ON THE MASS OF THE RR LYRAE STARS

ALLAN SANDAGE
Mount Wilson and Palomar Observatories
Carnegie Institution of Washington, California Institute of Technology
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ABSTRACT

Recent work has indicated that the mass of the individual RR Lyrae stars in M3 is $\mathfrak{M}/\mathfrak{M}_{\odot} = 1.25$, as required by current ideas of stellar evolution, only if (1) $\bar{M}_{pv} = -0.43$ or (2) $P(\bar{\rho}/\bar{\rho}_{\odot})^{1/2} = 0^{d}.055$ or (3) the RR Lyrae stars are bluer by $\Delta(B - V) = 0.11$ mag. than main-sequence stars of the same effective temperature. The third alternative is shown to be probably the case from a study of the change in the emergent flux from the stars as a function of surface gravity. The case for RR Lyrae itself is also discussed. It appears that RR Lyrae is reddened by about $\Delta(B - V) \approx 0.12$ mag. and that the considerations which are valid for the M3 variables apply also to RR Lyrae.

Evidence was presented in a recent paper by Roberts and the present writer (1955) that the mass of the individual RR Lyrae stars in the globular cluster M3 is $\mathfrak{M}/\mathfrak{M}_{\odot} = 0.70$. This result follows from the assumptions that (1) $\bar{M}_{pv} = 0.00$; (2) the value of the pulsation constant is $Q = 0^{d}.041$ (Epstein 1950); and (3) the $CI = f(T_e)$ relation for these variables is the same as for type I main-sequence stars. A mass of 0.70 is lower than is expected from current ideas concerning stellar evolution. It was pointed out that a more acceptable mass of $\mathfrak{M}/\mathfrak{M}_{\odot} = 1.25$ would be obtained (1) if $\bar{M}_{pv} = -0.43$ or (2) if $Q = 0^{d}.055$ or (3) if the effective temperature for the RR Lyrae stars is lower by 800° K than that for main-sequence stars of the same color index. The third possibility is equivalent to the statement that the RR Lyrae stars should be bluer than main-sequence stars by $\Delta(B - V) = 0.11$ mag. for the same effective temperature. The present paper suggests that the third alternative is probably correct, since it is shown that the emergent radiation flux for stars with a low surface gravity like the RR Lyrae stars differs by the required amount from the flux of the higher-gravity main-sequence stars of equal T_e.

CONTINUOUS RADIATION AS FUNCTION OF SURFACE GRAVITY

The emergent flux from a stellar atmosphere is obtained from the well-known expression

$$F_{\nu}(0) = 2\int_{0}^{\infty} B_{\nu}(T_{\tau}) E_2\left(\frac{\kappa_{\nu}}{\bar{\kappa}}\tau\right) d\left(\frac{\kappa_{\nu}}{\bar{\kappa}}\tau\right). \tag{1}$$

This integral has been evaluated by Burkhardt (1936) and more recently by Chandrasekhar and Breen (1947) for the case where κ_{ν}/κ is independent of optical depth. In the notation of these latter authors the emergent flux is given by

$$F_{\nu}(0) = B_{\nu}(T_0)\, \mathfrak{F}\left(\frac{h\nu}{kT_e}, \frac{\kappa_{\nu}}{\bar{\kappa}}\right). \tag{2}$$

The flux and consequently the color indices and spectrophotometric gradients for any star may therefore be computed, once $\kappa_{\nu}/\bar{\kappa}$ is known for various wave lengths, provided that $\kappa_{\nu}/\bar{\kappa}$ is assumed to be independent of τ. For the present calculations, the table of $\kappa_{\nu}/\bar{\kappa}(\theta_e, P_e)$ by Chandrasekhar and Münch (1946) is used.

Emergent fluxes and the resulting spectrophotometric gradients have been computed for a series of cases with θ_e ranging from 0.5 to 0.9 and with P_e taking the values of 10, 10^2, 10^3, and 10^4 dynes cm^{-2}. The computations were entirely analogous to those of Chandrasekhar and Münch, with the exceptions that the wave-length interval for the gradients was from λ 4000 to λ 6500 A in the present case and that Chandrasekhar and Breen's $\mathfrak{F}$-function was used instead of Burkhardt's tables. This latter circumstance permitted gradients to be computed for the $P_e = 10$ cases over the entire range of θ_e.

