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MASS DISTRIBUTION AND MASS-LUMINOSITY RATIO IN GALAXIES

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Abstract. The observational data bearing on the mass distribution and mass-luminosity ratio in galaxies are rediscussed and assembled. The observations on the mass distribution indicate, contrary to earlier suggestions, that the data now available permit the assumption of identical spacial distribution of mass and luminosity. The results for the mass-luminosity ratio are summarized in Table 7. In agreement with earlier investigations, the elliptical galaxies are found to have a mass-luminosity ratio much larger than that of pure population I systems. The Andromeda nebula is found to have an intermediate mass-luminosity ratio, suggesting that the body of the Andromeda nebula may consist of a mixture of population II and old population I stars.

1. *Introduction.* The differences between stellar populations appear to be one of the main tools for the study of the origin and evolution of stars. A variety of characteristics in which stellar populations differ can be observed. The majority of these characteristics, however, refer only to the stars of relatively high luminosity. On the other hand, one characteristic, namely the mass-luminosity ratio, does refer to the absolutely fainter stars. This circumstance appears to lend special interest to this characteristic.

The observational data on the mass-luminosity ratio of different stellar systems were summarized not long ago by Holmberg.[1] This summary, however, does not include a discussion of the possible variation of the mass-luminosity ratio within one galaxy. Furthermore, two new observational investigations, one on double galaxies[2] and the other on a globular cluster[3] have since been published. Hence a rediscussion of the data now available seems worth-while.

For the distances of extragalactic objects a scale twice the traditional scale has been used throughout. The luminosities here used are all based on photographic, not photovisual, magnitudes.

The following five sections contain the detailed data for individual galaxies. All of the results are summarized and discussed in the last two sections.

2. *Andromeda nebula, M 31.* Earlier work on the rotational velocity in the Andromeda nebula indicated a very rapid increase in the mass-luminosity ratio from the center outwards.[4] Subsequently, a much larger set of observations on the velocities of emission patches in the Andromeda nebula was published by Mayall.[5] These new data do not seem to have been analyzed as yet in detail.

Mayall has represented his new observations in a graph in which the radial velocity, directly as measured, for the various emission patches is plotted against the x-coordinate, parallel to the major axis, of these patches. Since this graph does not represent directly the run of the circular velocity with distance from the center of the system, the same observations here have been replotted in Figure 1 in terms of the circular velocity, V_c, and the distance from the center, r. To compute V_c and r from Mayall's radial velocities and coordinates, three assumptions had to be made: First, the radial velocity of the Andromeda nebula as a whole was taken to be 270 km/sec, in good agreement both with the Mount Wilson value for the nucleus of the Andromeda nebula and with the mean of the velocity values of Mayall for patches along the minor axis. Second, the tilt of the Andromeda nebula was taken to be 75°. Third, all the emission patches were assumed to lie in the plane of the Andromeda nebula and to move in purely circular orbits.

The points in Figure 1 still show an appreciable scatter. Thus the forming of normal points seems advisable. Furthermore, the figure indicates a discrepancy between the north following side of the Andromeda nebula (dots) and the south preceding side (crosses). The origin of this discrepancy will be discussed in Section 5. Here the discrepancy has only to be considered to avoid its influencing the normal points. Consequently, the observations were grouped for the normal points, as shown by the arrows at the bottom of Figure 1, so that as nearly as possible each group contained the same number of observations from both sides of the nebula. The outermost normal point contains only the one high-weight observation by Humason.[5]

Contrary to earlier indications, the five normal points in Figure 1 (circles) do not suggest solid body rotation anywhere in the range covered by the observations. Rather, they suggest fairly constant circular velocity over the whole interval from 25′ to 115′. Thus, these latest observations hint at a mass distribution that is not homogeneous, but rather concentrated toward the center.

Obviously even these new observations are insufficient to derive a definite mass distribution and thus prove whether or not the mass and light distributions in the Andromeda nebula are identical. One may, therefore, turn the problem around and ask whether the velocity observations are discordant with the assumption that the mass and light distributions are identical. This question may be investigated as follows.

Let us represent the surface brightness along the major axis of the nebula as a superposition of straight sections:

$$I(r) = \sum_n a_n \left(1 - \frac{r}{R_n}\right). \tag{1}$$

Each term in the sum contributes an intensity which starts at the center with a value a_n and decreases linearly to zero at R_n. Further, let us represent the mass distribution throughout the nebula by a surface density on a flat disk—following Wyse and Mayall[4]—and let us represent the surface density by

$$\sigma(r) = \sum_n A_n \left(1 - \frac{r}{R_n}\right). \tag{2}$$

If the mass and light distributions are identical then

$$A_n = f a_n, \tag{3}$$

where f is a constant and represents the mass-luminosity ratio of the nebula. In these equations one may use as units for I and a_n the solar luminosity per square minute of arc, for σ and A_n the solar mass per square minute of arc, and for r and R_n a minute of arc. This gives f directly in solar mass per solar luminosity. Further on, the distance of the nebula D will be used in terms of parsecs and the circular velocity V_c in terms of km/sec.

