

## CLASSIFICATION OF THE S-TYPE STARS

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## ABSTRACT

In this survey, spectra of type S are defined as those having bands of  $ZrO$  strong enough to be detected with low dispersion. This limitation is made because the group will be a useful one only if most or all of the members can be recognized on slitless spectrograms and if they are enough alike to give significance to their statistical properties. Stars with weaker  $ZrO$  bands are termed "MS" stars, and 14 of these are tabulated. Seven other stars showing neither  $ZrO$  nor  $TiO$  bands in their spectra are set aside as related more distantly to the S-type stars. The 69 S-type stars of the main catalogue are classified from the relative intensities of their  $ZrO$  and  $TiO$  bands into a two-dimensional system, in which the variables appear to be *temperature* and *chemical composition*. The lines of evidence bearing on the luminosities of the S stars—their motions and galactic distribution, as well as the classification by Feast of the faint companion of  $\pi'$  Gruis—suggest that these stars belong among the giants and bright giants (luminosity classes III and II), possibly spanning a considerable range in absolute magnitude. Preliminary counts of M, MS, S, and C stars in the giant branch of the cooler stars with banded spectra show that roughly 10 per cent of them show appreciable strengthening of the S characteristics—the lines of the heavier metals and the bands of their oxides.

## PRINCIPLES OF CLASSIFICATION

Spectral class S was introduced by P. W. Merrill (1922*a, b*) in 1922 to designate a group of curious red stars which had not fitted well into either class M ( $TiO$  stars) or classes R and N (carbon stars). The original type stars of the class were the slightly variable star  $\pi'$  Gruis and the two long-period variables R And and R Cyg. The history of the recognition of the peculiarities of these stars and of the early attempts to classify them has been well summarized by Merrill in his monograph *Spectra of Long-Period Variable Stars* (1940), where references to the pioneer papers will be found.

Although the members of class S have never been numerous, observers have generally agreed in assigning to it those stars whose spectra showed either or both of these features: (1) distinct bands of  $ZrO$  in the blue and visual regions; the strongest  $TiO$  bands may be practically absent (R Gem), relatively weak (AD Cyg), or stronger than the  $ZrO$  bands (HR 1105); (2) unusual strength of certain absorption lines in the blue-green, particularly  $\lambda$  4554 ( $Ba$  II) and  $\lambda$  4607 ( $Sr$  I), as compared to their intensities in normal K- or M-type stars.

There are other spectral features characteristic of the group, but characteristics 1 and 2 are the only signposts which have been consistently employed for recognition of S-type stars. An example of an additional criterion which is only occasionally useful is the strength of  $H\beta$  emission in long-period variables. In those of type S,  $H\beta$  is frequently much stronger relative to  $H\gamma$  and  $H\delta$  than in typical M-type long-period variables; but this is due to the weakening of  $H\beta$  emission by the strong  $\lambda$  4847 absorption band of  $TiO$  in the latter. That the effect is mainly a consequence of the absolute band strengths is shown by the failure of  $H\beta$  to be conspicuously strengthened in S-type variables like S Cyg, in which the  $TiO$  absorption remains strong.

The physical description of the stars forming type S has involved difficulties which have never been fully surmounted. In the first place, less than a hundred stars have ever been assigned to the group, and most of these have such faint apparent magnitudes that they have not been well observed spectroscopically. Consequently, we do not have reliable statistical information about such properties as mean luminosities or temperatures. Furthermore, even this small group is far from homogeneous. The differences between

individual spectra are sometimes more obvious than their similarities. Thus in R Cyg and R Gem strong bands of  $ZrO$  completely dominate the visual spectrum; but in some stars, especially those of the "R CMi group," which were put into class S entirely on the basis of criterion 2—enhanced atomic lines—it is difficult to detect any  $ZrO$  bands even on spectrograms of considerable dispersion.

The problem should become less complex if we can deal with a more homogeneous group of stars. For this reason the present investigation attempts, first, to sharpen the definition of type S and then to review critically the membership in the class. We should like to define spectral types by features which can be recognized on spectrograms of rather small scale, for the criteria will have wide usefulness only if they can be applied to surveys with slitless Schmidt spectrographs or samplings of distant fields with slit spectrograms of low dispersion. This condition is fulfilled fairly well by the strongest bands of  $ZrO$ , which in many known S-type stars are easily seen with a dispersion of 250 Å/mm or less.

Let us, accordingly, define type S by our previous criterion 1 only—the presence of distinct bands of  $ZrO$  in the easily observable spectral regions. Other features, such as the strengthening of the  $Ba\ II\ 4554$  line will be used only as supplementary criteria.<sup>1</sup>

Absorption from the ground state of the  $ZrO$  molecule gives rise to three band systems which are useful in classification (Lowater 1932; Afaf 1950). These bands all degrade to the red from triplet heads, of which those of the  $\Delta v = 0$  sequences are the strongest and, hence, the most sensitive for detection of  $ZrO$  when  $TiO$  is also strong. The most valuable bands are discussed.

1.  $\alpha$ -System in the blue;  $C^3\Pi \rightarrow X^3\Pi$ .—The strongest head is  $\lambda\ 4640.6$  ( $0, 0 R_1$ ), but it can be seen readily only when the overlapping  $\lambda\ 4626.1$  band of  $TiO$  is weak. In other cases the next strongest  $ZrO$  band,  $\lambda\ 4619.8$  ( $0, 0 R_3$ ), must be used. Since this head is separated by only about 6 Å from the  $TiO$  bands, spectrograms with a dispersion of at least 100 Å/mm are needed to distinguish the bands unless the  $ZrO$  is quite strong. Consequently, this system is of only limited usefulness on most objective-prism plates. On the other hand, the maximum speed of blue-sensitive emulsions, such as the Eastman type O, comes just in this part of the spectrum, and, consequently, it is feasible to take slit spectrograms showing these bands in many of even the red stars.

2.  $\beta$ -System in the yellow;  $B^3\Delta \rightarrow X^3\Pi$ .—The bands of this system do not seem to be quite so intense as those of the  $\alpha$ - and  $\gamma$ -systems, but the strongest head at  $\lambda\ 5551.7$  is far enough away from  $TiO\ \lambda\ 5569$  (a rather weak band) to be distinguished easily on spectrograms with a dispersion of 320 Å/mm or more (Figs. 1 and 2). This spectral region has been somewhat neglected in the past, partly because it falls within the minimum of the sensitivity-curve of many panchromatic emulsions; but the Eastman type-G sensitizing is very efficient at  $\lambda\ 5551$ , and type-D sensitizing also holds up well down to  $\lambda\ 5500$ . When all factors are taken into account, the  $\lambda\ 5551$  head appears to be the  $ZrO$  band most useful for the marginal detection of the molecule with low dispersion. A few of the adjacent bands, particularly  $\lambda\ 5718.1$  ( $0, 0 R_3$ ), are sometimes useful, but are much weaker than the main head.

3.  $\gamma$ -System in the red;  $A^3\Sigma \rightarrow X^3\Pi$ .—The  $\lambda\ 6473.7$  ( $0, 0 R_3$ ) band is one of the most persistent bands of  $ZrO$  and was found by Bobrovnikoff (1934) to give the best evidence for the presence of considerable amounts of  $ZrO$  even in such M-type stars as  $\beta$  Peg and  $\rho$  Per. A scale of about 130 Å/mm (corresponding to about 30 Å/mm at  $H\gamma$  on a typical prismatic spectrogram) is required to separate the  $\lambda\ 6473$  head from the  $TiO$  band at  $\lambda\ 6479.0$ . With this dispersion the strong S-type stars are easily recognizable. In stars with weak  $ZrO$ , however, the  $\lambda\ 6473$  band is difficult to detect with certainty even on

<sup>1</sup> Observed enhancement of  $\lambda\ 4554$  is not a necessary condition for an S-type star, for when the  $\lambda\ 4462$  band of  $TiO$  is strong in very cool stars, it extends far enough to the red to overlap and weaken the  $Ba\ II$  line. Nor is the enhancement of the line a sufficient condition for membership in the class, for  $\lambda\ 4554$  is quite strong in many of the red carbon stars also.

plates of greater dispersion, for the measurements on  $\beta$  Peg carried out by D. N. Davis (1947) on Mount Wilson coude spectrograms show that the near-by atomic lines ( $Ca$  I 6471.6 and  $Fe$  I 6475.6) are strong enough to contribute appreciably to the total absorption when blended with the still weak  $ZrO$  band.

These three band systems all arise from the normal electronic level of  $ZrO$ , and their strongest sequences originate in the lowest vibrational states also. Their behavior in absorption should, therefore, be consistent, and the observations indicate that this is so. Since the same situation holds for most of the  $TiO$  bands used in classification,<sup>2</sup> it is possible to compare the  $ZrO$  and  $TiO$  bands interchangeably in any or all of the spectral regions discussed earlier. This can be done by expressing estimates of band strengths in terms of the intensities of these bands in standard "type" stars, as is usually done in the case of line intensities in classifying stars of earlier type. In Table 1 the band intensities, in columns 8 to 10, have been carefully compared, usually on at least two good spectrograms of each star, and can be used as standards except where uncertain values are indicated by a colon. To minimize the embarrassment arising from the fact that we are necessarily dealing with a number of variable stars with variable absorption bands, we specify that in long-period variables the standard intensities and the classification refer to an average *normal maximum*.<sup>3</sup> Intensities at other phases also are recorded in Table 1 wherever observations are available. For semiregular and irregular variables the amplitude is small, and the standard intensity can be taken as applying to the star at *mean* brightness. In every case the normal range of brightness of the star is given in column 6, and the estimated brightness when the spectrum was observed is given in column 7. A range of magnitudes in column 7 indicates that intensity estimates from plates of different dates have been averaged.

The intensities of the bands of  $TiO$  are known to increase consistently with the subdivisions of type M (Adams 1925; Morgan *et al.* 1942), at least out to M8.<sup>4</sup> Consequently, it has been possible and convenient to define the intensity scale for  $TiO$  by adding one unit to the spectral subdivision of a normal M-type giant. Thus a  $TiO$  intensity of 4 in Table 1 indicates that these bands are as strong as in an M3 giant. The extra unit was added because at M0 the strongest bands are already strong enough to be seen clearly, even with low dispersion. For  $ZrO$  also the scale of intensities was chosen so that in the stars at intensity 1 the most sensitive bands are just distinctly visible on spectrograms having the dispersions suggested in the foregoing paragraphs.

In addition, intensities of the infrared bands of *lanthanum oxide* (Keenan 1948) have been added to Table 1 for the stars for which spectrograms on 1-N plates were taken. This has been done in order to bring the photographic infrared region of these very red stars into the plan of classification. In very strong S-type stars the  $LaO$  bands of the red system entirely dominate this part of the spectrum, and on the Perkins plates these bands have never been observed in a star which did not also show  $ZrO$ . Consequently, their presence appears to be another sufficient criterion for assigning a star to type S.

It happens, however, that the  $LaO$  bands fade out of the spectra of several long-period variables near the phase of maximum light, when the strongest sequence near  $\lambda$  7900 is replaced by some of the infrared bands of  $CN$ .<sup>5</sup> Figure 3 shows how the  $LaO$  bands stand

<sup>2</sup> The only exception among the important  $TiO$  bands is  $\lambda$  6479, which is a 4,2 band of the  $\gamma$ -system, originating in absorption by molecules in the second vibrational level.

<sup>3</sup> It is well known that when a long-period variable has an unusually faint maximum, the spectrum may resemble that normally observed at a phase far down the light-curve. Examples are the low maximum of Mira in 1923 as described by Joy (1926), and the one of R Gem in 1950 recorded in Table 1.

