AXIAL ROTATION AND STELLAR EVOLUTION

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ABSTRACT

Some of the current ideas of stellar evolution are tested by using Arne Slettebak's homogeneous data on the rotational velocities of stars. The values of $V \sin i$ for stars not now on the main sequence are shown to be consistent with the idea that these stars were once main-sequence objects but have moved into the giant and supergiant region of the M_{bol}, log T_e plane by an evolutionary process. In particular, the data are entirely consistent with the evolutionary tracks computed by M. Schwarzschild and his collaborators.

I. SCOPE OF THE PROBLEM

According to current ideas of stellar evolution, stars now in the giant and supergiant region of the M_{bol}, log T_e diagram were once on the main sequence. They have moved to their present positions because of changes of internal structure due to a chemical in homogeneity in the deep interior. The details of this process have been studied theoretically by a number of workers (Oke and Schwarzschild 1952; Roy 1952; Sandage and Schwarzschild 1952; Taylor 1954). These investigations give the evolutionary tracks followed by idealized stars as they move from the main sequence.

There are several ways to test observationally the general content of these ideas of stellar evolution. The study of color-magnitude diagrams of star clusters is one powerful test method, and progress has recently been made in exploiting this approach (Johnson 1954; Miczaika 1954; Sandage 1954; Johnson and Sandage 1955). Another method, which is dynamical in character, is a comparison of the rotational velocities of main-sequence stars with those of giant and supergiant stars. The assumption of the conservation of angular momentum is made. With this approach, we ask whether the observed rotational velocities of all stars not now on the main sequence are consistent with the hypothesis that each star has moved from the main sequence to its present position along the tracks of evolution we wish to test.

This problem has recently been studied in a fundamental paper by Oke and Greenstein (1954). These authors analyze the rotational velocities of thirty-four stars. Their sample was very small and unfortunately excluded several stars which were known to have rapid rotation. This occurred because their plates were taken for another purpose. The conclusion of Oke and Greenstein was that the evolutionary picture was consistent with their data. However, owing to the smallness of their sample and to its statistical bias, questions as to the generality of their results arise.

Adequately to exploit this approach, rotational velocities for a large number of stars well distributed over the M_{bol}, log T_e plane must be available. Furthermore, the position of each star in this plane must be known, so that the proposed evolutionary tracks may be used to predict the point along the main sequence from which each star came. Fortunately for this problem, several large investigations of the rotational velocities of stars have recently been completed. Among these, the work of Huang (1953), Herbig and Spalding (1955), and particularly Slettebak (1949, 1954, 1955) and Slettebak and Howard (1955) should be mentioned. Slettebak's extensive investigations are especially valuable, since they cover the entire relevant range of spectral types from B3 to G0 and include the most accurate two-dimensional spectral classifications on the revised MK system (Johnson and Morgan 1953) that are now available for the stars studied. This is important, since it permits M_{bol} and log T_e to be determined for each star in question. Furthermore, Slettebak's data are statistically unbiased because of his method of selection. Consequently, Slettebak's data will be used in this paper to test the compatibility of the observed V sin i with the theoretical evolutionary tracks. The tracks chosen for the test are those computed by Sandage and Schwarzschild. It should be pointed out that the results are rather insensitive to the exact details of the assumed tracks. Consistency of the data with the evolutionary picture would not necessarily imply that the assumed evolutionary models are correct in detail but only that the mapping of stars from the main sequence into the giant region along tracks of nearly constant luminosity would seem to work in a general way.

The validity of using these particular tracks in the present case may be questioned, since the evolutionary models were computed for the case of no mixing between the exhausted core and the virgin envelope. If rotation strongly affects mixing, then tracks for unmixed models cannot be applied. Recent theoretical work (Mestel 1953) indicates, however, that rotation has an almost negligible effect on mixing, if the angular velocity is nearly constant throughout the star, even for stars near dynamical instability. Consequently, there appears to be no contradiction in applying unmixed models to the present study.

II. METHODS OF ANALYSIS

There are three possible methods of treating the data. These methods differ in the severity of the analysis and in the amount of data needed to make the analysis. The first method asks the question: Are there any giant and supergiant stars rotating *faster* than would be expected from the fastest-rotating star now on the main sequence in the relevant spectral range? This is the method of Oke and Greenstein. A second and more comprehensive method tests not just the maximum rotational velocity but the entire distribution function of V sin i. Each star in the giant and supergiant region is projected back onto the main sequence, using the evolutionary tracks. From the observed V sin i and the known ratio of the present radius to the radius which the star had on the main sequence, R/R_0, the main-sequence value of V sin i may be computed. The distribution function of these "recovered" values of V sin i can then be compared with that of stars now on the main sequence. The second method requires many more data than the first method but is a more severe test. The third method is a still more comprehensive test. If data were available for a sufficient number of giants and supergiants, the distribution function of V sin i at several points along any given evolutionary track could be found and compared with the distribution predicted from the theoretical tracks. The number of data required for this test is so great that it cannot be used in the present case. The application of these three methods requires, of course, the assumption that the distribution function of V sin i along the main sequence has not changed in the last 10^8–10^9 years.