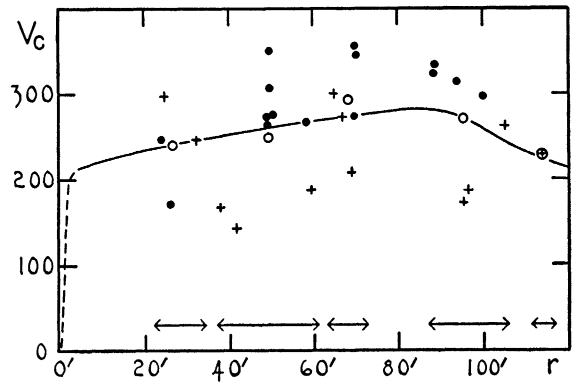

Figure 1. Circular velocity in the Andromeda Nebula in km/sec. Dots: observations on north following side. Crosses: observations on south preceding side. Circles: normal points. Curve: computed under assumption of identical mass and light distributions.

Now the circular velocity and also the mass have to be found in terms of the above representations. For the total mass of the nebula one finds directly from (2)

$$\mathrm{M} = \frac{\pi}{3} \mathrm{M}_\odot \sum_n A_n R_n^2. \tag{4}$$

For the gravitational force in the plane of the galaxy as a function of the distance from the center one finds from (2)

$$F(r) = 4G\mathrm{M}_\odot \left(\frac{1.115}{D\cdot 10^{15}}\right)^2 \sum_n A_n g\left(\frac{r}{R_n}\right). \tag{5}$$

Here the squared parenthesis takes care of the conversion from minutes of arc to centimeters and the function g represents the nondimensional factor for the gravitational force produced by a surface density σ as given by one term in (2). This function has already been computed by Wyse and Mayall.[4] In terms of their functions M and m

$$\left.\begin{aligned} g\left(\frac{r}{R_n}\right) &= M\left(\frac{r}{R_n}\right) \quad \text{for} \quad r \leq R_n, \\ g\left(\frac{r}{R_n}\right) &= m\left(\frac{R_n}{r}\right) \quad \text{for} \quad r \geq R_n. \end{aligned}\right\} \tag{6}$$

The circular velocity is related to the gravitational force by

$$V_c(r)^2 = rF(r) \cdot \frac{D \times 10^{15}}{1.115} \cdot \left(\frac{1}{10^5}\right)^2. \quad (7)$$

If one introduces here the expression for the gravitational force as given by (5) and further eliminates the A_n by (3), one obtains

$$V_c(r)^2 = \left[59.0 \frac{f}{D}\right] \cdot \left[r \sum_n a_n g\left(\frac{r}{R_n}\right)\right]. \quad (8)$$

The above equations may be used in the following sequence. First a set of a_n and R_n values is determined so that the observed surface brightness run can be represented by (1). Next the second bracket in (8) is computed as a function of the distance from the center. Third, this computed function is compared with the run of the observed circular velocities to see whether the assumption of equal mass and light distribution is permissible. If so, a value is determined for the first bracket in (8), which is a constant and the only free parameter in this procedure, so as to fit the computed function as well as possible to the observed velocities. Finally, with this value of the first bracket, together with the distance of the nebula, the value of f is found which in turn gives the values of A_n by (3) and total mass by (4).

TABLE I. SURFACE BRIGHTNESS IN ANDROMEDA NEBULA. NUMERICAL COEFFICIENTS OF REPRESENTATION BY STRAIGHT SECTIONS

n	R_n	a_n*
1	100′	46
2	50	27
3	25	75
4	12.5	100
5	6.25	375

* Unit = solar luminosity per square second of arc.

TABLE II. COMPUTATION OF GRAVITATIONAL FORCE IN ANDROMEDA NEBULA

	$a_n g(r/R_n)$						
r	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$\sum_n a_n g$	$(r\sum a_n g)^{\frac{1}{2}}$
5′	7.0	6.8	29.3	55.6	225.	324	40.2
10	11.5	10.6	41.7	59.9	42.0	166	40.7
20	18.0	15.0	44.9	11.2	9.8	99	44.5
30	22.4	16.8	16.9	4.8	4.5	65	44.1
40	25.6	16.2	8.4	2.6	2.6	55	46.9
50	27.6	11.2	5.2	1.6	1.5	47	48.4
60	28.6	6.1	3.6	1.2	0.8	40	49.0
70	28.6	4.2	2.6	0.8	0.6	37	50.9
80	27.6	3.0	2.0	0.7	0.4	34	52.1
90	25.0	2.3	1.5	0.5	0.2	30	52.0
100	19.1	1.9	1.2	0.4	0.0	23	48.0
110	13.3	1.6	1.1	0.3	0.0	16.3	42.3
120	10.4	1.3	1.0	0.3	0.0	13.0	39.5

