THE CIRCUMSTELLAR ENVELOPE OF ALPHA HERCULIS

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ABSTRACT

In the system of α Herculis, the M-type supergiant, and its visual companion, a giant G-type spectroscopic binary, are surrounded by a common circumstellar envelope. This envelope produces violet-displaced absorption cores in the zero-volt lines of the M star and stationary zero-volt absorption lines in the spectrum of the G star. The minimum radius of the envelope is $2 \times 10^5 R_\odot$; its minimum mass is probably about $1 \times 10^{-5} \mathfrak{M}_\odot$. It is condensed into clouds that fill only the fraction 10^{-7} of the whole spherical volume; within a condensation, the electron density is about 10^6 cm^{-3}. The envelope is expanding at a velocity of 10 km/sec; the matter in it has been ejected by the M star and is being lost to the system at a probable rate of at least $3 \times 10^{-8} \mathfrak{M}_\odot$/year. There is evidence for a comparable rate of mass loss from all other late-type supergiants. This process may be important in the evolution of all massive stars that have exhausted their hydrogen.

I. INTRODUCTION

The system of α Herculis is a well-known visual binary. It comprises a third-magnitude supergiant M star, which is an irregular variable, and a fifth-magnitude giant G star, which is a spectroscopic binary. Recent coudé spectrograms of the G star have shown a group of remarkable stationary absorption lines that must be attributed to a circumstellar cloud or envelope at a great distance above its photosphere. Since no comparable cloud is known to exist around any other G star and since the M star has long been known to have an extended atmosphere, the observations at once suggest that both stars are enveloped by a single circumstellar cloud.

Figure 1 illustrates the spectroscopic peculiarities of this system. The figure shows portions of three spectrograms, in which the original dispersion was 4.5 A/mm. The strips designated "*I*" are enlargements from a spectrogram of the G star, plate Pb 1684. The strips marked "*II*" also represent the G star, but at a different date; they are enlargements from plate Ce 9333. At the time of exposure of this plate, the G star and the M star were both held stationary on the slit. The strips marked "*III*" are from the M-star spectrum on this plate. Strips *II* and *III* have been reproduced together; in the reproduction the two spectra are seen in exactly the same configuration as on the original plate.

The absorption lines from the reversing layer of the G star are widened slightly, as though by a rotational disturbance with $V \sin i \simeq 15$ km/sec. They are also somewhat weaker than in the spectrum of a normal star of the same type. The displacement of these lines in *II* relative to *I* results from a 45-km/sec change in radial velocity, associated with the spectroscopic orbit of the G star. But a sharp line at λ 4227 and others at H and K indicate identical velocities on the two plates. These stationary lines are present on all plates of the G star. Other stationary lines that have been found in the G-type spectrum are indicated in Table 1. These lines clearly originate in circumstellar gas that envelops the whole spectroscopic orbit of the G star and that does not share in the orbital motion. All the stationary lines arise from the lowest levels of abundant atoms or ions. There is no trace even of some normally strong lines of Fe I that arise from levels with excitation potentials between 0.00 and 0.10 volt. The excitation temperature of the circumstellar gas is therefore very close to zero, as in interstellar gas.

The same circumstellar lines that appear in the G-star spectra are also present in the M-star spectrum, as may be seen in *III*. In addition, the spectrum of the M star shows

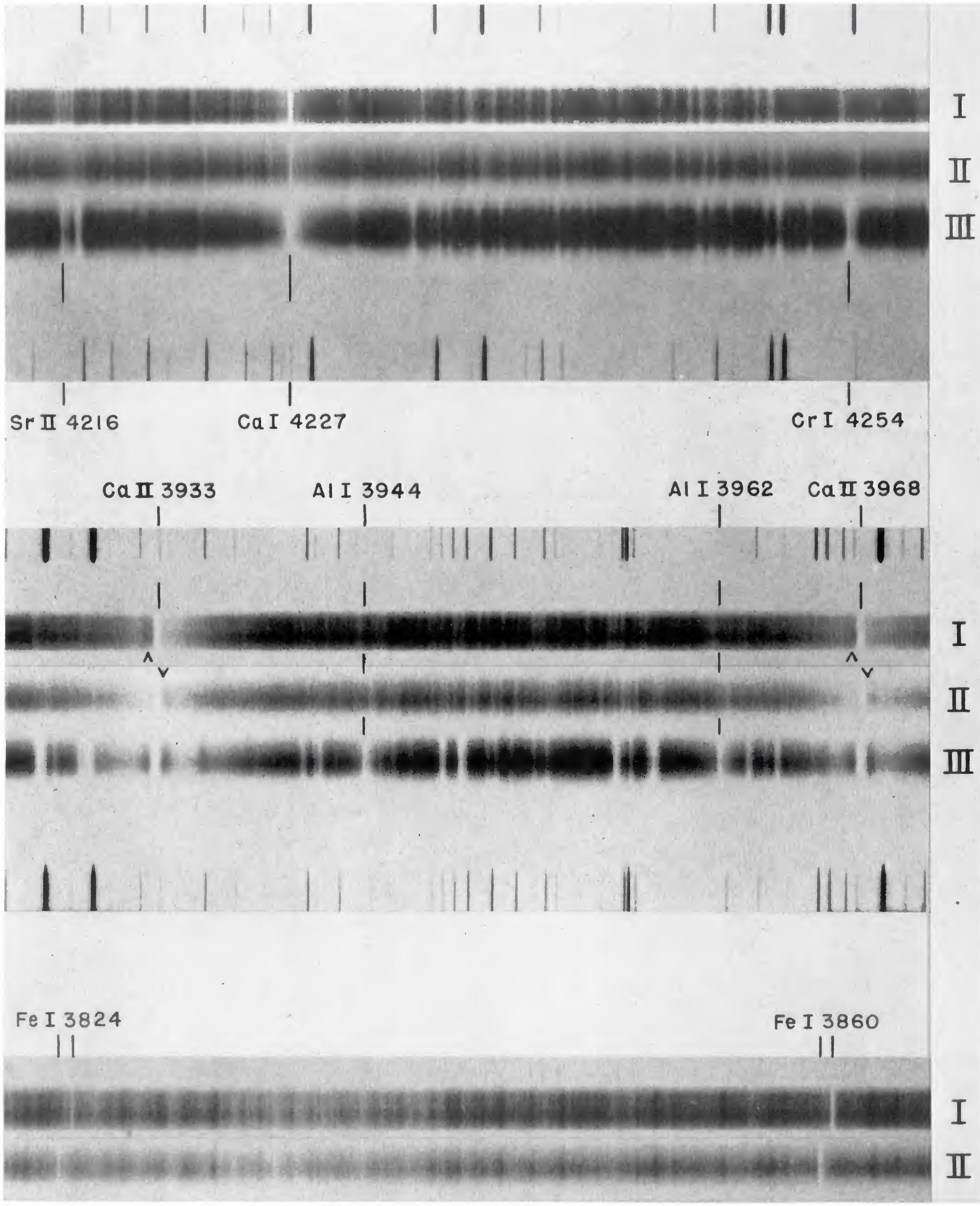

FIG. 1.—Coudé spectra of α Herculis. Strips *I* and *II* show α Herculis *B* (the G-type star) at two different phases in the 52-day orbit. Strips *III* show α Herculis *A* (the M-type star). In the lowermost pair the lines from the reversing layer of *B* have been lined up.

circumstellar lines arising from zero-volt levels in all other abundant atoms and ions (see Table 1). These abnormal absorption cores in the spectrum of the M star were discovered by Adams and Miss MacCormack (1935). They are a feature common to the spectra of all M stars that are more luminous than normal giants. They are well shown in the α Orionis spectrum that is reproduced in Plate VI of *Mount Wilson Contributions*, No. 638 (Adams 1941). Since they are invariably displaced shortward from the lines of the reversing layer, Adams and Miss MacCormack attributed them to an extended atmosphere in slow expansion. Spitzer (1939) supposed that gas ejected from the photosphere absorbs the lines during its ascent, then becomes ionized, and returns to the photosphere in an unobservable state. Comparison of strips *II* and *III* of Figure 1 shows that the circumstellar lines common to both stars indicate very nearly the same radial velocity. There is clearly every reason to believe that in the case of α Herculis the extended atmosphere of the M star actually envelops the G star. Although some of the absorption bands that dominate the visual spectrum of the M star probably arise from the ground state of the TiO molecule, none of the band lines exhibit shortward cores in the M spectrum, and none of them can be seen in the G spectrum. Apparently TiO is largely dissociated in the circumstellar envelope.

