LUMINOSITY FUNCTION AND SPACE MOTIONS OF G8–K1 STARS DERIVED FROM SPECTROSCOPIC PARALLAXES*

IAN HALLIDAY†
David Dunlap Observatory
Received March 31, 1955; revised May 20, 1955

ABSTRACT

Visual MK classifications and luminosity measures made with the help of an oscilloscopic microphotometer have been determined for 227 stars in the spectral range G8–K1, from spectrograms with a dispersion of 33 A/mm An analysis of the luminosities leads to a relative luminosity function which shows a distinct minimum in the region of the subgiants Spectroscopic parallaxes and space motions are computed for all the stars. For 24 stars the component of the space motion in the galactic plane is 65 km/sec or more.

INTRODUCTION

Recent theories of stellar evolution and galactic structure have indicated a need for more information on the distribution of stars in luminosity and on their motions in various parts of the Galaxy. A study of the luminosity function and space velocities of a group of stars of late spectral type within about 300 parsecs of the sun is reported here. The spectral range is bounded on the one side by the large increase in the frequency of giant stars at G8 and on the other by the complete disappearance of subgiant stars for types later than K1 (Roman 1952). The G8 and G9 stars are treated in one group, and the K0 and K1 stars in another.

Quite different luminosity functions for stars of this type have been published by Gratton (1933) and by Strömberg, whose results were converted to unit volume by Oort (1932). Gratton's function differs from Strömberg's in that it has a continuous increase in the frequency of stars between giants and dwarfs, while Strömberg's function has a pronounced minimum for the subgiants.

OBSERVATIONAL MATERIAL

All suitable spectrograms of dispersion 33 A/mm which have been taken with the 74-inch telescope of the David Dunlap Observatory were employed. Most of these spectrograms had been obtained for radial-velocity determinations. Altogether, 453 stars in the range from G8 to K1 were classified visually on the MK system (Johnson and Morgan 1953) for both luminosity class and spectral type. Of these, 82 had been observed because of their known parallaxes, so these stars were not included in the study of the luminosity function.

First, an analysis was made of the visual classifications of the remaining 371 stars, in an attempt to establish whether the observations would agree with the function by Gratton or by Strömberg. This analysis was inconclusive because of the difficulty in determining the range in absolute magnitude covered by luminosity class III–IV. Since the standard spectra are all for integral classes, a tendency exists to put doubtful stars in the integral rather than the half-integral classes. Luminosity class III–IV, then, covers a smaller range in magnitude than does class III or class IV. The numerical evaluation of the range is difficult, and the luminosity function changes appreciably as successive assumptions are tried.

* *Communications from the David Dunlap Observatory*, No 37.

† Now at the Dominion Observatory, Ottawa, Canada.

To eliminate this uncertainty, a more continuous variation in the luminosity parameter was needed so that the distribution could be treated statistically. At the time this problem arose, the oscilloscopic microphotometer designed and built by W. R. Hossack (1953) was being tested for luminosity-classification work. Because the method showed considerable promise for both speed and accuracy, the stars were reclassified with this microphotometer.

The microphotometer, the technique of measurement, and the method of reduction have been described by Hossack (1953, 1954). The calibration-curves from microphotometer readings to "luminosity number" (a decimal system of luminosity classes) were constructed, by a method described by Trumpler and Weaver (1953). The luminosity numbers may be taken to be on the MK system because a comparison of over one hundred visual classifications (to be published later) with those of Miss Roman (1952) had shown good agreement in both luminosity class and spectral type.

In general, four luminosity criteria were used, and the luminosity numbers derived from the separate calibration-curves were averaged without weights. The criteria used were: 4191/4196, 4202/4196, 4202/4215, 4250/4215. For the supergiants and bright giants, comprising about 2 per cent of the stars, the ratio 4233/4236 was used and given a larger weight than the other ratios. The ratio 4077/4071 was not used because of underexposure of many of the spectrograms in this region. The visual estimates of spectral type were retained.

The microphotometer requires spectrograms of more uniform quality than does the visual method of classification, and suitable spectrograms were available for 227 of the 371 stars which had been classified visually. The results are shown in Table 1

The first column is the HD number of the star, and the second column is the luminosity number, L. The third column is the visual classification of the star for spectral type and luminosity class. The fourth is the apparent visual magnitude. Any magnitude given to a hundredth is the average of the value in a list published by Mrs. Payne-Gaposchkin (1937) and the value in the *Potsdam Generalkatalog* (Müller and Kempf 1907), with a zero-point correction of -0.28 mag. applied to the Potsdam values Magnitudes given to a tenth of a magnitude are from the *Henry Draper Catalogue*.

Visual absolute magnitudes, M, are shown in the fifth column. These were determined for each spectral-type and luminosity-number combination from calibrations published by Miss Roman (1952) and by Keenan and Morgan (1951). Miss Roman's values for luminosity numbers 2.0, 2.5, and 3.0 were accepted, and Morgan's values for class 5.0. Linear interpolation was applied between these points, and values for spectral type G9 were interpolated between G8 and K0.

The spectroscopic parallaxes, π_{sp}, are shown in the sixth column. They were computed from the values of m and M by the relation

$$M = m + 5 + 5 \log \pi_{sp} .$$

No corrections for space absorption have been applied.

Only 22 stars in Table 1 have measured trigonometric parallaxes (Jenkins 1952). These parallaxes, π_{tr}, are shown in the seventh column. The agreement of the two parallaxes is reasonable, with a tendency for the larger spectroscopic values to exceed the trigonometric values. The spectroscopic parallaxes should have smaller percentage errors for the giants than for the dwarfs on account of the larger sample available for calibration purposes. The trigonometric parallaxes for giants are of little value, since they are generally comparable in size with their probable errors.

The last four columns of Table 1 show the position angle of the total proper motion, θ; the tangential velocity, T; the radial velocity, V; and the total space velocity, W. These are discussed in a later section.