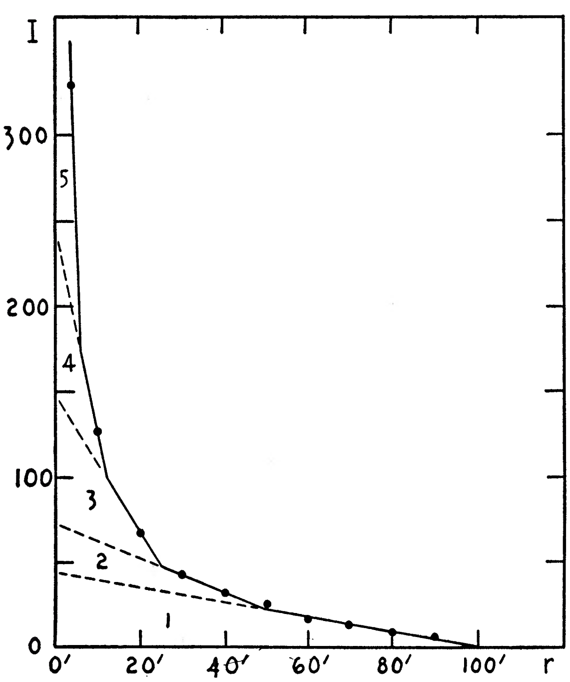

Figure 2. Surface brightness in the Andromeda nebula as represented by straight sections. The dots give the observations by Redman and Shirley as reduced by Wyse and Mayall.

To apply this procedure to the Andromeda nebula the brightness measurements by Redman and Shirley[6] as reduced by Wyse and Mayall[4] were used. Since these observations are given in solar luminosity per square second of arc and since in their derivation the traditional distance scale was used, the resulting a_n values are too small by a factor of 4×3600 and correspondingly the final value of f has to be divided by this factor to convert to our units. The representation of these observations by (1) is shown in Figure 2. The a_n and R_n as used in this representation are listed in Table I. The computation of the second bracket in (8) is given in Table II. This computed function had then to be fitted to the velocity observations shown in Figure 1 by choosing the appropriate value for the first bracket in (8). This value was found to be 29, in units corresponding to those in which the observed surface intensities were given. With this numerical value the right-hand side of (8) gave a computed velocity curve as shown in Figure 1. Considering the fact that this computed curve is arbitrary only in the one factor with which it can be proportionately raised or lowered, the fit of the curve to the five normal points seems surprisingly good.

We may then conclude that the best available observations of the rotational velocity in the Andromeda nebula are not discordant with the assumption of equal mass and light distribution.

In Figure 1 the dotted extension of the velocity curve into the center is not based on computation and is drawn only to indicate the possibly extremely steep rise of the circular velocity at small distances from the center. This steep rise in the circular velocity does not, however, imply necessarily an equally steep rise in the actual rotational velocity, since in the central bulge of the Andromeda nebula the stellar orbits may well be sufficiently disoriented, so that in this central portion the gravitationally defined circular velocity and the actual mean rotational velocity might differ greatly. If in the future the gravitational force within this central portion is to be determined, observations are needed not only of the rotational velocity as a function of distance, but also of the velocity dispersion.

Using the above value for the first bracket of (8) and accepting 460,000 parsecs for the distance of the Andromeda nebula, one obtains for the mass-luminosity ratio of this nebula

$$f = 16.$$

Finally, this value of f gives, according to (3) and (4), for the total mass of the Andromeda nebula

$$M = 1.4 \times 10^{11} M_{\odot}.$$

As was to be expected, this discussion has not altered the earlier derivation of the total mass of the Andromeda nebula noticeably. It does, however, suggest that the Andromeda nebula throughout its main body has the above constant value for the mass-luminosity ratio.

3. *Spiral galaxy in Triangulum, M 33.* The radial velocities of a number of emission patches in M 33 were measured by Mayall and Aller.[7] They summarized their data in six normal points, which together with their probable errors are here reproduced in Figure 3. These data were analyzed by Wyse and Mayall,[4] who drew a curve directly through the six normal points and derived from this curve the mass distribution. They obtained a fairly homogeneous mass distribution, in contrast with the surface brightness which rises steeply toward the center. This result was, however, emphasized as uncertain owing to the large probable errors of the velocity observations.[8]

Again it appears useful to turn the problem around and ask whether the velocity observations are seriously discordant with the assumption of identical mass and light distribution. Accordingly, the same procedure as described for M 31 in the preceding section was applied here to M 33.

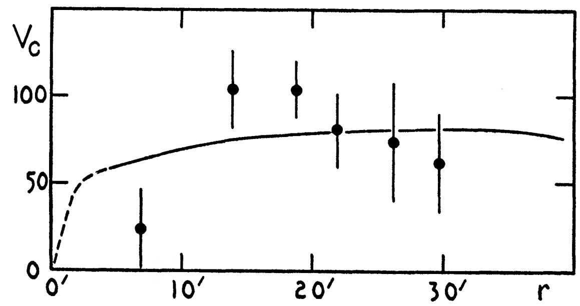

Figure 3. Circular velocity in the spiral galaxy M33. Dots: normal points listed by Mayall and Aller. Vertical lines: probable errors. Curve: computed under assumption of identical mass and light distributions.