The first and second methods are applied here to Slettebak's data. Each star of luminosity class other than V was plotted in the M_{bol}, log T_e plane by using Keenan and Morgan's (1951) calibration of the MK system as revised by Keenan (1955). A track of evolution was put through each star and carried back to the main sequence. The star's original spectral class and R/R_0 could then be determined. The rotational velocity of any star will decrease as R/R_0 increases, if angular momentum is conserved. The exact manner of the decrease will depend upon the distribution of the rotational velocities within the star. If the star rotates as a rigid body, the change of the moment of inertia, I, must be known as a function of R/R_0. For the theoretical models under consideration the moment of inertia may be found at each point of the evolutionary track, since the density distribution within the star is known at seven points corresponding to the seven computed models (Sandage and Schwarzschild 1952.) Hence I may be found as a function of R/R_0, and consequently the relation $(V \sin i)_0 = f(R/R_0)(V \sin i)_{\text{now}}$ obtained. The computations of the moments of inertia have been made by Oke and Greenstein (1955), and the results are taken from their paper. Following these authors, this case of rigid-body rotation is called "case A." At the other extreme we may consider the case where every infinitesimal spherical shell maintains its own angular momentum during the expansion. Here $(V \sin i)_0 = R/R_0(V \sin i)_{\text{now}}$. Oke and Greenstein call this "case B." The true situation may be expected to lie somewhere between case A and case B.

III. RESULTS

To facilitate the tests, the data were divided into the four spectral intervals B0–B9, A0–A3, A4–F0, and F1–G0. These particular intervals were chosen because the distribution of $V \sin i$ along the main sequence seemed to be similar within each range. Applica-

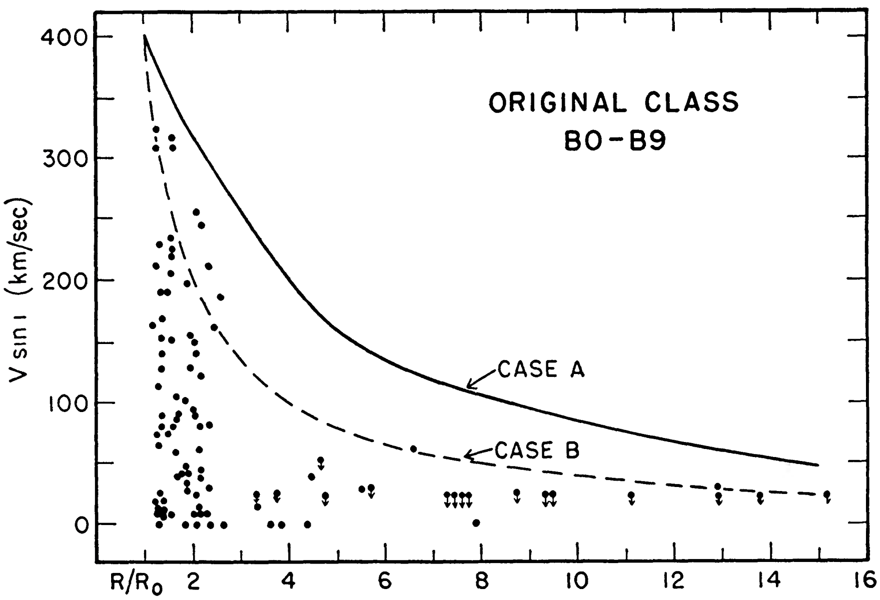

FIG 1.—Observed $V \sin i$ for stars not now of luminosity class V plotted against the ratio of their present radius to their radius on the main sequence. Stars whose original spectral class was B0–B9 are plotted The solid line, case A, shows how $V \sin i$ changes for a star rotating as a rigid body if it follows the Sandage-Schwarzschild evolutionary models. Case B is for stars in complete differential rotation

tion of the first test method requires that the maximum value of $V \sin i$ be known for stars now along the main sequence. Histograms of the $V \sin i$ distributions for stars in Slettebak's lists of present luminosity class V were constructed for each of the four intervals, and the maximum $V \sin i$ was determined (see Figs. 3 and 4). These maximum $V \sin i$ are for B0–B9, 400 km/sec; for A0–A3, 360 km/sec; for A4–F0, 280 km/sec; and for F1–G0, 120 km/sec. These values represent the upper limits for the stars studied. Although the lists are completely homogeneous and statistically unbiased, there are stars not included in the present material, such as ϕ Per (B2, $V \sin i \approx 500$ km/sec) and HR 2142 (B2 IV, V; $V \sin i = 450$ km/sec) with higher rotations than the maxima given. The adopted maxima refer only to those stars in the present material whose luminosity class is known. These values must be used to keep the sample statistically unbiased.