TABLE 1

CIRCUMSTELLAR LINES IN SPECTRA OF α HERCULIS *A* AND *B*

Line	Star	Line	Star	Line	Star
Na I 5889.95	M, G	Ca II 3933 66	M, G	Mn I 4034.49.	M
Na I 5895.92	M, G	Ca II 3968 47	M, G	Fe I 3679 92.	M, G
Al I 3944 01	M	Cr I 4254 35	M	Fe I 3719 94.	M, G
Al I 3961.52.	M	Cr I 4274 80	M	Fe I 3824.44.	M, G
K I 7664.91	M	Cr I 4289 72	M	Fe I 3859 91..	M, G
K I 7698.98	M	Mn I 4030 76	M	Sr II 4077.71.	M
Ca I 4226 73	M, G	Mn I 4033 07	M	Sr II 4215 52.	M

Figure 1 illustrates two other points of interest. In both stars the circumstellar lines at H and K are superposed on emission lines. The gas that produces these lines is apparently not to be identified with the circumstellar envelope, for the lines do not appear in the sky between the G and M stars—the space between strips *II* and *III*. Moreover, the emission lines of the G star move with this star in its orbital motion. In *I* the emission lines are displaced to the violet, with the lines from the reversing layer of the G star; in *II* both sets of lines are displaced to the red. A spectrogram of the G star by O. C. Wilson, that extends well into the ultraviolet, shows no trace of the Fe II emission lines noted by Herzberg (1948) in the region $\lambda\lambda$ 3150–3300 of the M-star spectrum. These lines, which occur in M stars over a wide range of luminosities (Wildt 1951), are probably not to be attributed to the circumstellar envelope.

Finally, the carets near *I* and *II* indicate the positions where the reversing-layer H and K lines should be centered, as determined by interpolation among the other reversing-layer lines. After due allowance is made for the disturbance produced by the emission lines and the circumstellar absorption lines, it seems that the underlying reversing-layer lines are not centered in the expected positions. Microphotometer tracings of these spectrograms indicate that this effect is certainly real. In all probability, it is to be attributed to the companion of the G star in its spectroscopic orbit. The light of this star is probably also responsible for similar disturbances in the profiles of a few other strong lines and for the general weakness of the reversing-layer lines in the spectrum of the G star. Although a careful search has been made for them, none of the lines of this spectrum have been otherwise identified.

II. THE RADIAL VELOCITIES

Dr. Adams and Dr. Merrill have very kindly put at my disposal their velocity measures of α Herculis A (the M-type star) from nine coudé spectrograms. Table 2 lists their results, together with two of my recent determinations. The velocities for the reversing layer depend on measures of between 12 and 189 subordinate lines per plate; none of these lines are disturbed by circumstellar cores. In the typical cases of Ce 2302 (29 lines) and Ce 9333 (18 lines), the probable errors are 0.2 km/sec. The measures indicate a significant velocity variation, with a total range of 4.9 km/sec. Such small velocity variations appear to be typical of the irregular M-type variables (Joy 1942; McLaughlin 1946). In a study of α Orionis, Sanford (1933) found some correlation between the long-period light and velocity variations, but little or no correlation between the shorter-period fluctuations of these quantities. Adams (1945, 1955) has discussed the velocity variation of the α Orionis reversing layer and has found an irregular variation with a total range of 8 km/sec.

TABLE 2

RADIAL VELOCITIES FOR α HERCULIS A (km/sec)

Ce Plate	Disp (A/mm)	JD	Obs.	Reversing Layer	Circumstellar									c_2
					Na I	Al I	K I	Ca I	Cr I	Mn I	Fe I	Ca II	Sr II	
619	5 6	2427146 00	WSA	−35 6	−48 2	..								
670	5 6	7264 81	WSA	−37 1	−46 5	..								
2061	2 9	9445 77	WSA	−34 8					−44 8				−46 1	−24 22 (10)
2302	2 9	9738 94	WSA	−32 2					−42 8	−42 5			−42 4	−23 71 (7)
3421	10 3	2431221 96	PWM	−32 5				−43 2						
3775	5 7	1578 83	WSA	−36 3			−48 0							
3778	5 7	1579 84	WSA	−35.2			−46 0							
3787	5 7	1593 96	WSA	−35 3			−44 7							
3845	2 9	1630 73	WSA	−35 2				−45 9	−46 8	−43 8	−43 0	−38 6	−46 6	−25 16 (5)
9318[2]	10 2	4919 98	AJD	−36 7				−49 3				−41 5		
9333	4 5	4937 85	AJD	−35 8		−43 9		−45 6	−46 7			−41 2	−44 6	
Mean				−35 16	−47 4	−43 9	−46 2	−46 0	−45 1	−44 3	−43 0	−40 8	−44 5	

Shortward-displaced absorption cores have been measured in the accessible resonance lines of all abundant atoms and ions in α Herculis A. These circumstellar lines are listed in Table 1. Inspection of Table 2 shows that these lines indicate constant radial velocity, within the errors of measurement. There are probably no significant differences from element to element, except in the case of Ca II. The mean radial velocity from all cores, excluding Ca II, is −45.0 km/sec, as compared with −35.2 km/sec for the reversing layer. The mean velocity from the H and K absorption cores is −40.8 km/sec. We shall see that a similar effect occurs in the velocities from the circumstellar lines of the G star, where H and K are discrepant with the other cores. The discordance can be attributed to the true interstellar H and K lines, as was first suggested by G. H. Herbig.

On three of Adams' spectrograms of the M star, he has measured at each of the strong resonance lines a weak absorption feature that lies longward from the circumstellar core. This component he designates c_2. It occurs also in many of the resonance lines of α Orionis, and he has discussed it at some length in his recent paper on this star (Adams 1955). The interpretation of this feature is still in some doubt, but it is probably only the red wing of the reversing-layer line, which is unocculted by the circumstellar line.

The velocities for α Herculis B (the G-type star) are given in Table 3. I am indebted to H. W. Babcock, J. L. Greenstein, G. Münch, and D. M. Popper for obtaining some of the spectrograms of this star. On each plate I have measured between 9 and 31 subordinate lines for the determination of the radial velocity of the reversing layer. The fiducial wave lengths were taken from the *Revised Rowland Catalogue* (St. John *et al.*

1928). The circumstellar lines that I have measured are among those designated "G" in Table 1; for these lines, the fiducial wave lengths were taken from the *Revised Multiplet Table* (Moore 1945). Within the errors of measurement, these lines are stationary, and Ca I and Fe I indicate the same velocity, namely, −39.5 km/sec, as compared with −38.4 km/sec for the γ-velocity of the G star. H and K are discrepant and in the same sense as in the M-star spectrum.

The elements of the spectroscopic orbit of the G star are given in Table 4. Sanford's elements are based on his (1921) least-squares solution from 28 prism spectrograms with a dispersion of 37 A/mm at Hγ. The adopted elements have been derived from a least-squares adjustment of the preliminary elements indicated to the velocities of Table 3. I have used Sterne's (1941) Method 2, together with the least-squares algorithm of Kopal (1950). The velocity-curve is illustrated in Figure 2. The general agreement between Sanford's orbit and mine is good. The circumstellar component of Ca I 4227 can-

TABLE 3

RADIAL VELOCITIES FOR α HERCULIS *B* (km/sec)

Plate	Dispersion (A/mm)	JD 2430000+	*M*	Obs.	Reversing Layer	Circumstellar		
						Ca I	Fe I	Ca II
Ce 9199. ..	4 5	4849 01	338°1	AJD	−12.0±0 2	−41 7	. .	.
Pb 1506. .	4 5	4870 96	131.3	AJD	−72 7±0.4	−40.9	.. .	.. .
Ce 9273 .	10 2	4906 95	22 5	DMP	−36 1±0 4	−39 6	...	..
Ce 9307b .	10 2	4916 88	91.8	GM	−71 0±0 4	−37 0	... .	... .
Ce 9318[2]	10 2	4919.98	112 4	AJD	−75.1±0 4	−40.6	−36.8	−33.7
Ce 9333 .	4 5	4937 85	238 2	AJD	−18.7±0.2	−40 4	−40 3	−35.1
Pb 1684 .	4 5	4977 66	156 0	JLG	−64 1±0 2	−40.0	−40 4	−34 8
Ce 9373 .	4.5	4993 64	267 6	GM	− 5 8±0 3	−39 2	−40.2	−34 5
Pb 1771.	4 5	5027 65	144 9	AJD	−69.0±0 3	−39.9	. .	.
Pc 1803	13 5	5030 68	166 1	AJD	−56 6±1 3		.	. .
Pd 1832	17 8	5054 60	333 1	JLG	− 9 5±0 5	. ..	...	....
Pc 1845.	10 2	5056 62	347 1	AJD	−20 2±0 7	−39 1	−38.6	−35.2
Ce 9514 .	10 2	5060 62	15 1	HWB	−33 9±0 3	−37 4	.	.
Mean	.. .		.	.	.. ..	−39.6	−39.3	−34.7