<sup>4</sup> Conspicuous exceptions seem to occur only near the minima of long-period variables, where overlapping band heads and the general "veiling" of the spectrum described by Merrill (1940, pp. 51 and 98) sometimes reduce the apparent intensities of even the strong  $TiO$  bands.

<sup>5</sup> The  $CN$  bands, just as in supergiants of types K and M, are accompanied by strong atomic lines which contribute appreciably to the blended absorption feature observed with low dispersion.



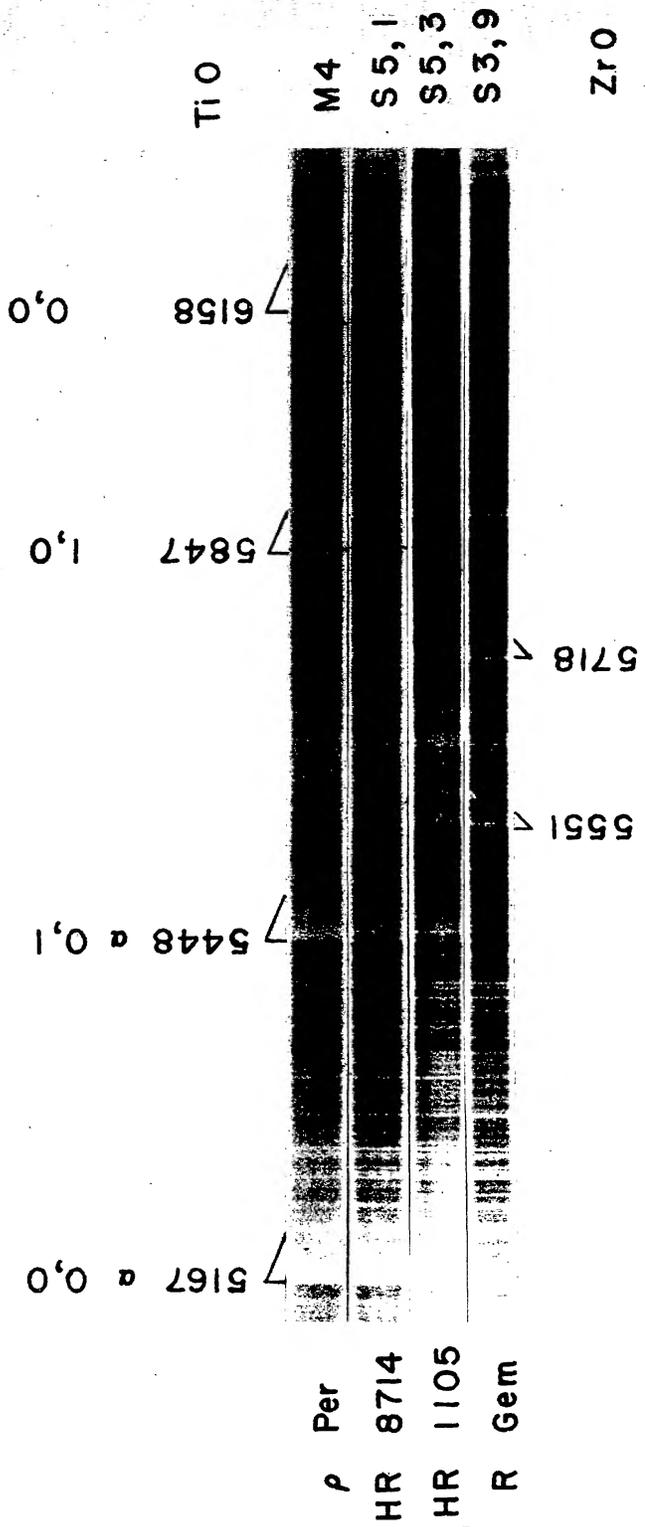


FIG. 2.—Transition from type M to type S in stars of about the same temperature. *f*/4 camera, giving scale of 80 Å/mm at λ 5600 on original negative.

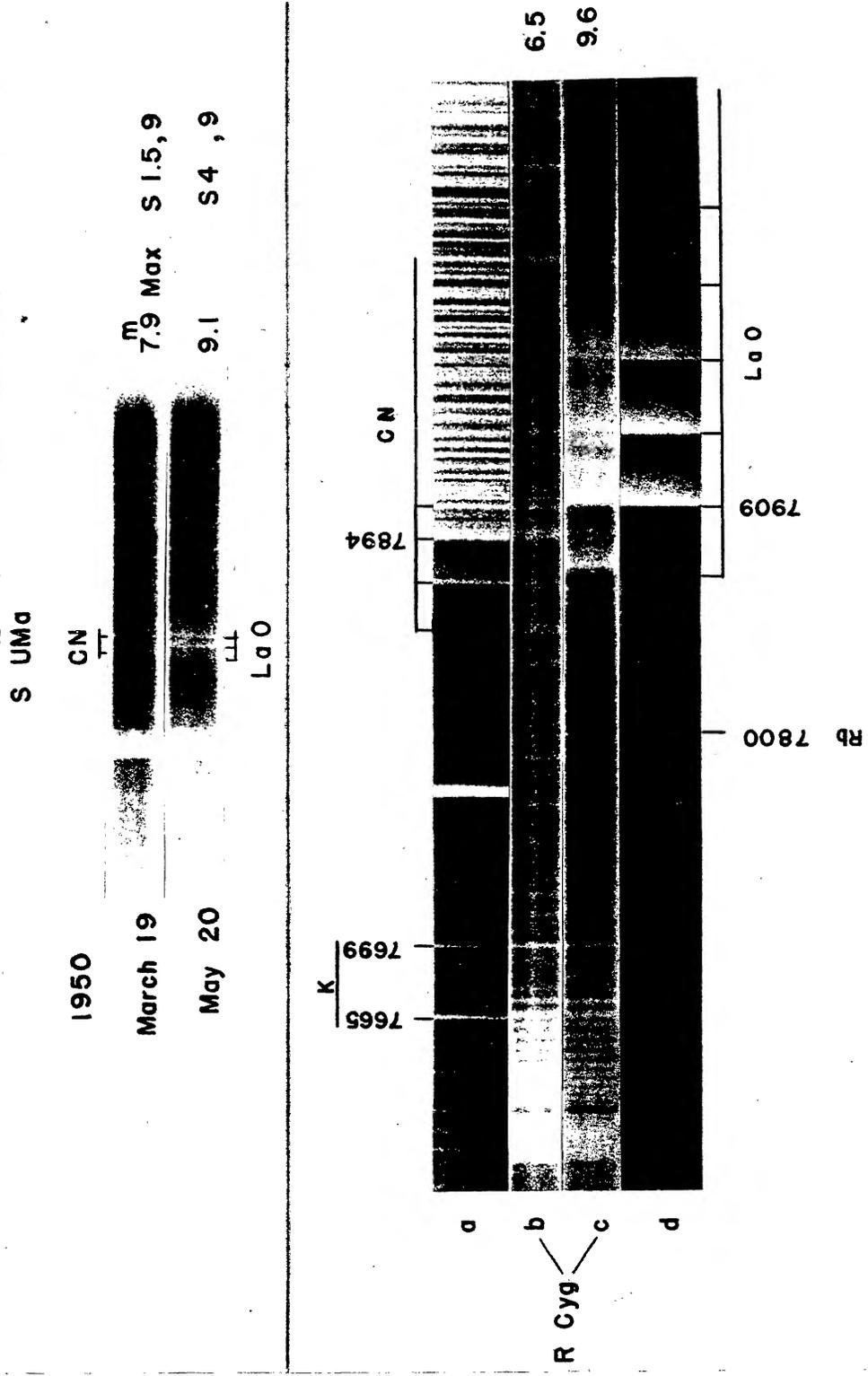


FIG. 3.—Variation of infrared  $L\alpha O$  bands with temperature in long-period variables. Upper exposures on S UMa with prismatic camera, giving scale of 255 A/mm at  $\lambda$  8000. Lower exposures with grating spectrograph, giving scale of 48 A/mm on original negative. Exposure *a* made with carbon arc, showing CN bands; *d*, with  $L\alpha O_3$  in iron arc. Exposures *b* and *c* show R Cyg on July 9, 1950, and August 10, 1949.

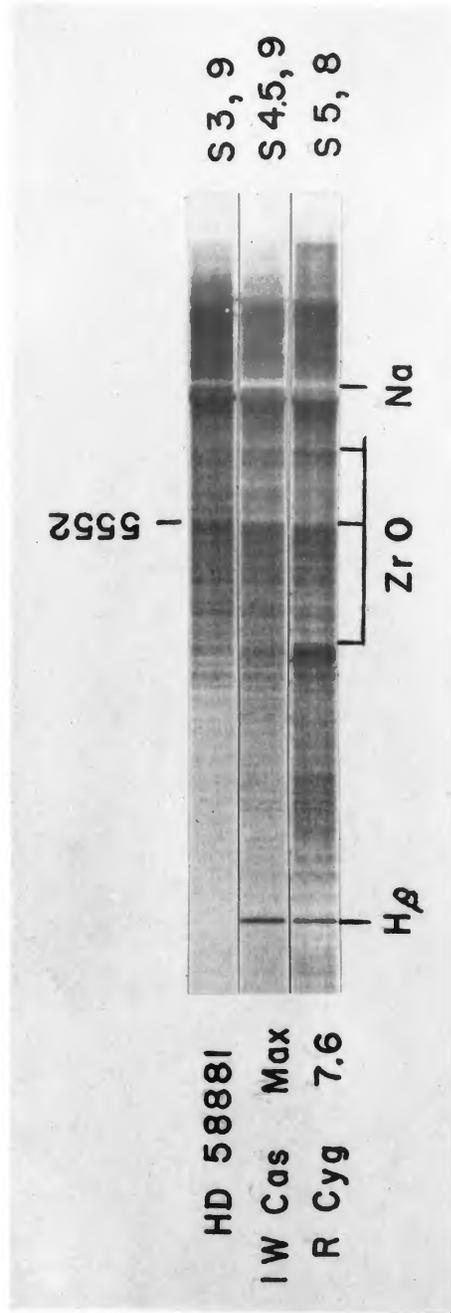


FIG. 1.—Variation of *ZrO* bands with temperature in nearly pure S-type spectra. *f*/1 camera, giving scale of 275 Å/mm at  $\lambda$  5600 on original negative. *H $\beta$*  emission is conspicuous in the two long-period variables.