The surface brightness along the major axis of M 33 has been measured by Patterson.[9] Her observations are shown in Figure 4 together with the representation by straight sections. The coefficients of this representation are listed in Table III. The computation of the second bracket in (8) is given in Table IV. It was found that the computed second bracket could be fitted to the

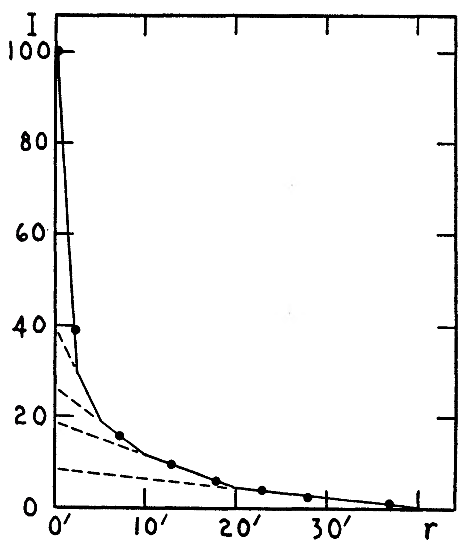

Figure 4. Surface brightness in the spiral galaxy, M33, as represented by straight sections. The dots represent the observations of S. Patterson.

mean of the observations shown in Figure 3 by choosing for the first bracket of (8) the value 30. With this choice the right-hand side of (8) gives a curve as shown in Figure 3. It is true that this curve does not go directly through the six normal points, as did the curve used by Mayall and Wyse.[4] Considering, however, the discrepancy between the normal points and the curve in

TABLE III. SURFACE BRIGHTNESS IN SPIRAL GALAXY M33. NUMERICAL COEFFICIENTS OF REPRESENTATION BY STRAIGHT SECTIONS

n	R_n	a_n (arb. unit)
1	40′	9.3
2	20	10.0
3	10	6.7
4	5	24.0
5	2.5	50.0

TABLE IV. COMPUTATION OF GRAVITATIONAL FORCE IN SPIRAL NEBULA M33

r	$a_n g(r/R_n)$ $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$\sum_n a_n g$	$(r\Sigma a_n g)$
4′	2.32	3.91	3.73	14.38	5.60	29.9	11.0
8	3.64	5.56	4.01	2.69	1.30	17.2	11.7
12	4.54	6.22	1.51	1.15	0.60	14.0	13.0
16	5.17	5.99	0.75	0.62	0.35	12.9	14.4
20	5.58	4.16	0.47	0.38	0.20	10.8	14.7
24	5.79	2.24	0.32	0.29	0.10	8.7	14.5
28	5.79	1.54	0.23	0.19	0.07	7.8	14.8
32	5.56	1.12	0.17	0.17	0.04	7.1	15.0
36	5.05	0.87	0.13	0.12	0.02	6.2	15.0
40	3.87	0.70	0.11	0.10	0.00	4.8	13.8

proportion to the probable errors of the normal points, one finds from Figure 3 that three points are represented by the curve within their probable error, and the three other points are represented with discrepancies between 1 and 2 times their probable error. It seems, therefore, that the discrepancy between the observed normal points and the computed curve can hardly be considered significant.

We conclude, therefore, that the present velocity observations in M 33 do not disagree with the assumption of identical mass and light distribution. On the other hand it is obvious that the observational data in M 33 are in fact insufficient to prove such constancy of the mass-luminosity ratio.

Since the surface brightness measurements here used were given only in arbitrary units, the above value for the first bracket in (8) gives a value of f in terms of the reciprocal of the same arbitrary unit. In the computation of the A_n values by (3), however, the arbitrary unit cancels out and correspondingly the total mass computed by (4) is obtained in our regular units. The result for M 33, using a distance of 480,000 parsecs, is

$$M = 5 \times 10^9 M_\odot.$$

This value is somewhat higher than those given earlier, since in the present picture more mass is put between 20′ and 40′ from the center than was done in the earlier analysis.

Finally, the mass-luminosity ratio f can be obtained in the regular units by employing Holmberg's determination of the apparent photographic magnitude of M 33 for which he finds +6.32 mag.[10] Combining this determination with the above values for distance and mass, one finds for the mass-luminosity ratio of M 33

$$f = 4.$$

4. *Elliptical galaxy NGC 3115.* NGC 3115 is an E7 galaxy which has been studied in detail by Oort, whose analysis was based on his own photometric observations and on the rotational velocities obtained by Humason.[12] Humason's velocity measurements cover only the central portion out to 45″ from the center. This central portion contains only a fraction of the total luminosity of the galaxy. One can therefore not expect to derive from these observations the total mass of the galaxy. One may hope however to obtain an estimate of the mass-luminosity ratio of the central portion.

The measured velocities fitted very well a solid body rotation which indicated a fairly homogeneous mass distribution. This combined with the steep rise of the luminosity toward the center indicated a great increase of the mass-luminosity ratio from the center outwards. This result could not be avoided in Oort's analysis since an early estimate of the velocity dispersion in the central part of NGC 3115 had given so small a value that, according to Oort's equation (1),[11] the measured rotational velocity and the gravitationally defined circular velocity must be practically identical.

