SPECTROPHOTOMETRY OF THE CENTRAL STARS OF FOUR PLANETARY NEBULAE

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ABSTRACT

An analysis of coudé spectrograms of the nuclei of four planetary nebulae with predominantly absorption spectra, viz., IC 418, IC 2149, NGC 2392, and IC 4593, yields spectral classes of O6–O7 on R. M. Petrie's system. The profiles of the hydrogen lines are compared with those of the O9 star 10 Lacertae. Electron densities somewhat lower than those previously obtained for other planetary nuclei with absorption-line spectra seem to be indicated. It is suggested that the *He/H* ratio can be greater in the planetary nuclei than in normal O stars and that the excitation temperatures can be systematically lower than those derived from Petrie's scale. A final decision will require a much more detailed analysis, involving the calculation of model atmospheres.

I. INTRODUCTION

Since all the energy emitted by a planetary nebula is ultimately derived from its central star, theoretical studies of the physical processes underlying the production of the nebular spectrum require a knowledge of the radiation emitted from this central star. Unfortunately, the radiation relevant to the excitation of the nebular spectrum all lies in the inaccessible ultraviolet. It is necessary to observe the spectrum of the central star in the accessible wave lengths and then try to infer the character of the radiation in the far ultraviolet. The problem is a difficult one, both observationally and theoretically. The central stars of the planetaries are invariably faint. Descriptions of the spectra have been published by a number of observers, but quantitative studies of the emission and absorption lines have been few.[1]

Some planetary nuclei, such as BD+30°3639, show Wolf-Rayet type spectra with relatively narrow lines, while others, such as the nucleus of NGC 6751, show broad emission Wolf-Rayet lines. We are concerned here, however, with objects that show a predominantly absorption spectrum with only a few, if any, relatively sharp emission lines. Some of these central stars show spectra similar to classical O and Of stars. Figure 1 compares the spectrum of the nucleus of NGC 2392 with two Of stars HD 14947 and HD 16691. Notice the prominent absorption lines of hydrogen and ionized helium and the strong emissions of λ 4686 *He* II and λ 4634, λ 4640 *N* III. Microphotometer tracings show that the emission lines in the nucleus of NGC 2392 are somewhat sharper than in Of stars (see Fig. 2, which compares the region from λ 4542 *He* II to λ 4686). The emission lines are somewhat weaker in IC 4593, stronger in IC 418, and missing in IC 2149.

Some years ago, one of us made a detailed study of the absorption-line spectra of the brighter central stars of the planetary nebulae.[2] The spectrograms were obtained with the quartz optics in the Cassegrain spectrograph at the McDonald Observatory and the highest available dispersions. Measures of the equivalent widths of the absorption lines yielded spectral classes by R. M. Petrie's method,[3] as well as estimates of the excitation temperatures and electron densities, and even permitted some speculations concerning the surface gravities and masses of these stars.

[1] See, e.g., the papers by Struve and Swings, *Proc. Nat. Acad. Sci.*, **26**, 454, 548, 1940; **27**, 225, 1941; *Ap. J.*, **92**, 289, 1940.

[2] *Ap. J.*, **108**, 462, 1948.

[3] *Pub. Dom. Ap. Obs. Victoria*, **7**, 321, 1947.

The fact most securely established by this investigation was that a satisfactory study of the central stars of the planetaries would require a much higher dispersion. For example, the profiles of the absorption hydrogen and helium lines should be obtained, so that the possible broadening agencies could be identified and inferences be drawn concerning the structure of the stellar atmosphere. The circumstance that makes observations of the planetary nuclei particularly difficult is that the star is involved in a nebula which is often rather bright. Not only does the presence of the bright nebula make guiding awkward, but the superposition of the strong nebular emissions on the stellar absorption lines makes it difficult to estimate the true profiles of the latter. With high dispersion at a large telescope, many of these difficulties can be overcome.

II. THE OBSERVATIONS

The present study was undertaken to provide improved data on the profiles and total intensities of the absorption and emission lines in four bright planetary nuclei. We employed the 100-inch coudé spectrograms taken by one of us (O. C. W.) in connection with a slit-spectrographic study of the internal motions in the planetaries.[4] The dispersion of

TABLE 1

OBSERVATIONS OF THE CENTRAL STARS OF PLANETARY NEBULAE

Planetary Nebula	α	δ	Photographic Magnitude (Berman)	Photovisual Magnitude (Liller)	Spectral Class	Plate No.
IC 418	5^h22^m8	$-12°45'$	10.8	. . . .	O7	Ce 5991, 6072
IC 2149	5 43.5	+46 07	. . . .	. . . .	O7.5	Ce 5402, 5547
NGC 2392	7 23.3	+21 07	10.5	10.6	O6	Ce 5093, 5094, 5546, 6071, 6975
IC 4593	16 7.0	+12 20	10.2	11.2	O7	Ce 4266, 5209, 5211, 5653

the 32-inch camera and grating combination is 10 A/mm. Owing to the extreme faintness of the central stars, it was possible to secure observations for only a small number of them. The spectrograms employed are listed in Table 1, which gives the planetary nebula, its position, the approximate magnitude and spectral class of the central star, and the plate number. Two columns of magnitudes are given. One set is from the compilation by Berman.[5] For NGC 2392 and IC 4593 we give the results obtained more recently by William Liller,[6] who employed photoelectric methods. Berman's value of 13.3 for the magnitude of the central star of IC 2149 is certainly too faint.