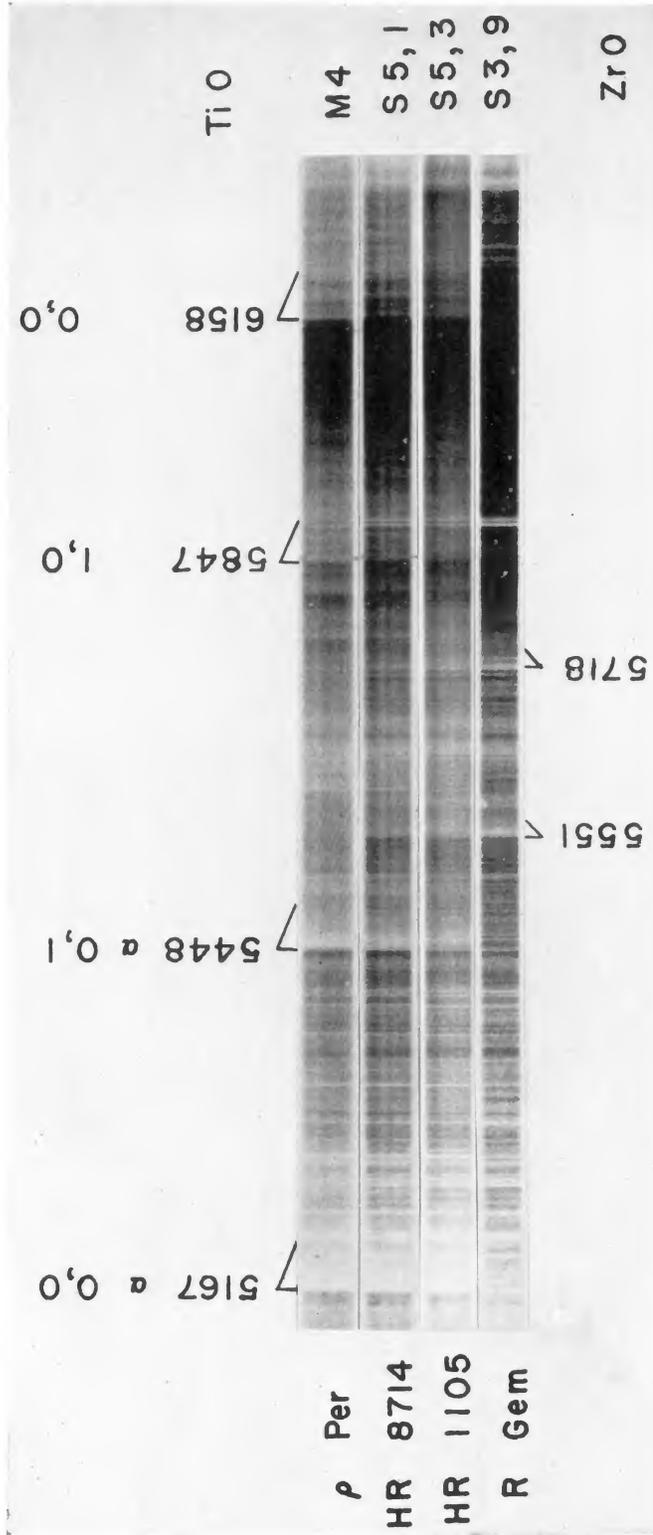


FIG. 2.—Transition from type M to type S in stars of about the same temperature. *f*/4 camera, giving scale of 80 Å/mm at  $\lambda$  5600 on original negative.

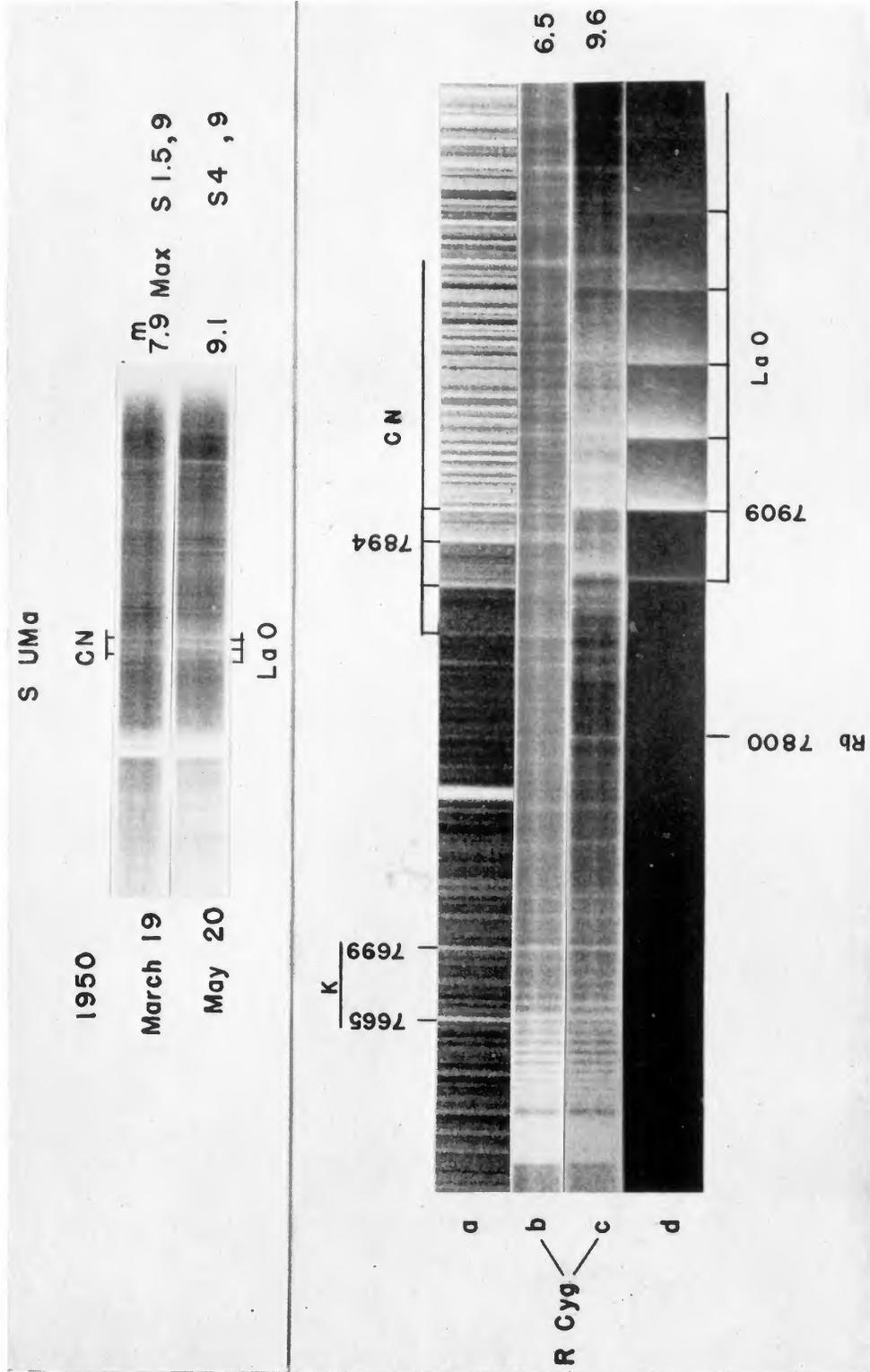


FIG. 3.—Variation of infrared  $LaO$  bands with temperature in long-period variables. Upper exposures on S UMa with prismatic camera, giving scale of 255  $\text{\AA}/\text{mm}$  at  $\lambda$  8000. Lower exposures with grating spectrograph, giving scale of 48  $\text{\AA}/\text{mm}$  on original negative. Exposure *a* made with carbon arc, showing CN bands; *d*, with  $La_2O_3$  in iron arc. Exposures *b* and *c* show R Cyg on July 9, 1950, and August 10, 1949.

TABLE 1

S-TYPE STARS BRIGHTER THAN  $m_V = 11$ 

STAR	HD	BD	$\alpha(1950)$	$\delta(1950)$	$m_V$		INTENSITIES			TYPE	NOTE
					NORMAL	OBSERVED	TiO	ZrO	LaO		
IW Cas	-	-	$0^{\text{h}} 00^{\text{m}}.2$	$+48^{\circ} 26'$	<u>11.5-(14.7</u>	Max.(8.6)	0	4.5	3:	S4.5,9e	1
						Pg.					
X And	1167	$+46^{\circ} 41a$	0 13.6	$+46^{\circ} 43'$	8.9-14.7	8.8-9.5	0	3	Strong	S3,9e	
R And	1967	$+37^{\circ} 58$	0 21.4	$+38^{\circ} 18'$	6.8-13.9	Max.(6.8)	3	4	1-:	S6,6e	
						10.5	5:	4:	-	S7,5e	
U Cas	4350	$+47^{\circ} 19a$	0 43.6	$+47^{\circ} 59'$	8.0-14.6	Max.(8.2)	3.5	3.5	0	S5,5e	
						8.9	4	4	0	S6,6e	
						9.0-9.6	5.5	4.5	-	S8,6e	
RR And	4895	$+33^{\circ} 116a$	0 48.7	$+34^{\circ} 02'$	9.3-14.1	9.5 >	6.5	2:	-	S7,2e:	2
S Cas	7769	$+71^{\circ} 66$	1 16.0	$+72^{\circ} 21'$	8.4-14.4	Max.(7.7)	2	3	-	S4,6e	
RZ Per	-	-	1 26.0	$+50^{\circ} 36'$	9.5-14	11.0: <	0	4	-	S4,9e	
-	-	$+51^{\circ} 471$	1 58.9	$+52^{\circ} 19'$	Var.?	9.4-9.9	5	2	Weak	S6,e	3
Vys 9	-	$+31^{\circ} 392$	2 15.2	$+31^{\circ} 31'$	Var.?	9.0-9.5	4.5	1.5	-	S5,1	4
BI And	-	-	2 22.9	$+37^{\circ} 55'$	9.5-11.5	9.7-9.9	4	6	6	S8,8	5
HR 1105	22649	$+62^{\circ} 597$	3 37.8	$+63^{\circ} 03'$	5.32	5.3	4	2.5	Very	S5,3	
									Weak		
WX Cam	-	-	3 45.3	$+53^{\circ} 03'$	10-11:	10.0-10.5:	1.5	4	-	S5,8	6
-	-	$+24^{\circ} 620$	4 02.6	$+24^{\circ} 36'$	9.2:	9.2:	3.5	2	-	S4,2	7
SX Cam	-	-	4 19.1	$+68^{\circ} 07'$	<u>11.0-14.3</u>	?	5:	3:	?	S6,3:	8
						Pg.					
Vys 2	-	$+22^{\circ} 700$	4 24.4	$+22^{\circ} 15'$	<u>11.5</u>	Pg. 10:	5	1.5	-	S6,1	9
WY Cam	-	-	4 30.5	$+79^{\circ} 02'$	9.5-11.0:	-	-	-	-	-	10
Dearborn 808	-	$+12^{\circ} 612$	4 32.9	$+12^{\circ} 33'$	10.0:	10.0-10.5:	5	1.5:	-	S6,1:	11
T Cam	29147	$+65^{\circ} 422a$	4 35.2	$+66^{\circ} 03'$	7.7-13.5	9.0:	1.5	3	-	S4,7e:	
						10.0	2:	-	7	S7,8e:	
-	-	$+79^{\circ} 156$	4 46.8	$+79^{\circ} 55'$	9.9:	9.9:	3.5	2	-	S4,2	12
TV Aur	-	$+48^{\circ} 1187$	4 54.4	$+48^{\circ} 29'$	9.2-10.3	9.7:	1:	4.5	-	S5,8:	
Vys 3	34738	$-22^{\circ} 1070$	5 16.6	$-22^{\circ} 16'$	8.8:	8.5	2.5	3	-	S4,5	13
-	35155	$-8^{\circ} 1099$	5 19.9	$-8^{\circ} 43'$	7.0	7.0	3.5	1.5	-	S4,1	
-	-	$+15^{\circ} 1200$	6 23.9	$+15^{\circ} 57'$	9.4:	9.4:	4.5	2	-	S5.5,2	14
DY Gem	-	$+14^{\circ} 1350$	6 33.1	$+14^{\circ} 14'$	<u>11-12:pg</u>	10:	5.5	4	-	S8,5	15
CX Mon	-	-	6 34.7	$+0^{\circ} 58'$	<u>11.8-12.7</u>	-	-	-	-	-	16
						Pg.					
-	49368	$+5^{\circ} 1414$	6 45.7	$+5^{\circ} 36'$	8.7:	8.7:	4	1.5	-	S5,1	17

TABLE 1 (Con't)