This was not the case with Chandrasekhar and Münch (1946), since Burkhardt's tables are not extensive enough. Blanketing by the metal lines, with the resulting change in the continuum, has not been considered in these computations. The computed gradients, $G(4000/6500)$, and the resulting reciprocal color temperatures, θ_c, are given in Table 1. Figure 1, which is analogous to Figure 2 of Chandrasekhar and Münch, shows the $\theta_c = f(\theta_e, P_e)$ relation. It is evident that the color temperature depends critically upon the electron pressure in stars whose reciprocal effective temperature at the effective photosphere ranges from $\theta_e = 0.6$ to $\theta_e = 0.8$.

Figure 1 is not in convenient form to interpret data obtained by wide-band filter photometry. A relation analogous to Figure 1 is needed in which the reciprocal color temperature is replaced by the directly measured color index. Once the emergent fluxes are known, the color indices can be computed from the sensitivity functions of the photometric system. Computations for this result were made in an investigation of another problem by nine graduate students of the California Institute of Technology astronomy

TABLE 1

PHOTOMETRIC GRADIENTS AND COLOR TEMPERATURES

P_e		θ_e 0.5	0.6	0.7	0.8	0.9
10	$G(40/65)$	0 77	.	1 01	1 87	2 07
	θ_c	0 15		0 28	0 64	0 71
10^2	$G(40/65)$	0.775	0 85	1 56	1.93	2 07
	θ_c	0 16	0 19	0 51	0 66	0 71
10^3	$G(40/65)$	0 839	1 31	1.69	1 96	2 07
	θ_c	0 19	0 41	0 56	0 67	0.71
10^4	$G(40/65)$	1 08	1 49	1 71	1 96	2 07
	θ_c	0 31	0 48	0 58	0 67	0 71

department in Greenstein's class on stellar atmospheres. These authors have kindly permitted the present use of their relation before publication. The emergent fluxes computed by these workers covered models with θ_e ranging from 0.1 to 0.8 for P_e's of 10^2, 10^3, and 10^4. They used the values of $\kappa_\nu/\bar{\kappa}$ computed by Vitense (1951), together with Burkhardt's tables. Their results, slightly smoothed, are shown in Figure 2. The curve for $P_e = 10$ was estimated by the present writer from the $P_e = 10$ curve of Figure 1. The zero point of the $B - V$ values probably does not correspond with observational data, since the effect of line blanketing is neglected. This is not important, however, since we are interested here only in the differences in $B - V$ for different P_e's.

Figure 2 is very similar to Figure 1 and shows that stars with a low electron pressure at the effective depth for the formation of the continuum will be bluer than stars with high P_e. Since the electron pressure is a function of the surface gravity through some form $P_e(A, \theta_e, g)$, where A is the hydrogen-to-metal ratio and g is the surface gravity, it is evident that the results of Figure 2 are implicitly dependent upon g. It is this circumstance which predicts a difference in the color indices between the RR Lyrae stars and main-sequence stars of the same θ_e.

Ideally, the $P_e(A, \theta_e, g)$ relation should be obtained from detailed tables of model atmospheres, but such tables are laborious to compute. A first approximation for $P_e(A, \theta_e, g)$ at the effective optical depth of the continuum may, however, be made by Unsöld's method of preliminary analysis (see, e.g., Aller's *Astrophysics*, **1**, 222). The steps necessary to arrive at $P_e(A, \theta_e, g)$ are (1) plot the relation $P_g(P_e, \theta_e, A)$ obtained, for example, from Strömgren's tabulation (1944); (2) convert $\bar{\kappa}(\theta_e, P_e)$, obtained, for example, from Chandrasekhar and Münch, to $\bar{\kappa}(\theta_e, P_g, A)$ by use of the $P_g(P_e, \theta_e, A)$ relation and plot the lines of constant θ_e in the $\bar{\kappa}, P_g$-plane for different values of A; and (3) plot the straight lines $\log P_g = \log g - \log \bar{\kappa} + \log \tau_0$ for different values of $\log g$ in the $\bar{\kappa}, P_g$-plane. This equation results from the integration of $dP_g = (g/\bar{\kappa})d\tau$ with the