TABLE 4

ORBITAL ELEMENTS FOR α HERCULIS *B*

Orbit	Sanford (1921)	Preliminary Deutsch (1955)	Adopted Deutsch (1955)
P (days) . .	51 590 ± 0 002	51 578	51 578 (adopted)
e . .	0.028 ± 0.026	0	0.0220±0.0022
ω. . . .	27°89 ±19°3	.. .. ..	67°5 ±5°7
K . .	29 64 ± 0 77	35 8	36 122 ±0.053
(T_0−JD 2400000)*..	22468 581 ± 3 030	34790 90	34791 026 ±0 012
γ (km/sec) . .	−37 2	−37 8	−38 427 ±0 036
$a \sin i \times 10^{-6}$ (km) .	21 027†	. . . .	25.618
f(*m*).	0 1395	. . ...	0 2583

* Sanford's T_0 is the epoch of periastron; Deutsch's T_0 is the epoch of the ascending node.
† There is a misprint in the value of this quantity in Sanford's paper.

not be resolved on Sanford's plates, but a recent examination of his measures by Sanford himself has shown that the residuals from this line are systematically affected.

A summary of the radial-velocity data is given in Table 5. In the following sections it will be shown that the difference between the mean reversing-layer velocities of components A and B can be understood in terms of the motion in the long-period, visual orbit and that the difference between the true circumstellar velocities is a projection effect depending simply on the geometry of the envelope.

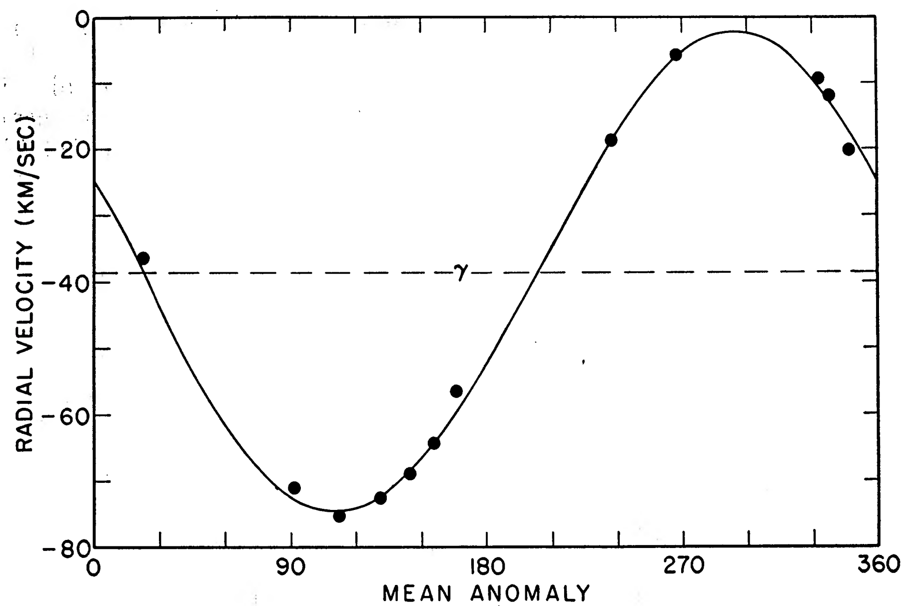

FIG. 2.—The velocity-curve of α Herculis B. The period is 51.6 days

TABLE 5

RADIAL-VELOCITY SUMMARY

		A	B
Star		A	B
Spectral type		M	G
Radial velocity (km/sec)	reversing layer	−35 2	−38 4(γ)
	circumstellar Ca II lines	−40 8	−34 7
	other circumstellar lines	−45 0	−39 5

III. DISTANCE, SPECTRAL TYPES, AND DIMENSIONS

The trigonometric parallax is given as negative (Jenkins 1952), and no really accurate spectroscopic parallax is known for either the M or the G star. In the case of the M star the chief uncertainty lies in the calibration of the luminosity classes among the giants and supergiants of very advanced type. In the case of the G star, there is doubt about both the classification and the apparent magnitude.

Table 6 summarizes the existing classifications and spectroscopic absolute magnitudes. The absolute magnitudes M_v given for A (the M-type star) have been adjusted, where necessary, to refer to the mean brightness of A, with $m_v = 3.5$ (Pickering 1908). Keenan's value of $M_v(A)$ has been found by extrapolating the Keenan-Morgan (1951) luminosity calibration from M3 II to M5 II. Wilson's $M_v(A)$ is obtained from his measures of the width $\Delta\lambda$ of the Ca II emission at K; a relation between M_v and $\Delta\lambda$ has been empirically established from observations of many stars of different types and absolute magnitudes (Wilson 1954). Wilson's $M_v(B)$ is obtained from my measures of $\Delta\lambda$ in the spectrum of B, together with his M_v versus $\Delta\lambda$ relation. The tabulated distances r are found from $M_v - m_v$, with $m_v(A) = 3.5$ and $m_v(B) = 5.6$. The second of these is not well determined, since it rests on measures (Pickering 1908; Wallenquist 1954) of $\Delta m = m(B) - m(A)$; and most of these were made without regard to the variability of A.

Another indication of distance, independent of those in Table 6, can be obtained from a consideration of the effect of the interstellar H and K lines on the measured velocities of the absorption cores in both A and B. We shall assume that the measured velocity from H and K in A, for example, is the weighted mean of the envelope velocity $V_E(A)$ and the normal interstellar gas velocity V_I: $V = (W_E V_E + W_I V_I)/(W_E + W_I)$. For the

TABLE 6

SPECTRAL TYPES AND ABSOLUTE MAGNITUDES

Star	Type	M_v	Distance (pc)	Reference
A. . .	M5	−2 0	125	Adams *et al.* (1935)
	M5	−1 8	115	Joy (1942)
	M5 II	−2 4	150	Keenan (unpub.)
	. . .	−1 5	100	Wilson (1954)
B	F8	2 1	55	Adams *et al.* (1935)
	F8p	. . . . .	. . .	Leonard (1923)
	G8 II+F	−2 3±1 0	380	Bidelman (letter)
	. . .	0 2	120	Wilson (1954)
	G0 II–III	−1	210	Deutsch

weights, we take the equivalent widths of the circumstellar and interstellar Ca II lines, respectively. According to Binnendijk's compilation (1952), at the co-ordinates of α Herculis the average strength of interstellar K is given by

$$W_I(\text{mA}) = 410\, r\ (\text{kpc})\,,$$

for distances less than 0.5 kpc. Therefore, if $W = (W_E + W_I)$, we find that

$$r\ (\text{kpc}) = \frac{W\,(\text{mA})}{410}\,\frac{V - V_E}{V_I - V_E}\,. \qquad (1)$$

An estimate of the core equivalent widths, W, has been made from spectra and intensity tracings of H and K in each star. For the position of α Herculis, we compute $V_I = -18.8$ km/sec. For A, then, we have $W = 600$ mA; and, from Table 4, $V = -40.8$ km/sec, $V_E = -45.0$ km/sec. Equation (1) then gives $r = 0.24$ kpc. Repeating the calculation for B, with $W = 380$ mA, $V = -34.7$ km/sec, and $V_E = -39.5$ km/sec, we find $r = 0.22$ kpc. Actually, the interstellar K line probably falls partly within the circumstellar line, so that the total equivalent width W is somewhat less than $(W_E + W_I)$.

In consideration of all these results, we adopt $r = 150$ pc as a basis for the subsequent discussion. The true distance may be appreciably larger than this, as Bidelman has found from the spectroscopic parallax of B. However, the argument from the strengths and velocities of the interstellar lines would seem to rule out any distance larger than, say, 250 pc.

The angular separation of the visual pair AB is 4".7. At the adopted distance of 150 pc, the projected linear separation of the two stars is therefore $\Delta = 700$ a.u. If the circumstellar envelope that surrounds A and B is spherical, with its center at A, its radius must then be at least $1.5 \times 10^5 R_{\odot}$.