STAR	HD	BD	$\alpha(1950)$	$\delta(1950)$	$m_v$		INTENSITIES			TYPE	NOTE
					NORMAL	OBSERVED	TiO	ZrO	LaO		
R Lyn	51610	+55° 1154	6 <sup>h</sup> 57 <sup>m</sup> 2	+55° 24'	7.8-13.6	8.1-8.3 8.7 9.9	0 0.5: 1:	3 5: 5:	-	S3,9e S5,9e: S6,8e:	
R Gem	53791	+22° 1577	7 04.3	+22° 47'	7.1-13.1	6.8-7.2 7.3-7.8 8 (Ft.Max) 8.9	0 0 0 0	3 3.5 6 4	0 4.5 7.5 -	S3,9e S3.5,9e S6,9e S4,9e	
RR Mon	56567	-	7 15.0	+ 1° 11'	9.6-15.0	10.2:	8:	1.5	-	S7,2e:	18
-	58881	-11° 1941	7 24.7	-11° 37'	<u>10.0-10.7</u> Pg.	9.3:	0	3	-	S3,9	19
KZP 1098	-	-15° 1953	7 36.0	-15° 57'	<u>11-12</u> Pg.	10.3	3:	3.5	-	S5,6	20
SU Mon	62164	-10° 2171	7 39.9	-10° 46'	7.7-9.0	8.4-8.5	1	2.5	-	S3,6	
T Gem	63334	+24° 1778	7 46.1	+23° 59'	8.7-13.7	8.7 9.0 9.9 10.2	3 4 6 7	3 4 4 4	-	S4.5,4e S6,6e S8,6e S9,5e	
-	63733	-18° 2040	7 47.5	-18° 53'	8.5	8.5	2.5	2	-	S3.5,2	
-	64332	-11° 2121	7 50.6	-11° 30'	8.1	8.1	5	2.5	-	S6,3	
BD Mon	-	-	7 58.6	- 5° 29'	10.3-(12.4	-	7:	-	-	Se?	21
V Cnc	70276	+17° 1825	8 19.1	+17° 27'	7.5-13.0	7.9-8.1	0	2:	-	S2,9e:	
-	-	-28° 6970	9 06.9	-28° 48'	8.4	8.8:	-	4:	-	-	22
S UMa	110813	+61° 1313	12 41.8	+61° 22'	7.7-11.7	8.0: 8.7-9.0 10.0-10.6	0 0 0:	1.5 3 5:	0 4-: -	S1.5,9e S3,9e S5,9e:	
KZP 2016	-	+44° 2267	13 19.1	+44° 15'	<u>9.5-10.1</u> Pg.	10.2:	0:	2.5	-	S2.5,9: 23	
R Cam	127226	+84° 323	14 21.3	+84° 04'	8.3-12.6	8.1	0	2:	0:	S2,9e:	
ST Sco	149511	-30° 13283	16 30.2	-31° 02'	7.8-9.7	-	2:	3.5	-	S4,7:	
V635 Sco	156957	-41° 11533	17 15.3	-41° 39'	<u>9.5-10.7</u> Pg.	8.4:	5	5:	-	S7,6:	
-	-	+23° 3093	17 20.1	+23° 21'	10.1	10.1	4	3	-	S5,4	
VS12 Oph	-	-	17 39.0	+ 6° 45'	<u>11.8-12.9</u> Pg.	10.9:	5	2.5	-	S6,3	
V679 Oph	172804	+ 6° 3898	18 39.5	+ 6° 45'	9.0-9.3	9.0-9.3	1	4	1.5:	S4.5,8	
ST Sgr	176592	-	18 58.7	-12° 50'	9.4-(15	8.9-9.2	3.5	3:	-	S5,5e:	
-	177175	+12° 3780	19 00.8	+12° 11'	8.7	8.7	6	2.5	0:	S7,2	
W Aql	-	-	19 12.7	- 7° 08'	8.2-13.6	7.9-8.0	0	4	4	S4,9:	
T Sgr	180196	-17° 5546	19 13.4	-17° 04'	8.0-(13.1	8.0-8.1 8.8-9.5	1+: 2:	4.5 4.5	4.5 7:	S5,8e: S5,8e:	

TABLE 1 (Con't)

STAR	HD	BD	$\alpha(1950)$	$\delta(1950)$	$m_v$		INTENSITIES			TYPE	NOTE
					NORMAL	OBSERVED	T10	ZrO	LaO		
CE Vul	-	-	19 <sup>h</sup> 31. <sup>m</sup> 0	+23°30'	<u>12-14Pg.</u>	10.8:	4	6	Strong	S8,7	24
R Cyg	185456	+49°3064	19 35.5	+50°05'	7.3-13.8	6.4-6.5 7.0 7.1-7.9 8.0-8.9 9.0-9.9	0 0 Tr: Tr: 1:	3.5 0 5 6 6:	0 0 7: - 8	S3.5,9e S4,9e S5,8e: S6,8e: S6,8e	
Vys 12	-	-	19 37.1	+67°09'	9.7	9.7	0.5:	4	3:	S4,8	
$\chi$ Cyg	187796	+32°3593	19 48.6	+32°47'	4.9-13.3	5: 6: 8.4	7 8 9	0-2 0-2 1-2	- 0: 0:	S7,1e: S9,1e: S10,1e:	25
AA Cyg	190629	+36°3852	20 02.6	+36°40'	8.4-9.2	8.5-8.6 9.0-9.9	5 5	4 5	5 -	S7,5 S7.5,6	
S Cyg	-	-	20 04.4	+57°50'	9.8-15.2	10.0	4	2	-	S5,2e	
SZ Cep	193028	+76°784	20 11.2	+77°02'	9.3-14	-	-	-	-	Se:	26
Dearborn 6715	-	+0°4492	20 21.5	+0°47'	9.3-10.0:	9.3	6	2.5:-	-	S7,2	27
AD Cyg	195665	+32°3850	20 29.6	+32°24'	8.5-9.5	9.3	1.5	4	4	S5,8	
Z Del	195763	+16°4290	20 30.4	+17°17'	9.0-14.0	8.6-8.9 9.5 10.4	4 4.5 6	2 2 2:	- - -	S5,2.5e S5.5,2.5e S7,2e:	
BV Vul	-	+28°3944	20 55.1	+29°08'	10.5-11.5	10.6:	1.5:	6	-	S7,8	
V 471 Cyg	-	-	21 06.1	+38°22'	<u>11.8-13.2</u> Pg.	-	-	-	-	-	28
X Aqr	211610	-	22 16.0	-21°09'	8.2-(14	8:	-	-	-	S6,3e:	29
$\pi$ Gru	212087	CD -46°14292	22 19.7	-46°12'	5.8-6.4	-	2	3.5:-	-	S5,7:	30
SX Peg	-	-	22 47.9	+17°38'	8.7-13	9:	0	4	-	S4,9e	31
HR 8714	216672	+16°4833	22 52.1	+16°40'	6.48	6.48	4.5	1.5	Tr:	S5,1	32
BG And	-	+42°4690	23 28.7	+42°59'	<u>11.3-13.3</u> Pg.	9.3:	4.5	4	2	S6.5,5e	
WY Cas	-	-	23 55.4	+56°13'	8.8-14.6	Near Min.	-	-	Strong	Se	33
W Cet	224960	-15°6531	23 59.6	-14°57'	7.4-14.5	8:	5.5	3	0:	S7,3e	

## NOTES TO TABLE 1

1. IW Cas Vyssotsky 7 (1942).
2. RR And Vyssotsky 8 (1943), confirmed by Merrill.
3. BD +51<sup>o</sup>471 Suspected S by Merrill, Sanford and Burwell (unpublished). Position furnished by Sanford.
4. BD +31<sup>o</sup>392 Vyssotsky 9 (1943).
5. BI And Vyssotsky 15 (1946). In the First Supplement to the General Catalogue of Variable Stars by Kukarkin and Parenago, this star is classed as a long-period variable ( p = 220 days) of small amplitude. The Perkins spectrograms show no evidence of emission at H $\beta$  (1 plate) or the infrared Ca II lines.
6. WX Cam MSB 5 (1933).
7. BD +24<sup>o</sup>620 Vyssotsky unpublished (1953).
8. SX Cam Long-period variable usually below 11th mag. Recognized class S by Bidelman; classification from his McDonald red spectrogram of March 17, 1951.
9. BD +22<sup>o</sup>700 Vyssotsky (1942).
10. WY Cam Vyssotsky 10 (1943). No slit spectrograms available. Infrared CN suspected on slitless spectrogram by Blanco.
11. BD +12<sup>o</sup>612 Lee, Baldwin and Hamlin (1943).
12. BD +79<sup>o</sup>156 Vyssotsky unpublished (1953).
13. HD 34738 Vyssotsky (1943).
14. BD +15<sup>o</sup>1200 Vyssotsky unpublished (1953).
15. DY Gem Vyssotsky unpublished (1950:).
16. CX Mon MSB 24 (1933). No additional observations.
17. HD 49368 MSB unpublished. (See MSB 25 in Table 3).
18. RR Mon Not previously assigned to type S. Merrill (1941) described Sr II 4077 and 4215 as strong.
19. HD 58881 Harvard variable 100.
20. KZP 1098 Vyssotsky unpublished (1953).
21. BD Mon Vyssotsky 16 (1946). Vyssotsky assigned Davis class S5e, implying TiO stronger than ZrO. Objective-prism plate in infrared by Blanco shows very TiO bands.
22. BD -28<sup>o</sup>6970 ZrO strength estimated on McDonald spectrogram taken by Bidelman.
23. KZP 2016 Vyssotsky unpublished (1953). Galactic latitude +72<sup>o</sup>.
24. CE Vul ZrO bands discovered independently by three observers. Classified S by Rust (1938; MSB 67 (1942); Vys 19 (1946). No. IV: 3 in list of red stars by Hetzler (1937), who estimated IR index as 8<sup>m</sup>0.

## NOTES TO TABLE 1 (Con't)

25.  $\chi$  Cyg  
There are strong indications of differences in strength of the ZrO bands at the same phase of different maxima. T. Mao-Lin (1950) suggested that ZrO was relatively more prominent in 1948 than in 1944-5, when Merrill had observed the star. Perkins spectrograms show ZrO of approximately intensity 2 near the maximum of 1948. ZrO was apparently weaker in 1949 and almost absent at the 1947 maximum.
26. SZ Cep  
Infrared LaO observed as strong by Blanco.
27. ED +0°4492  
Lee et al (1943). Visual magnitude observed at Perkins as 9.3 on July 5, 1953, 10.0 on Sept. 14, and between 10.5 and 11.0 on Sept. 28. Variability of this star seems not to have been noticed previously.
28. V 471 Cyg  
Vyssotsky 6 (1942). Variability discovered by Ross and by Zinner.
29. X Aqr  
Vyssotsky (private communication, 1953) states that on a McCormick spectrogram of Sept. 28, 1945, ZrO 4640 was quite strong and that the general appearance of the bands of TiO and ZrO resembled that of HD 64332, classified as S6,3 here. On this date the visual magnitude of X Aqr was about 8 according to the AAVSO observations. The only Perkins spectrogram was taken on Aug. 19, 1953, at  $m_v = 10.1$  decreasing, and is a rather weak plate showing no clear evidence of ZrO, but very strong TiO. Possibly this star resembles  $\chi$  Cyg in variability of ZrO.
30.  $\pi'$  Gru  
Feast (1953) observed the Balmer Lines and several metallic lines in emission at times of decreasing brightness. Classified from McDonald and Radcliffe spectrograms.
31. SX Peg  
ZrO found by Bidelman, unpublished.
32. HR 8714  
Not previously assigned to type S.
33. WY Cas  
ZrO bands by Nassau et al, unpublished (1952).  
LaO bands observed as strong by Blanco.

out in the spectrum of R Cyg at  $m_v = 9.6$ , only to disappear almost completely as the star brightens by 3 mag. Actually, most of the change takes place within the last magnitude of the rise to maximum of such stars as R Cyg and S UMa (Fig. 3). Thus even an S-type star with very little *TiO* may or may not show the bands; and, although these bands have been shown by Nassau and his collaborators (1948, 1954) at the Warner and Swasey Observatory to be a powerful means of detecting faint S-type stars, they do not provide so complete a census of these stars as can be made by means of the more persistent bands of *ZrO* in the visual region. The usefulness of *LaO* is further limited by the fact that in stars with *TiO* intensities of 8 or more, the *TiO* is accompanied by bands of vanadium oxide, of which the strong infrared heads near 7900 Å (Keenan and Schroeder 1952) effectively blot out any *LaO* bands which might be present. This apparently happens in the spectrum of  $\chi$  Cyg during part of its normal light-cycle.