TABLE 1

SPECTRAL CLASSIFICATIONS, PARALLAXES, AND SPACE MOTIONS

H D.	L	Visual	m	M	π_{sp}	π_{tr}	θ	T	V	W
166	5 0	K0 V	6 06	+6 00	0 097	0 069	122	1	− 3	3
167	3 2	G9 III	6 58	+0 90	0073		276	46	+ 9	47
1419	2 9	K0 III	5 95	+0 40	0078		254	42	+11	43
1527	3 2	K1 III	6 14	+1 34	0110		278	30	−31	43
1605	4 0	K1 IV	7 47	+3 50	0161		350	27	+14	30
2767	3 7	K1 III	5 70	+2 69	0250		300	11	+15	19
2952	2 9	K0 III	5 86	+0 40	0081		123	27	−28	39
3856	2 9	G9 III–IV	5 68	+0 10	0077		279	24	+ 7	25
4006	3 1	G9 III–IV	7 9	+0 62	0035		204	57	−18	60
4372	3 8	K1 IV	7 21	+2 96	0141		249	25	+17	30
4686 .	3 2	G9 III	7 12	+0 90	0057		270	37	+ 1	37
4831	3 0	G8 III	7 23	0 00	0036		239	57	−10	58
5449 .	3 3	G9 III	8 6	+1 17	0033		295	24	+ 7	25
7229.	3 1	G8 III	6 11	+0 28	0068		209	13	+37	39
7426 .	3 0	G9 III	8 7	+0 35	0021		18	51	+ 3	51
7578	3 6	K1 III	5 95	+2 42	0197		278	12	+ 8	14
8375	3 7	G8 IV	6 15	+1 96	0145		45	82	+ 4	82
8747	3 7	K0 III	6 58	+2 55	0156		256	18	− 6	19
8949.	3 2	K1 III	6 22	+1 34	0106		73	36	− 3	36
10588 .	3 3	G8 III–IV	6 28	+0 84	0082		289	37	− 5	37
11650.	2 6	G9 III	7 59	−0 65	0022		105	74	− 5	74
11721	3 2	G8 III–IV	8 1	+0 56	0031		33	47	+28	55
12402.	2 6	K1 III	6 49	−0 48	0040		77	42	+14	44
12638	2 9	G8 III	7 04	−0 20	0036		39	26	−22	34
12897.	3 0	K1 III	7 20	+0 80	0052		58	80	− 3	80
13013	3 0	G8 III	6 28	0 00	0055		178	29	+26	39
13747	3 2	K0 III	6 28	+1 23	0098		120	75	+15	76
13836.	5 0	G8 V	8 2	+5 60	030		101	37	− 1	37
14067	2 9	G9 III	6 38	+0 10	0055		255	57	−17	60
+26°392*	3 0	G9 III	8 0	+0 35	0030		262	138	−68	154
15328	3 3	K0 III	6 36	+1 49	0106		291	17	+ 9	19
15464	3 0	K1 III	6 15	+0 80	0085		127	32	+ 5	32
16467	3 1	G9 III	6 16	+0 62	0078		42	18	− 7	19
17228.	3 0	G8 III	6 18	0 00	0058	. . . .	68	31	+18	36
17382 .	4 6	K1 V	8 2	+5 12	024	0 077	102	42	+ 6	42
18202.	3 0	G8 III	6 42	0 00	.0052		68	10	+25	27
19066	3 5	K0 III	5 92	+2 02	0166		299	30	−36	47
19121	3 5	K0 III	5 97	+2 02	0162		342	13	−12	18
19525. .	3 5	G9 III	6 28	+1 71	.0122		334	47	+28	55
23526	2 9	G9 III	5 84	+0 10	0071		182	34	−38	51
24154	3 0	K0 III	6 64	+0 70	0065		117	2	+54	54
24768	2 9	G8 III	7 50	−0 20	0029		320	26	− 2	26
25296	2 8	G8 III	7 20	−0 40	0030		22	25	−29	38
25602	3 1	K0 III–IV	6 18	+0 96	0090		146	43	− 8	44
25877	2 6	G8 II	6 16	−0 80	0041		348	11	−13	17
26076	3 3	K1 III	5 98	+1 61	0134		347	8	+ 1	8
26605	3 2	G9 III	6 42	+0 90	0079		294	47	+24	53
28191.	3 1	K1 III	6 13	+1 07	0097		117	28	+ 7	29
28322.	3 1	G9 III	6 08	+0 62	0081	. .	30	16	+16	23
28505 . .	3 2	G8 III	6.42	+0 56	0 0067	.	210	33	−76	83