The usual strip-calibration system provided the photometric standards for the plates. We traced the spectrograms in the usual way and derived the profiles and total intensities of the absorption and emission lines. In many instances the central portions of the profiles of the hydrogen lines cannot be determined because of the overlapping of the strong nebular emissions. We indicate these portions of the profiles by dotted lines.

Figure 3 shows the measured profiles for $H\gamma$, $H\delta$, and $H\epsilon$. Some of the profiles show a suggestion of an asymmetry, in the sense that the wings are deeper on the violet side. This tendency is particularly marked on $H\gamma$ and $H\delta$ on plate Ce 5211 of IC 4593 but

[4] *Ap. J.*, **111**, 279, 1950.

[5] *Lick Obs. Bull.*, **18**, 57, 1937.

[6] Thesis, University of Michigan, 1953. We are indebted to Dr. Liller for allowing us to quote his results in advance of publication.

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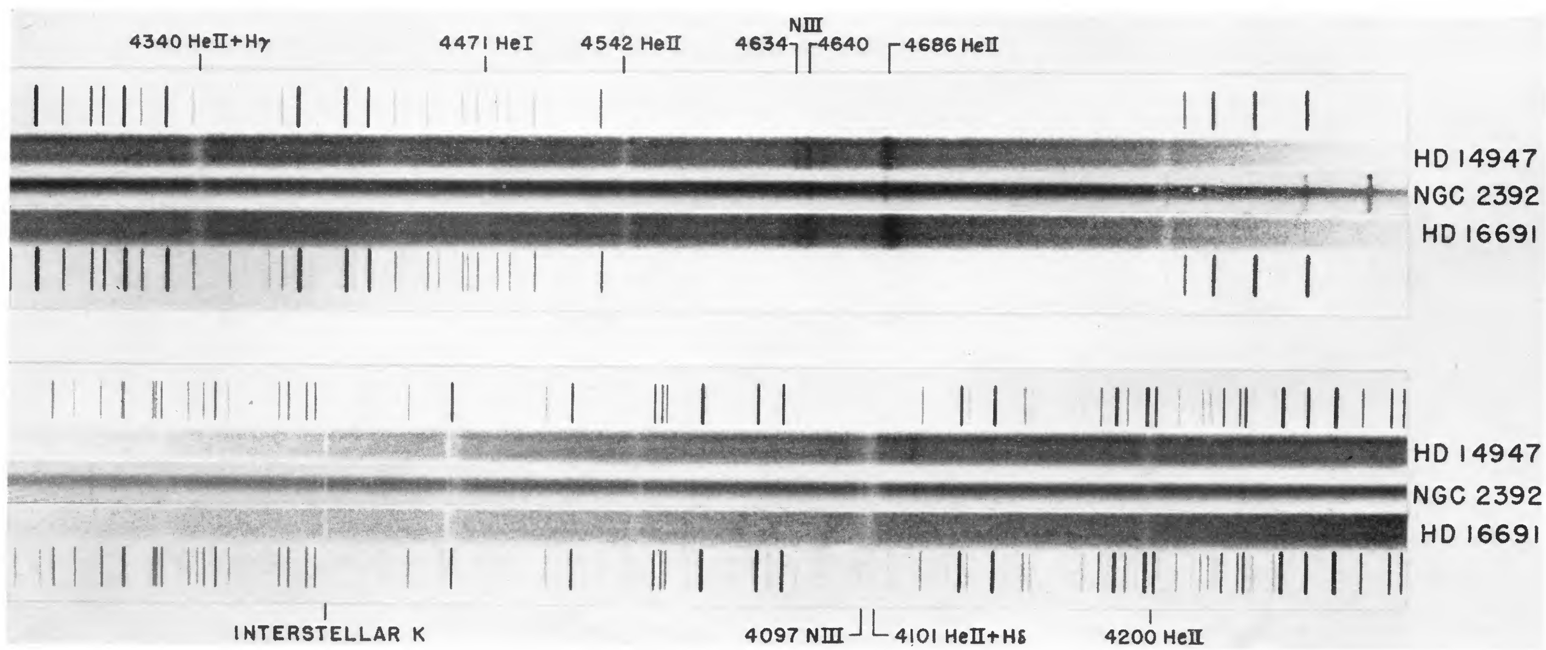

FIG. 1.—Comparison of the spectrum of the nucleus of NGC 2392 with those of the Of stars HD 14947 and HD 16691