Whenever the *LaO* bands *can* be observed, however, their sensitive response to small changes in brightness makes them very valuable as indicators of *temperature*. Evidently, *LaO* is starting to decompose rapidly at the pressure and temperature ( $T_e \approx 2800^\circ \text{K}$ ) of an atmosphere such as that of S UMa just below maximum light, while *ZrO* is stable up to somewhat higher temperatures. The relative behavior of the bands of these two molecules is inconsistent with the published laboratory data, which indicate that the dissociation energy of *ZrO* is appreciably lower than that of *LaO*, but the laboratory estimates for these molecules may be in error by as much as 2 electron volts (Gayden 1952; Herzberg 1950).

The definition of type S can now be completed in terms of the intensity scales of Table 1. Necessarily the choice of the lower limit of intensity of *ZrO* is somewhat arbitrary. For the convenience of observers working with moderately low prismatic dispersions (100–200 Å/mm at  $H_\gamma$ ) we include in *class S all stars which have a ZrO intensity greater than 1*. There is also a marginal group of stars in which the *ZrO* bands are weaker (and usually overshadowed by the *TiO* bands) but can be seen distinctly with slightly higher dispersion. These stars are clearly intermediate in character between those of types M and S. They will be assigned the classification MS. Thus  $\sigma'$  Ori, which has *TiO* bands corresponding to type M4 with the addition of faint *ZrO* bands (intensity approximately 1), can be called “M4S” to indicate clearly its character. Fourteen MS stars in which the *ZrO* bands have intensities estimated between 0.2 and 1.0 are collected in Table 2.

Table 1 contains the 69 stars definitely assigned to class S. With the exception of a few southern stars for which spectrograms from McDonald or Mount Wilson were available, the table is limited to stars north of  $-23^\circ$  which attain visual magnitudes brighter than 11.0. The magnitudes in column 6 show the normal range in brightness for the stars that are known to vary in light. Column 7 gives the estimated brightness at the time of spectrographic observation. Where a range of values is given in this column the band intensities of the next three columns are means from several spectrograms taken on different dates.

For *ZrO* and *TiO* the band intensities are based chiefly upon Mount Wilson or Perkins spectrograms, but Yerkes spectrograms of the red region of several important stars were generously made available by Dr. W. B. Bidelman. The Mount Wilson plates used were those taken by Merrill between 1919 and 1945 ( $\lambda$  4600 region) and a few covering the red region (taken mostly by Sanford), together with about 45 spectrograms of the yellow region taken by the author (chiefly with the Cassegrain spectrograph on the 60-inch reflector) during the winters of 1951–1952 and 1952–1953. The Perkins material consisted of prismatic spectrograms having scales of 27, 54, or 104 Å/mm at  $H_\gamma$  and taken between 1946 and 1954. Most of the band intensities were estimated on exposures of the yellow region (chiefly on Eastman 103a-D plates) taken with the shortest camera, but a number of plates of the blue and red regions were taken to check doubtful cases.

Table 1 contains 24 stars not previously published as showing *ZrO* bands in their spectra. The inclusion of most of these was made possible by the generous co-operation of

TABLE 2  
STARS OF TYPE MS

STAR	HD	BD	$\alpha$ (1950)	$\delta$ (1950)	$m_p$	Observed	INTENSITIES	TYPE	REMARKS
					Normal		TiO		
							ZrO		
T Cet.....	1760	-20°50	0 <sup>h</sup> 19 <sup>m</sup> 2	-20°20'	5.2-6.0	.....	6-7	M5-6Se	
HR 363.....	7351	+28°196	1 11.3	+28 15	6.63	6.63	3.5	M2S	Ba II 4454 strong
W And.....	14028	+43°461a	2 14.3	+44 04	7.5-13.7	8.0:	8	M7-8Se	
.....	.....	- 2°891	4 21.8	- 2 39	9.3:	9.3:	3.5	M2S	*
4o' Ori.....	30959	+14°777	4 49.7	+14 10	5.19	5.19	4	M3S	
Vys. 5.....	49683	-20°1589	6 46.7	-20 22	8.9:	8.9:	5.5	M4S	*
AA Cam.....	54587	+69°413	7 09.5	+68 53	9.0-9.6/Pg.	7.8	6	M5S	
Y Lyn.....	58521	+46°1271	7 24.6	+46 06	6.3-7.4	.....	7	M6S	
HR 2967.....	61913	+14°1729	7 39.2	+14 20	5.81	5.81	4	M3S	
Dearborn 2660.....	.....	+ 6°2063	8 53.0	+ 6 24	8.9:	8.9:	4	M3S	
RS Cnc.....	78712	+31°1946	9 07.6	+31 10	5.6-6.8	6:	7	M6S	
ST Her.....	142143	+48°2334	15 49.3	+48 38	6.8-8.5	7.5-8	7-8	M6-7S	
HR 6702.....	163990	+45°2627	17 55.3	+45 21	6.22	6.22	7	M6S	
57 Peg.....	218634	+ 7°4981	23 07.0	+ 8 24	5.41	5.41	5	M4S	Dearborn S $\lambda$

NOTES TO TABLE 2

-2°891 ZrO bands found by Vyssotsky (1953 unpublished).  
 Vys 5 ZrO bands found by Vyssotsky (1942).

their discoverers, W. P. Bidelman, at Yerkes; J. J. Nassau and V. Blanco, at Warner and Swasey; R. F. Sanford, at Mount Wilson; and A. N. Vyssotsky, at Leander McCormick, who furnished positions (and in some cases field charts) in advance of publication. For these stars the notes at the end of the table attempt to indicate the observer who first noted the S-type character of the spectrum, but this could not be done with certainty for some of the stars which were independently discovered by several observers.

The  $LaO$  intensities in column 10 of Table 1 could not be estimated for all the stars, and where values are given, they are based mostly upon Perkins infrared spectrograms. These were taken either with the grating spectrograph giving a normal dispersion of 50 Å/mm or with a prismatic dispersion of about 250 Å/mm at 8000 Å. It was not found possible to obtain slit spectrograms of all the stars in the catalogue. This was especially true of long-period variable stars which happened to have their recent maxima at times

TABLE 3  
STARS OF TYPES K AND M WHICH WERE FORMERLY CLASSIFIED AS TYPE S

Star	HD	BD	$\alpha(1950)$	$\delta(1950)$	$m_v$	Revised		Remarks
						Type		
Dearborn 454...	.....	+ 7°393	2h 26m 4	+ 8° 11'	8.9	M4:		Dearborn S ?
Dearborn 794....	.....	+ 2°725	4 28.6	+ 2 14	9.8	M3		Dearborn S ?
MSB 25.....	264416	+ 5°1422	6 46.3	+ 5 39	9.1	M		MSB S ?
Dearborn 2029...	.....	.....	7 11.6	+ 0 10	9.7:	K		Dearborn S ?
Dearborn 2062...	.....	- 0°1665	7 13.7	- 1 08	9.9	M2.5		Dearborn S ?
Dearborn 3655...	.....	.....	15 08.7	+ 3 33	10.3:	M6:		Dearborn S ?
RW Lib.....	136734	.....	15 20.1	-23 53	9:	M4e		
Dearborn 3746...	.....	+10°2859	15 26.1	+10 39	9.7	M4		Dearborn S ?
Dearborn 4244...	155819	+ 5°3352	17 11.3	+ 5 51	8.7	M3		Dearborn S ?
Dearborn 5495...	.....	+ 4°4036	19 12.5	+ 4 45	10.2:	M3.5:		Dearborn S ?
Dearborn 8089...	.....	+ 8°5128	23 49.3	+ 8 48	9.5	M5.5		Dearborn S ?

NOTES TO TABLE 3

MSB 25 (HD Ex. 264416) Classified as M1 in *HD Extension* (*Harvard Ann.*, Vol. 100). 0.6 minutes east is the MS star HD 49368, which was suspected of S characteristics by Merrill, Sanford, and Burwell, but not included in their published lists.

Dearborn 3655 Both a Perkins slit spectrogram in the orange region and an objective-prism plate taken by Blanco in the infrared agree in showing the strong  $TiO$  bands, with no clear indication of either  $ZrO$  or  $LaO$ .

RW Lib Reclassified as M4e by Merrill (1952b) from coudé spectrogram taken with 200-inch reflector. RW Lib is of particular interest because it has the highest radial velocity ( $\approx 135$  km/sec) of any of the stars previously suspected of belonging to class S. A careful examination of the Mount Wilson prismatic spectrograms supports Merrill's reclassification; only  $TiO$  bands were definitely present on any of the plates of good quality.

Dearborn 5495 (BD+4°4036) Magnitude reported as variable by Dearborn observers. On the one Perkins spectrogram of the visual region the star appears rather red for its type.

when they could not be observed. For a number of these faint stars Dr. Victor Blanco kindly took slitless spectrograms on I-N plates with the 24-inch Schmidt telescope at the Warner and Swasey Observatory and lent them to the author for examination. The few remaining stars for which no band intensities are available are all stars for which published descriptions make their membership in class S fairly definite, and they have been included in order to make Table 1 a reasonably complete census of the brighter S stars.

A number of stars which have appeared in previous lists of suspected S-type stars showed no trace of  $ZrO$  bands on our program plates. Those which appear to belong to types K or M have been collected in Table 3, which, for completeness, includes also a few which had already been eliminated from class S by other observers. Most of the stars in this table are faint ones for which the earlier estimates of type were either based upon slitless spectrograms of extremely small scale or known to be doubtful for other reasons.

Consider, next, the problem of a more exact classification of the S-type stars of Table

1. The first introduction of subdivisions was made by D. N. Davis (1934). She divided the S stars into five groups (S1, . . . , S5) on the basis of the absolute and relative strengths of the *ZrO* and *TiO* bands. If we set aside her class S1 for the moment, the remaining four groups start with S2 for the stars with strong *ZrO* and almost no *TiO* and progress in order of decreasing relative prominence of the S-type characteristics to S5, in which the spectra have strong *TiO* and weak *ZrO*. That classification, though described by her as “necessarily preliminary,” has proved of practical value because it does provide a convenient description of the most conspicuous features visible on small-scale spectrograms of the blue or visual regions.

The usefulness of Miss Davis' arrangement is limited, however, by the fact that it is a one-dimensional scheme, in which the subclasses are probably sensitive to more than one physical variable, with *chemical composition*, perhaps, the dominant one. This last characteristic is important and should not be lost in any revised classification; but, since the main subclasses for all other kinds of stars are arranged to progress in order of decreasing *temperature*, it is worth while to attempt a classification within type S which will be consistent with the others and will take into account the available evidence bearing on relative temperature differences among these stars.

Since stars can be found representing every step in the transition from pure type M (strong *TiO*, negligible *ZrO*) to extreme type S (strong *ZrO*, negligible *TiO*), it is reasonable to try an ordering which will be consistent with the subclassification of type M. This is feasible, for the *ZrO* bands also strengthen with decreasing light in long-period variables and thus are consistent with the bands of *TiO* in their dependence upon temperature.<sup>6</sup>

Accordingly, the *temperature class* for a star of type S will be defined by *summing* the intensities of *ZrO* and *TiO*, expressed on our standard scale. A weighted sum is desirable because the simple sum gives total intensities which are inconsistently high for spectra in which both sets of bands are strong. The exact manner of weighting is not critical, for anyone using the final classification will naturally make direct comparisons with the type stars. The formula adopted for convenience is:

Temperature class = Intensity of the stronger set of bands + half the intensity of the weaker set.