TABLE 1—*Continued*

H D.	L	Visual	m	M	π_{sp}	π_{tr}	θ	T	V	W
28930	2 9	G8 III	6 00	−0 20	0 0058		225	31	−40	51
30111	2 9	G9 III	6 98	+0 10	0043		140	26	+10	28
30138	2 9	G9 III	5 94	+0 10	0068		243	5	+29	29
30545	3 0	K1 III	5 86	+0 80	0097	. .	285	5	−34	34
32518	3 2	K1 III	6 28	+1 34	0103	0 006	106	28	− 3	28
34810	2 7	K0 III	6 17	−0 20	0053		259	9	−12	15
35238	3 1	K1 III	6 20	+1 07	0094		288	26	+32	41
36040	3 2	K1 III	5 82	+1 34	0127		277	9	+ 8	12
36041 .	3 1	G9 III	6 28	+0 62	0074		126	15	+ 5	16
37329	3 3	G9 III	6 34	+1 17	0092		45	13	+ 3	13
37956	3 0	K1 III	6 44	+0 80	0074		288	10	+17	20
38527	3.2	G8 III	5 76	+0 56	0091	. .	229	29	−41	50
38645	2 9	G9 III	6 18	+0 10	0061	003	135	16	+ 2	16
38765	3 8	K1 III	6 14	+2 96	0231		71	35	+23	42
39632	2 3	G9 II	6 04	−1 58	0030		9	28	− 1	28
39743	2 9	G8 III	6 26	−0 20	0051		90	18	− 6	19
40280	2 9	G9 III	6 43	+0 10	0054		273	44	− 8	45
40460	2 8	G9 III	6 52	−0 15	0046		189	64	+85	106
41467	2 7	K0 III	6 00	−0 20	0058		128	10	0	10
41636	2 7	G9 III	6 28	−0 40	0046		197	45	−93	103
42351	2 9	K1 II–III	6 28	+0 48	0069		180	24	−16	29
42466	3 0	K1 III	5 92	+0 80	0095		188	15	+ 8	17
44391	2 0	G9 II	7 46	−2 60	00097		270	127	−21	129
44780	3 0	K1 III	6 26	+0 80	0081		25	10	− 3	10
44867	3 1	G9 III	6 22	+0 62	0076		118	32	+60	68
45824	3 0	G8 III	8 0	0 00	0025		57	51	−15†	53
46277	2 6	G9 III	7 8	−0 65	0020		311	25	−13	28
46336	3 2	G9 III	8 0	+0 90	0038		121	15	+24	28
46509	2 7	K0 III	5 72	−0 20	0065		39	26	−20	33
47220	2 8	K1 III	6 16	+0 16	0063		218	35	−24	42
47270	3 0	K1 III	6 25	+0 80	0081		65	25	−35	43
47358.	2 9	G9 III	6 14	+0 10	0062		145	9	−22	24
50371	3 3	G9 III	6 13	+1 17	0102		169	48	−48	68
51814	2 6	G8 III	5 82	−0 80	0047		270	10	+ 1	10
53925	2 8	K1 III	6 15	+0 16	0063		0	3	+ 3	4
54825	2 3	K0 III	6 62	−1 52	0024		339	37	+31	48
56989	2 9	G9 III	5 82	+0 10	0072	. .	171	4	+ 8	9
58683	2 7	G8 III	8 1	−0 60	0018	0 014	212	291	+46	295
59878	2 9	G9 III	6 32	+0 10	0057		281	14	+19	24
62141	2 6	K0 III	6 16	−0 50	0047		324	33	−14	36
62264	3 7	K0 III	6 10	+2 55	0196		50	11	− 7	13
63410	3 2	G8 III	6 76	+0 56	0058		272	23	+72	76
63799	3 1	K1 III	6 14	+1 07	0097		100	33	−62	70
64938	2 6	G8 III	6 08	−0 80	0042		11	5	+ 2	5
65066	2 9	K0 III	5 98	+0 40	0077		111	9	−49	50
65735	3 2	K1 III	6 24	+1 34	0105		49	10	+17	20
65757	3 7	K1 III–IV	6 36	+2 69	0184		11	8	+16	18
67402.	2 8	G9 III	6 88	−0 15	0039		209	22	+ 2	22
68461‡	3 2	K0 III	5 94	+1 23	0114		56	6	−31	32
68776	3 2	G8 III	6 20	+0 56	0 0074		256	5	+14†	15

TABLE 1—*Continued*

H D	L	Visual	m	M	π_{sp}	π_{tr}	θ	T	V	W
69478	2 3	G8 III	6 14	−1 64	0 0028		222	21	+16	26
72908	2 4	G9 III	6 25	−1 24	0032		355	16	−19	25
73593	3 3	G8 IV	5 23	+0 84	0132	0 014	25	52	−38	64
73599	3 3	K1 III	6 34	+1 61	0113		205	9	+ 5	10
74669	3 3	K0 III	7 22	+1 49	0071		202	33	+21	39
76508	2 8	K1 III	6 10	+0 16	0065		247	24	+10	26
76629	2 7	G8 III	6 10	−0 60	0046		38	12	−24	27
78633	3 1	G8 III–IV	6 38	+0 28	0060		134	32	+12	34
79517	3 0	G8 III	6 36	0 00	0053		201	51	+62	80
85505	2 7	G9 III	6 20	−0 40	0048		232	28	+10	30
88476	3 2	G8 III	6 75	+0 56	0058		270	12	+ 4	13
89268	3 3	K1 III	6 25	+1 61	0118		101	8	−19	21
95233	3 1	G9 III	6 35	+0 62	0071		101	3	+ 5	6
100030	3 9	G8 IV	6 24	+2 52	0180		105	38	+45	59
100655	3 5	G9 III	6 31	+1 71	0120		313	9	− 5	10
101112	3 0	K1 III	6 14	+0 80	0086		48	26	+11	28
103736	3 2	G8 III	6 13	+0 56	0077		180	22	+26	34
103953	2 8	K0 III	6 56	+0 10	0051		89	55	−14	57
108471	3 1	G8 III	6 28	+0 28	0063		87	33	− 2	33
108861	3 3	G8 III–IV	5 96	+0 84	0095		109	35	− 7	36
109996	3 1	K1 III	6 30	+1 07	0090		252	14	−21	25
111591	2 8	K0 III	6 32	+0 10	0057		108	38	+13	40
112570	2 9	G9 III	5 94	+0 10	0068		182	34	+18	38
113847	3 2	K1 III	5 48	+1 34	0149		41	15	− 9	17
114724	2 8	K1 III	6 26	+0 16	0060		182	19	−15	24
114793	3 4	G8 III	6 34	+1 12	0090		290	17	−13	21
116594	2 9	K0 III	6 18	+0 40	0070		10	37	+ 3	37
118266	2 7	K1 III	6 34	−0 16	0050		119	100	+42	108
118536	3 0	K1 III	6 24	+0 80	0082		140	18	+ 3	18
122742	4 6	G8 V	6 14	+4 48	047	061	132	34	− 3	34
122744	3 0	G9 III	6 24	+0 35	0066		265	16	− 9	18
133485	3 2	G8 III–IV	6 38	+0 56	0069		146	16	− 9	18
134493	2 8	K0 III	6 14	+0 10	0062		170	23	−12	26
136274	5 0	G8 V	7 99	+5 60	033	005	258	72	−12	73
138085	3 1	G8 III–IV	6 35	+0 28	0061		336	57	+37	68
140438	3 7	G8 III–IV	6 37	+1 96	0131		42	9	+ 6	11
144046	2 4	G9 III	6 04	−1 24	0035		240	59	−27	65
144287	5 0	G9 V	7 08	+5 80	056	041	328	70	−25	74
145457	3 4	G9 III	6 56	+1 44	0095		359	24	+13	27
146603	2 5	G8 III	6 06	−1 00	0039		208	71	+ 6	71
152224	2 8	K0 III	6 12	+0 10	0063		25	36	−10†	37
153226	4 3	K0 IV	6 26	+4 14	038		335	15	−11	19
153312	3 2	K0 III	6 18	+1 23	0102		151	11	− 2	11
154391	2 8	K1 III	6 00	+0 16	0068		310	50	+ 1	50
154619	2 7	G8 III–IV	6 20	−0 60	0044		64	57	− 4	57
155500	3 4	K0 III	6 12	+1 76	0134		51	16	+14	21
159925	3 0	G9 III	6 02	+0 35	0073		144	15	+24	28
160822	3 0	K0 III	6 24	+0 70	0078		223	63	+15	65
161178	3 0	G9 III	5 72	+0 35	0084	−0 010	86	8	+23	24
162468	3 5	K1 III–IV	6 02	+2 15	0 0168		265	6	+29	30