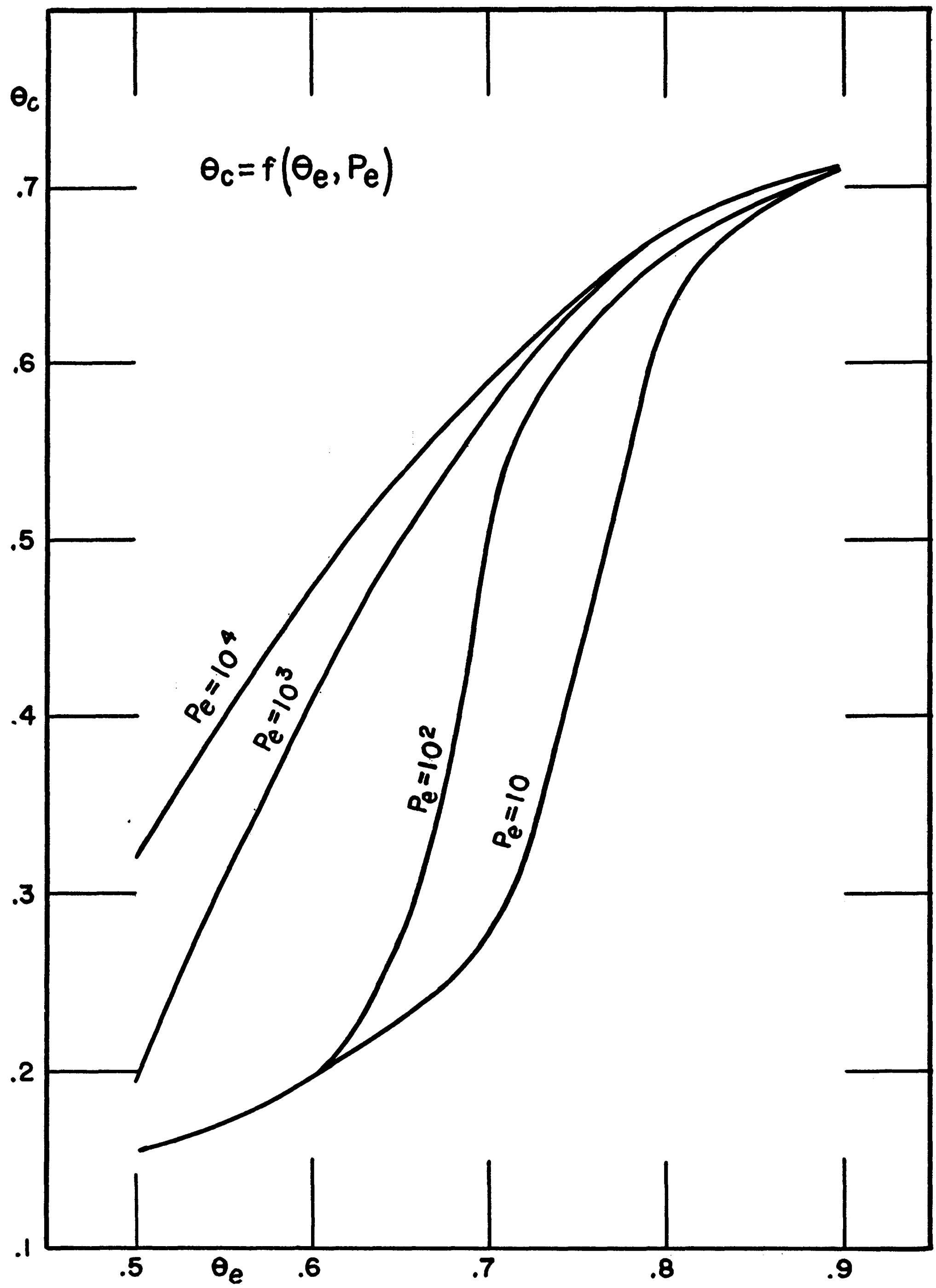

FIG. 1.—The reciprocal color temperature as a function of electron pressure and reciprocal effective temperature.

assumption that $\bar{\kappa}$ is independent of τ. The effective depth for the formation of the continuum is taken as $\tau_0 = 0.63$. The intersections of these lines with the curves of constant θ_e in the $\bar{\kappa}$, P_g-plane give values of P_g for various values of θ_e and g. These values of P_g may be converted to P_e by the $P_g(P_e, \theta_e, A)$ relation and the resulting set of P_e, θ_e, and g plotted as in Figure 3. The curves shown in Figure 3 were computed with Strömgren's $P_g(P_e, \theta_e, A)$ tables and Chandrasekhar and Münch's $\bar{\kappa}(\theta_e, P_e)$ values for two values of log A.