Recently Dr. Minkovoski has kindly made available a new, though still very preliminary, estimate of the random velocities in NGC 3115 based on spectrograms of higher dispersion. From this it appears that the velocity dispersion in the central portion of NGC 3115 might be as high as 300 km/sec. With this high value, Oort's equation (1) gives large differences between the observed rotational velocity and the circular velocity in the central portion of NGC 3115. In consequence it does not seem possible to derive the mass distribution in this galaxy until detailed measurements of the velocity dispersion are available.

On the other hand, the mass-luminosity ratio may now be estimated as follows: At the outermost point to which Humason's velocity measurements reach, the rotational velocity is so high and the system relatively so flat that the random velocities presumably play only a minor role and the rotational and circular velocities should be nearly equal. To obtain this equality one has to choose $(\kappa f)^{\frac{1}{2}} = 12$ in Oort's terms,[11] according to his Table 2 (Line 5, columns 5 and 6). Assuming κf to be constant and to have the above value, one can compute the run of the circular velocity and of the velocity dispersion from Oort's data (Table 2, equation 1, and the numbers given in the last paragraph of page 300). The results are listed here in Table V. They show that the value of κf could not have been chosen noticeably smaller since if so chosen the circular velocity at 45″ from the center would have turned out smaller than the observed rotational velocity. On the other hand, the κf value could not have been taken much larger since then around 16″ from the center an improbably large value for the velocity dispersion would have been found.

TABLE V. VELOCITIES IN THE INNER PARTS OF NGC 3115

Distance from Center $\bar{\omega}$	Circular Velocity θ_c	Rotational Velocity θ_0	Velocity Dispersion $\bar{\Pi}$
8″.1	510	79	320
16.1	580	158	340
32.2	530	316	240
45.5	480	447	100

Finally, to obtain f, one has to estimate κ, which in Oort's terms is the factor by which his distance estimate has to be corrected. According to Dr. Hubble, it appears likely that Oort in his early work may have overestimated the distance of NGC 3115 by perhaps 30 per cent so that on the new distance scale $\kappa = 1.4$. From the above value of κf one then obtains

$$f = 100.$$

This value for the mass-luminosity ratio of the inner portion of NGC 3115 is much larger than that found for the solar neighborhood, as was already emphasized by Oort.

Stebbins and Whitford have measured photoelectrically the apparent magnitude of NGC 3115.[13] With a correction of 0.4 mag. for galactic absorption, with an estimated correction of 0.3 mag. for insufficient diaphragm size, and with a distance of 2,100,000 parsecs one finds from their measurement

$$L = 9 \times 10^8 L_{\odot}.$$

If one assumes that the above value of the mass-luminosity ratio holds throughout the main body of NGC 3115, one obtains finally for its total mass

$$M = 9 \times 10^{10} M_{\odot}.$$

5. *Elliptical galaxy M 32.* The rotational velocities in the Andromeda nebula as measured by Mayall appear to show a marked difference between the two sides of the galaxy.[5] As Figure 1 shows, this difference between the south preceding side (crosses) and the north following side (dots) appears over a range of central distances from 40′ to 90′ and averages approximately

$$\Delta V = 80 \text{ km/sec.}$$

Dr. Baade has pointed out that the velocity asymmetry in the Andromeda nebula has a corollary in an asymmetry of form. Figure 5 is a reproduction of a plate taken with the 48-inch Schmidt telescope of the Palomar Observatory. The major axis of the Andromeda nebula is drawn into this picture in such a way that it properly represents the central core and the north following half. On the south preceding side one now finds that the spiral structure is not properly centered around the major axis, but rather seems to be centered on the far side of the major axis.

It is tempting to assume that it is the gravitational pull of the nearer companion M 32 which causes the asymmetry in velocity and form in the Andromeda nebula. Under this assumption one may derive a rough estimate of the mass of the companion.

Regarding the position of the companion, Dr. Baade has pointed out that the companion cannot be in front of the plane of the Andromeda nebula since an increasing lack of resolution into stars is noticed in M 32 where it is covered by the spiral arms of the Andromeda nebula. We shall here assume that the companion in fact lies in or near the plane of the Andromeda nebula. If in reality the companion lies well behind the plane, the mass here derived would be a lower limit. Since little can be said about the orbit of