TABLE 1—*Continued*

H.D.	*L*	Visual	*m*	*M*	π_{sp}	π_{tr}	θ	*T*	*V*	*W*
162734	3 3	K0 III	6 30	+1 49	0 0109		356	17	−23	27
163993	2 6	G8 III	3 56	−0 80	0134	0 018	102	30	+18	35
165473	3 3	G9 III	7 6	+1 17	0052		196	29	+40	49
166207	2 9	K0 III	6 12	+0 40	0072		356	62	−38	73
167304	3 0	K0 III	6 12	+0 70	0082		203	32	−28	43
169221	2 7	K1 III	6 27	−0 16	0052		315	19	+ 2	19
170619	3 5	G8 IV	7 7	+1 40	0055		357	31	− 5	31
170829	4 4	G8 IV	6 42	+3 92	032	028	184	40	−39	56
171994	4 0	G8 IV	6 20	+2 80	0209		358	17	−26	31
172424	4 0	G8 III	6 24	+2 80	0205		214	3	−22	22
173398	2 5	K0 III	5 88	−0 80	0046		351	57	− 9	58
174414	2 9	K0 III	6 76	+0 40	0053		304	16	+35	38
175679	2 5	G8 III	6 05	−1 00	0039		162	4	+ 3	5
175743	3 1	K1 III	5 58	+1 07	0125	003	200	62	+64†	89
176776	3 1	K1 III	6 26	+1 07	0092		32	5	− 8	9
177199	3 3	K1 III	5 98	+1 61	0134		286	5	+13	14
179094‡	3 5	K1 IV	5 89	+2 15	0179	004	236	38	+23	44
181122	2 7	G9 III	6 34	−0 40	0045		194	35	+ 7	36
181597	3 3	K1 III	6 12	+1 61	0125	002	342	15	+ 5	16
181655	5 0	G8 V	6 09	+5 60	080		216	17	+22	28
182272	2 8	K0 III	5 98	+0 10	0067		189	24	+ 4	24
182488	5 0	K0 V	6 28	+6 00	088	057	352	8	0	8
182617	3 4	K0 III	7 62	+1 76	0067		225	13	+30	33
182635	2 9	K1 III	6 32	+0 48	0068		358	53	−13	55
183399	2 7	K0 III	6 52	−0 20	0045		229	31	+ 7	32
185264	3 4	G9 III	6 38	+1 44	0103		311	19	+27	33
185436	3 0	K0 III	6 30	+0 70	0076		240	51	+25	57
187614	2 7	G8 III	6 40	−0 60	0040		79	18	+ 8	20
189671	2 3	G8 II	6 34	−1 64	0025		312	30	− 3	30
190252	3 0	G8 III	6 18	0 00	0058		29	47	+ 4	47
193094	2 9	G9 III	6 16	+0 10	0063		323	37	− 1	37
194220	2 9	K0 III	6 12	+0 40	0072		44	31	− 2	31
194616	2.7	K0 III	6 30	−0 20	0050		56	14	−12	18
194937	3.2	G9 III	6 02	+0 90	0095		21	18	+ 6	19
194953	2 7	G8 III	6 25	−0 60	0043		80	45	− 6	45
195820	3 4	K0 III	6 06	+1 76	0138		2	19	+ 7	20
196642	3 1	K0 III	6 00	+0 96	0098		197	22	−19	29
196787	2 4	G9 III	5 20	−1 24	0051		66	11	+ 8	14
197249	3 2	G8 III	6 14	+0 56	0077		1	33	+15	36
198236	3 1	G8 III	6 29	+0 28	0063	001	235	47	+ 6	47
200578	3 3	G8 III	6 76	+0 84	0065		239	19	− 9	21
201669	3 2	G8 III	7 8	+0 56	.0036		269	72	+22	75
203358	4 5	G8 IV–V	6 32	+4 20	038		260	10	−12	16
203630	3 3	K1 III	5 98	+1 61	0134		306	9	− 9	13
203733	2 9	K0 III	8 0	+0 40	0028		67	96	+33	102
203886	2 8	K0 III	6 28	+0 10	0058		66	15	− 8	17
205688	2 9	G8 III–IV	6 26	−0 20	0051		310	99	− 5	99
206040	3 3	K1 III	6 00	+1 61	0132		265	22	+16	27
206374	4 9	G8 V	7 42	+5 32	038	0 031	103	28	−28	40
206509	3 3	K0 III	5 90	+1 49	0 0131		259	14	+19	24

