

PHOTOMETRIC STUDIES OF ASTEROIDS. V. THE LIGHT-CURVE AND PHASE FUNCTION OF 20 MASSALIA

THOMAS GEHRELS

Yerkes Observatory

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ABSTRACT

Light-curves were obtained of 20 Massalia at phase angles ranging from near 0° to 20° . The amplitude of the light-curve appears to have changed from 0.17 to 0.23 mag. over the observed interval, March 27–May 15, 1955. For phases between 7° and 20° the phase factor was 0.03 mag/degree, but at phase angles smaller than 7° the factor was larger. The synodic period of rotation was found to be $8^h5^m53^s \pm 1^s$.

I. THE OBSERVATIONS

This study differs from those in the previous papers of the series, in that the primary purpose was to find an accurate magnitude-phase relation, especially at small phase angles. The asteroid 20 Massalia was considered suitable because it could be measured with precision with the available telescope, the 24-inch reflector, and because it went through nearly 0° phase angle at opposition. Even so, every opportunity had to be used to observe the object, and cloudy weather prevented observation on the night of smallest phase. Two comparison stars were used on each night, a "principal comparison star" and a second star used as a check on the constancy of the first, observed about six times per night. In all, nine such comparison stars were used; they were subsequently intercompared. The intercomparisons were made on two nights by the writer at the Yerkes Observatory and on two nights by Mr. Robert Weitbrecht at the McDonald Observatory, with the 13-inch telescope. I am greatly indebted to Mr. Weitbrecht for this help and also for his assistance in making certain improvements on the Yerkes photometer.

The Yerkes and McDonald photometers both contain a 1P21 phototube, used with dry-ice refrigeration, and the standard filters of the (U , B , V) system. The red leak of the ultraviolet filter was assumed to be negligible, as no very red objects are involved.

At the 24-inch, diaphragms were used of 0.5 and 1 mm in diameter, corresponding to 43" and 86", respectively. With the 24-inch, an MIT amplifier and a Brown recorder were employed, and the calibration of the amplifier was checked against a precision potentiometer. At the 13-inch at McDonald, Mr. Weitbrecht used a direct-current integrator of his own design, which fed into a Brown recorder voltmeter system. Mr. Weitbrecht calibrated the integrator independently and used it throughout with a fixed time interval of 25 seconds. This interval is about the same as that employed with the MIT-Brown recorder at Yerkes.

Observations of the asteroid and the two comparison stars could be made on six nights. Because short-period variation caused by rotation was to be expected and was actually found, each run was made as long as conditions permitted. The detailed procedures of observation, reduction, and correction for differential extinction are similar to those described in Paper I of the series (Groeneveld and Kuiper 1954). The observations are listed in Table 1. The second column gives the quality of the night, mainly evaluated from the plot of the photometer reading of the principal comparison star against time. The figure number and the comparison stars are listed in the third and fourth columns, respectively.

II. THE LIGHT-CURVES

The light-curves are reproduced in Figures 1–6. The abscissae give universal time, the ordinates give the magnitude differences with the primary comparison star, in terms of V on the (U , B , V) system. Open dots are observations that were of lesser quality

than the average of the night, so that a mean light-curve could be constructed with more precision. From the figures it appears that the maxima and minima became progressively sharper from March 27 to May 15, 1955. The mean light-curve was therefore made of the intermediate observations, those of April 1 and April 17, drawn as a full line in Figure 7. The maxima and minima as far as observed on March 27 and on May 15 are also indicated in Figure 7, in positions which they have when the best fit is made with the full curve. The horizontal line in the figure indicates the mean magnitude, such that the areas inclosed by the mean curve above and below the line are equal.

