. 1, HD 95735.—Note that the Ca II emission lines have nearly the same width as the comparison lines of similar strength. This fact doubtless accounts for the discrepant point representing this star in Figures 1 and 2.

No. 7, HD 191026.—The H_3 and K_3 absorption components appear here for the first time. On the original plates they are sometimes seen in stars of about K0.

Nos. 9–13 inclusive.—The emission intensities in Nos. 9, 10, and 11 are 2. For No. 12 it is 1, the only star with this intensity shown. All these stars except No. 13 are included by Miss Roman in her catalogue of high-velocity stars.

No. 17, λ And.—An example of exceptionally strong emission intensity On plates of higher dispersion, narrow H_3 and K_3 absorption components are visible.

No. 19, 75 Leo.—In this M-type star there is little or no shortward displacement of the H_3 and K_3 components.

Nos. 22–25 inclusive.—These four M-type stars show interesting differences in structure of H and K emissions.

No. 28, γ Aql—Note fairly broad additional absorption on shortward edge of H and K emission lines.

No. 30, ω Gem—The violet emission component appears stronger than the red. In this star the emission is displaced shortward and the absorption longward, with a total difference of 15 km/sec.

No. 33, ζ Cep.—The weak, sharp absorption component is probably interstellar.

No. 35, α Aqr.—All absorption lines are sharp. Compare with preceding star, α Sco.

No. 36, β Dra.—Absorption lines have a veiled or filled-in appearance. Compare with preceding star, α Aqr. See note to Table 3 concerning β Dra.