We are now ready to compute the expected $\Delta(B - V)$ between RR Lyrae and main-sequence stars of the same θ_e. The surface gravity of an RR Lyrae star when it is at mean radius is about 7.0×10^2 cm sec^{-2}. This follows from a mean radius of 7 $R_\odot$ (Stebbins 1953) and $\mathfrak{M}/\mathfrak{M}_\odot = 1.25$. (If $\mathfrak{M}/\mathfrak{M}_\odot$ had been assumed to be 0.70, the final results would have given 1.25 by an iteration process through the period-density relation with

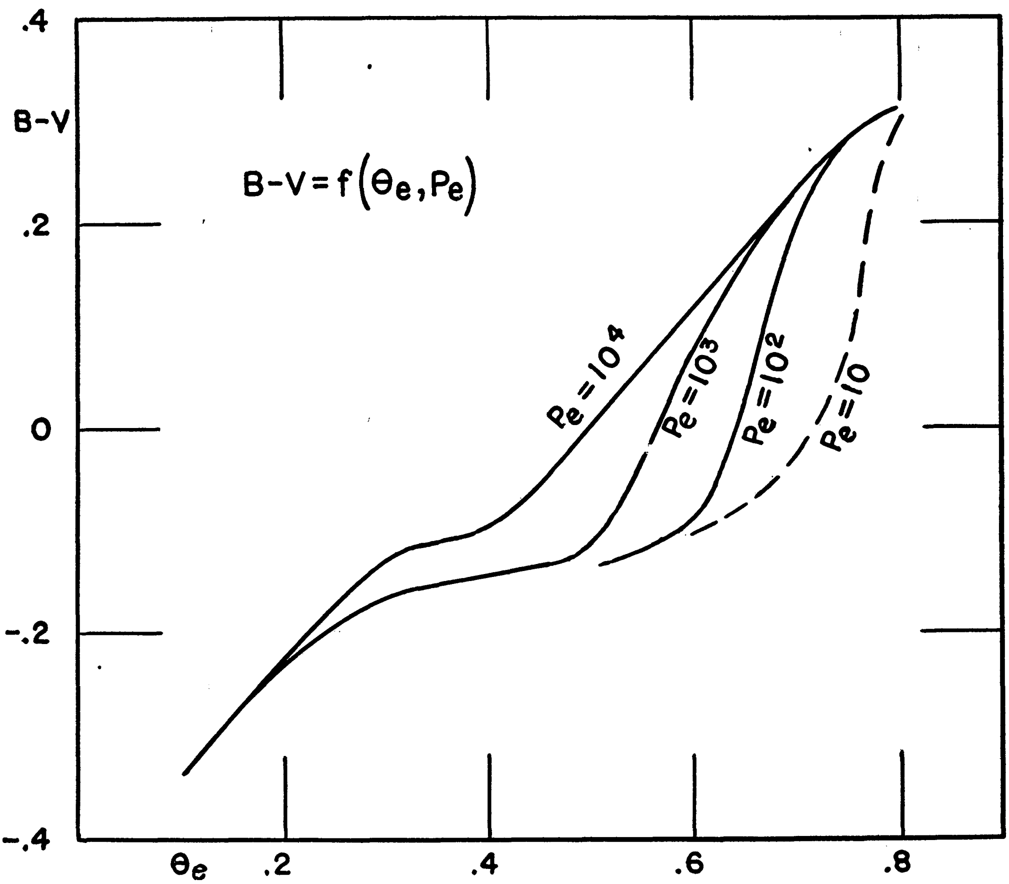

FIG. 2.—The color index, $B - V$, as a function of the electron pressure and reciprocal effective temperature. The zero point of $B - V$ is arbitrary.

the valid $CI = f[T_e]$ relation.) The value of g for main-sequence stars at $T = 7000°$ K and $M_{\text{bol}} = +3.3$ is $g = 2.6 \times 10^4$ cm sec^{-2}. Figure 3 shows that, at $\theta_e = 0.72$, the effective log P_e for the RR Lyrae stars and for the comparable main-sequence stars is 1.10 and 1.80, respectively. Entering these values in Figure 2 gives $\Delta(B - V) = 0.12$ mag. This is in agreement with $\Delta(B - V) = 0.11$ mag. required from the observational data if $\mathfrak{M}/\mathfrak{M}_\odot = 1.25$. The conclusion from these computations, therefore, is that the mass of the RR Lyrae stars in M3 is close to 1.25, provided that (1) $\bar{M}_{\text{pv}} = 0.00$ and (2) $P(\bar{\rho}/\bar{\rho}_\odot)^{1/2} = 0^{\text{d}}041$.