From February 27 to May 15 a change in the amplitude of the light-curve is observed of 0.06 mag., from 0.17 to 0.23 mag. approximately; during this interval the change of aspect was 6° . If the variation in amplitude were caused by the small variation of aspect, much larger differences between light-curves of the same asteroid would have been

TABLE 1
OBSERVING RUNS ON ASTEROID

U.T. 1955	Quality of Night	Figure	Comp. Stars
March 27	Poor after 4:30 U.T.	1	A I and A II
March 28	Good until 7:30 U.T.	2	A I and A II
April 1	Very good; haze at end	3	B I and B II
April 17	Irregular	4	C I and C II
April 26...	Good after 4:10 U.T.	5	D I and D II
May 15.	Very good	6	E I and D I

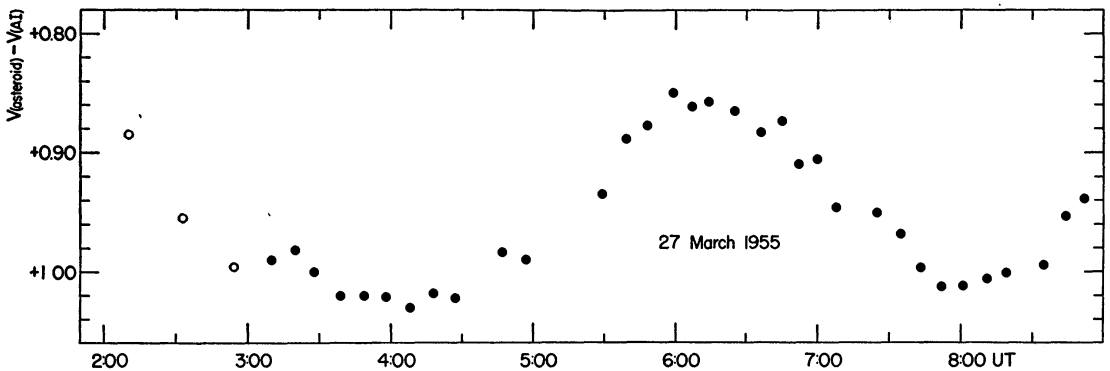


FIG. 1.—Observations on March 27, 1955. $V(A I) = 8.056$ mag.

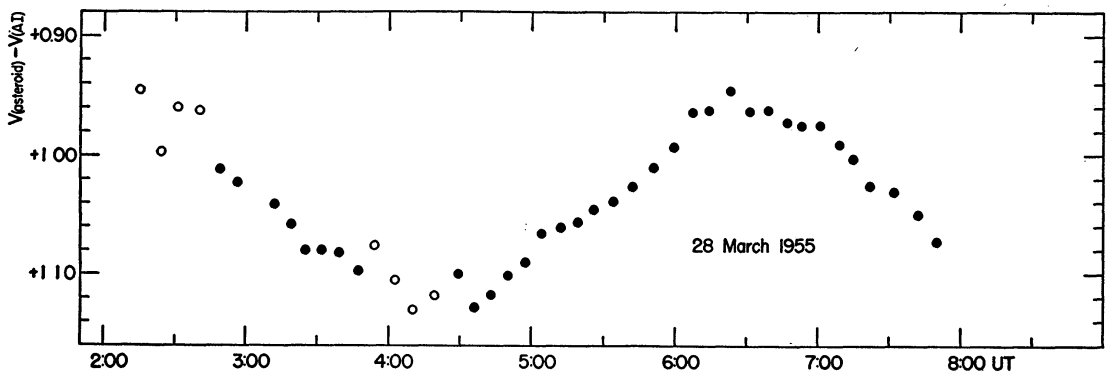


FIG. 2.—Observations on March 28, 1955. $V(A I) = 8.056$ mag.

expected than were found in the previous papers of this series for asteroids observed at different oppositions. The same reasoning probably applies for changes of the light-curve caused by changes in direction of the rotational axis with respect to the earth and sun. The parameter that did change by a relatively large amount is the phase angle. It is therefore probable that the observed changes are a function of phase, in the sense that, with increasing phase angle, the amplitude increased and the light-curve became sharper.