For a star with *TiO* and *ZrO* intensities both equal to 3, for example, the temperature class is 4.5. In column 11 of Table 1 the temperature class is given by the *first* figure following the letter S.

The second parameter of classification should be a measure of the relative prominence of the lines and bands due to the heavy metals (such as *Zr* and *La*) to that of the lighter metals (such as *Ti* and *V*). This parameter can be called the *abundance class* and defined by the relative intensities of *ZrO* and *TiO*. The simple ratio, Intensity *ZrO*/Intensity *TiO*, generally decreases in the spectrum of any particular long-period variable as the light of the star drops from maximum toward minimum. This effect is largely due to the fact that the bands of *TiO* are more sensitive to decreasing temperature (within this range of temperatures) than are those of *ZrO*. This temperature effect can be approximately compensated by multiplying the *ZrO/TiO* ratio by the temperature class. It is then convenient to relate the abundance class to this product by Table 4.

The abundance class for each of the stars in Table 1 is given by the *second* figure following the S in column 11. It will be noticed that the abundance class shows no systematic change with phase for the long-period variables in Table 1. This is what we might expect if the class is really an indicator of composition of the star's atmosphere, and it suggests that the correction for the effect of temperature works well enough to be worth keeping.

<sup>6</sup> The veiling effects mentioned earlier occur only near the minima of long-period variables and do not seriously affect the classification of these stars at other phases of their light-curves.

It should be emphasized, however, that in calling the second index figure an abundance class we are merely giving it a convenient label—the questions of just which elements differ in abundance and of how large the differences are remain to be investigated on spectrograms of high dispersion.

If the types of Miss Davis are plotted against either the temperature classes or the abundance classes, there is evident some correlation with each of them, except that her S4 stars show a wide scatter in the new classes. This is an added indication that her ordering involved both variables—temperature and composition.

There are two tests that can be applied immediately to our temperature classes. For two long-period S-type variables, radiometric temperatures at both maximum and mini-

TABLE 4

## ABUNDANCE CLASSES

$ZrO/TiO \times$ Subtype	Abundance Class	$ZrO/TiO \times$ Subtype	Abundance Class	$ZrO/TiO \times$ Subtype	Abundance Class
1.0–1.9.....	1	4.0–4.9.....	4	7.6–9.9.....	7
2.0–2.9.....	2	5.0–5.9.....	5	10.0–50.....	8
3.0–3.9.....	3	6.0–7.5.....	6	>50.....	9

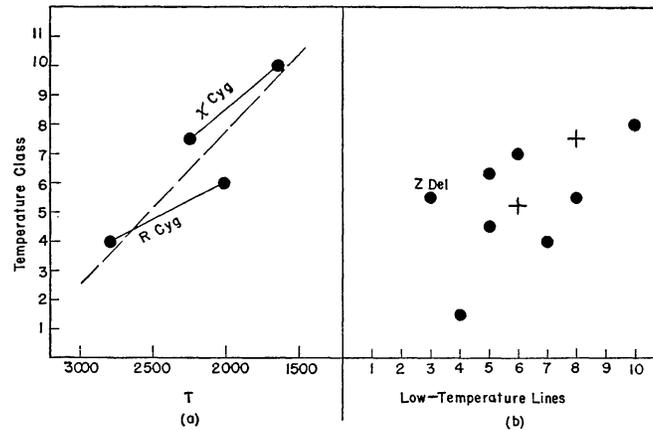


FIG. 4.—Temperature class plotted against radiometric temperatures (*a*), and intensities of low-level atomic lines (*b*).

mum were determined by Pettit and Nicholson (1933). These are plotted against temperature classes in Figure 4, *a*. A slight extrapolation was needed to give the temperature classes at minimum magnitude.

The figure shows an appreciable discrepancy between the values for  $\chi$  Cyg at maximum and R Cyg at minimum; this is not surprising, for it might be expected that the calibration of the classes in terms of absolute temperatures would show some variation in passing from a nearly pure S-type star (R Cyg) to one like  $\chi$  Cyg in which  $TiO$  is very strong and the S characteristics are quite weak. Nevertheless, the correlation is good enough to suggest use of the dashed line in the figure to estimate approximate temperatures for the stars in Table 1. Radiometric temperatures for more of the brighter S stars are badly needed.

The other test is made in Figure 4, *b*, by comparing temperature classes with the intensities of a group of “low-temperature” lines of neutral atoms as estimated by Merrill (1952) from coude spectra of 10 S-type stars. Intensities of the blue bands of  $TiO$  and  $ZrO$  were given by Merrill for these same spectrograms and have been transferred to the

intensity scales of Table 1 in assigning temperature classes. Again the correlation is fair, with Z Del the most discrepant star in the plot.

Further refinement of the temperature classification appears unprofitable until more is known of the effective temperatures of the S stars. Ordinary colors measured through absorption filters are not reliable indicators of temperature for any of the red stars with banded spectra; a glance at Figures 2 and 3 is enough to show that an "infrared index," for example, of such stars is likely to be as much a measure of differing patterns of absorption bands as of temperature gradients in the continuous background. Radiometric temperatures, on the other hand, are significant because they are based upon integrations of such wide stretches of spectrum that even whole systems of absorption bands are averaged out.

At the other extreme it would be possible to determine fairly significant temperatures by measuring monochromatic colors at wave lengths chosen to avoid the strong bands. For this purpose "monochromatic" could be defined in practice as referring to a strip of spectrum not more than, say, 50 Å in extent. Some of the most promising of such windows between terrestrial and stellar absorption bands are listed in Table 5. Even within

TABLE 5  
SPECTRAL REGIONS USABLE FOR MEASUREMENT OF MONOCHROMATIC  
COLOR TEMPERATURES IN LATE-TYPE STARS

Region	$\lambda\lambda$	Remarks	Region	$\lambda\lambda$	Remarks
Violet.....	4000-4080	Cont. absorption in carbon stars later than C4	Yellow.....	5790-5840	Some CN in late carbon stars
Blue-green...	4900-4950	Some ZrO in strong S stars; C <sub>2</sub> in strong carbon stars	Infrared.....	7500-7580	Some CN in carbon stars; VO in M7-M9 stars
Green.....	5650-5700	Some band absorption in late S and M stars	Infrared.....	8680-8750	Fairly strong CN in carbon stars

these selected strips nothing approaching a continuum is to be found, and the remarks in the third column of the table mention the sources of the molecular absorption most likely to be present. Nevertheless, these regions are far more transparent than the average and should allow significant estimates of the very large color differences which accompany moderate temperature changes in the range below 3000° K. The regions have been chosen to avoid also the strongest emission lines occurring in the spectra of long-period variables—the lines of the Balmer series and of Ca II.

Up to this point the intensities of the infrared bands of LaO have not been used in assigning types. The extreme sensitivity of LaO to temperature (Fig. 3) suggests that the weighted sum LaO + TiO should correlate well with ZrO + TiO. The plot of the two sums in Figure 5, *a*, shows that LaO + TiO can be used to estimate temperature classes for any of the S-type stars in which LaO is observable. Reduction of the LaO intensity sums (strong +  $\frac{1}{2}$  weak) to temperature classes is given in Table 6, though again, in practice, observers will probably find it more convenient to classify by direct comparisons with spectra of standard stars.

The subdivisions of type S in which the LaO bands are strong enough to be observable with moderate dispersion are shown by the crosshatched area in Figure 5, *b*. The bordering zone with single hatching indicates roughly the region of ambiguity—spectra in which the LaO bands are close to the threshold of visibility and might or might not be seen, depending partly upon the quality of the spectrograms. The area of visibility of LaO has about the shape that would be expected, for in spectra with low abundance classes the LaO becomes increasingly hard to detect under the overlapping bands of TiO and VO, which characterize late M-type stars. In even a pure S-type star LaO is not observed if the temperature is greater than about 2900°.

Figure 5, *b*, was plotted from estimates on Perkins slit spectrograms, taken with either grating or prism. Fortunately, the zone of doubtful observability is relatively narrow, because the intensity of *LaO* does change so rapidly with temperature. For the same reason the area of visibility has nearly the same extent even on slitless spectrograms of very small scale. This is concluded from inspection of the Warner and Swasey I-N spectrograms taken by Nassau and Blanco. This fact is important, for it means that Figure 5, *b*, can be used by observers conducting infrared surveys of red stars with

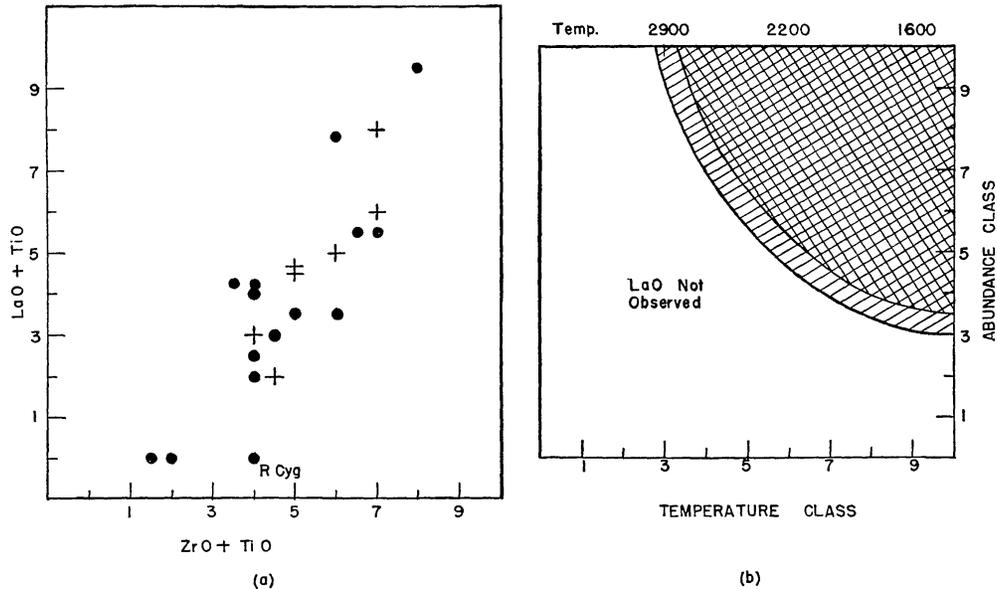


FIG. 5.—Behavior of infrared *LaO* bands. *a*, Correlation of *LaO* + *TiO* intensity with *ZrO* + *TiO*. *b*, Crosshatched area shows range of temperature and abundance classes within which *LaO* is easily observed.

TABLE 6

ESTIMATION OF TEMPERATURE CLASSES FROM INTENSITIES  
OF INFRARED *LaO* BANDS

<i>LaO</i> + <i>TiO</i>	Temp. Class	<i>LaO</i> + <i>TiO</i>	Temp. Class	<i>LaO</i> + <i>TiO</i>	Temp. Class
0.5–2.2	3	3.7–5.2	5	6.8–8.2	7
2.3–3.6	4	5.3–6.7	6	8.3–9.5	8

Schmidt telescopes to estimate how many subdivisions of type S they can detect. When the frequencies of occurrence of the different subdivisions have been well established, the counts on slitless spectrograms can then be used to give the total numbers of S stars in the areas surveyed.

LUMINOSITIES

The spectroscopic criteria of luminosity which are used for M-type stars cannot be applied to S-type stars, at least not until some independent means of recalibrating them are found, for the most sensitive lines (particularly *Sr* II 4077 and 4215) are among those which appear to reflect most strongly the abundance differences among the metals. Trigonometric parallaxes fail likewise to give even group means for the luminosities of these giant stars, none of which is close enough to have an apparent magnitude brighter than 5. Nor has it proved practicable to use the method of interstellar-line intensities,

for the radial velocities of the S stars have not been found to be large enough to separate the interstellar and stellar D lines, as R. F. Sanford (1944) was able to do for several N-type stars of large velocity.