THE CASE OF RR LYRAE ITSELF

In 1953 Savedoff showed that the observational data of Stebbins and Whitford (Stebbins 1953) for RR Lyrae itself gave a mass consistent with evolutionary theories, provided that the main-sequence $CI = f(T_e)$ relation is used. The six-color measures for RR Lyrae give $V - I = -0.73$, which corresponds to a temperature of $T_1 = 6800°$ K (Stebbins and Whitford 1945). At median light M_{bol} is assumed to be -0.04. Substitution of these values in

$$\frac{\mathfrak{M}}{\mathfrak{M}_\odot} = \left(\frac{0.041}{P}\right)^2 \left(\frac{L}{L_\odot}\right)^{3/2} \left(\frac{T_\odot}{T}\right)^6 \tag{3}$$

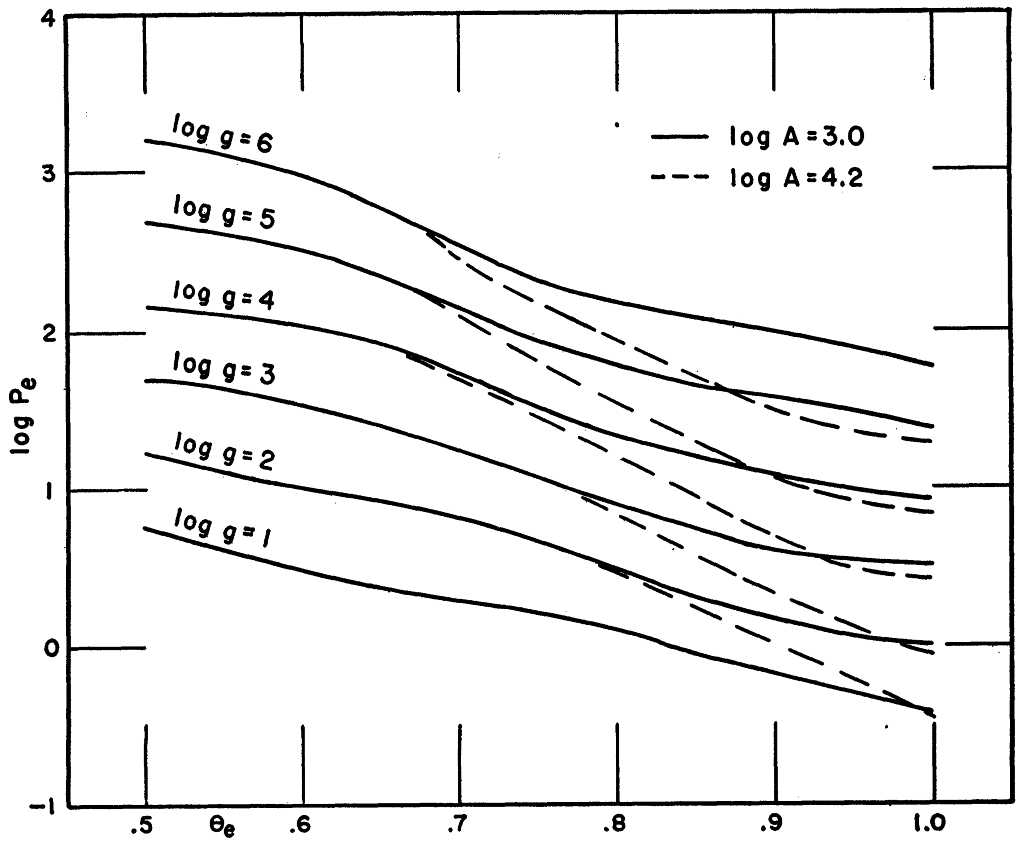

FIG. 3.—The electron pressure as a function of surface gravity, hydrogen-to-metal ratio, and reciprocal effective temperature.