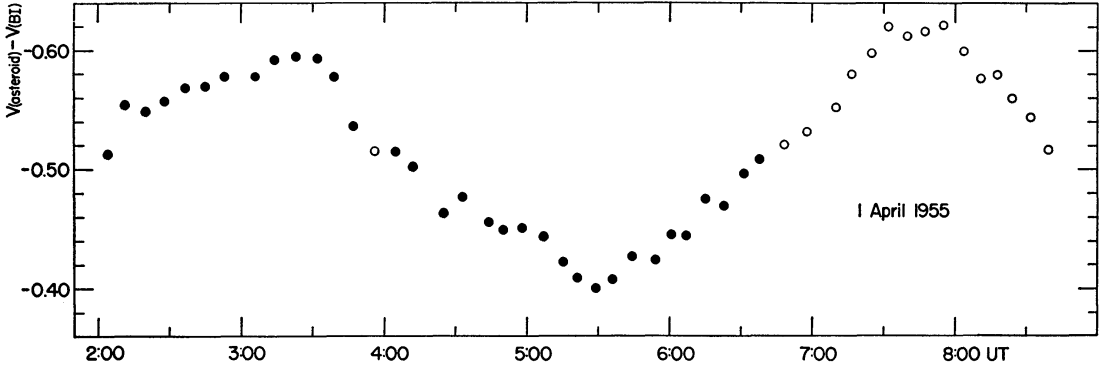


FIG. 3.—Observations on April 1, 1955. $V(BI) = 9.769$ mag.

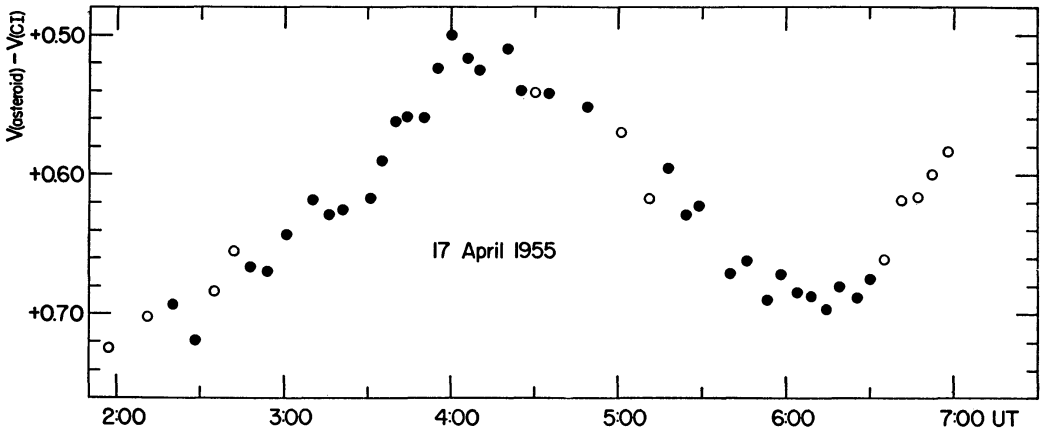


FIG. 4.—Observations on April 17, 1955. $V(CI) = 9.139$ mag.

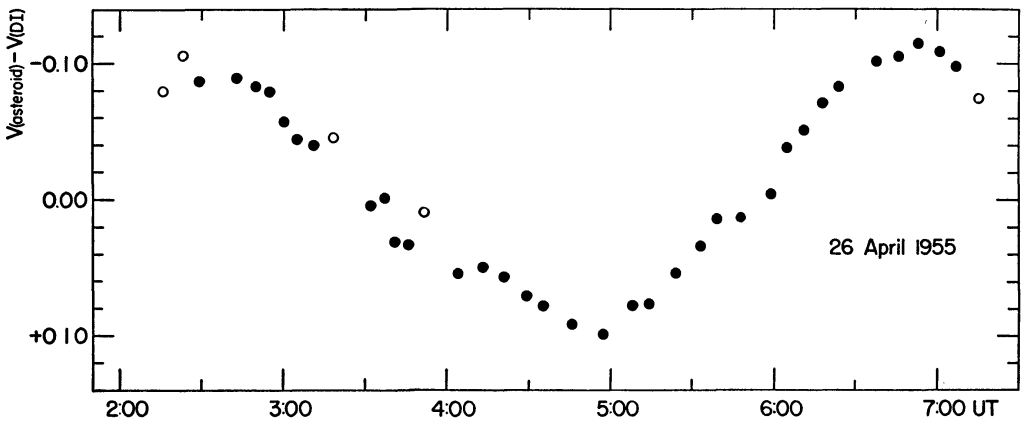


FIG. 5.—Observations on April 26, 1955. $V(DI) = 9.963$ mag.