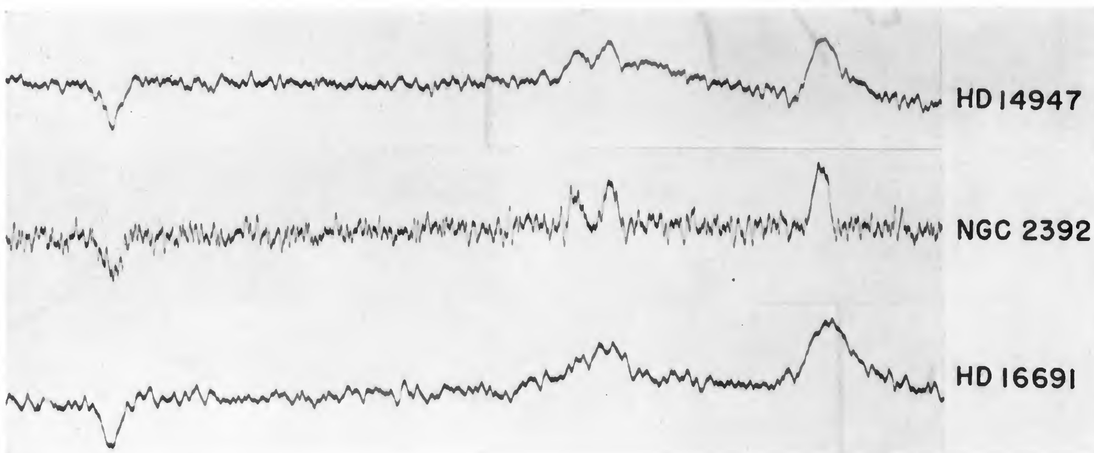

FIG. 2.—Comparison of tracings of HD 14947, NGC 2392, and HD 16691

may be shown by other plates as well. This asymmetry is probably due to blending with the *He* II lines, for it appears to be less marked, if present at all, in the higher members of the Balmer series. The strong, somewhat diffuse, nebular lines cannot be expected to exert much influence on the absorption-line profiles away from the line centers.

In Figure 4 we compare the profiles of the hydrogen lines in the central star of NGC 2392 with those observed in the spectrum of 10 Lacertae.[7] The planetary nuclei hydrogen lines are all weaker than the corresponding lines in 10 Lacertae, although the profile shapes are generally similar.

The equivalent widths, measured in the usual way and expressed in equivalent angstroms, are given in Table 2. The equivalent widths of the absorption lines are com-

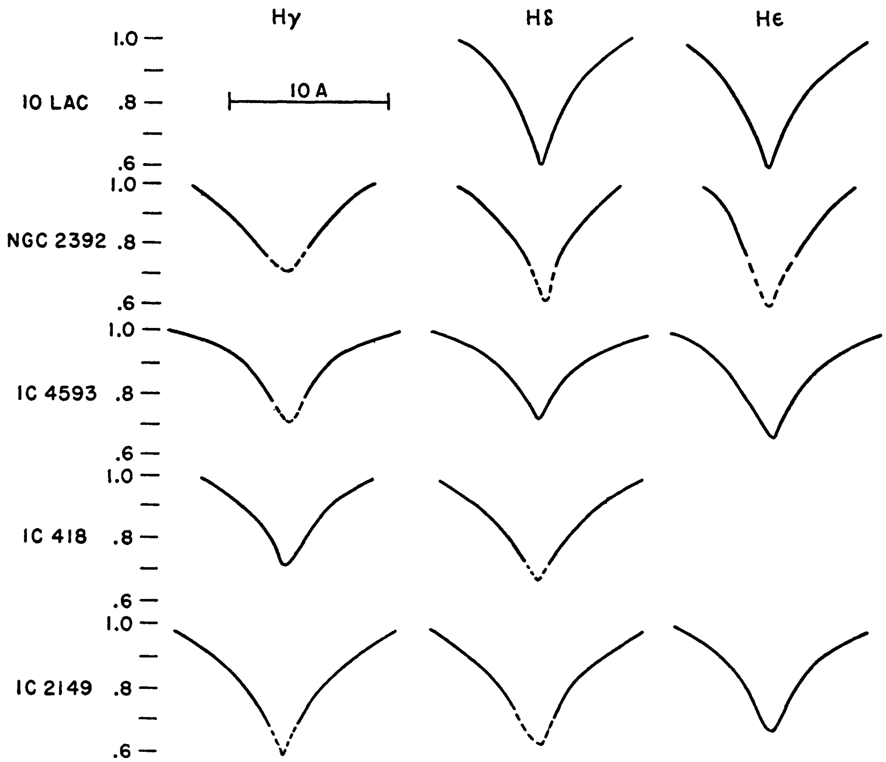

FIG. 3.—Comparison of the profiles of the Balmer lines in planetary nuclei with those in 10 Lacertae

parable with those measured in the classical Of stars. The intensities of the emission lines (denoted by "*E*") are expressed in equivalent angstroms in terms of the underlying continuum. These emission-line profiles are illustrated in Figure 5. They may show time variations such as those found by J. B. Oke for the classical Of stars.[8] Our data are not sufficient to establish the amplitude and period of such variations if they exist.

The accuracy of the tabulated equivalent widths depends strongly on the quality of the plate and the extent of blending with nebular emissions. On a well-exposed plate the average error of the intensity of an unblended line of moderate strength (e.g., $W_\lambda \sim 0.7$) is probably about 20 per cent. A comparison with previous intensity measures for three stars common to the two lists—the IC 418, IC 4593, and NGC 2392 nuclei—shows a number of discrepancies which are to be attributed to the inadequate dispersion em-

[7] L. H. Aller, *Ap. J.*, **104**, 347, 1946.

[8] Thesis, Princeton University, 1953.