TABLE 1—*Continued*

H D	L	Visual	m	M	π_{sp}	π_{tr}	θ	T	V	W
206731	2 1	G8 II–III	5 92	−2 28	0 0023		236	9	+11	14
207088	2 7	G8 III	6 27	−0 60	0042		83	88	+10	89
207740	4 6	G8 V	8 0	+4 48	020		277	30	+21	37
209813	2 5	K0 III	6 37	−0 80	0037		46	78	− 9	79
210026	2 9	K0 III	7 9	+0 40	0032		294	27	+ 1	27
211555	2 8	K0 III	7 02	+0 10	0041		18	44	−12	46
212750	3 1	K0 III	7 12	+0 96	0059	. .	284	55	+12	56
212943	2 7	K1 III	4 66	−0 16	0109	0 021	174	123	+62	138
213014	3 4	G9 III	7 32	+1 44	0067		263	6	−30	31
213025	3 2	G8 III	6 35	+0 56	0069		256	46	−31	55
213178	3 1	K0 III	7 02	+0 96	0061		349	20	+ 3	20
214265 .	3 1	K0 III	7 00	+0 96	0062		66	37	− 7	38
215030	3 1	G9 III	5 77	+0 62	0093	000	56	62	− 2	62
217019	2 5	K1 III	6 16	−0 80	0041		44	84	+18	86
217673	2 7	K1 III	6 16	−0 16	0054		248	24	+ 6	25
218103	2 5	G9 III	6 36	−0 90	0035		130	31	− 6	32
218416	2 5	K0 III	6 02	−0 80	0043		317	17	+16	23
218660	3 0	K1 III	6 46	+0 80	0074		10	18	+21	28
219736	3 5	K1 III	6 84	+2 15	0115		255	39	+ 6	39
221113	2 8	G9 III	6 36	−0 15	0050		215	97	+27	101
221115	2 8	G9 III	4 38	−0 15	0124	009	42	22	−10	24
221364	3 0	K0 III	6 42	+0 70	0072		260	7	+ 3	8
221861	Ib	K0 Ib	5 70	−4 50	00091		333	12	+ 7	14
222618.	3 0	G8 III	6 10	0 00	0060	. .	288	18	+ 1	18
224085‡	4 9	K0 V	7 02	+5 73	055	0 034	70	33	−13	35
224895	3 2	K1 III	6 76	+1 34	0082		234	44	− 7	45
225292	3 1	G9 III	6 46	+0 62	0 0068		81	31	+18	36

* BD number, not in *Henry Draper Catalogue*

† The radial velocity is variable, but no orbit has been computed The value given is the average of all measures

‡ The following stars have characteristics:

HD 68461: *H* lines strong, possibly composite spectrum. Spectroscopic binary
179094: *Ca* H and K emission known Spectroscopic binary
224085: *Ca* H and K emission known, H lines very weak Spectroscopic binary

DERIVATION OF LUMINOSITY FUNCTIONS

The distribution of the stars in Table 1 according to luminosity number is shown in Table 2. The G8 and G9 stars are combined in one group, and the K0 and K1 stars in another group. The tabular interval chosen is one-fifth of a luminosity class, as shown in the first column. The observed number of stars in each luminosity interval is given in the second and seventh columns, and in the third and eighth columns these distributions have been smoothed of minor fluctuations.

Before the smoothed frequency functions are converted to luminosity functions they must be corrected for the effects of error dispersion. The luminosity measures are subject to errors from (*a*) effects of the exposure density of the spectrograms, (*b*) matching the line intensities in making the readings, and (*c*) calibrating to luminosity numbers. As a result of these errors, the observed distributions can be expected to have less sharp maxima and minima than the true distributions.

An estimate of the probable error of the luminosity-number determinations was made in the following way. For each star four luminosity-number estimates were available, one from each of the spectral criteria used. From the internal agreement of these four numbers, the estimated probable errors of the mean turned out to be ± 0.11 luminosity class for the G8 and G9 stars and ± 0.14 class for the K0 and K1 stars. This is in good agreement with the value of ± 0.16 class obtained by Hossack (1954) from measuring three ratios on each of several spectrograms of a single star of similar spectral type. This indicates that there were no serious errors affecting all the measures on each spectrogram. If such errors existed, the internal agreement of the four ratios would have yielded a smaller probable error than would the comparison of the average luminosity numbers from different spectrograms. There were not sufficient spectrograms of good quality on the 33 A/mm dispersion to use the second method for the present material.

The smoothed frequency functions of the third and eighth columns of Table 2 were corrected by a numerical method for assumed probable errors of ± 0.11 class for the third column and ± 0.14 class for the eighth and a Gaussian distribution of the errors. The corrected distributions are shown in the fourth and ninth columns.

TABLE 2

Luminosity Functions for G8, G9, and K0, K1 Stars

L	G8, G9					K0, K1				
	Obs	Smooth	Corrected	Final	Luminosity Function	Obs	Smooth	Corrected	Final	Luminosity Function
1 0,2 1	2	1 2	0 94	0 94	0 035	1	0 6	0 27	0 27	0 009
2 2,2 3	3	2 8	1 9	1 9	0 17	1	1 4	0 27	0 27	0 025
2 4,2 5	6	7 0	5 8	5 8	1 4	4	4 0	1 7	1 7	0 45
2 6,2 7	15	15 0	15 3	15 3	10	11	11 0	9 4	9 4	6 6
2 8,2 9	22	22 0	23 1	23 1	29	23	23 0	26 9	26 9	45
3 0,3 1	26	26 0	28 4	28 4	59	26	26 0	30 4	30 4	131
3 2,3 3	24	24 0	28 3	28 3	120	21	21 0	23 9	23 9	225
3 4,3 5	7	7 0	3 9	3 9	35	8	9 0	6 6	6 6	128
3 6,3 7	2	2 2	0 54	0 54	10	5	4 0	2 2	2 2	91
3 8,3 9	1	1 5	1 2	1 3	54	2	1 9	1 1	1 2	98
4 0,4 1	2	1 2	1 1	1 3	110	1	1 0	0 60	0 77	139
4 2,4 3	0	1 1	1 0	1 4	260	1	0 8	0 67	0 98	370
4 4,4 5	2	1 2	1 1	1 6	650	0	0 7	0 61	1 1	860
4 6,4 7	2	1 5	1 4	1 9	1700	1	0 8	0 71	1 2	2000
4 8,4 9	1	2 0	1 9	2 5	4600	1	1 0	0 96	1 6	5400
5 0	4	3 3	4 6	5 2	20500	2	1 8	2 6	3 2	23000

Corrections to the frequencies of dwarf stars in Table 2 were required because there was a selection effect in the sample studied which involved the proper motions of the stars. About 65 per cent of the stars in the group were observed for radial velocity at the David Dunlap Observatory (Young 1945), in order to complete the list of radial velocities of northern stars in Schlesinger and Jenkins' (1940) *Catalogue of Bright Stars*. Since stars with large proper motions had been observed for radial velocity at an early date, the sample observed at the David Dunlap Observatory is deficient in high proper motions. Corrections for this selection effect are now considered.