Besides the distance and the radius of the envelope, we shall need to know certain other characteristics of the system in the discussion to follow. Table 7 gives the values that have been adopted. Like the distance of the system, nearly all the figures of Table 7 are appreciably uncertain. The spectral type of A is by Keenan (unpublished material). The type of B is my own, from a comparison with coudé plates of Morgan-Keenan standards. The type of C is inferred from the mass function of the system BC; it and the other properties of C will be discussed below. The apparent magnitudes of A and B have already been discussed; their absolute magnitudes have been found on the assumption that $r = 150$ pc. The absolute bolometric magnitudes of A and B follow from bolometric

TABLE 7

DIMENSIONS, ETC., IN THE SYSTEM OF α HERCULIS

	STAR		
	A	B	C
Spectral type	M5 II	G0 II–III	A3 V
Apparent visual magnitude (m_v)	3 5*	5 6	7 7
Absolute visual magnitude (M_v)	−2 4	−0 3	1 8
Absolute bolometric magnitude (M_B)	−5.9	−0.6	1.4
Effective temperature (T_e)(° K)	2700	5200	9100
Radius ($\odot$ = 1)	580	12	1 7
Log dilution factor (D)	−5 43	−8 80	−10 50
Log energy density [$Du_\lambda(T_e)$] at λ 2040 A	−7 4	−5 5	− 4 7
Mass $\mathfrak{M}$ ($\odot$ = 1)	15	4 1	2 5

* m_v for α Her A is the mean magnitude.

corrections by Kuiper (1938); the value for A represents a slight extrapolation. The effective temperatures are from Keenan and Morgan (1951); the value for A is again extrapolated.

The radii in solar units have been computed in the relation

$$\log R = 8.44 - 2 \log T_e - 0.2 M_B ,$$

which follows from the definition of T_e and the adopted values $T_e(\odot) = 5713°$, $M_B(\odot) = 4.62$. The interferometric determination by Pease (1931) yielded, for the angular diameter of A, $d = 0''.030$; since the star was relatively faint for this technique, the result was given as a "preliminary value." Our adopted values for the distance and radius indicate a value of $d = 0''.036$, in very good agreement with Pease's measurement. The radiometric observations of Pettit and Nicholson (1928), in conjunction with their water-cell absorption temperature, $T = 2400°$, gave, for the angular diameter, $d = 0''.065$. Their heat-index temperature was somewhat lower and yielded $d = 0''.090$. Both these radiometric determinations are affected by the TiO absorptions in the red, which produce large deviations from the Planck spectrum, for which the method was developed. Gabovits (1936) has attempted to overcome this difficulty by using an effective temperature determined from the difference between photographic and bolometric apparent magnitudes. For most of the M stars he has discussed, including α Herculis A, his method significantly reduces the discrepancy between Pettit and Nicholson's two temperatures. For α Herculis A, he is led to a temperature of 2390° (Gabovits and Öpik 1935). He also takes a brighter M_v than we have adopted and derives a radius of 1280$\odot$, or 2.2 times as large as we have found above.

The dilution factors D of Table 7 will be needed in the discussion of the ionization in the envelope; they are computed for a representative point in the envelope, where $D = (R*/2\Delta)^2$. The corresponding radiation densities follow from the respective Planck densities $u_\lambda(T_e)$. The masses of A and B are computed in the empirical mass-luminosity relation of Russell and Moore (1940),

$$\log \mathfrak{M} = -0.1048\,(M_B - 5.23)\,.$$

In order to approximate the properties of C, its mass has first been found from my mass function (Table 4) on the assumption that $\sin^3 i = 0.67$ and that $\mathfrak{M}(B) = 4.1$. Its bolometric magnitude then follows from the mass-luminosity law; and its absolute visual magnitude, temperature, and spectral type from the condition that it lie on the main sequence. Finally, the apparent magnitude is derived from its absolute visual magnitude by application of the adopted modulus of the system. These inferred characteristics of the third star in the system are particularly uncertain. They imply a difference $(C - B)$ of about 1 mag. in the photographic region, which is roughly compatible with the observed weakness of the lines in the spectrum of B and with the disturbances that have been noted in the wings of its strong lines.

It now remains to be shown that the observed difference between the mean reversing-layer velocity of the M star and the γ-velocity of the G star can be attributed to the motion in the visual orbit. Assuming that the close pair BC describes a circular orbit around A, with radius equal to the projected separation AB, $\Delta = 700$ a.u., and that the total mass of the system (Table 7) is $21.6\odot$, we find, for the orbital velocity, 5.2 km/sec. This is to be compared with the observed velocity difference (Table 5) of 3.2 km/sec. The difference can clearly be ascribed to projection effects and/or eccentricity of the long-period orbit. If the orbit were circular and in the plane of the sky, the position angle would change at the rate of $0^\circ\!.090$ year^{-1}. The observed rate of change, according to an unpublished compilation by Eggen, has just this value.

IV. SPECTROPHOTOMETRY OF THE CIRCUMSTELLAR LINES

From spectrophotometric tracings of the spectrum of the G star near Ca I 4227 on plates Ce 9373 and Pb 1684, the profile of the reversing-layer absorption line was reconstructed, and the contour of the circumstellar line was then referred to this reconstructed profile as the effective continuum. I find that the equivalent width, W, of the circumstellar λ 4227 is 175 mA. An approximate correction of the observed profile for the instrumental blurring function indicates that the circumstellar line has a true central absorption of about 0.90 and a Gaussian parameter $a = 110$ mA. Taking a as the true Doppler parameter b of the circumstellar line, I find the corresponding Doppler velocity v_0 to be 7.8 km/sec, corresponding to a kinetic temperature $T = 1.5 \times 10^5\,^\circ$ K. Entering Strömgren's (1948) curve of growth with $\log (W/b) = 0.20$, I then find $\tau_0 = 1.4$. The corresponding number of Ca I atoms over the disk of the G star is N_G (Ca I) $= 7.6 \times 10^{11}$ cm^{-2}.

The much greater strength of the circumstellar H and K lines and their extremely low central intensities have complicated their measurement. As an estimate of the strength of these lines, which is compatible with the tracings, I have assumed that the strength of each is the same as that of a rectangular profile with zero central intensity and width equal to the value found at the measuring engine. The result is 380 mA for each line. On the assumption (see Sec. II) that the interstellar component contributes 60 mA in each case, I adopt $W = 320$ mA for the mean strength of H and K of the envelope alone. Using the same Doppler velocity as before, 7.8 km/sec, I obtain $b = 103$ mA, $\log (W/b) = 0.49$, and, from Strömgren's curve of growth, $\tau_0 = 7.1$, corresponding to N_G (Ca II) $= 2.1 \times 10^{13}$ cm^{-2}. In the mean, therefore, N_G (Ca II)/N_G (Ca I) $= 28$.

Before going on to discuss the ionization equilibrium, we shall have to convert the surface densities just found into volume densities. To accomplish this conversion, we shall suppose that the G star lies in the plane of the sky, as illustrated in Figure 3, and that the radius of the envelope $R_E = \sqrt{2}\Delta$. This represents a very conservative estimate of the actual radius of the envelope, for it moves outward through the distance R_E in only 5000 years. The line of sight to the G star then passes through a thickness Δ of the envelope. The observations suggest that there is no appreciable velocity gradient over the envelope; the continuity equation for the ejected gas therefore requires that the density vary as $1/r^2$, where r is the distance from the M star. We shall see in Section VI that the gas in the envelope is very probably escaping from the system. If the ejection has occurred continuously over an interval of the order of a million years, the density and velocity of the outermost parts must merge continuously with these properties of the interstellar medium. If the ejection period has been appreciably shorter than a million years, the density may still follow an inverse-square law out to a distance at least an order of magnitude greater than our assumed R_E.

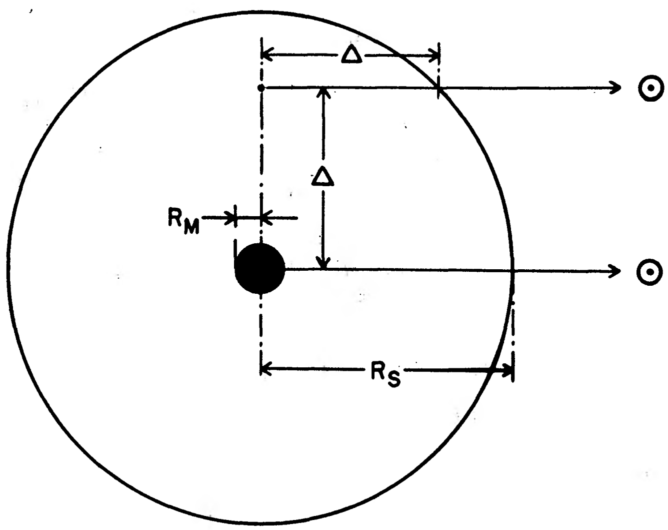

FIG. 3.—The assumed configuration of the visual binary and its common envelope. The stars are not drawn to scale.