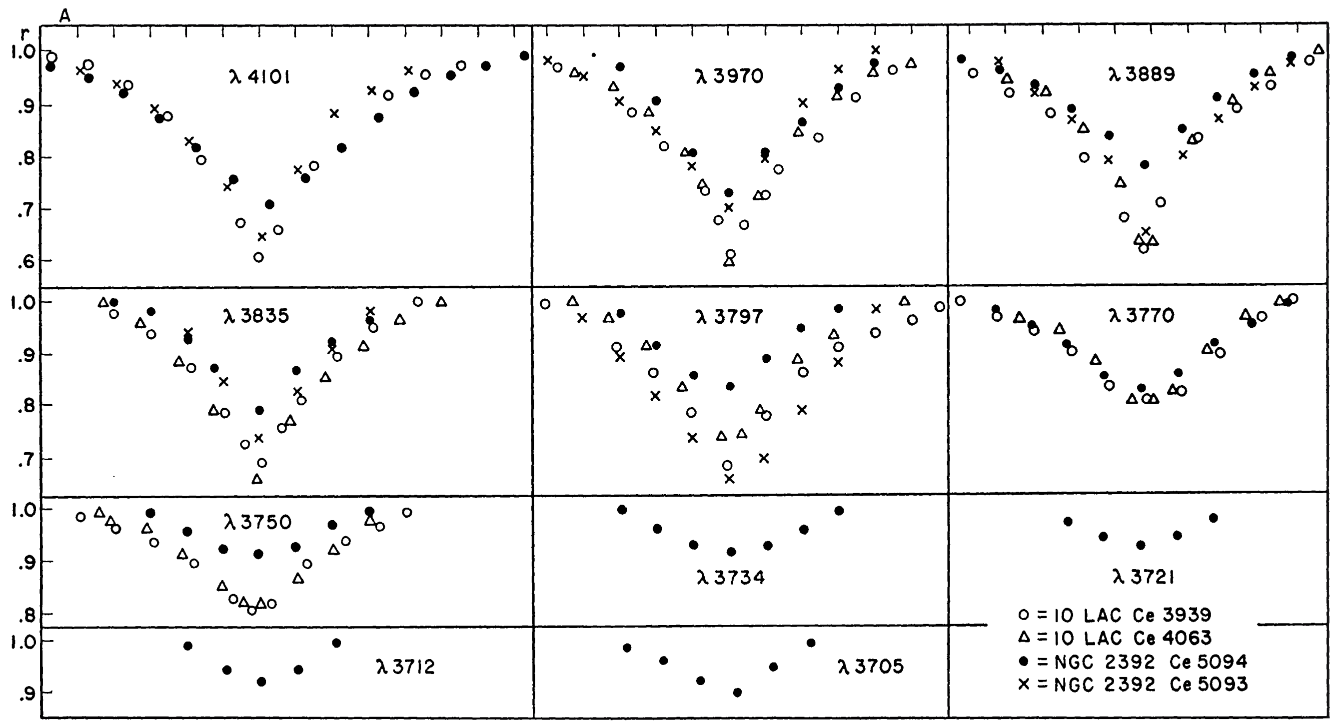

FIG. 4.—Detailed comparison of the hydrogen line profiles in 10 Lacertae with those in the nucleus of NGC 2392

ployed in the earlier work. In particular, it should now be possible to make improved estimates of such quantities as the spectral type, the excitation temperature, and the electron pressure.

III. TEMPERATURES AND SPECTRAL CLASSES OF PLANETARY NUCLEI

The photometrically measured line intensities permit a determination of the spectral classes by the method of R. M. Petrie,[3] who uses the *He* I/*He* II, *He* II/*H*, and *Si* IV/*He* II ratios. We have used the *He* I/*He* II ratio, i.e., $I\ (\lambda\ 4471)/\frac{1}{2}[I\ (4200) + I\ (4542)]$, and the *He* II/*H* ratio, i.e., $\frac{1}{2}[I\ (4200) + I\ (4592)]/\frac{1}{4}[I_c\ (H\gamma) + I_c\ (H\delta)]$, to fix the spectral classes. Here $I_c\ (H\gamma)$ and $I_c\ (H\delta)$ denote the intensities corrected for blending with the helium lines. If the helium/hydrogen ratio in the planetary nuclei differs from that of normal O stars, the spectral class estimated from the *He* I/*He* II ratios will differ systematically from that estimated from the *He*/*H* ratio. In order to avoid the influence of abundance differences, we might prefer the *He* I/*He* II ratio; but in the low-excitation