Figures 1 and 2 show how the $V \sin i$ of a star rotating at the maximum permissible value for $R/R_0 = 1$ will change with increasing R/R_0. Cases A and B are shown. The observed values of $V \sin i$ are plotted at the R/R_0 appropriate for each star considered.

If any star is rotating faster than is consistent with the evolutionary hypothesis, it would lie above the limiting line of case A in Figures 1 and 2. These figures show that, to within the error of measurement, no stars for which the necessary data are available in Slettebak's lists are rotating faster than would be expected from the evolutionary tracks. The six rapidly rotating stars excluded by Oke and Greenstein are included in the present study and have no higher $V \sin i$ values than would be expected. There are, however, several rapidly rotating stars contained in Slettebak's list of B2–A2 stars which are excluded from Figures 1 and 2. These are stars for which estimates of the luminosity

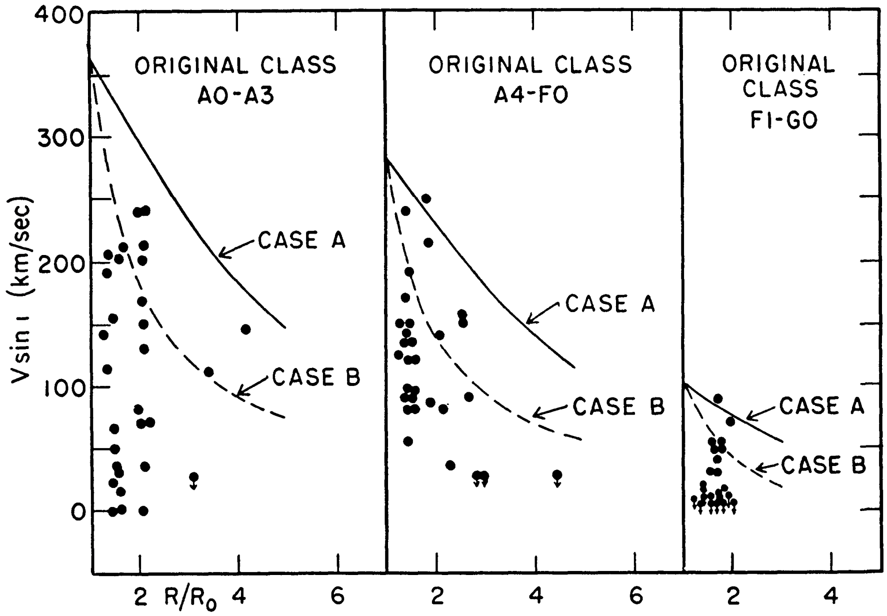

FIG 2 —Same as Fig 1 for original spectral types A0–A3, A4–F0, and F1–G0

TABLE 1

STARS WITH NO LUMINOSITY CLASS AVAILABLE

Star	Sp	$V \sin i$ (Km/Sec)	R/R_0*	Star	Sp.	$V \sin i$ (Km/Sec)	R/R_0*
ψ Per	B5e	390	1 39	7 Cep	B7	300	1 56
120 Tau	Bp	280	(1 4)	HR 8731	B2pe	340	1 21
HR 2142	B2 IV, V	450	1 21	o And	B6pe	330	1 51
κ Dra	B7p1	250	1 56	β Psc	B5pe	145	1 39
θ Cr B	B7nne	400	1 56	ψ Cyg	A3 IV, V	300	1 33
48 Lib	Bpe	400	(1 4)				

* Computed by assuming stars are of luminosity class IV.