We shall also assume that the ionization does not vary appreciably over most of the envelope. The mean volume density of atoms or ions along the line of sight to the G star will be $\bar{n}_G = N_G/\Delta$. By integration, we readily find that the density law within the envelope may then be put in the form

$$n(r) = 0.64\, \bar{n}_G \left(\frac{R_E}{r}\right)^2 . \tag{2}$$

Along the line of sight to the G star, the density falls by only a factor of 2 from the G star to the rim of the envelope.

Since $\bar{n}_G$ is known for Ca I and Ca II, integration of equation (2) will enable us to find the number N_M of each of these per cm^{-2} of the M star. We find

$$N_M = 0.90 \left(\frac{R_E}{R_M}\right) N_G ; \tag{3}$$

i.e., the surface density at the M star should be greater than that at the G star by nearly the ratio of the envelope radius to the M-star radius. Through the curve of growth, we should now be able to predict the strengths of circumstellar lines in the spectrum of the M star and to compare these predictions with observed line strengths. However, none of the existing plates are suitable for the photometry of the circumstellar lines in the M-star spectrum, for these lines are closely blended with the reversing-layer lines. A qualitative test is provided by the lines of Sr II. If we assume that the ionization is about the same for strontium as for calcium and that the abundance ratio Ca/Sr has its normal value of 2.2×10^3 (Aller 1953), then N_G (Sr II) $= 1.0 \times 10^{10}$ cm^{-2}. With f assumed to be about $\frac{1}{2}$, this leads to $\tau_0 = 3 \times 10^{-3}$ for λ 4216 in the circumstellar spectrum of the G star. From the curve of growth, we find the corresponding line strength to be only 0.6 mA. That Sr II 4216 is not observed in the circumstellar spectrum of the G star is therefore to be expected. On the other hand, from equation (2) it follows that N_M(Sr II) $= 3 \times 10^{12}$ cm^{-2}; hence $\tau_0 = 1$, and, from Strömgren's curve of growth, $W = 140$ mA for λ 4216. This is roughly twice the strength of the line observed in the circumstellar spectrum of the M star (see Fig. 1), but this order of agreement is all that can be required from so suppositious an argument.

Another quantity for which we shall have use in our later discussion is the total number of calcium atoms or ions in the envelope. By integration of equation (2) over the volume of the envelope, we find that this is given by

$$N_E = 8R_E^3 \bar{n}_G . \tag{4}$$

We may also compute the mean velocity of expansion along the line of sight to the G star. From Figure 3 we see that this is

$$\bar{v}_G = \frac{v\int_0^\Delta n(r)\sin\theta\, ds}{\int_0^\Delta n(r)\, ds},$$

where ds is the element of length along the line of sight; θ is the angle between the radius vector to ds and the plane of the sky; and v is the whole velocity of expansion, which we have taken to be constant throughout the envelope. We find, then, that $\bar{v}_G = 0.38\, v$. Taking $v = 9.8$ km/sec from the data of Table 5, we compute that $\bar{v}_G = 3.7$ km/sec. But Table 5 indicates that $\bar{v}_G = 39.5 - 35.2 = 4.3$ km/sec. The agreement is again as good as can be expected.

Our results for N_M, N_E, and $\bar{v}_G$ depend, of course, on the assumptions we have made about the geometrical configuration and the density gradient within the envelope. It will be found, however, that no other model compatible with the observations will lead to values of N_M and $\bar{v}_G$ that differ in order of magnitude. The quantity N_E can hardly be much smaller than we have concluded here; if we have underestimated R_E, it may be appreciably larger.

V. THE IONIZATION IN THE ENVELOPE

The ionization of Ca I in the envelope will depend on the radiation density u_λ in its ionization continuum, $\lambda < 2040$ A. We have already seen (Table 7) that if star C is really an A star, it dominates the radiation field in this spectral range. For the equation of dissociative equilibrium, we shall therefore use the approximate form

$$\log \frac{\bar{n}_G(\text{Ca II})\,\bar{n}_G(e)}{\bar{n}_G(\text{Ca I})} = -\chi_0 \frac{5040}{T_i} + 1.5 \log T_i + \log \frac{2g_1}{g_0} + 0.5 \log \frac{T_e}{T_i} + 15.38 + \log D\,, \tag{5}$$

where the radiation temperature, T_i, and the geometrical dilution factor, D, have the values of T_C and W_C, respectively, of Table 7. In this form of the equation, no account is taken of the physical dilution by absorption in lower layers; also neglected are the effects of recombinations on excited levels. To justify the neglect of physical dilution, we note that the absorption coefficient k_ν, as derived by Bates and Massey (1941), gives, for the total optical thickness of the envelope in the Ca I continuum, $t = N_M(\text{Ca I})\, k_\nu = 6 \times 10^{-3}$ along the line of sight to the M star. The other abundant elements—sodium, aluminum, and potassium—with first ionization potentials less than 6.09 volts will also contribute to t, but the four elements together will produce an optical thickness $t < 1$. The recombinations on higher levels have the effect of reducing D below the geometrical value; but Strömgren has shown that in representative cases the reduction factor lies between 0.5 and 0.1 for the Ca I-Ca II equilibrium.

We shall first assume that the envelope is an H II region, with the electron temperature $T_\epsilon = 10000°$. Equation (5) then yields $\bar{n}_G(e) = 3 \times 10^6$ cm^{-3}. In the equation governing the next stage of ionization, the effects of physical dilution and recombinations on excited levels may no longer be negligible. By neglecting these effects, however, we are able to find the upper limit of log $[\bar{n}_G(\text{Ca III})/\bar{n}_G(\text{Ca II})]$ as -2.3. It follows, then, that most of the circumstellar calcium resides in the singly ionized state and that along the line of sight to the G star the average space density of all calcium is very nearly $\bar{n}_G(\text{Ca II})$, or 2×10^{-3} cm^{-3}.

However, if the circumstellar cloud is an H II region, as assumed above, then the density of electrons must very nearly equal the density of hydrogen. Assuming a normal abundance ratio Ca/H of 3×10^{-6} (Aller 1953), it follows that, corresponding to the derived value of $\bar{n}_G(e)$, we should have $\bar{n}_G(\text{Ca}) = 10$ cm^{-3}. But this is 5000 times greater than the density we have inferred from the observed strength of the calcium lines. To remove this discrepancy, it would be necessary to conclude that the circumstellar gas is condensed into clouds that occupy only 1/5000 of the total volume of the envelope.

It now remains to be seen whether the A star is capable of maintaining the ionization of hydrogen throughout the circumstellar mass. If the ultraviolet radiation were that of a black body at 10000°, the radius of the H II region in a medium of constant number density n(H) of hydrogen would be

$$S_0 = 0.5\, [n\,(\text{H})]^{-2/3}\,,$$

where S_0 is in parsecs and n(H) is in cm^{-3} (Strömgren 1939). In a cloudy medium with packing fraction a and density n(H) within the clouds, the radius would clearly, then, become

$$S_0 = 0.5\, a^{-1/3}\, [n\,(\text{H})]^{-2/3}\,.$$

Taking $a = 1/5000$, and $n(\text{H}) = \bar{n}_G(e) = 3 \times 10^6$, we find that $S_0 = 4 \times 10^{-4}$ pc. But this is less than one-tenth the minimum radius of the envelope. Moreover, the calculated radius is already an overestimate for the size of the Strömgren sphere around star C, for no account has been taken of the depletion of the Lyman continuum by the stellar atmosphere. In addition, star C is probably later than A0, and S_0 decreases rapidly with advancing type. It follows that star C can probably not maintain the ionization of hydrogen throughout the shell, as was first assumed.

It becomes necessary, therefore, to explore the possibility that hydrogen is largely neutral in the envelope, with the electrons supplied by the metals. If we adopt, for the electron temperature, $T_\epsilon = 2500°$, then we find that $\bar{n}_G(e) = 2 \times 10^6$ cm^{-3}. Assuming that the metals are nearly all once ionized and that the ratio of metals-to hydrogen has its normal value of about 1/6000, we conclude that $\bar{n}_G(\text{H}) \simeq 1 \times 10^{10}$ cm^{-3}. For a normal abundance of Ca, this implies $\bar{n}_G(\text{Ca}) \simeq 3 \times 10^4$ cm^{-3}. Comparing this with the value derived for a uniform distribution of gas, we find a packing fraction of the order of 7×10^{-8}. The envelope must then be composed of condensations separated by distances that are about 240 times their linear dimensions.