An analysis of all those stars in the bright-star catalogue with northern declinations, spectral types between G5 and K5, and visual magnitudes from 5.8 to 7.0 indicated that the stars observed at the David Dunlap Observatory comprise an unbiased sample for proper motions less than 0''.21 per year. For larger proper motions the deficiency of stars in the David Dunlap Observatory group is evident. The deficiency was evaluated for each of six ranges in proper motion. About 70 stars in the G8 and G9 group and another 70 in the K0 and K1 group are from the observing program affected by the selection effect. The corrections amount to a total of nearly 3 stars for each group.

The distribution of these corrections was converted from proper motion to absolute magnitude by assuming that the large proper motions correspond to tangential velocities of 50 km/sec. This rather large value for the tangential velocity insures that the correction is not underestimated for the brighter stars. The average apparent magnitude of the group is $m = 6.4$, so that proper motions of 0".21 per year or more correspond to absolute magnitudes of $M = +2.9$ or fainter By means of the relation between luminosity numbers and absolute magnitudes, the corrections for the proper-motion selection effect were applied, and this gave the final corrected distributions of the fifth and tenth columns of Table 2. Only the last seven rows were affected by the correction.

The absolute magnitudes determine the relative volumes of space observed for each luminosity interval in Table 2. These ratios are independent of the apparent magnitudes of the stars if, as must be true for all but the most luminous stars in this survey, space absorption is negligible. The absolute magnitudes corresponding to the division points between successive rows in Table 2 were computed by averaging the values for the four spectral-type–luminosity-number combinations involved—e.g., G8, 2.9; G9, 2.9; G8, 3.0; and G9, 3.0—between the fifth and sixth rows. If the successive values of this average be indicated by M_i, then the relative volumes are proportional to the values of

$$(R_i)^3 - (R_{i+1})^3 ,$$

where

$$\log R_i = -0.2 M_i .$$

When the relative volumes of space were computed, division of the frequencies in the fifth and tenth columns of Table 2 by the appropriate volume yielded the relative luminosity functions shown in the sixth and eleventh columns. These indicate the relative numbers of stars of different luminosities determined from a sample of stars, almost all of which are within 300 parsecs of the sun. No attempt has been made to convert to absolute luminosity functions in order to indicate the number of stars of each luminosity per cubic parsec. The stars used do not comprise a sufficiently homogeneous sample to permit ready determination of completeness factors, and the interesting features of the functions are shown just as well in the form of relative functions.

The final functions of Table 2 are plotted in Figure 1 for the G8 and G9 stars and in Figure 2 for the K0 and K1 stars. The luminosity scale has been converted here to absolute magnitude, and allowance has been made for the slight change in slope at class 3.0 of the linear relation between luminosity class and absolute magnitude. Only a limited region of the absolute-magnitude scale is shown. The frequency scales in the two figures are independent and do not give the relative frequencies of G8 and G9 giants compared to K0 and K1 giants. The dashed curves indicate the functions obtained if the error-dispersion correction is not applied.

The corrected and uncorrected curves cross one another in two places in the region of the giant stars. These are not symmetrically situated with respect to the maximum, because division by the relative volumes of space has greatly changed the shape of the original frequency functions. One point of intersection is evident in each figure, and the other point is in the wing of each curve between $M = 0$ and $M = -1$.

DISCUSSION OF LUMINOSITY FUNCTIONS

The important result is the presence of a distinct frequency minimum for both groups of stars near luminosity number 3.6 or near absolute magnitude +2. These subgiant stars appear to be less numerous than either normal giants or dwarf stars in the spectral range from G8 to K1. This is in agreement with the results obtained by Strömberg. For bright giant stars the frequencies are less significant, since no account has been taken of galactic concentration for these stars.

To be sure of the reality of the frequency minimum near absolute magnitude +2, the

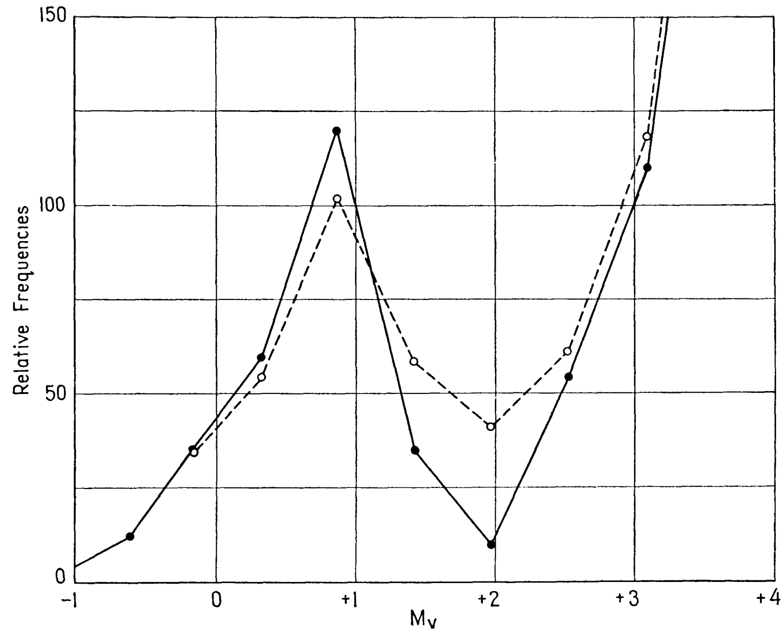

FIG 1.—Relative frequencies of G8 and G9 stars as a function of visual absolute magnitude. The solid curve is the corrected function for a probable error of 0 11 luminosity class The dashed curve is the uncorrected function