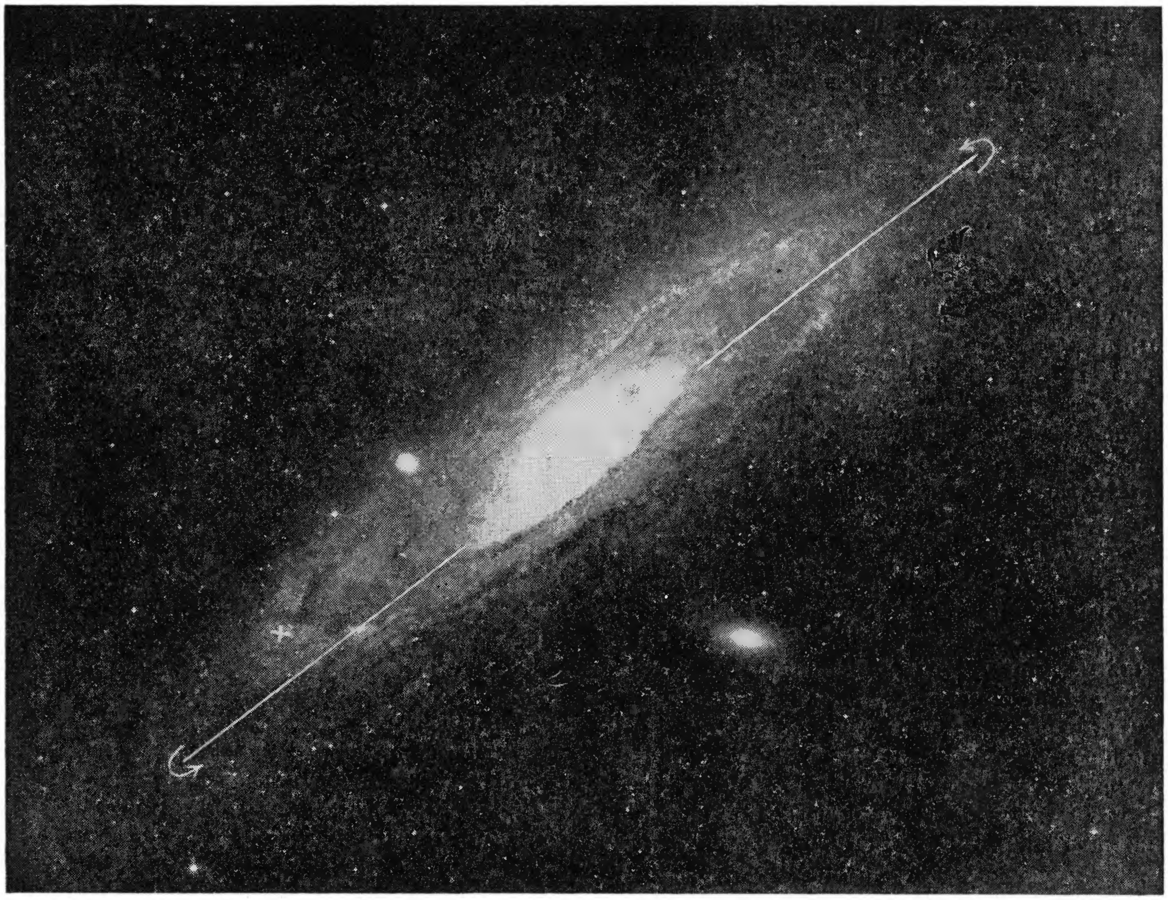

Figure 5. Asymmetry in form of the Andromeda nebula. The sense of the rotation is toward the observer at the lower left, south preceding, and away from the observer at the upper right, north following. The forward motion at the lower left is somewhat slower than the backward motion at the upper right. The major axis is drawn, according to Dr. Baade, through the nucleus, through all the arms at the upper right, and through the outermost arms at the lower left. The perturbed area, marked by a cross, is noticeable by its asymmetric form relative to the major axis. This perturbation may be caused by the close companion, M 32 (to the left of the axis).

the companion relative to M 31, the dynamical situation can be set up only in the roughest of approximation:

$$\Delta V = gt, \quad g = \frac{GM}{d^2}. \qquad (9)$$

Here g is the average gravitational acceleration exerted by the companion on the perturbed region and t is the length of time during which this acceleration has been effective. If this effective time had been half a rotation period or longer, presumably a semistationary state would have ensued which would have been symmetrical with respect to the line connecting the centers of the Andromeda nebula and the companion. Thus the observed asymmetry in the rotational velocity could not have been explained. If, on the other hand, the effective time was only about a quarter of the rotation period, the perturbed area has receded from the companion throughout this time and hence has been decelerated in its motion, in agreement with the observed velocity asymmetry. If one, therefore, takes t to be of the order of a quarter of the period of rotation and if one takes for the distance of the perturbed region from the companion, d, the present value, one has

$$t = 5 \times 10^7 \text{ yrs.}, \quad d = 8000 \text{ pc}.$$

With this and the above value for ΔV equations (9) give for the mass of M 32

$$M = 2.5 \times 10^{10} M_\odot.$$

Holmberg's apparent magnitude for M 32,[10] together with the new distance of the Andromeda nebula, gives for the luminosity of M 32

$$L = 1.1 \times 10^8 L_\odot,$$

which results in a mass-luminosity ratio for M 32 of

$$f = 200.$$

The uncertainty of this mass-luminosity ratio is obviously great. The approximate agreement with the value found for the elliptical nebula NGC 3115, however, seems encouraging.