TABLE 2

EQUIVALENT WIDTHS OF ABSORPTION AND EMISSION LINES IN THE SPECTRA OF PLANETARY NUCLEI

λ	Identification	IC 418	IC 2149	NGC 2392	IC 4593
4861	$H\beta$	2.06	....	....	2.46
4686 *E*	*He* II	0.81	....	1.35	0.78
4651.35 *E*	*C* III	0.38	....	....	0.40
4650.16 *E*	*C* III	0.70	....	....	....
4647.40 *E*	*C* III	1.04	....	....	0.36
4640.64 *E*	*N* III	0.78	....	0.61	0.38
4634.16 *E*	*N* III	0.46	....	0.45	0.33
4541.59	*He* II	0.52	0.69	0.64	0.53
4771.50	*He* I	0.35	0.34	0.47	0.29
4340	$H\gamma$+*He* II	1.20:	2.02	1.36	1.49
4325 *E*	*C* III	0.30	....	....	....
4200	*He* II	0.46	....	0.60	0.52
4101.74	$H\delta$+*He* II	1.40	1.80	1.21	1.35
4097	*N* III	....	....	0.34	....
4026	*He* I, *He* II	0.35	0.30	0.58	0.46
				0.24	
3969.5	$H\zeta$, *He* II	1.45	1.42	1.00	1.26
3923.48	*He* II	0.11:	....	0.48	0.14
3889	*H*8, *He*, *He* II	0.90	1.30	1.23	1.36
3858.07	*He* II	....	....	0.30	....
3835.3	*H*9+*He* II	0.86	1.27	0.92	1.02
3819.68	*He* I	....	....	0.28	....
3797.9	*H*10+*He* II	0.80	1.01	0.92	1.14
3770.63	*H*11	0.60	0.73	0.86	0.72
3761:	*O* III, *Si* IV	0.28	....	....	0.14
3757.26	*O* III	....	....	....	0.14
3754.7	*N* III, *O* III	....	....	....	0.21
3750.15	*H*12	0.56	0.58	0.61	0.44
3734.37	*H*13	0.43	0.58	0.42	0.38
3721.94	*H*14	0.34	0.38	0.25	0.47
3711.97	*H*15	0.14	0.30	0.20	0.34
3705	*H*, *He* I	0.18	0.28	0.26	0.23
3697.15	*H*17	....	0.29	....	....
3691.56	*H*18	....	0.21	....	....
3686.83	*H*19	....	0.14	....	....
3484.90	*N* IV	....	....	0.13	....
3482.98	*N* IV	....	....	0.11	....
3478.69	*N* IV	....	....	0.19	....

planetaries IC 418, IC 2149, and IC 4593, the stellar λ 4471 line tends to be filled in by the nebular emission. Hence the spectral class tends to be estimated systematically too early for these stars. The second, third, and fourth columns of Table 3 give for each nucleus the basic intensity data: I (4471), I (*He* II) $= \frac{1}{2}[I\ (4200) + I\ (4542)]$, and $I\ (H) = \frac{1}{4}[I_c\ (H_\gamma) + I_c\ (H\delta)]$. The fifth, sixth, seventh, and eighth columns give the *He* I/*He* II and *He* II/*H* ratios with the corresponding spectral types. The ninth, tenth, and eleventh columns list the adopted spectral classes, the excitation temperature according to Petrie's calibration for normal O stars, and the previously estimated spectral classes. Notice that the *He* I/*He* II estimates tend to be earlier than those from the *He* II/*H* ratios because the intensity of 4471 tends to be estimated too low. A comparison of the temperatures with those estimated by Berman[5] from the Zanstra method is of interest (see accompanying tabulation). The excitation temperatures of IC 418 and

Nucleus	T (Sp. Class) (° K)	T (Zanstra) (° K)	Nucleus	T (Sp. Class) (° K)	T (Zanstra) (° K)
IC 418	33,200	25,000	NGC 2392	34,500	35,000
IC 2149	32,500	40,000	IC 4593	33,400	25,000

IC 4593 are definitely well in excess of the Zanstra temperatures. These are bright, presumably optically thick, nebulae, for which the Zanstra method should be valid. Deviations from a black-body energy distribution in the ultraviolet might account for some of