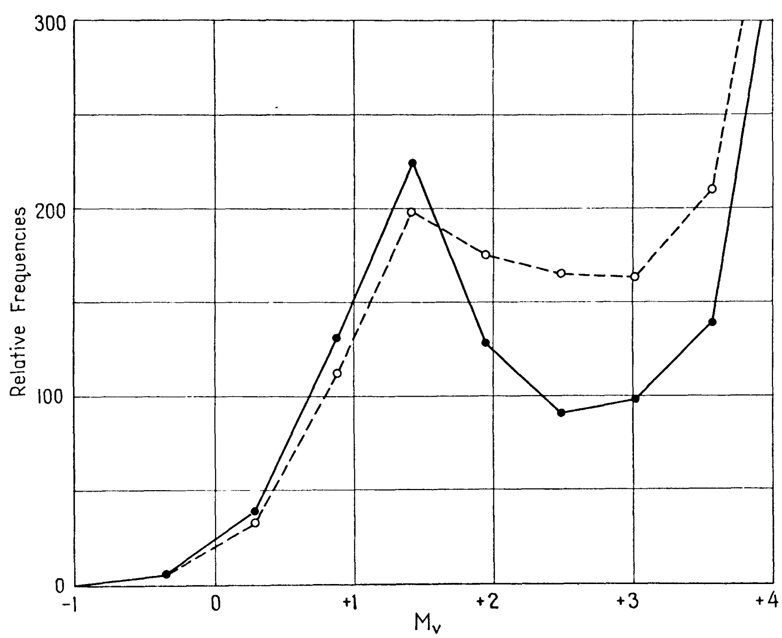

FIG. 2.—Relative frequencies of K0 and K1 stars as a function of visual absolute magnitude The solid curve is the corrected function for a probable error of 0 14 luminosity class The dashed curve is the uncorrected function.

assumptions which might affect it are now examined. In forming the calibration-curves to derive luminosity numbers from microphotometer readings (Trumpler and Weaver 1953), the sharp peak of the visual classifications at class III was diffused by assuming that one-quarter of these should be class III–IV and one-quarter should be class II–III. Many individual readings near class 3.5 result from this assumption, which tends to diffuse the final curve in this region rather than create an artificial minimum.

On the other hand, the error-dispersion correction tends to create a minimum; hence it is important not to overcorrect for this effect. The solid curves in the figures are based on the best available estimates of the errors involved, while the dashed curves are based on the assumption of no errors in the luminosity numbers, and they show that the minima exist even if the error-dispersion correction is omitted entirely.

The correction due to the proper-motion selection effect is small for stars as bright as the subgiants. The correction applied was based on a rather large assumed tangential velocity of 50 km/sec, which means that the corrections probably are too large for the brighter stars involved, again tending to decrease the prominence of the frequency minimum.

The frequency minimum could be strongly affected by the method of calibrating from luminosity numbers to absolute magnitude. Linear interpolation was applied between Miss Roman's values for class III and Morgan's values for class V. If Miss Roman's values for classes III–IV and IV had been used (which were derived from small samples), the minimum near class 3.6 would have been more pronounced, since the range in absolute magnitude from class III to class III–IV given by Miss Roman is smaller than that used here. The relative volumes of space in this range would then be smaller, which would give rise to a larger maximum near class 3.2, while a lower minimum near class 3.6 (or fainter) would result from the larger relative volumes associated with the correspondingly larger range in absolute magnitude. The effect would be further emphasized in converting from the luminosity-number intervals of Table 2 to the absolute-magnitude scales of Figures 1 and 2. The linear interpolation of absolute magnitude, then, also tends to decrease the extent of the minimum for subgiants.

Linear interpolation is probably the better procedure to adopt in any case, for any group of stars visually classified as class III–IV is likely to be closer to class III in its properties than to class IV. This is because there is an unavoidable error in the classifications and a great preponderance of stars near class III in the entire group, so that more stars near class III than near class IV will be classified as III–IV. While the definition of class III–IV could be based on some such group of stars, it would be more meaningful to have a uniform change of both spectral characteristics and absolute magnitude with the luminosity designation.

If the relation between luminosity number and absolute magnitude is not unique, so that there is a slight scatter in the absolute magnitudes of stars for which the correct luminosity numbers are identical, then the true luminosity function will have slightly more pronounced maxima and minima than the one derived here on the assumption of a unique relation between L and M.

All the assumptions which might affect the reality of the frequency minimum for subgiants in the luminosity function are, therefore, such as to decrease its size. Its presence in the final functions and its general similarity in the two spectral groups leave little doubt that the effect is real.

SPACE MOTIONS

Space motions were computed for each star, the procedure being as follows. The GC proper motions were first corrected by the systematic corrections given in the N30 catalogue (H. R. Morgan 1952). For some of the fainter stars the proper motions from the Yale catalogue (Barney 1953) were used. The spectroscopic parallax values were used to remove the parallactic component due to solar motion from each proper-motion component. A solar motion of 20 km/sec toward an apex at $\alpha = 18^h04^m$, $\delta = +28°$ was assumed. The position angles, θ, of the total proper motions are given in the eighth column of Table 1. Total proper motions were computed and converted to kilometers per second to give the tangential velocities, T, shown in the ninth column. The radial velocities, V (corrected for the same solar motion), are shown in the tenth column. The velocities were derived from those in *Publications of the David Dunlap Observatory*, Volume **1**, Nos. 3, 13, and 16, and from an additional list soon to be published. The space velocity, W, in the eleventh column is the vector sum of T and V and is then the velocity of the star relative to the local standard of rest.