class were impossible because of the large rotational broadening of the lines. In most cases it is probable that the luminosity class is either IV or V, but a closer decision is not possible. Table 1 lists these excluded stars.The fourth column of this table gives R/R_0 on the assumption that the luminosity class is IV. We may test, therefore, whether the rotations of these stars are also consistent with the evolutionary hypothesis, even if $R/R_0 > 1$. Three of the stars listed do exceed the criterion lines of case A by values which are significant. These are HR 2142 (B2 IV, V), θ Cr B (B7nne), and 48 Lib (Bpe). However, there is as good a chance that these stars are of class V as of class IV. Consequently, the high $V \sin i$ cannot be regarded as objectionable to the theory. Four stars in Slettebak's list of A3–G0 stars (1955) had indefinite luminosity classes for the same reason. They were α Aql (A7 IV, V), ψ Cyg (A3 IV, V), 33 Cyg (A3 IV, V), and α Cep (A7 IV, V). Here a decision on the correct luminosity class was possible for three of these from trigonometric parallaxes: α Aql is of class V, while 33 Cyg and α Cep are of class IV. These last two stars are plotted in Figure 2. The luminosity class of ψ Cyg is still unknown. This star is listed in Table 1. The conclusion from the results of the first test method is that all data are consistent with the evolutionary hypothesis.

The results of the second test method are shown in Figures 3 and 4. Values of $(V \sin i)_0$ were computed for each star from the known value of R/R_0 according to case A or case B. The distribution function of these recovered $V \sin i$ is compared with the distribution

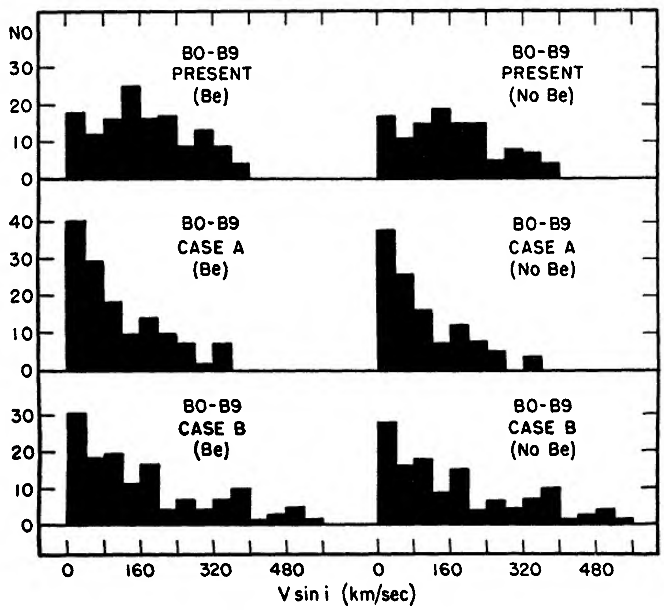

FIG. 3.—Histogram of the velocity distribution B0–B9 stars. Top diagrams are for stars currently on the main sequence. Distributions with and without Be stars are given. The middle histograms are the recovered distribution functions for case A. The bottom histograms are the recovered functions for case B.

function of stars of present luminosity class V. In Figures 3 and 4 the histograms of the recovered distribution for cases A and B have been normalized to the same total number as class V stars. Two cases are treated in Figure 3, one including and one excluding the Be stars.

Visual inspection is adequate in Figure 3 to see that the recovered B0–B9 distribution functions for both cases A and B contain more stars with small $V \sin i$ than they should. The observed values of $V \sin i$ for the giants and supergiants which originated as B0 V–B9 V stars are not high enough. A partial explanation for this apparent anomaly lies in the method of treating the data. Many of the giant and supergiant stars have observed values of $V \sin i$ which are tabulated as less than 25 km/sec. This value is the lower limit of detection with Slettebak's technique. The recovered $(V \sin i)_0$ on the main sequence is then known only to less than $R/R_0 \times 25$ for case B, with a similar situation for case A. For large R/R_0 this upper-limit value of $(V \sin i)_0$ can be quite high. The data were statistically treated as if the true velocity were half the upper-limit value. This may not be the case. If the stars with large R/R_0 have true $V \sin i$ near the upper limit, part of the low-velocity portion of the distribution function will move to higher $(V \sin i)_0$. This will explain part, but not all, of the discrepancy. If the remaining discrepancy is statistically significant, speculation concerning a possible change in the velocity distribution along the B0–B9 portion of the main sequence in the last 10^7–10^8 years, or, what is more likely, a loss of mass of these stars as they expand is possible. The data are too scarce, however, to make this venture profitable.

Comparison of the distribution function in the A0–G0 interval is shown in Figure 4. Case B seems definitely not to fit the data. This may also be seen from Figures 1 and 2, where many stars violate the case B upper line. Case B therefore seems unrealistic. The stars must rotate more nearly as rigid bodies than as stars in complete differential rotation.