6. *Coma cluster of galaxies.* Additional data on the masses of galaxies may be obtained from the dynamics of clusters of galaxies.[14] For an approximately stationary cluster the virial theorem gives

$$\tfrac{1}{2}M_c\bar{V}^2 = -\tfrac{1}{2}\Omega. \quad (10)$$

Here M_c is the mass of the cluster, V is the total velocity of a member galaxy, and Ω is the potential energy of the cluster.

The assumption of approximate stationariness may be questionable for a loose and extended system like the Virgo cluster but appears fairly safe for a compact system like the Coma cluster. In the latter case, with a cluster radius about 8×10^5 parsecs and an average total velocity of a member galaxy about 1400 km/sec (see below), a galaxy traverses the distance of the cluster radius in half a billion years on the average. Consequently, if the gravitational potential had a value much less than that required by the virial theorem the cluster would expand with unaltered velocities so that its radius would more than double in a billion years. Or, conversely, the cluster would have had to have been formed within the last billion years in its present compact form, contraction not being possible if Ω is much less than the kinetic energy—which seems hard to imagine. On the other hand, the virial theorem need be used here only in rough approximation, which would not be vitiated by such minor deviations from stationariness as pulsations with a moderate amplitude or slow expansions or contractions.

To apply (10) to the Coma cluster, observations of the velocity dispersion in the cluster are needed. Mr. Humason has generously made available his radial velocity observations for 22 member galaxies of the Coma Cluster. These 22 values give for the root mean square radial velocity relative to the mean of the cluster 1080 km/sec. This value may be somewhat too high since among the 22 galaxies observed there might be a few non-members, which in most cases would appear with rather high relative velocities. Among the 22 relative radial velocities there are five which exceed 1500 km/sec in absolute value. If these are eliminated the remaining 17 values give a root mean square radial velocity 630 km/sec. This elimination of all five of the high values is nearly certainly an overcorrection for possible non-members. In the following computation, therefore, the root mean square radial velocity in the Coma cluster is taken to be

$$(\tfrac{1}{3}\bar{V}^2)^{\frac{1}{2}} = 825 \text{ km/sec}.$$

This value is estimated to have an uncertainty of not more than 30 per cent.

To apply the virial theorem (10) to a cluster one needs further to evaluate the gravitational potential in terms of the distribution of the members in the cluster. For a spherically symmetrical system, which should be a sufficient approximation to the Coma cluster, this evaluation is conveniently done in terms of strip counts, i.e., counts of members in strips of unit width, $S(q)$, as a function of the perpendicular distance of the strip from the cluster center, q. With the usual relation between the strip counts and the actual density in the cluster as a function of the central distance, $\rho(r)$,

$$S(q) = \int_q^R \rho(r) 2\pi r dr \quad (11)$$

one obtains for the gravitational potential and the total mass of the cluster

$$\Omega = -\int_0^R \frac{GM}{r} 4\pi r^2 \rho\, dr = -2G\int_0^R S^2(q)\, dq, \quad (12)$$

$$M_c = \int_0^R 4\pi r^2 \rho\, dr = 2\int_0^R S(q)\, dq. \quad (13)$$

By introducing these two expressions into (10) one obtains the virial theorem in the form

$$\bar{V}^2 = \frac{GM_c}{\bar{r}} \quad \text{with} \quad \bar{r} = 2\frac{\left(\int_0^R S\,dq\right)^2}{\int_0^R S^2\,dq}. \quad (14)$$

Thus the strip counts determine only the effective mean distance, $\bar{r}$, and high accuracy or smoothness is not needed in them since they occur only in simple quadratures. The units of the strip counts cancel out in (14) and hence can be chosen arbitrarily. The employment of the virial theorem in the form of (14) makes direct use of the observationally obtainable counts and thus avoids the necessity of using a model for the internal density distribution in the system.

For the Coma cluster Zwicky has given counts in terms of number of galaxies per square degree, $n(\text{I})$, as a function of the projected distance from the center, I.[15] These counts were here reduced by eight galaxies per square degree throughout to bring them into agreement with the results of a recent investigation by Omer who found a total membership of 800 galaxies for the Coma cluster and a radius of approximately 100′.[16] The corrected counts were then transformed into strip counts by the usual relation

$$S(q) = 2\int_q^R n(\text{I})\, d\sqrt{\text{I}^2 - q^2}. \qquad (15)$$

The strip counts of stars per minute of arc thus obtained are listed in Table VI.

TABLE VI. COMPUTED STRIP COUNTS IN COMA CLUSTER

q	$S(q)$*	q	$S(q)$*
1′.25	15.09	52′.5	1.82
3.75	11.96	57.5	1.51
7.5	8.70	62.5	1.13
12.5	7.17	67.5	1.07
17.5	5.38	72.5	0.44
22.5	4.36	77.5	0.29
27.5	3.88	82.5	0.28
32.5	2.78	87.5	0.21
37.5	2.91	92.5	0.20
42.5	2.34	97.5	0.26
47.5	2.27	102.5	0.00

* Unit = stars per minute of arc.