To find whether the radiation from star C is able to maintain the ionization of the metals throughout the envelope, as has now been assumed, we must still compute the physical dilution of the ionizing radiation. Following Wilson and Abt (1954), we adopt 7.5 volts as a first ionization potential representative of the abundant metals. Our approximation to the ionization equation then yields $n_1/(n_0 + n_1) \simeq \frac{5}{6}$. To discover whether this value is appreciably diminished by physical dilution, we compute the optical thickness, t, of the envelope on the assumption that our representative metal is 50 times as abundant as calcium and has the same continuous absorption coefficient. We find that $t = 1.4$. Physical dilution must therefore be appreciable only at distances from the A star of the order of R_E. Over most of the envelope the star C is, indeed, able to maintain most of the metals in the singly ionized state. The ionization is not so high, however, as to produce any difficulty in accounting for the neutral lines observed.

In summary, our calculation of the ionization indicates that the envelope is an H I region in which the bulk of the gas is condensed into clouds that fill only about one part in 10^7 of the volume. Since the H and K lines are saturated and are involved to an unknown extent with the true interstellar lines, the abundance determination for Ca II is subject to relatively large errors. The ionization calculation is further impaired by the uncertainty in the temperature of the ionizing radiation from star C. However, it seems

TABLE 8

STARS SHOWING CIRCUMSTELLAR ABSORPTION LINES

Star	Variability	Type	Star	Variability	Type	Star	Variability	Type
R And	L. P.	Se	χ Cyg	L. P.	Mpe	R Lyr	S. R.	M5
ϵ Aur	E.	F0 I*ap*	6 Gem	. . . .	M1 I*a*	α Ori	S. R.	M2 I*b*
R Cas	L. P.	M7e	η Gem	S R.	M2 III	ϵ Peg	.	K2 I*b*
RW Cep	I.	M0: I*a*-0	g Her	S. R.	M6 III:	ρ Per	S. R.	M3 III
VV Cep	E.	M1p0	α Her *A*	I.	M5 II	19 Psc		N0
μ Cep	S. R.	M2 I*a*	R Hya	L. P.	M6e	α Sco		M1: I*b*
o Cet	L. P.	M6e	R Leo	L. P.	M8e			

impossible to escape the conclusion that the ionization equilibrium indicates a density far higher than the average density along the line of sight; this discordance will become still greater if, as is possible, we have underestimated the radius of the envelope. The condensations in the circumstellar envelope of α Herculis are probably related to those that have already been noted in the extended chromospheres of ζ Aurigae (Wilson and Abt 1954), 31 Cygni (McLaughlin 1952; McKellar *et al.* 1952), and VV Cephei (McLaughlin 1951). As Wilson and Abt have pointed out, it is not at all understood how such condensations can endure.

VI. THE CIRCUMSTELLAR ENVELOPES OF LATE-TYPE SUPERGIANTS

Violet-displaced absorption cores, like those discovered by Adams and MacCormack in α Herculis A and several other M-type supergiants, have now been found on coudé spectrograms of all the stars listed in Table 8. In most cases these lines are sufficiently weak to require a dispersion of at least 10 A/mm for their detection. There is a strong presumption that these circumstellar lines occur in most or all late-type stars that are more luminous than ordinary giants. Very high luminosity is apparently not a necessary condition for the occurrence of these lines. Thus they appear to be present in the star β Pegasi, which Keenan classifies M2 + II–III. In this star the cores are not fully resolved, but Miss Davis (1947) has noted that the Na I D lines are asymmetrical at a dispersion of 5.7 A/mm. These lines and the resonance lines of K I give significant negative velocity residuals. The absorption cores in H and K give a velocity residual of -12.5 km/sec. This effect in β Pegasi is apparently typical for late-type giants. Thus Joy and

Wilson (1949) write of the absorption cores seen superposed on the Ca II emission lines in many stars: "... K_3 does not appear in dwarfs or subgiants but is readily seen in practically all giants and is wide and strong in supergiants. The greater intensity of the red components of emission as compared with the violet components in most of the stars indicates an outward motion of the high absorbing layer."

Another manifestation of the expanding envelopes around late-type supergiants is the gradient of radial velocity with respect to excitation potential that has been noted in a number of investigations. For the Fe I lines in α Orionis, Sanford and Adams (1930) found $\Delta RV/\Delta EP = +0.35$ km/sec volt; in ρ Cassiopeiae, Greenstein (1948) found $\Delta RV/\Delta EP = +3$ km/sec volt. This velocity gradient is especially well known in the long-period variables (Adams 1941; Merrill 1945, 1946*a* and *b*, 1947*a* and *b;* Joy 1954). In a few long-period variables the strongest circumstellar lines are actually resolved on existing coudé spectrograms; in Plate XXV of Merrill's 1945 paper, the cores can be barely discerned in the resonance lines of Mn I in the spectra of R Aquilae and R Cassiopeiae. In the postmaximum spectrum of χ Cygni, these lines appear in emission, and, as may seen in Figure 2 of Merrill's paper on this star (1947*b*), they may be split by overlying absorption cores. The behavior of the circumstellar absorption lines in long-period variables should prove a rewarding problem for future investigation with the highest possible dispersion. If, as in α Herculis, these lines originate in an extremely distended envelope, one would expect them to change little or none during the cycle of stellar variation.

In the case of α Herculis we have been led to a model of the envelope in which the gas is condensed into clouds that fill only the fraction 7×10^{-8} of the volume of the envelope. Within each cloud, most of the metal atoms are singly ionized by the radiation of the A-type star, and the corresponding electron density is of the order of $n_e = 2 \times 10^6$ cm^{-3}. If the abundance ratio of hydrogen to metals has its normal value, the smoothed average density of atoms along the line of sight is $\bar{n}_G(\mathrm{H}) = 600$ cm^{-3}; the hydrogen is mainly neutral. Using equation (4), we find for the total number of atoms in the envelope $N_E = 1.3 \times 10^{52}$. The mass of the envelope is $\mathfrak{M}_E = 2 \times 10^{28}\ g = 1 \times 10^{-5}\ \mathfrak{M}_\odot$. Since we may have seriously underestimated R_E, this is in the nature of a lower limit.

The observations indicate (see Table 5) that at distances of the order of R_E, this matter is streaming outward with a velocity of 10 km/sec relative to the M star. Taking account of the three stellar masses that comprise the system, we find, for the escape velocity at the rim of the envelope, $U(R_E) = 6.3$ km/sec. It appears, then, that most of the matter in the envelope is being lost to the system. Our model leads to a rate of mass loss equal to $3 \times 10^{-8}\ \mathfrak{M}_\odot$/year; if the envelope is larger than we have assumed, this rate may be appreciably larger. The rate of mass loss is therefore rapid enough to be comparable with the time scale for evolution of a massive star (Tayler 1955).

The dynamics of this mass loss are highly obscure. The velocity of escape from the surface of the M star is 100 km/sec. As in all M-type supergiants, the reversing-layer lines are appreciably broadened as though by turbulence. In his study of α Herculis *A*, Spitzer (1939) found that the profiles of weak lines approximate a dispersion law, with a half-width at half-intensity corresponding to a turbulent velocity of about 4 km/sec. Spitzer suggests that "the atoms with the largest outward velocity would naturally rise highest in the atmosphere." One could imagine that prominence-like masses are ejected from the reversing layer with a velocity spectrum approximating a dispersion law. If each of these masses were able to retain its identity and to move like a free particle under the gravity of the M star, then at any height above the reversing layer one would find a slight predominance of ascending masses because of the escape of the masses that were ejected with speeds exceeding the escape velocity. Since the observed line represents an integral of the velocity spectrum along the radius vector, one could then expect to find the line to show an asymmetry in the observed sense. A quantitative evaluation of this effect (see the appendix) indicates, however, that it is far too small to account for the observed asymmetries. The asymmetry has also been found to be negligible for the case of a Maxwellian distribution of ejection velocities with a dispersion of 4 km/sec.

In the most favorable case the escape velocity at the surface of the M star is lowered only 0.5 km/sec by the gravitational field of the G star and its close companion. Even to move from the M star to the G star, a particle at the surface of the M star must have a velocity greater than 99 km/sec. There is no evidence whatever for the ejection of gas from the M star at such high velocities. The observations rather suggest that gas is ejected at a velocity of the order of only 10 km/sec and then streams outward under an effective gravity that is virtually zero. Under dynamical gravity alone, such a velocity of ejection would carry a particle to a height of only 1 per cent of R_A, or to a distance of less than $10^{-4}\Delta$. It is obvious, then, that the enormous distention of the envelope cannot be understood in terms of particle dynamics under gravity alone. For the case of a gas with a density law that is a smooth function of r, it can be shown that there exist non-adiabatic compressible hydrodynamic flows having the character of a slowly accelerated expansion starting from the reversing layer with a very small velocity. It is not clear, however, how these ideas can be adapted to the cloudy medium that is indicated by the spectroscopic observations.