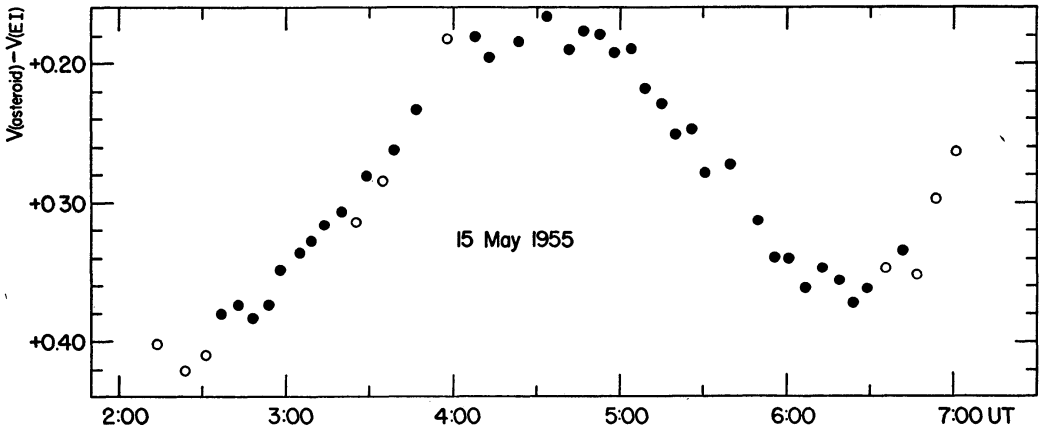


FIG. 6.—Observations on May 15, 1955. $V(EI) = 10.151$ mag.

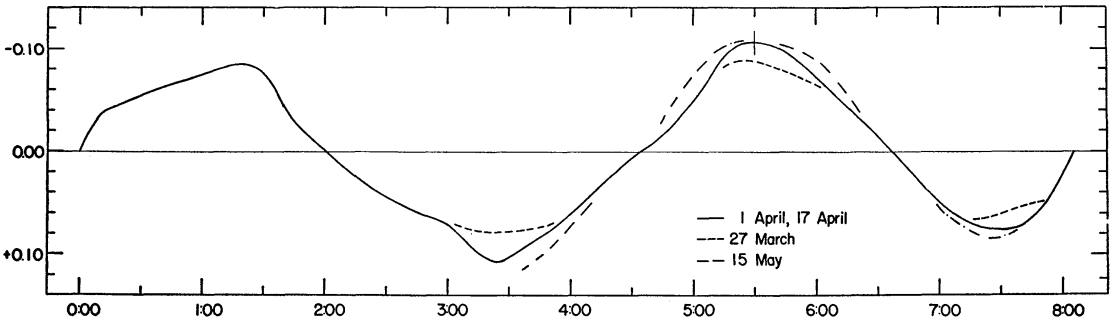


FIG. 7.—The mean light-curve of 20 Massalia (spring, 1955)

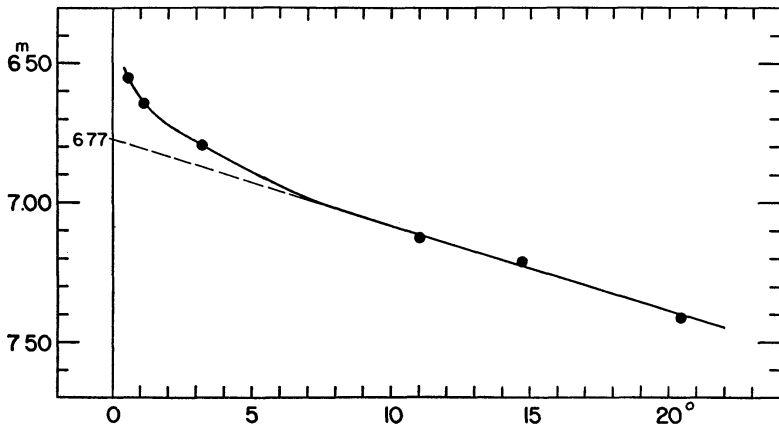


FIG. 8.—The phase function of 20 Massalia (spring, 1955). *Abscissae*, the phase angle in degrees. *Ordinates*, the observed magnitudes, V on the (U, B, V) system, reduced to unit distances from the sun and earth.