gives $\mathfrak{M}/\mathfrak{M}_\odot = 1.15$. This result suggests that RR Lyrae gives results inconsistent with the stars in M3. There is evidence, however, that RR Lyrae is reddened and consequently that the measured $V - I$ of -0.73 does not correspond to the normal color. Stebbins (1953) states: "In the colors [of RR Lyrae] between V and I a good match can be made [with other stars] but, as in the case of η Aquilae, at all phases both the violet and the infrared are stronger than in other stars"; and later: "There is no possibility [from the measured colors] of getting a color class for RR Lyrae at maximum corresponding to a regular spectrum at A2." These data are suggestive of Harris' explanation (1954) by interstellar reddening of the discrepancy in the colors of η Aquilae. Considerations similar to those of Harris suggest that RR Lyrae is also reddened by about $\Delta(V - I) \approx 0.32$ mag. Table 2 shows the data. The measured six colors for RR Lyrae are those quoted by Stebbins (1953), while those of the comparison stars are from Stebbins and Whitford (1945). Comparison with little-reddened stars is made at phases of maximum, minimum, and median light of RR Lyrae. Subtraction of a reddening of $\Delta(V - I) = 0.32$ mag. from the measured colors gives colors which match the mean of α Cyg and γ UMi (mean spectral class A2) at maximum light, NPS 3s and HD 118216 (mean spectral class F3) at minimum light, and 39 Aur and 78 UMa (mean spectral class A7) near median light. There is still an excess in V for RR Lyrae at median and minimum light, but this may be a result of the bluer continuum near the V point due to the effect shown in Figures 1 and 2. These "corrected" color classes for RR Lyrae are now in good agreement with the spectral class determined from the metal lines (Münch and Terrazas 1946; Struve and Blaauw 1948). A reddening of $\Delta(V - I) = 0.32$ is equivalent to $\Delta(C_{\rm int}) \approx 0.12$

TABLE 2

COMPARISON OF SIX-COLOR DATA

	U	*V*	*B*	*G*	*R*	*I*	Remarks
RR Lyrae .	−0 56	−0 65	−0 27	−0 03	+0 30	+0 53	Max. light
Reddening	+ .19	+ 13	+ 07	+ .01	− .08	− 19	
RR "normal"	− 75	− 78	− 34	− 04	+ 38	+ 72	
Mean A2 .. .	−0 78	−0 81	−0 36	−0 03	+0 38	+0 72	α Cyg, γ UMi
RR minus ..	+0 03	+0 03	+0 02	−0 01	0 00	0 00	
RR Lyrae .	−0 33	−0 44	−0 11	−0 03	+0 13	+0 29	Med. light
Reddening ..	+ 19	+ 13	+ 07	+ 01	− 08	− 19	
RR "normal"	− 52	− 57	− 18	− 04	+ 21	+ 48	
Mean A7	−0 45	−0 45	−0 19	−0 03	+0 22	+0 47	39 Aur, 78 UMa
RR minus	−0 07	−0 12	+0 01	−0 01	−0 01	+0 01	
RR Lyrae	−0 30	−0 30	−0 07	−0 02	+0 09	+0 12	Min. light
Reddening ..	+ 19	+ 13	+ 07	+ 01	− 08	− 19	
RR "normal"	− 49	− 43	− 14	− 03	+ 17	+ 31	
Mean F3	−0 44	−0 37	−0 14	−0 02	+0 15	+0 29	NPS 3s, HD 118216
RR minus....	−0 05	−0 06	0 00	−0 01	+0 02	+0 02	

mag. The unreddened color for RR Lyrae would then be $C_{\rm int} \approx 0.10$ mag., since Stebbins gives $C_{\rm int}$ (med.) = 0.22 mag. as the observed color. This normal color now agrees with the color-period relation for the M3 variables (Roberts and Sandage 1955, Fig. 7), which the uncorrected color of 0.22 violates. Thus RR Lyrae is similar to the M3 variables, and the argument of the last section applies to it as well. A reddening of $\Delta(CI) = 0.12$ mag. just compensates for the effect of low surface gravity, so that use of the incorrect relation between the color index and the effective temperature does lead to the nearly correct $\mathfrak{M}/\mathfrak{M}_\odot$. This circumstance explains Savedoff's result (1953).

Added December 13, 1955.—The recent discovery by H. W. Babcock (*Pub. A.S.P.*, February, 1956) of a magnetic field in RR Lyrae suggests that the field should be taken into account when the period is computed (cf. Chandrasekhar and Limber, *Ap. J.*, **119**, 10, 1954). This may alter the value of Q calculated by Epstein. However, the conclusion of the present paper will not be changed unless the emergent radiation flux is significantly affected by the magnetic field. The present problem may be inverted so that Q is considered as unknown but determined, once $\bar{M}_{\rm pv}$ and $\mathfrak{M}/\mathfrak{M}_\odot$ are known. With this approach, Q has the value of $0^{\rm d}041$, provided that $\bar{M}_{\rm pv} = 0.00$, $\mathfrak{M}/\mathfrak{M}_\odot = 1.25$, and the $\theta_e = f(B - V, P_e)$ relation of Figure 2 applies.

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