The probable errors of the space velocities are very different from star to star and depend on the apparent magnitude, the absolute magnitude, and the relative sizes of the components V and T. With individual probable errors of 0.4 mag. for M, 0".008 per year for the proper motion, and 2 km/sec for V, the probable errors for W for a sixth-magnitude star with $V = 15$ km/sec and $T = 25$ km/sec are: 16 km/sec for luminosity number 2.0, 5 km/sec for 3.0, and 4 km/sec for 5.0. If the space motion is large, the probable error in W will be large if T is much larger than V, while the error in W is always small if V is much larger than T. The errors will be larger for fainter stars because of the proper-motion errors.

TABLE 3

SUSPECTED HIGH-VELOCITY STARS

H.D.	*L*	*G*	Long.	H.D.	*L*	*G*	Long
11650	2 6	74	198	118266	2 7	108	332
13747	3 2	74	196	136274	5 0	68	212
+26°392*	3 0	151	345	144046	2 4	65	216
28505	3 2	83	322	144287	5 0	72	146
40460	2 8	100	180	166207	2 9	73	170
41636	2 7	91	298	175743	3 1	89	334
44867	3 1	65	186	203733	2 9	93	121
50371	3 5	65	306	207088	2 7	75	144
58683	2 7	213	213	209813	2 5	78	159
63410	3 2	75	155	212943	2 7	80	290
63799	3 1	70	347	217019	2 5	86	122
79517	3 0	67	152	221113	2 8	84	329

* BD number, not in *Henry Draper Catalogue*

The space velocities in Table 1 reveal a number of high-velocity stars. Spectral differences between such stars and normal stars have been described by other investigators (Roman 1952; Keenan and Keller 1953) but were not apparent on the spectrograms used for this investigation. Some of the differences are best seen with lower dispersion, and others depend on criteria which are outside the region of good exposure density of the spectrograms.

A list of the suspected members of the high-velocity group is of value. A total of 32 stars have space velocities of 65 km/sec or larger. For each of these stars the component in the galactic plane was computed, with the galactic pole taken as $\alpha = 12^h40^m$, $\delta = +28°$. Table 3 lists the 24 stars for which this component is 65 km/sec or more. Successive columns give the HD number; the luminosity number, L; the component of W in the galactic plane, G; and the galactic longitude toward which the component is directed.

None of the 24 stars in Table 3 has its velocity vector G directed between galactic longitudes 347° and 121°, i.e., none is within 66° of galactic longitude 55°, the direction of galactic rotation in the solar neighborhood. This is in accord with the known avoidance of this direction by high-velocity stars. Two of the stars in Table 3, HD 144287 and HD 212943, have been listed previously as high-velocity stars (Miczaika 1940).

CONCLUSIONS

Relative luminosity functions and individual space motions have been deduced from spectroscopic parallax data for 227 stars, obtained with the help of an oscilloscopic microphotometer. The luminosity functions have a pronounced minimum in the region of the subgiants, and an examination of all pertinent assumptions has shown that the effect must be real. It should be emphasized that the results apply to the narrow range in spectral type from G8 to K1.

A comparison of the luminosity function for the late G stars with the curve for the early K stars suggests that the frequency minimum is more pronounced for the late G stars. This conclusion is strongly dependent on the calibration which has been adopted to convert from luminosity numbers to absolute magnitude. This difference may have considerable evolutionary significance.

The determination of space velocities has revealed a number of suspected high-velocity stars which are listed in Table 3. While the accuracy of the space-velocity calculations is not sufficient to insure that all the stars of Table 3 are high-velocity stars (or that some other stars in Table 1 are not high-velocity stars), the magnitudes and directions of the motions in Table 3 indicate that a high proportion of these stars must be members of the high-velocity group. As such, their spectra deserve further study.

The writer wishes to express his thanks to Dr. W. W. Morgan and Dr. Daniel L. Harris III, of the Yerkes Observatory, for valuable suggestions on the classification criteria and the luminosity-function problem and for advice on the sources of apparent magnitude. He would also like to thank Dr. W. R. Hossack for his co-operation on the program with the oscilloscopic microphotometer and Dr. Donald A. MacRae for advice on the statistical methods employed. Finally, the author is indebted to Dr. John F. Heard for his helpful suggestions on all phases of the problem and for his general supervision of the research.

REFERENCES

Barney, I. 1953, *Yale Trans.*, Vol. **24**.
Gratton, L 1933, *B.A.N.*, **7**, 85.
Hossack, W. R. 1953, *J.R.A.S. Canada*, **47**, 195.
———. 1954, *Ap. J.*, **119**, 613.
Jenkins, L. F. 1952, *General Catalogue of Trigonometric Stellar Parallaxes* (New Haven: Yale University Observatory).
Johnson, H. L., and Morgan, W. W. 1953, *Ap. J.*, **117**, 313.
Keenan, P. C., and Keller, G. 1953, *Ap. J.*, **117**, 241.
Keenan, P. C., and Morgan, W. W. 1951, *Astrophysics*, ed. J. A. Hynek (New York: McGraw-Hill Book Co.), p. 23.
Miczaika, G. 1940, *A.N.*, **270**, 249.
Morgan, H. R. 1952, *Astronomical Papers American Ephemeris and Nautical Almanac*, Vol. **13**, Part III.
Müller, G., and Kempf, P. 1907, *Pub. ap. Obs. Potsdam*, Vol. **17**.
Oort, J. H. 1932, *B.A.N.*, **6**, 289.
Payne-Gaposchkin, C. 1937, *Harvard Mimeograms*, Ser. III, No. 1.
Roman, N. G. 1952, *Ap. J.*, **116**, 122.
Schlesinger, F , and Jenkins, L. F. 1940, *Catalogue of Bright Stars* (2d ed.; New Haven: Yale University Observatory).
Trumpler, R. J., and Weaver, H. F. 1953, *Statistical Astronomy* (Berkeley: University of California Press), p. 155.
Young, R. K. 1945, *Pub. David Dunlap Obs.*, **1**, 309.