III. THE PHASE EFFECT

The measures of the comparison stars are recorded in Table 2. Columns 1 and 2 give the identification; 3, the date; and 4, the difference in magnitude on the (U , B , V) system with the star that is most central of the group, C I. For each star the weighted mean value of column 4 is found in column 5. The weights depend on the quality of the night and the number of observations; the weights are 1, 3, 2, and 1 for May 13, 16, 21, and 25, respectively; the first and the last of these refer to 13-inch observations.

Since many observations of star C I were available, its magnitude and color could be determined accurately from the observations of standard stars that were made during the same night. They were found to be $V = 9.139$ mag. ± 0.004 (p.e.) and $B - V = +0.58$ mag. ± 0.01 (p.e.). With this knowledge, the sixth column was computed from

TABLE 2
OBSERVATIONS OF COMPARISON STARS

STAR		MAY 1955 U.T.	$V(\text{Star}) - V(\text{C I})$	Average of $V(\text{Star}) - V(\text{C I})$	$V(\text{Star})$	$B - V$
Ident. (1)	BD (2)	(3)	(4)	(5)	(6)	(7)
A I. . . .	$-1^{\circ}2639$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} -1.079 \\ -1.082 \\ -1.079 \\ -1.097 \end{array} \right.$	-1.083	8.056	+1.01
A II	$-1^{\circ}2648$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +0.211 \\ +0.202 \\ +0.194 \\ +0.202 \end{array} \right.$	+0.201	9.340	+0.70
B I . . .	$-0^{\circ}2550$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +0.630 \\ +0.629 \\ +0.637 \\ +0.621 \end{array} \right.$	+0.630	9.769	+0.71
B II. . . .	$-1^{\circ}2633$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +0.756 \\ +0.750 \\ +0.761 \\ +0.750 \end{array} \right.$	+0.754	9.893	+0.72
C I. . . .	$+0^{\circ}2875$	9.139	+0.58
C II. . . .	$+0^{\circ}2884$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +0.889 \\ +0.902 \\ +0.891 \\ +0.890 \end{array} \right.$	+0.895	10.034	+0.97
D I	$+0^{\circ}2864$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +0.821 \\ +0.825 \\ +0.825 \\ +0.820 \end{array} \right.$	+0.824	9.963	+0.96
D II	$+0^{\circ}2861$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +1.916 \\ +1.894 \\ +1.895 \\ +1.856 \end{array} \right.$	+1.892	11.031	+0.97
E I	$+1^{\circ}2625$	$\left\{ \begin{array}{l} 13 \\ 16 \\ 21 \\ 25 \end{array} \right.$	$\left\{ \begin{array}{l} +0.996 \\ +1.005 \\ +1.018 \\ +1.039 \end{array} \right.$	+1.012	10.151	+0.93

the fifth. The colors of the comparison stars, $B - V$, are listed in column 7; they were determined in a similar manner as V . The conversion to V for all the stars is made merely for uniformity of presentation; the phase function as it is found below is independent of errors in V for star C I, a result of the differential intercomparisons used.