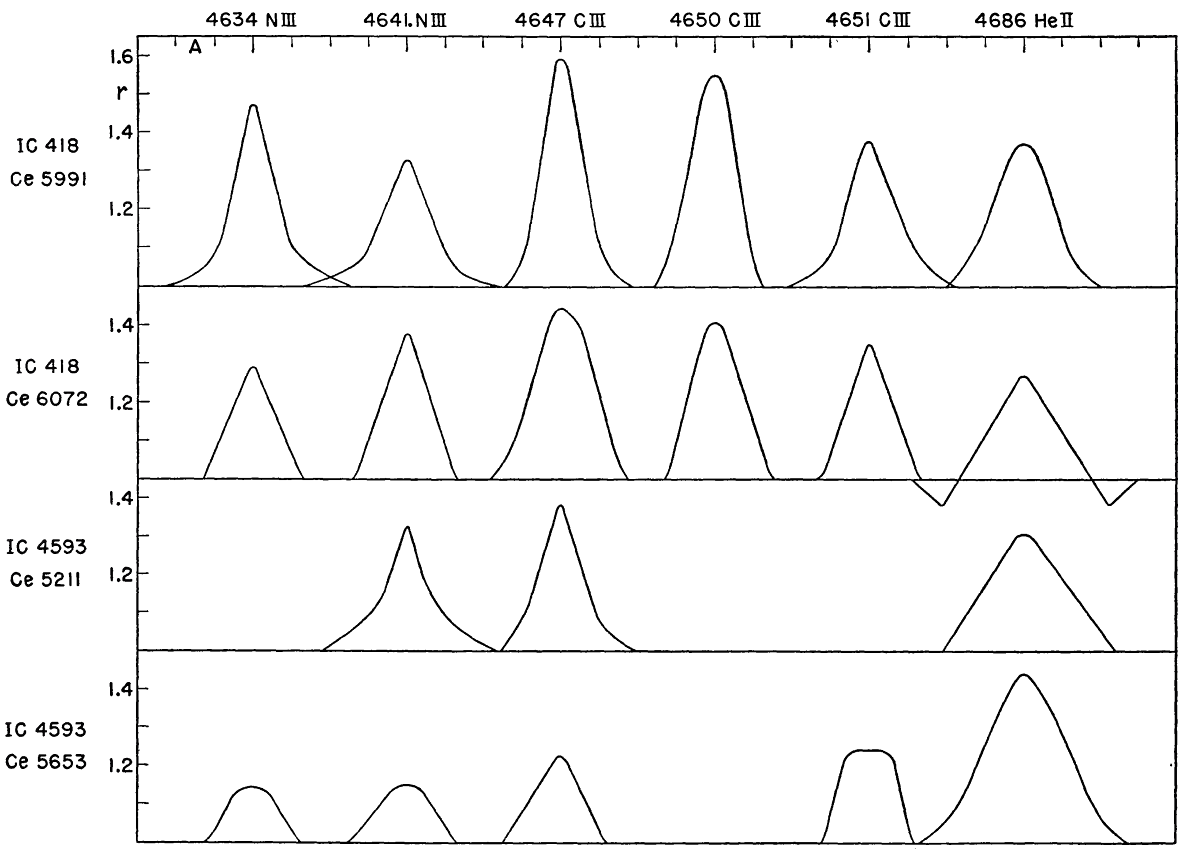

FIG. 5*a*.—Profiles of emission lines observed in the nuclei of planetary nebulae

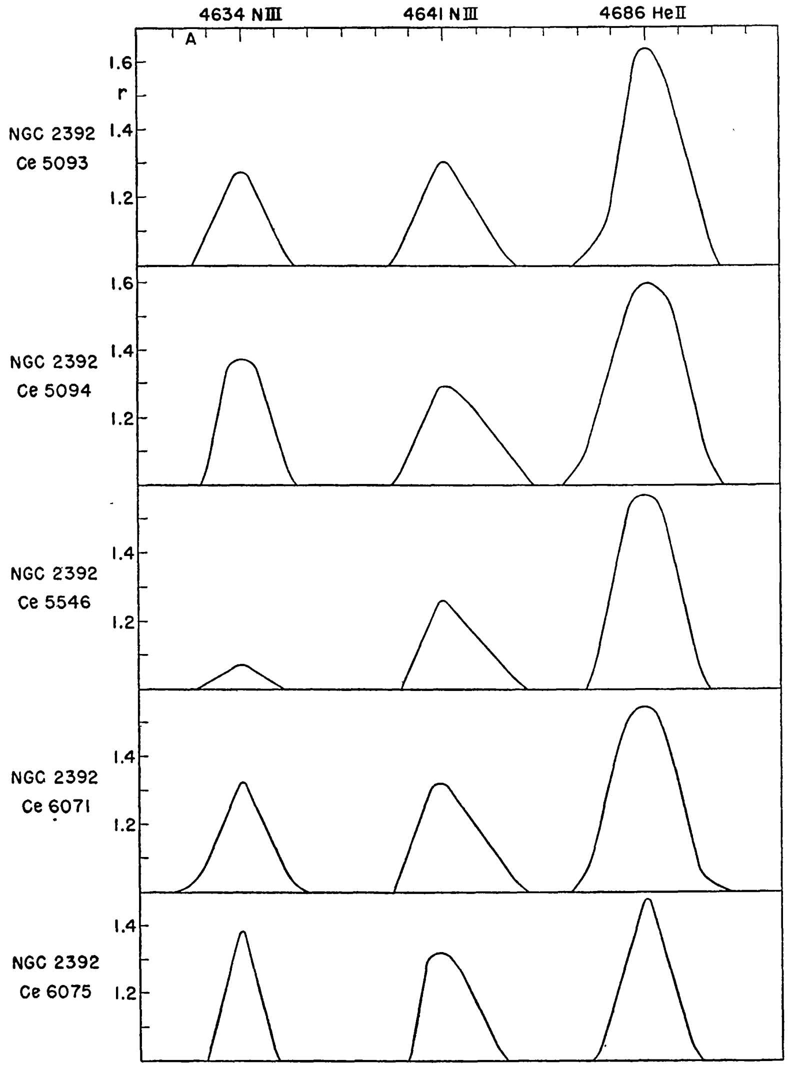

FIG. 5b.—Profiles of emission lines observed in the nuclei of planetary nebulae

TABLE 3

SPECTRAL CLASSES OF PLANETARY NUCLEI

Nucleus	*He* I	*He* II	*H*	*He* I/ *He* II	Sp.	*He* II/ *H*	Sp.	Adopted Spectrum	*T* (° K)	Previous Estimates
IC 418......	0.35	0.49	0.44	0.71	O6	1.11	O7	O7	33,200	O5, +WC 7*
IC 2149.....	.34	.65:	.72	.52	O5.5	0.90	O7.5	O7.5	32,500	O7*
NGC 2392...	.47	.62	.33	.76	O6.2	1.87	O5.9	O6	34,500	O5.5 †
IC 4593.....	0.29	0.53	0.50	0.55	O5.5	0.96	O7.3	O7	33,400	O6.5 †