Spitzer has noted the importance of radiation pressure on atoms and ions that have resonance lines in the brighter parts of the continuous spectrum. For an unexcited ion of Ca II exposed to the residual radiation in the H and K lines of the M-star reversing layer, the force due to radiation pressure exceeds that due to gravity by some 200 times. For an unexcited atom of K I, which has its ultimate lines in the relatively bright near infrared, this ratio of forces increases to more than 10^5. Nevertheless, the forces due to radiation pressure cannot support the whole envelope if the hydrogen-to-metal ratio has its normal value. In this case the integrated gravitational force on the whole envelope is readily computed to be $G\mathfrak{M}_A\mathfrak{M}_E/R_AR_E = 5 \times 10^{24}$ dynes. If we suppose that the envelope is optically thick over a range $\Delta\lambda = 0.3$ A at H and over a similar range at K and that the scattered light is all returned to the reversing layer, then we find the integrated force of radiation pressure on the Ca II in the envelope to be $(32\pi^2/3c)R_A^2B_\lambda\Delta\lambda = 4 \times 10^{20}$ dynes. In this calculation I have computed the Planck intensity B_λ for the effective temperature of the M star; no account has been taken of the depletion of B_λ by the M-star reversing layer or of the radiation from the other two stars in the system. Resonance scattering by other abundant atoms and ions—some of which have ultimate lines in the red, where B_λ is large—will increase the net force of radiation pressure at most by a factor of the order of 10^2. It follows, then, that the gravitational force on the envelope exceeds that of ordinary radiation pressure by a factor of the order of 10^2.

We must suppose either that momentum is transferred to the envelope by some unknown process or that the envelope is deficient in hydrogen and therefore much less massive than we have thought. In support of the first alternative, we note that Babcock (1953) has found evidence for large-scale magnetic fields in the reversing layers of four luminous M stars. All these objects exhibit forbidden lines in emission and therefore must be considered to be enveloped in circumstellar nebulae. The spectrum of α Herculis *A* has not been observed for the Zeeman effect, but one may surmise that it and all other supergiant M stars may support intensely magnetic areas in their reversing layers. If this is so, magnetic forces may play a dominant role in the dynamics of the circumstellar envelopes, as indeed they are known to do in the dynamics of solar prominences.

In connection with the other alternative, that the envelope consists mainly of metals subject to radiation pressure, it is very hard to understand how the metals could be separated from the much more abundant lighter elements. Using the theory developed by Unsöld (1938), we compute that, at a density of 1×10^{10} hydrogen atoms/cm^3, the net outward acceleration of Ca II ions would, in the steady state, maintain a differential velocity of only 0.5 km/sec relative to hydrogen. On the other hand, the outward acceleration of K I atoms would maintain a differential velocity of nearly 10 km/sec, which agrees with the observations. Moreover, Struve (1947) has given evidence for a metal-enriched nebula around α Scorpii that may closely resemble the circumstellar envelope of α Herculis.

The star α Scorpii *A* is an M1 supergiant; its visual companion, at a distance of 3″, is of type B4 V (Stone and Struve 1954*a*). The nebula that Struve finds enveloping the B star has a diameter of 5″; at a distance of 185 pc (Stone and Struve 1954*b*), its radius is 460 a.u. Struve's observations appear not to rule out the possibility that the B star excites emission lines in part of a larger envelope that is centered at the M star. If this is the case, the over-all radius of the envelope would be 1000 a.u.

That α Scorpii *A* exhibits the absorption cores typical of M-type supergiants was discovered by Adams and MacCormack (1935). A recent coudé plate of α Scorpii B, at a dispersion of 4.5 A/mm, shows that these circumstellar absorption lines are not present in that spectrum. Probably the absence of these lines is to be attributed simply to a relatively high degree of ionization in the part of the envelope seen projected on the B star. If the density, composition, and packing fraction of the α Scorpii envelope were comparable with their values in our H II model for the envelope of α Herculis, we should conclude that the α Scorpii envelope would be an H II region throughout. Its emission measure, however, would be of the order of only 600, so that we should not expect to observe the Balmer lines in emission. In fact, they are not observed; but emission lines of Fe II and [Fe II] are observed. In comparison with ordinary nebulae, therefore, the α Scorpii envelope would seem to be metal-enriched.

Struve (1947) has suggested that the α Scorpii emission nebula may result from the volatilization by the B star of metal-enriched solid particles. These smoke particles are thought to form a cloud roughly analogous to the zodiacal light and/or comet swarm of the solar system. Such a smoke cloud is not likely to represent stellar ejecta, however, but rather a residue of prestellar material from which the more volatile elements have escaped at an early stage of stellar evolution. It would appear very difficult to account for such a separation of elements on the short time scale of the observed ejection process. For the absorption-line envelope of α Scorpii, like that of other late-type supergiants, is expanding so rapidly that it can persist only a few thousand years unless it is continuously replenished from the material of the star.

The system of α Scorpii is probably representative of M-type supergiants that are attended by B-type companions. In most of the other binaries of this kind, the existence of the companion is inferred from its contribution to the ultraviolet spectrum. Examples are three of the stars observed by Babcock: WY Geminorum, HD 60414, 5, and VV Cephei. Other members of the group are W Cephei and HD 203338, 9 (Struve and Swings 1940*a*). The binary long-period variables Mira and R Aquarii are probably similar systems. All these objects exhibit emission lines of [Fe II]. Struve and Swings (1940*b*) write: "The normal appearance of the B spectrum of α Scorpii and of other members of this group suggests that the emission lines originate not in the reversing layers of these stars but in nebulosities whose presence is somehow connected with the existence of a cool supergiant in the system."

The common spectroscopic peculiarities of all late-type supergiants lead us now to believe that each may be surrounded by an escaping envelope of radius approximating 1000 a.u. When the envelope is not excited by the light of an early-type companion, it impresses upon the spectrum of the cool star the violet-displaced absorption cores originally discovered by Adams and MacCormack. If the system contains an early-type star, it excites the envelope to produce the nebula that is observed in such cases.

The most critical problem that is posed by these escaping circumstellar envelopes concerns their hydrogen-to-metal ratio. If this is much lower than normal, we may conceivably interpret the ejection process as a radiation-pressure phenomenon acting selectively on the metals. In this case the rate of mass loss is too small to be of evolutionary significance. But if, as seems more likely, unknown forces are acting to eject all elements together, then the rate of mass loss may be of fundamental importance in the evolution of late-type supergiants. On the basis of present theory, it seems improbable that there exist any equilibrium configurations for a massive star that has reached the limiting shell-source configuration and has burned essentially all its hydrogen. The possibility suggests itself that such a star may then shed most of its mass, so that the remnant could come to equilibrium as a white dwarf. On this picture, it may be possible to understand a system like Sirius, for example, in which the less massive star is the degenerate one and presumably the most highly evolved. Sirius *B* would then be the remnant of a star originally much more massive than Sirius *A*; it has evolved more rapidly and has finally shed most of its mass as a late-type supergiant before lapsing into its present configuration.

Dr. O. C. Wilson and Dr. W. S. Adams have read the manuscript of this paper, and both have offered helpful comments on it. It is a pleasure to record my indebtedness for their kind assistance.

APPENDIX

Particles are ejected radially from the surface of the M star, with radius r_0, with the distribution function $\phi(r_0, v_0)$ of initial speeds $v_0 > 0$. The distribution function for the returning particles will be given by the relations

$$\phi(r_0, -v_0) = \phi(r_0, v_0), \qquad (-u_0 < -v_0 < 0),$$

$$= 0, \qquad (-v < -u_0),$$

where u_0 is the velocity of escape at r_0. Consider the particles outbound at r, with speeds in dv. The rate at which these cross the sphere of radius r is

$$4\pi r^2 v\phi(r, v)\, dv .$$

At r_0, these particles had outbound speeds v_0, given by

$$v^2 - v_0^2 = 2\,G\mathfrak{M}\left(\frac{1}{r} - \frac{1}{r_0}\right),$$

and the rate at which they crossed the sphere of radius r_0 is

$$4\pi r_0^2 \phi(r_0, v_0)\, dv_0 .$$

For continuity, these two rates must be equal (collisions are neglected). Since

$$v\,dv = v_0\, dv_0 ,$$

it follows that

$$\phi(r, v) = \left(\frac{r_0}{r}\right)^2 \phi(r_0, v_0), \tag{1a}$$

for $v > 0$. But, by the same argument, the inbound particles with speeds in dv_0 must cross the r_0 sphere at the same rate as they cross the r-sphere with speeds in dv. Hence the last equation is valid for all v.