The only individual S-type star for which the luminosity has been determined is the bright southern star  $\pi'$  Gruis. M. W. Feast (1953) obtained spectrograms of the companion of magnitude 10.9 at a separation of  $2''.7$  and showed that the primary and companion had the same radial velocity within the uncertainty of measurement. The absence of any observed relative motion between the two stars over the preceding half-century was further evidence that the pair form a physical system. From Feast's classification of the companion as a G0 V star, he was able to estimate the visual absolute magnitude of  $\pi'$  Gruis as between  $-1$  and  $0$ . Since no other favorable cases of S stars as members of binary systems are known, it is necessary to turn to statistical methods for average luminosities of groups of these stars.

Wilson and Merrill (1942) compared the radial velocities and proper motions of 17 long-period variables to derive their mean luminosity of  $M_v = -1.0$ . Their solution can now be repeated with slightly improved data. Two of their stars, R Ori and R CMi, do not show enough evidence of  $ZrO$  to be classed as type S under our definition. On the other hand, RR And is now included in type S, and the proper motion of W Cet is available from the Yale catalogue, giving again a total of 17 stars. Mean parallaxes were computed separately from  $\tau$ - and  $\nu$ -components by the usual expressions. For the  $\tau$ -components the mean tangential velocity of 19.3 km/sec was obtained from the radial velocities of the same stars after the contribution due to galactic rotation was removed. The results are:

$$\begin{aligned} \bar{p}_\tau &= 0''.00154, & \bar{p}_\nu &= 0''.00215, & \bar{p} &= 0''.00184; \\ \bar{m}_\nu &= 8.1, & \bar{M}_\pi &= -0.5, & \bar{r} &= 543 \text{ psc}. \end{aligned}$$

After corrections for galactic absorption of  $-0.3$  mag. and for dispersion of 5 ( $\log \bar{p} - \log p$ )  $\approx -0.2$  mag. are applied, we obtain

$$\bar{M} = -1.0,$$

with an uncertainty of several tenths of a magnitude, due as much to the possible systematic errors in the corrections as to scatter in the measured motions. The agreement with the mean value found by Wilson and Merrill thus implies only that the mean visual absolute magnitude of the S-type long-period variables lies in the neighborhood of  $-1$  and does not appear to differ significantly from the luminosity of the average long-period variable of type M5–M8. The mean residual radial velocity of our group of S-type variables is  $19.3 \pm 4.1$  km/sec (m.e.).

In addition to the 31 long-period variables, Table 1 contains 29 S-type stars which are either variables of small amplitude or stars for which the variability of light has not been established. For only 6 of the stars in this group are radial velocities and proper motions available. Until velocities can be measured accurately for more of these faint red stars, the best clues to their luminosity appear to be offered by their galactic distribution. In Table 7 the mean galactic latitudes for the long-period Mira-type variables and for the variables of small amplitude are shown separately for several ranges of apparent visual magnitude. The magnitudes refer to maximum light for long-period variables and to mean light for the remaining stars. The figures in parentheses are the number of stars in each group. Corresponding figures for the carbon stars, separated into types R and N, were derived for the stars in Sanford's catalogue (Sanford 1944), which have fairly well-determined visual magnitudes. Since the S stars are definitely a low-velocity group (none is known to have a radial velocity exceeding 100 km/sec), the well-known group of high-velocity carbon stars were excluded from the comparison by counting only those with  $V_r < 60$  km/sec.

Although Table 7 represents a very crude way of expressing galactic concentration, it does permit suggestive comparison, and the numbers of S-type stars are too small to justify more elaborate analysis.<sup>7</sup> The mean latitudes of the variables of small amplitude are generally lower than those of the long-period variables. This merely shows that type S shares the known tendency of long-period variables to scatter widely from the galactic plane and is a reminder that the luminosities may differ considerably in stars showing different kinds of variability. The more significant figures are in the last three columns, which show that the galactic concentration of the small-amplitude variables of type S is probably slightly less than that of similar N-type stars but appreciably greater than the concentration of the R-type stars. Sanford (1944) brought together the evidence from radial velocities and from interstellar D-line absorption to estimate the luminosities of

TABLE 7  
MEAN GALACTIC LATITUDES

TYPE	LONG-PERIOD VARIABLES				SMALL-AMPLITUDE VARIABLES			
	5 <sup>m</sup> -8 <sup>m</sup>	8 <sup>m</sup> 1-9 <sup>m</sup> 0	9 <sup>m</sup> 1-10 <sup>m</sup> 0	10 <sup>m</sup> 1-11 <sup>m</sup> 0	5 <sup>m</sup> -8 <sup>m</sup>	8 <sup>m</sup> 1-9 <sup>m</sup> 0	9 <sup>m</sup> 1-10 <sup>m</sup> 0	10 <sup>m</sup> 1-11 <sup>m</sup> 0
S.....	27° (9)	22° (11)	16° (7)	21° (1)	22° (3)	+8° (7)	17° (13)	14° (7)
R.....	28 (2)	30 (4)	....	....	17 (8)	26 (13)	21 (20)	27 (13)
N.....	28 (6)	22 (11)	4 (4)	....	21 (31)	9 (33)	7 (50)	8 (9)

the carbon stars. His data indicate that for type R,  $\bar{M}_v \approx -0.5$ , and for type N,  $\bar{M}_v \approx -2.0$ . The galactic concentration of the group of S stars suggests that their mean absolute magnitude lies between these two values.

A further inference which may be drawn from the similarity in the general distribution of the S stars and the late carbon stars is that the S-type variables of small amplitude are likely to be associated with the spiral arms of our galactic system. The absence of large space motions among these stars is consistent with this conclusion. Another test can be made by adopting a preliminary estimate of  $-1.5$  for their mean absolute magnitude and plotting their corresponding distances as projected upon the galactic plane. In Figure 6 this has been done for all stars in the group except the two which have galactic latitudes exceeding  $30^\circ$ .

On the diagram can be traced roughly the spiral arm coming in between  $0^\circ$  and  $50^\circ$  longitude and extending past the sun to longitudes  $180^\circ$  and  $220^\circ$  which has been mapped by Morgan *et al.* (1953) from aggregates of supergiant stars and by Oort and his collaborators (Van de Hulst 1953) from the velocities of the 21-cm hydrogen line. In addition, there is a remarkable gap between longitudes  $60^\circ$  and  $170^\circ$ , where no S stars occur at distances between 200 and 900 parsecs. Beyond this gap lies a group of 13 or 14 stars, and it looks as though at least some of these were in the outer branch of the local arm, which, from Morgan's data, lies in the direction of the anticenter, at distances of from 900 to 1400 parsecs from the sun. The gap in Figure 6 then would correspond to the space between the two branches of the local arm. If this identification is correct, no major correction to our tentative value of  $-1.5$  for  $\bar{M}_v$  is indicated.

The alternative possibility that this group lies in the outer spiral arm known to cross the anticenter at a distance of about 2400 parsecs appears to be ruled out by the distribution of the S stars in galactic latitude. None of the supergiant aggregates in that arm, which have been studied by Morgan and others, lies at galactic latitudes much exceeding

<sup>7</sup> The statistical discussion of the distribution of S stars which was carried out by Ikaunieks (1950) is not useful for our purpose, since, out of the 88 possible S stars in his list, 17 have already been removed from the group by our classification and at least another half-dozen are of very doubtful membership. His methods may prove of value, however, when the number of known S stars becomes much larger as a result of surveys with Schmidt telescopes.

5°. Several of the S stars in our group, however, are considerably farther from the galactic plane; and a few, such as the three stars in Taurus (+24°620, +22°700, +12°612), at latitudes of  $-17^\circ$  to  $-21^\circ$ , may be in the nearer parts of the local arm. These latter stars, then, would have luminosities appreciably below the mean and close to the estimates made by Feast for  $\pi'$  Gru. This is the first evidence indicating the real dispersion in absolute magnitudes which probably exists within type S. Another star which may have a low luminosity is BD+44°2267 ( $m_v \approx 10$ ; type S recognized by Vyssotsky 1953, unpublished), which is remarkable for its situation only  $18^\circ$  from the galactic pole. It will be important to determine the space motion of this exceptional star, which would lie more than 1300 parsecs above the galactic plane if its absolute magnitude were  $-1.5$ . A plot of the long-period variables of type S projected on the galactic plane as in Figure 6 shows little more than a random scatter.

The evidence bearing on the luminosities of the S-type stars as a whole is still too scanty for final conclusions but suggests that most of them belong among the giants and

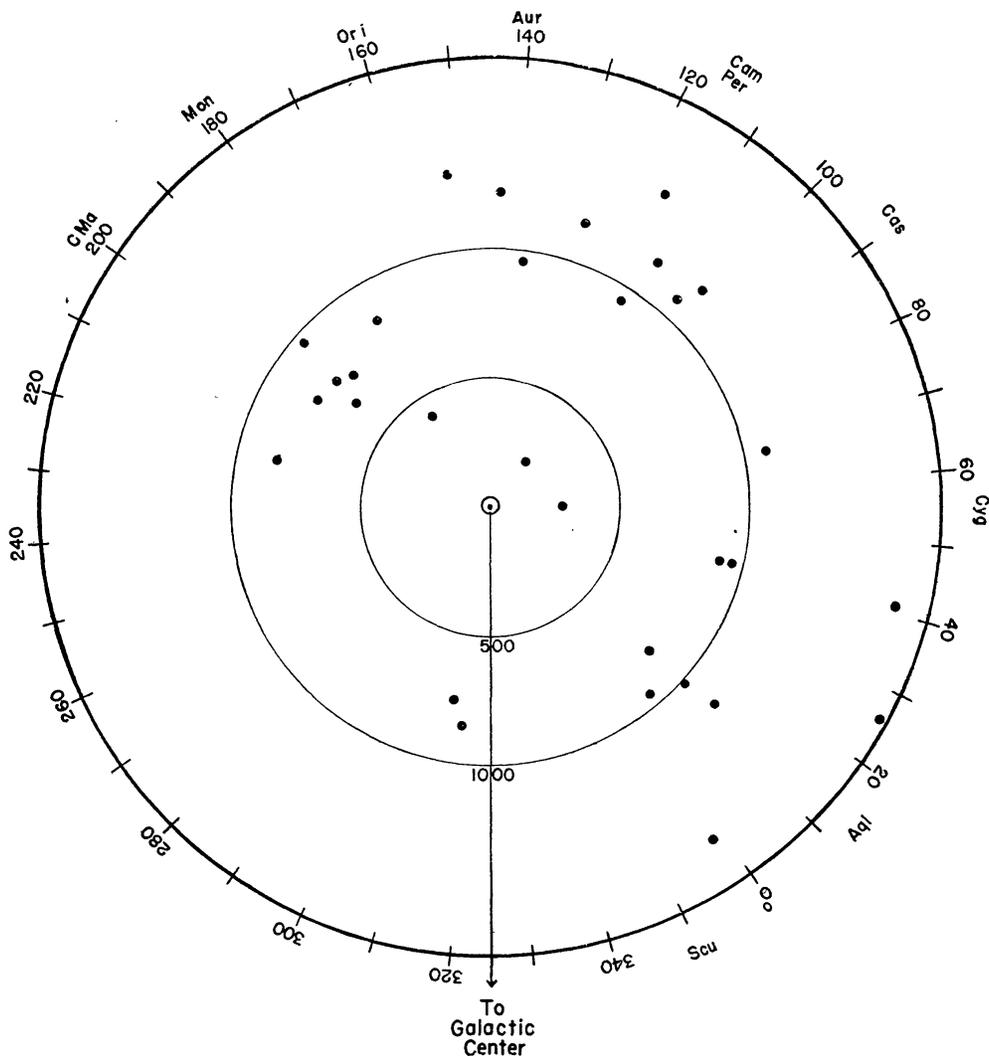


FIG. 6.—Projected distribution of S stars on galactic plane (long-period variables omitted). The blank sector from  $230^\circ$  to  $310^\circ$  is the part inaccessible to observation from northern latitudes. Distances are given in parsecs on the assumption that  $\bar{M}_v = -1.5$ .

bright giants (luminosity classes III and II) rather than among the supergiants, where earlier ideas (present author's included) had placed them.