The next step in obtaining the phase function is to determine the magnitude differences between the averages defined by the light-curves and the star C I. This is done in Table 3, where column 1 gives the date of observation. Column 3 is taken from Table 2, while column 4 gives the magnitude difference between the secondary and the primary comparison star as found on the night of column 1. This value is independent of the separate calibrations recorded in Table 2. Column 5 contains the difference between the mean magnitude of the asteroid and the magnitude of the primary comparison star for that

TABLE 3
COMPUTATION OF MEAN MAGNITUDES OF ASTEROID

DATE 1955 U.T. (1)	COMPARISON STAR		$V(I) - V(II)$ (4)	$V(\text{ASTR.}) - V(I)$ (5)	$V(\text{ASTR.})$ (6)	$\bar{V}(\text{ASTR.})$ (7)	$\bar{V}_1(\text{ASTR.})$ CORR. TO UNIT DIST. (8)
	Ident. (2)	$V(\text{Star})$ (3)					
March 27 .	{ A I	8 056	+0 943	8 999}	8 989	6 555
	{ A II	9 340	-1 304	. .	8 979}		
March 28	{ A I	8 056	+1 037	9 093}	9 085	6 646
	{ A II	9 340	-1 300	. ..	9 077}		
April 1... .	{ B I	9 769	-0 510	9 259}	9 254	6 793
	{ B II	9 893	-0 135	. .	9 248}		
April 17....	{ C I	9 139	.. .	+0 614	9 753}	9 737	7.130
	{ C II	10 034	-0 927	. .	9 721}		
April 26....	{ D I	9 963	-0.008	9 955}	9 938	7.210
	{ D II	11 031	-1 102	. .	9 921}		
May 15	{ E I	10 151	+0 286	10 437}	10 441	7 414
	{ D I	9 963	+0 195	10 444}		

night, found by fitting the mean curve of Figure 7 to the observed light-curve of the night in question. This procedure was checked by fitting each consecutive pair of light-curves and deriving the difference in the ordinates. In combining the two types of derivation, corrections were found and applied to the first set of differences (using the mean light-curve) of not larger than 0.006 mag. The direction of these corrections is such that the broken lines in Figure 7 for March 27 and 28 would have to be moved slightly downward and those of May 15 slightly upward.

The mean magnitude of the asteroid in column 6 is found via the two comparison stars separately, for star I by adding the values of columns 3 and 5 and for star II by adding the values of columns 3, 4, and 5. The average of the two values in column 6 for each date is entered in column 7. The last column gives the mean magnitude of the asteroid, \bar{V}_1 , after correction to unit distance from the sun and the earth.

In Table 4 the positions and planetary relations for 20 Massalia are given. Column 1 gives the dates of observations in universal time, and columns 2 and 3 the observed (not the ephemeris) position of the asteroid. The next three columns give the values of the heliocentric distance, r , the geocentric distance, ρ , and the phase angle, α ; they were computed from the mean anomaly and the angular distance to the opposition point at time of observation. A description of this new method to compute r , ρ , and α , together

with a discussion of its precision, will be published later. The distances of the planet to the sun and to the earth and the phase angle at the time of observation could not be obtained from the "extended ephemeris" in the 1955 volume of *Ephemerides of Minor Planets*, since the O - C values of position were as large as -3^m9 and $+24'$ near opposition. In column 7 of Table 4 the light-time is tabulated, which is needed in the next section.

In Figure 8 the results of the last column of Table 3 are plotted against those of the sixth column of Table 4. The estimated probable error of the points is ± 0.009 mag., which is less than half the radius of the dots in Figure 8.

The phase function of Figure 8 appears to be essentially linear between 7° and 22° , and thus one could very well speak of a phase factor of 0.031 mag/degree, as is customary in asteroid work. However, close to opposition there is a pronounced nonlinear increase in brightness. This *opposition effect* had been suspected by the writer before on the basis of several photographic magnitude determinations of lesser precision, made as part of the current Yerkes-McDonald asteroid survey; but the present observations establish its reality beyond doubt.