If we average the velocity distribution along the radius vector, we obtain

$$f(v) = c\int_{r_0}^{\infty} \phi(r, v)\, dr ,$$

where c is a normalization constant. Using equation (1a), we then find, for the outbound particles,

$$f(v) = c\int_{r_0}^{\infty}\left(\frac{r_0}{r}\right)^2\phi(r_0, v_0)\,dr \qquad (v>0)\,,$$

and, for the inbound particles,

$$f(v) = c\int_{r_0}^{r_0(u_0/v)^2}\left(\frac{r_0}{r}\right)^2\phi(r_0, v_0)\,dr \qquad (-u_0<v<0)\,.$$

In the second integral the upper limit is the distance where $-v$ is the velocity of escape.

For the velocity distribution at r_0, we first take a truncated Maxwell law,

$$\phi(r_0, v_0) = \frac{1}{\sqrt{\pi}\,a}\exp\left[-\left(\frac{v_0}{a}\right)^2\right] \qquad (v_0>-u_0)\,.$$

Introducing this expression into the equation for $f(v)$, we find, after performing the integrations,

$$f(v) = \frac{c r_0}{\sqrt{\pi}\,a}\left(\frac{a}{u_0}\right)^2\exp\left[-\left(\frac{v}{a}\right)^2\right]\begin{cases}\left[1-\exp\left\{-\left(\frac{u_0}{a}\right)^2\right\}\right] & (v>0)\\ \left[1-\exp\left\{-\frac{(u_0^2-v^2)}{a^2}\right\}\right] & (-u_0<v<0)\,.\end{cases}$$

In the application to α Herculis, we must take $a \simeq 4$ km/sec, $u_0 \simeq 100$ km/sec. The term $e^{(v/a)^2}$ that produces the asymmetry in $f(v)$ is therefore negligibly small unless $v \simeq -u_0$.

We next consider a truncated dispersion law at r_0, of the form that Spitzer (1939) has obtained for the velocity distribution in the reversing layer,

$$\phi(r_0, v_0) = \frac{1/\pi a}{1+(v_0/a)^2} \qquad (v>-u_0)\,.$$

Performing the integrations to obtain $f(v)$, we then find

$$f(v) = c\log\begin{cases}\dfrac{1+(v/a)^2}{1+(v^2+u_0^2)/a^2} & (v>0)\\[2ex] \dfrac{1+(v/a)^2}{1+(u_0/a)^2} & (-u_0<v<0)\,.\end{cases}$$

Because the root-mean-square speed appropriate to the function $\phi(r_0, v_0)$ is infinite, these distributions are not amenable to normalization. For $|v| \ll u_0$, which is the case of interest, the asymmetry is again negligible.

This argument obviously retains its validity for the case where the ejection velocities are not purely radial, provided only that the distribution function for the ejection velocities is constant over the surface of the star.

REFERENCES

Adams, W. S. 1941, *Ap. J.*, **93**, 11; *Mt. W. Contr.*, No. 638.
———. 1945, *Pub. A.S.P.*, **57**, 95, 1945.
———. 1956, *Ap. J.*, **123**, 189.
Adams, W. S., Joy, A. H., Humason, M. L., and Brayton, A. M. 1935, *Ap. J.*, **81**, 187; *Mt. W. Contr.*, No. 511.
Adams, W. S., and MacCormack, E. 1935, *Ap. J.*, **81**, 119; *Mt. W. Contr.*, No. 505.
Aller, L. H. 1953. *Astrophysics* (New York: Ronald Press Co.), **1**, 327.
Babcock, H. W. 1953, *M.N.*, **113**, 357; *Mt. W. and P. Repr.*, No. 92.
Bates, D. R., and Massey, H. S. W. 1941, *Proc. R. Soc. London, A*, **177**, 329.
Binnendijk, L. 1952, *Ap. J.*, **115**, 428.

Davis, D. 1947, *Ap. J.*, **106**, 28; *Mt. W. Contr.*, No. 733.
Gabovits, J. 1936, *Tartu Pub.*, Vol. **28**, No. 5.
Gabovits, J., and Öpik, E. 1935, *Tartu Pub.*, Vol. **28**, No. 3.
Greenstein, J. L. 1948, *Ap. J.*, **108**, 78, 1948; *McDonald Contr* , No. 151.
Herzberg, G. 1948, *Ap. J.*, **107**, 94; *McDonald Contr.*, No. 143.
Jenkins, L. F. 1952, *General Catalogue of Stellar Parallaxes* (New Haven: Yale University Observatory).
Joy, A. H. 1942, *Ap. J.*, **96**, 344; *Mt. W. Contr.*, No. 668.
———. 1954, *Ap. J. Suppl.*, **1**, 39; *Mt. W. and P. Repr.*, No. 112.
Joy, A. H., and Wilson, R. E. 1949, *Ap. J.*, **109**, 231; *Mt. W. Contr.*, No. 758.
Keenan, P. C., and Morgan, W. W. 1951, *Astrophysics*, ed. J. A. Hynek (New York: McGraw-Hill Book Co.), p. 12.
Kopal, Z. 1950, *Harvard Obs. Mono.*, No. 8.
Kuiper, G. P. 1938, *Ap. J.*, **88**, 429.
Leonard, F. B. 1923, *Lick Obs. Bull.*, No. 343.
McKellar, A., Odgers, G. J., Aller, L. H., and McLaughlin, D. B. 1952, *Nature*, **169**, 990.
McLaughlin, D. B. 1946, *Ap. J.*, **103**, 35.
———. 1951, *ibid.*, **114**, 47.
Merrill, P. W. 1945, *Ap. J.*, **102**, 347; *Mt. W. Contr.*, No. 713.
———. 1946*a*, *Ap. J.*, **103**, 6; *Mt. W. Contr.*, No. 717.
———. 1946*b*, *Ap. J.*, **103**, 275; *Mt. W. Contr.*, No. 720.
———. 1947*a*, *Ap. J.*, **105**, 360; *Mt. W. Contr.*, No. 730.
———. 1947*b*, *Ap. J.*, **106**, 274; *Mt. W. Contr.* No. 735.
Moore, C. E. 1945, *Princeton Contr.*, No. 20.
Pease, F. G. 1931, *Erg. d. exakt. Naturwiss.*, **10**, 84; *Ann. Rept. Mt. W. Obs.* (1925).
Pettit, E., and Nicholson, S. B. 1928, *Ap. J.*, **68**, 279; *Mt. W. Contr.*, No. 369.
Pickering, E. C. 1908, *Harvard Ann.*, Vol. **50**.
Russell, H. N., and Moore, C. E. 1940, *The Masses of the Stars* (Chicago: University of Chicago Press), p. 112.
St. John, C. E., Moore, C. E., Ware, L. M., Adams, E. F., and Babcock, H. D. 1928, *Carnegie Inst. Washington Pub.*, No. 396.
Sanford, R. F. 1921, *Ap. J.*, **53**, 201; *Mt. W. Contr.*, No. 201.
———. 1933, *Ap. J.*, **77**, 110; *Mt. W. Contr.* No. 464.
Sanford, R. F., and Adams, W. S. 1930, *Pub. A.S.P.*, **42**, 255.
Spitzer, L., Jr. 1939, *Ap. J.*, **90**, 494; *Mt. W. Contr.*, No. 619.
Sterne, T. E. 1941, *Proc. Nat. Acad. Sci.*, **27**, 175.
Stone, S. N., and Struve, O. 1954*a*, *Pub. A.S.P.*, **66**, 191.
———. 1954*b*, *ibid.*, p. 257.
Strömgren, B. 1939, *Ap. J.*, **89**, 526.
———. 1948, *ibid.*, **108**, 242.
Struve, O. 1947, *Science*, **106**, 149.
Struve, O., and Swings, P. 1940*a*, *Ap. J.*, **91**, 546; *McDonald Contr.*, No. 22.
———. 1940*b*, *Ap. J.*, **92**, 316; *McDonald Contr.*, No. 26.
Tayler, R. J. 1955, *Ap. J.*, **120**, 332.
Unsöld, A. 1938, *Physik der Sternatmosphären* (Berlin: Julius Springer).
Wallenquist, B. 1954, *Uppsala Ann.*, Vol. **4**, No. 2.
Wildt, R. 1951, *A.J.*, **56**, 51.
Wilson, O. C. 1954, *Proc. Nat. Sci. Foundation Conf. on Stellar Atmospheres*, ed. M. H. Wrubel (Bloomington: Indiana University Press), p. 147.
Wilson, O. C., and Abt, H. A. 1954, *Ap. J. Suppl.*, Vol. **1**, No. 1; *Mt. W. and P. Repr.*, No. 108.