#### FREQUENCY OF OCCURRENCE

The S-type stars are recognized as relatively young on the galactic time scale because of their similarity to other "population I" objects in distribution and motions. In addition, the considerable strength in their spectra of the lines of  $Tc\ I$  (Merrill 1952*a*, p. 23), which has a half-life of the order of a quarter of a million years, suggests that they represent a relatively short-lived stage of evolution. The presence of MS transition stars shows their close relationship to type M. The relative frequency of occurrence of types M and S will therefore provide an important datum for testing any evolutionary theory seeking to explain their presence.

Since we lack precise knowledge of the masses and luminosities of the S-type stars, the best that can be done at this time is to compare them with the whole giant branch of type M. This is the more necessary, since individual luminosities are known for only a portion of the M-type stars.

TABLE 8  
RELATIVE NUMBERS OF LATE-TYPE STARS

TYPES	LONG-PERIOD	OTHERS	
	VARIABLES ( $m_v < 10$ )	$m_v < 8$	$8.0 \leq m_v < 10$
M3-M8.....	76	121	61
MS.....	4	8	3
M.....	82 (59)	308 (84)	155 (45)
MS.....	4 (3)	20 (5)	8 (2)
S.....	25 (18)	3 (1)	26 (8)
C.....	27 (20)	37 (10)	156 (45)

A search for MS stars was carried out on the prismatic spectrograms of the Mount Wilson and Perkins collections. The survey of the Mount Wilson plates covered the first 18 hours of right ascension and was limited to those negatives on blue-sensitive emulsions which had moderate density in the 4600 Å region, plus a few additional spectrograms covering the yellow part of the spectrum.

In making the search for MS stars, it soon became apparent that they could be detected efficiently only in stars cool enough to develop the more intense  $TiO$  bands to at least the strength corresponding to an M3 giant. Otherwise, the still fainter  $ZrO$  bands would remain below the threshold of visibility on these spectrograms of only moderate dispersion. For this reason the counts of types M and MS in the first two rows of Table 8 were limited to the range M3-M8. In order to compare these figures with all the known S-type stars, which, of course, include many with very weak  $TiO$  bands, they must be corrected for the fraction of M0-M2 stars in type M. This has been done for the next two rows of Table 8, where the percentages of the totals in each column have been given also (in parentheses). The correction factor for the first two columns was 2.55, obtained by counting the numbers in each subdivision of type M in the Mount Wilson catalogue of spectroscopic absolute magnitudes (Adams *et al.* 1935). For the long-period variables the much smaller correction factor, 1.08, obtained directly from Table 4 of Merrill (1941), reflects the relative scarcity of long-period variables in the early subdivisions of type M. Counts of carbon stars in Sanford's catalogue (1944) have been added in the last row of the table.

In the last two rows of Table 8 no distinction has been made between nonvariable stars and variables of small amplitude, since among these cool stars many not yet listed as variable will eventually be found to show changes in their light. Below the eighth magnitude the percentage of variables included in the sample is greater, for many of the spectrograms in both collections were taken in the course of studies of the semiregular and irregular variable stars.

Serious selection effects undoubtedly remain in Table 8. The last column has little significance, for the ease of discovery of even the faint carbon stars has made the count relatively complete for them, while the spectrograms of only a small fraction of the M-type stars were available for examination. The same overemphasis is present in the figures for type S, but to a lesser extent, since these stars are not so easily discovered on slitless search spectrograms. These effects are much less pronounced in the second and third columns of Table 8; though the percentages of carbon stars are somewhat too high even there.

The percentages for the stars brighter than the eighth magnitude are more significant. Besides confirming the known infrequency of S stars compared to carbon stars, they show that an appreciable fraction of the stars have the intermediate MS characteristics. The actual figures for the MS group depend upon our somewhat arbitrary definition of the limiting *ZrO* intensity and would be larger if all the spectra in which the *ZrO* bands can be definitely seen with moderately high dispersion were included; but the important point is that the tendency toward strengthening of the bands of the heavy oxides is not rare among the cool stars.

In striking contrast, the long-period variables, among which definite S-type stars are relatively common, as has been shown by other counts (Merrill 1940), include only about the same percentage of MS stars as are found in the other groups, as nearly as can be judged from such a small sample population.

## STARS WITH SPECTRA RELATED TO TYPE S

In Table 9 are collected seven red stars whose spectra show some similarity to type S in the intensity of their atomic lines, but without any positive indications of bands of

TABLE 9

## STARS WITH SPECTRA RELATED TO TYPE S

STAR	HD	BD	$\alpha(1950)$	$\delta(1950)$	$m_s$		$V_r$	<i>Ba</i> II 4454	<i>Sr</i> 4607	<i>Na</i> D	<i>CN</i> (IR)
					Normal	Observed					
W Cas.....	5235	+57°165	0 <sup>b</sup> 51 <sup>m</sup> 9	+58° 17'	8.5-11.8	8.0	-39	5	3	...	...
						8.5	...	...	...	...	8
						9.9	...	...	...	7:	...
R Ori . . . . .	31798	+ 7°768	4 55.7	+ 8 03	8.7-13.5	...	...	...	...	...	...
GP Ori . . . . .	.....	+15°726	4 59.9	+15 14	11.8-13.6	9.8-9.9	+79	4:	12:	10	5.5
					/Pg.						
FU Mon.....	44544	+ 3°1214	6 19.8	+ 3 28	11.6-12.7	8.6-9.0	....	5:	10:	8	...
					/Pg.						
R CMi . . . . .	54300	+10°1428	7 06.0	+10 06	7.5-10.8	8.2:	+48	5:	3:	4	...
						8.7	...	...	...	...	6
						9.4	...	...	...	10	...
.....	121447	-17°3961	13 53.1	-18 03	8.1	...	...	7:	4	5	4:
CY Cyg.....	198164	+45°3271	20 45.2	+45 52	9.9-11.7	8.6-8.9	+ 4	4:	4	7	5
					/Pg.						

## NOTES TO TABLE 9

- W Cas Infrared *CN* bands and Swan bands of  $C_2$  observed by Bidelman at Yerkes and by Keenan at Perkins (1950).
- R Ori Carbon features, including the  $C_2$  band at 4737 Å, noted as strong by Vyssotsky in report to Tonanzintla symposium (1942).
- GP Ori Remarkably strong D lines noted by Baldwin and Hamlin (1940, Pl. 4). A low-dispersion slit spectrogram taken by Morgan, reproduced in Pl. 55 of the Yerkes *Atlas of Stellar Spectra*, shows the differences between this spectrum and typical ones of types S and N. The observations of its spectrum have been summarized by Bidelman (1950), who tentatively assigned it to type S. McKellar and Stillwell (1944) and Sanford (1949), however, considered it a late carbon star on the basis of the strong *CN* bands. A Perkins infrared grating spectrogram (scale 100 Å/mm) taken at  $m_s = 10.0$ : on December 7, 1952, shows no evidence of *LaO*. The band absorption between  $\lambda$  7500 and  $\lambda$  8700 appeared to be entirely due to *CN*, except for an absorption feature near  $\lambda$  8610, which showed some similarity to the unidentified band observed at that position in very cool S-type stars.
- FU Mon Strong D lines observed by Baldwin and Hamlin (1940, Pl. 3). Similarity to GP Ori discussed by Bidelman (1950). Prismatic spectrogram on 103a-G emulsion, February 22, 1953, with 18-inch camera on Mount Wilson 60-inch reflector, shows a number of strong lines in the region of  $\lambda$  5500, but the *ZrO* band was not seen.
- R CMi *CN* bands and Swan bands of  $C_2$  observed by Bidelman at Yerkes and by Keenan at Perkins (1950).
- BD 121447 Similarities to R CMi noted by Merrill (1927). Considered by Bidelman and Keenan (1951) as an unusually red member of the *Ba* II group, which show relatively strong bands of *CN* and *CH*. My examination of the Mount Wilson spectrograms, including two additional ones taken in 1952, indicated that *ZrO* had never been strong enough to be definitely established as present. Five Perkins plates of the yellow region also fail to show *ZrO*.
- CY Cyg Nearly all recent observers of this red star, including Vyssotsky, Sanford, and Keenan (all unpublished), have doubted that it belongs with the S-type stars. The only bands which have been definitely recognized are those of the red and infrared systems of *CN*. On a coude spectrogram of the red region, which Dr. Sanford kindly permitted me to measure, no *ZrO* bands were identified. When this plate was matched in a comparator against a spectrogram of the M2 Ib star 119 Tau, nearly all the metallic lines between 5850 and 6720 Å were found to be slightly stronger in CY Cyg. The enhancement was least for the *Fe* lines and greatest for *Y*, *Zr*, and the *Li* resonance line at 6707. The radial velocity of +3.8 km/sec was measured by Sanford on this plate (1943, unpublished).

$ZrO$  (or of  $LaO$ ) on the plates at Mount Wilson or Perkins. They all show some affinity to the carbon stars, in that their infrared  $CN$  bands are stronger than those of the S stars but weaker than those of most of the recognized carbon stars. The table gives estimated intensities of the infrared  $CN$  bands and of several of the atomic lines which are most strongly enhanced in these spectra. The intensities for the  $Na$  D lines were estimated near the longward end of the prismatic spectrograms, where the dispersion is much less than for the lines in the blue, and the intensity scale is correspondingly compressed for sodium.

The  $Ba$   $\Pi$  line at 4554 is as prominent in this group as in the S stars; this is one of the reasons that each of these stars had been assigned to type S at one time or another. The absence from their spectra, however, of the  $TiO$  and  $ZrO$  bands which overlie the neutral lines of  $Sr$  and  $Na$  in types M or S brings out the extreme sensitivity of these resonance lines to temperature. Near minimum phase of the variables in Table 9, both  $Sr$  4607 and the D lines become stronger than in even the coolest of the normal S-type stars. In GP Ori the measurements of McKellar and Stillwell (1944) gave an integrated total absorption for the D lines of 56 A, equaled only by WZ Cas among the carbon stars covered by their survey.

The three stars R CMi, W Cas, and R Ori are known also to have the Swan bands of  $C_2$  present with enough strength to be recognizable with moderate dispersion (see the notes to Table 9). The resemblances of these three R CMi stars had been remarked by Merrill (1927). They were grouped by Davis (1934) to form her entire subclass S1. They are closely allied to the carbon stars; W Cas in particular has such strong  $CN$  bands that it can well be counted among the carbon stars.

The question of the nature of the stars of Table 9 is part of the broader problem of the relationship of the stars of type S to the other classes of cool stars. We can hope for answers to these questions only when estimates of the relative abundances of the atoms and molecules in their atmospheres become available. It will only be mentioned here that measurements of the infrared bands indicate that, in an S-type star,  $CN$  is more abundant than in one of type K or M but less abundant than in the transitional stars of Table 9.

Grateful acknowledgment is made to all the observers who have been mentioned as allowing their spectrograms or data to be used in assembling the tables of this paper. To Dr. Bowen and the staff of the Mount Wilson Observatory my thanks are given for the opportunity of spending two quarters as a guest observer there. The first of these, in the winter of 1951–1952, was supported in part from funds granted to the Ohio State University by the OSU Research Foundation for aid in fundamental research.

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