TABLE 4
POSITIONS AND PLANETARY RELATIONS

Date 1955 U.T. (1)	1955 R.A. (2)	1955 Dec. (3)	r (4)	ρ (5)	α (6)	Light- Time (7)
March 27 2	12:15 8	$-2^\circ 10'$	2.320	1 322	$0^\circ 57'$	10 ^m 99
March 28 2	12:14 8	$-2 04$	2 322	1 324	1 09	11 00
April 1 2.	12:11.2	$-1 41$	2 328	1 334	3 20	11 08
April 17 2	11:59 1	$-0 17$	2 353	1 412	10 99	11 73
April 26 2	11:54 6	$+0 20$	2 368	1 483	14 69	12 32
May 15 2	11:52 5	$+0 37$	2 398	1 681	20 42	13 96

IV. ABSOLUTE MAGNITUDE, COLORS, AND PERIOD

Since the greater part of the phase function is linear, it is proposed that the intercept of the extrapolated *linear* relation with the ordinate at 0° be defined as the absolute magnitude, g . Such a definition would agree with that of g in use up to the present time, because very few observations of asteroids with phase angles less than 4° have been made, partly because of their considerable orbital inclinations. The absolute magnitude for Massalia on the (U , B , V) system is then found to be

$$g(V) = 6.77 \text{ mag.} \pm 0.02 \text{ (p.e.)}.$$

Observations of the colors of Massalia show that

$$B - V = +0.816 \pm 0.006 \text{ (p.e.)}$$

and

$$U - B = +0.42 \pm 0.03 \text{ (p.e.)}.$$

The instantaneous synodic period can be determined for each time interval between the various light-curves; this is done in Table 5. On the mean light-curve of Figure 7 the "epoch of the primary maximum" is defined by the short vertical line. With this mean curve fitted to the observed light-curves of Figures 1-6, the epochs of the primary maximum were determined for each run. The results are given in Julian days in column 1 of Table 5. Column 2 gives the epoch of primary maximum, corrected for light-time. The intervals between consecutive observations of maximum are entered in column 3.

The time differences were also found by fitting the individual consecutive light-curves; the resulting values are given in column 4 after correction for light-time. Fitting the consecutive curves in some cases is not so precise as fitting each curve to the mean because consecutive curves may have only a small part of the cycle in common. The average of columns 3 and 4, listed in column 5, is therefore a weighted mean. Column 6 tabulates the number of cycles between consecutive observations; the quotient of columns 5 and 6 determines the period, which is entered in column 7.

TABLE 5
COMPUTATION OF THE INSTANTANEOUS SYNODIC PERIOD

EPOCH OF PRIMARY MAXIMUM		INTERVAL			No. OF CYCLES	P
JD 2435000 + (1)	JD(c) (2)	Mean Light-Curve (3)	Indiv. Light-Curves (4)	Weighted Average (5)		
193 ^d 7574	193 ^d 7498	1 ^d 0133	1 ^d 0118	1 ^d 0125	3	0 ^d 3375
194.7707	194.7631	4 0455	4 0471	4 0463	12	.33719
198.8163	198 8086	15.8587	15 8559	15 8578	47	.33740
214.6754	214 6673	9.1118	9.1076	9.1104	27	.33742
223.7877	223.7791	18 8972	18 8964	18.8969	56	0 33744

The average period may now be computed from the separate values of column 7 by assigning weights proportional to the numbers of cycles used in each case. The result is

$$\bar{P}_1 = 0^d337404 \pm 0^d000015 \quad \text{or} \quad 8^h05^m52^s \pm 2^s.$$

The probable errors were found in two independent ways: (a) from the internal agreement of the five values of Table 5 and (b) from the estimated uncertainty of one epoch of maximum, being ± 0.001 day.

If the first and the last epochs of maximum are used, corrected for light-time, one has a total of 145 cycles, lasting 48.9265 days. This defines a mean period of

$$\bar{P}_2 = 0^d337424 \pm 0^d000010 \quad \text{or} \quad 8^h05^m53^s \pm 1^s.$$

This value has a greater weight than the earlier mean value, as is easily shown; the two results may be combined into the synodic mean period,

$$\bar{P} = 0^d337417 \pm 0^d000009 \quad \text{or} \quad 8^h05^m53^m \pm 1^s.$$

In conclusion I would like to thank Dr. G. P. Kuiper for his advice.

REFERENCE

Groeneveld, I., and Kuiper, G. P. 1954, *Ap. J.*, 120, 200.