* Verontsov-Velyaminov, *Russ. A.J.*, 24, 88, 1947.
† L. H. Aller, *Ap. J.*, 108, 462, 1948.

this discrepancy. Furthermore, the excitation temperatures are not necessarily the same as the effective temperatures. Finally, if the *He/H* ratio is not the same as in normal stars, there will be an error in the temperature scale. On the other hand, for no accountable reason, IC 2149 has an excitation temperature which is lower than the Zanstra temperature by about the same amount as the other two were higher. For NGC 2392 the agreement seems to be good, but, alas, even here there is trouble. The other three nebulae are all low-excitation objects with rather weak [*Ne* III] and rather prominent λ 3727 [*O* II] emissions. Ionized helium lines are not present in the nebula. On the other hand, NGC 2392 exhibits a very high level of excitation, with strong λ 4686 *He* II and [*Ne* V]. Comparison with other high-excitation nebulae and the Zanstra temperature from ionized helium would suggest a very high temperature[9]—in the neighborhood of 100,000° K! It is strange that nebulae showing such a difference of excitation as IC 418 and NGC 2392 should have nuclei differing in temperature by only a couple of thousand degrees. The nucleus of 2392 is also outstanding, in that it shows evidence of ejections of matter because of a systematic difference in velocity as a function of excitation of the absorption lines. We have estimated the systematic velocity differences between the nebular and stellar hydrogen lines from the microphotometer tracings. IC 2149 and IC 4593 give no conclusive evidence for such velocity differences, but there is some suggestion of a systematic velocity difference in the nebular and stellar hydrogen lines in IC 418, although the magnitude of the displacement fluctuates from line to line. We should expect the broad nuclear lines in IC 418 to be produced by ejected matter. It is possible that the nucleus of NGC 2392 has a somewhat different ultraviolet energy distribution than do other O-type nuclei of apparently the same effective excitation. Another way out of the difficulty is to suppose that the NGC 2392 nucleus is a double star,[10] the hot companion being invisible in the ordinary photographic range. Radial-velocity measurements might reveal such a component, but no observations have yet been made to test this hypothesis.

IV. ESTIMATES OF ELECTRON DENSITY AND NUMBERS OF ATOMS ABOVE PHOTOSPHERE

In order to compare our results with those previously obtained, we shall estimate the number of hydrogen and helium atoms above the photosphere by the method used by Unsöld.[11] It is necessary to correct the Balmer-line intensities for the ionized helium lines. The procedure is exactly the same as that described in the earlier papers. We give here only the results, which are compared with those previously obtained for IC 4593 and NGC 2392. The second and third columns of Table 4 give log $N_{0,\,2}H$, the number of hydrogen atoms in the second quantum level above the photosphere. In our present series of observations, higher members of the Balmer series have been measured than in the earlier work, and the resultant log $N_{0,\,2}H$ should be more reliable. Notice that the number of atoms above the photosphere is increased for both IC 4593 and NGC 2392. If we now make an extremely crude estimate of N_ϵ from the intensities of $H\gamma$ and $H\delta$ (corrected for the ionized helium contribution) by the procedure suggested by Unsöld, we find the values of N_ϵ entered in the fourth and fifth columns. This estimate is based on the assumption that the line is formed in an isothermal layer; the contribution of Doppler broadening is neglected. The two estimates of N_ϵ are in poor agreement for IC 418 but are in reasonable accord for the other stars. The number of resolvable lines of the Balmer series gives an additional estimate of the electron density. If n_m is the principal quantum number of the last still-resolved line of the Balmer series, the concentra-

[9] See, e.g., the discussion by K. Wurm, *Die planetarischen Nebel* (Berlin: Academie-Verlag, 1951), p. 96.

[10] E.g., The nucleus of NGC 246 is a binary. The nucleus of NGC 1514 must be double, as an A0 star seems to be definitely too late in spectral class to excite a nebular spectrum.

[11] *Zs. f. Ap.*, **21**, 38, 1941.

tion of free electrons (or ions) per cubic centimeter is given by the Inglis-Teller formula[12]

$$\log N_\epsilon = \text{Const.} - 7.5 \log n_m .$$

Inglis and Teller give a numerical value of 23.26 for the constant. Experimental work by W. Lochte-Holtgreven and W. Nissen[13] at Kiel shows that the formula will give a rough determination of N_ϵ unless the density is so high that n_m is smaller than 7. At the lower densities appropriate to the atmospheres of early-type stars, they find that the empirical value for the constant, 23.46, gives a better approximation to the electron density than that found by other methods.

Except for IC 2149, in which the Balmer lines can be followed to $n_m = 19$, we find $n_m = 16$. Hence we obtain the values for log N_ϵ given under the heading "n_m" in Table 4.