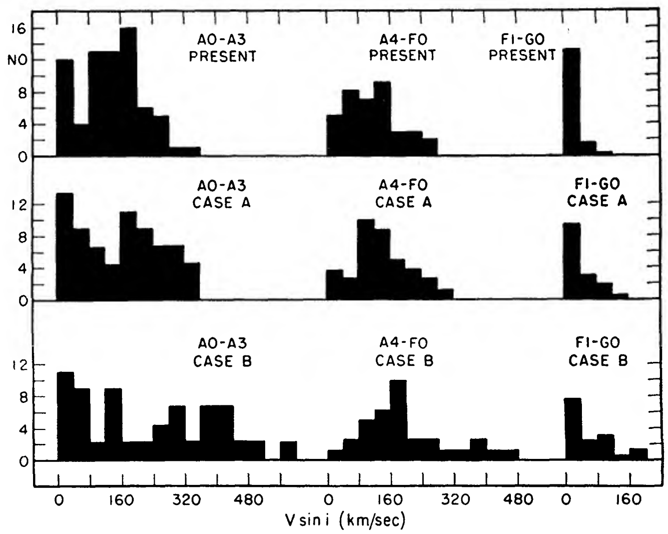

FIG. 4 —Same as Fig 3 for the original spectral types A0–A3, A4–F0, and F1–G0

The distribution functions for case A and for the present main sequence in this spectral interval are very similar. Statistical methods of comparison are needed, since the functions are so similar. Table 2 lists the relevant characteristics for these two distributions. Given are the mean values of $V \sin i$, the square of the standard deviation of the distribution, the total sample number n, twice the standard deviation for the distribution of the differences in the mean $V \sin i$ if repeated samples could be taken, and the difference in the mean $V \sin i$ between the distributions.

The mean values of $V \sin i$ differ between the distributions by about 20 km/sec. Is this difference statistically significant? Following the standard treatment (e.g., Hoel 1947), we let $(\overline{V \sin i})_{\mathrm{V}}$ and $(\overline{V \sin i})_{\mathrm{A}}$ be the means of the two distribution functions whose comparability we wish to test. These two means may be considered as the first pair of samples taken from a large population in repeated sampling, i.e., they may be treated as variables for which one pair of values is available. If the number of individual stars n is sufficiently large, the distribution functions of $(\overline{V \sin i})_{\mathrm{V}}$ and $(\overline{V \sin i})_{\mathrm{A}}$ which would be obtained from repeated sampling of the entire population will be normally distributed. If this is the case, it can be shown that the *differences* in the means are normally distributed, with a standard deviation given by

$$\sigma_{(\overline{V \sin i})_{\mathrm{V}} - (\overline{V \sin i})_{\mathrm{A}}} = \left(\frac{\sigma_{\mathrm{V}}^2}{n_V} + \frac{\sigma_{\mathrm{A}}^2}{n_A} \right)^{1/2} .$$

The statistical significance of the difference of the $\overline{V \sin i}$ between the two distributions may now be tested. We place the confidence limits of the statistical test at 0.05. This means that a difference of $2\sigma_{V-A}$ is just statistically significant. This value is tabulated in Table 2. For the A0–A3 and A4–F0 cases, the difference of $V \sin i$ between the distributions is not significant. Therefore, both distributions could have come from the same statistical population. The case F1–G0 is just significant, which indicates that the two means are farther apart than is expected if the true distributions are indeed the same. This case, however, does not represent the true situation, since nearly half the stars in this group have only upper limits to $V \sin i$, i.e., the tabulated $V \sin i$ are < 25 km/sec. The true statistics of the distribution are therefore not known.

The conclusion from the second test method is that the recovered velocity distribution in the range A0–G0, case A, is sufficiently like the present main-sequence distribution to be in agreement with the evolutionary hypothesis. The lack of agreement in the

TABLE 2

STATISTICS OF THE DISTRIBUTION FUNCTIONS

	A0–A3		A4–F0		F1–G0	
	Present V	Case A	Present V	Case A	Present V	Case A
$\overline{V \sin i}$ (km/sec) .	137	159	115	139	21	42
σ^2	6 00×10³	1 09×10⁴	4.55×10³	5 13×10³	3 7×10²	1 3×10³
n .	71	32	37	30	38	24
$2\sigma_{V-A}$	41		34		16	
$(\overline{V \sin i})_A - (\overline{V \sin i})_V$ (km/sec) ..	22		24		21	

B0–B9 range is not considered serious because of the unknown values of $(V \sin i)_0$ for stars of large R/R_0 with $V \sin i > 25$ km/sec. The first and second test methods therefore lead to the conclusion that the general evolutionary picture is tenable on the basis of the rotational-velocity data.

It is a real pleasure to acknowledge several discussions with Arne Slettebak concerning the observational aspects of this problem and to thank him for the privilege of using his data before the final list of rotational velocities was published.

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