TABLE 4

ELECTRON DENSITIES AND H/He RATIOS

Central Star	log $N_{0,\,2}H$		log N_ϵ				log $N_{1,\,4}H$ (He II)	log [$N(H^+)$/$N(He^{++})$]	H/He
	Present Series	McDonald Series (1945–1946)	$H\gamma$	$H\delta$	n_m	Adopted			
IC 418	15.37	....	13.21	13.82	13.46	13.5	14.70	1.27	13
IC 2149	15.44	....	13.96	14.07	12.86	13.3	14.60	1.44	20
IC 4593	15.50	15.35	13.48	13.64	13.46	13.5	14.65	1.45	20
NGC 2392	15.46	14.96	13.07	13.18	13.46	13.4	15.20	0.86	5

The adopted values of log N_ϵ are given in the column headed "Adopted." Notice the striking discrepancy between the estimates from the higher series members and from $H\delta$ and $H\gamma$ for IC 2149; perhaps the higher members of the series are actually formed in a shell. The estimate of log N_ϵ for IC 4593 is only slightly lower than that previously published, but the new value for NGC 2392 is lower by about a factor of 10! This result applies to the region of the atmosphere where the cores of the higher members of the Balmer series are formed.

If the method previously described for the estimation of effective surface gravities is employed,[2] somewhat smaller surface gravities than were earlier suggested are indicated. Our data are consistent with the suggestion that the masses of the planetary nuclei are not very much larger than that of the sun.

Finally, the same type of analysis as was applied to the Balmer lines to get the number of hydrogen atoms in the second-quantum level may be employed with the Pickering lines of ionized helium to get log $N_{1,\,4}H$, the number of ionized helium atoms in the fourth level. The determination is less certain; alternate lines of the Pickering series (those not blended with hydrogen) have to be used. The estimated values are given in the eighth column of Table 4. From a comparison of the second and eighth columns we obtain with the aid of the combined Boltzmann and Saha equations for hydrogen and ionized helium

$$\log \frac{N(H^+)}{N(He^{++})} = \log \frac{N_{0,\,2}H}{N_{1,\,4}H} + 0.60 .$$

Now, employing the electron pressures and excitation temperatures previously derived, we get the hydrogen/helium ratios found in the last column. Except for NGC 2392, the abundance ratios are not very different from those found recently for normal stars, i.e., ~20.

[12] *Ap. J.*, **90**, 439, 1939.

[13] *Zs. f. Phys.*, **133**, 124, 1952.

It should, however, be obvious that this estimate is meaningless. The derived He/H ratio depends very critically upon the temperature. We assigned an excitation temperature on the basis essentially of the H/He II ratio and Petrie's calibration of the temperature scale for normal O stars, which assumes a normal H/He ratio. Hence it is inevitable that the same H/He ratio would be found as for normal stars.

Suppose that the He/H ratio is actually greater for the planetary nuclei than for the normal type I stars. At the higher temperatures the Pickering lines will be greatly strengthened, simulating the spectrum of a yet hotter star. Thus if the excitation temperatures are really a bit lower than we have supposed, the stars turn out to be helium-rich! In the earlier paper attention was called to the nucleus of NGC 246, whose spectrum corresponds to that of a high-temperature helium star and cannot be explained by a high-temperature atmosphere with a He/H ratio of $\sim\frac{1}{30}$.

At the moment, a clear-cut decision concerning the He/H ratio in the planetary nuclei cannot be given. A complete analysis of the spectrum by the method of model atmospheres appears to be the most promising approach, but the path is not an easy one. In the NGC 2392 nucleus, different lines have different radial velocities.[14] Under such circumstances the basis for the construction of a definite model is not clear. It is necessary to reproduce the observed Balmer-line profiles and the total intensities of other lines, with the temperature, surface gravity, and He/H ratio as free parameters. Furthermore, the lines may be broadened by rotation. We must defer this calculation to a second paper.

Since the Balmer-line profiles in NGC 2392 show a general similarity to those in 10 Lacertae, we might expect the excitation temperature to be lower than we had supposed. The dependence of the H/He ratio on the excitation temperature in this star is exhibited in the accompanying table, calculated for an isothermal atmosphere and $\log P_\epsilon = 2.16$.

$\theta = 5040/T$	T (° K)	$\log [N(He^{++})/N(He^{+})]$	H/He
0.145	34,800	+0.83	5.7
.150	33,600	+ .52	5.2
.155	32,400	+ .21	4.2
.160	31,500	− .10	3.0
.165	30,500	− .40	1.9
0.170	29,600	−0.71	1.1

The planetary nucleus observed in the globular cluster M15 falls about 1 mag. above the horizontal branch of blue stars in the color-magnitude array. If the nuclei represent fringe stars in such an array, they may show a fair spread in luminosity, diameter, and atmospheric chemical composition. It is possible that the He/H ratio varies from one such star to another, as Münch and Greenstein have found for the blue Humason-Zwicky stars.[15] The character of the spectra of the planetary nuclei is not such as to permit detailed abundance studies to be made.

If the upper part of the color-magnitude array of a globular cluster represents different evolutionary stages of stars that were once on the main sequence,[16] the great age of the type II population means that planetary nuclei—like other stars in the globular clusters—cannot have masses much greater than about 1.5 solar masses. The character of the spectra of the planetary nuclei tends to support this conclusion. The plan of the planetary nuclei in the scheme of stellar evolution cannot be assigned until the upper part of the color-magnitude array has been explained by theory.

[14] See the discussion by O. C. Wilson, *Ap. J.*, **108**, 201, 1948.

[15] J. L. Greenstein, private communication.

[16] See A. Sandage and M. Schwarzschild, *Ap. J.*, **116**, 463, 1952.