With these data the second of equations (14) gives

$$\bar{r} = 89' = 6.6 \times 10^5 \text{ pc}.$$

Here the distance of the Coma cluster was taken to be 2.5×10^7 parsec which was derived from the recession constant on the new distance scale and from the recession velocity of 6680 km/sec found from Humason's radial velocities for the Coma cluster. Finally, with this value for $\bar{r}$ and the value for the velocity dispersion discussed earlier, the virial theorem as represented by the first of equations (14) gives for the total mass of the Coma cluster

$$M_c = 3 \times 10^{14}\, M_\odot.$$

If this total mass is divided among the 800 member galaxies, one obtains for the average mass of a galaxy in the Coma cluster

$$M = 4 \times 10^{11}\, M_\odot.$$

If account is taken of the new distance scale, this result is entirely in agreement with the much earlier investigations by Sinclair Smith and Zwicky[14] on clusters of galaxies and with the recent investigation by Page on double galaxies.[2]

To estimate the total luminosity of the Coma cluster or the average luminosity of the 800 member galaxies, one may use the early counts by Hubble and Humason in this cluster.[17] From these counts one finds +16.6 mag. for the average apparent photographic magnitude (averaged in terms of luminosity, not magnitude). This value has to be corrected by 0.25 mag. for galactic absorption and by very roughly 0.75 mag. for an underestimate of the brightnesses in the early accounts. This latter correction is extremely uncertain, being based on modern photoelectric measurements of only the brighter cluster members.[13] With these corrections and a modulus +32.0 mag. one obtains for the average absolute photographic magnitude −16.4 mag. which is only slightly fainter than the value −16.7 mag. which one finds for Holmberg's 28 nearby galaxies.[10] Thus

$$L = 5 \times 10^8\, L_\odot.$$

With the above value for the average mass one finally finds for the mass-luminosity ratio in the Coma cluster

$$f = 800.$$

This bewilderingly high value for the mass-luminosity ratio must be considered as very uncertain since the mass and particularly the luminosity of the Coma cluster are still poorly determined.

7. *Summary.* In the preceding five sections observational data have been discussed which bear on both the mass distribution and the mass-luminosity ratio in galaxies. Regarding the first point, the mass distribution, the data referring to the Andromeda nebula M 31, the spiral galaxy in Triangulum M 33, and the elliptical galaxy NGC 3115, may be summarized as follows. The observations now available permit the assumption that in any one galaxy the mass distribution and the luminosity distribution are identical. On the other hand the present observations are not accurate enough to prove this assumption. The earlier deduction that—contrary to the present conclusion—the mass density had to be much more homogeneous than the light density, seems to have arisen from a) an underestimate of the

random velocities in NGC 3115, b) large uncertainties in the early determinations of the rotational velocities in M 31 and M 33, and c) the lack of a detailed analysis of the most recent velocity observations in M 31.

Regarding the second point, the mass-luminosity ratio, the results are summarized in Table VII. In this table the following data have been added to those discussed in the preceding sections. The mass-luminosity ratio for the solar neighborhood has been taken from the early discussion by Oort.[18] His unit of luminosity, $M_{ph} = +6$, has been transformed here to the luminosity of the sun. The data on the Large Magellanic Cloud have been taken directly from the analysis by Holmberg,[1] after transferring them to the new distance scale. The mass of the globular cluster M 92 has been taken from the recent determination by O. C. Wilson.[3] This determination is based on the velocity dispersion within the cluster. Since the observed velocity dispersion is only somewhat larger than the error dispersion, the actual value of the mass of the cluster seems still very uncertain. However a secure upper limit to the mass can be computed by using the observed velocity dispersion without any correction for the error dispersion. Only this upper limit is listed in Table VII. Finally, the data on double galaxies have been taken from the recent paper by Page,[2] after transferring them again to the new distance scale. Page's twenty pairs were divided into two groups, the first containing nine pairs with elliptical galaxies of which two pairs consist each of an elliptical and a late spiral galaxy in which presumably the elliptical galaxy dominates in mass, and the second group containing the ten remaining pairs with spiral galaxies, among which are two pairs each combining a late spiral with an irregular galaxy in which the spiral dominates in luminosity and hence presumably also in mass; one pair consisting of two irregular galaxies was not used. For each group, the average luminosity and mass were derived separately, as shown by Table VII. The division of the data into two groups makes the material in each group dangerously small; indeed, in each group, the omission of the pair with the biggest velocity difference would approximately halve the resulting average mass. Hence the data given in Table VII for double galaxies are easily uncertain by a factor 2. Nevertheless, the grouping of the material into elliptical and spiral galaxies seemed useful for the comparison with the other data.

TABLE VII. SUMMARY OF MASS-LUMINOSITY RATIOS

Objects	Distance (in kpc)	Luminosity (in sol. lum.)	Mass (in sol. mass)	Mass/Lum. f
Solar Neighborhood	—	—	—	4
Triangulum Nebula, M33	480	1.4×10^9	5×10^9	4
Large Magellanic Cloud	44	1.2×10^9	2×10^9	2
Andromeda Nebula	460	9×10^9	1.4×10^{11}	16
Globular Cluster, M92	11	1.7×10^5	$<8 \times 10^5$	<5
Elliptical Galaxy, NGC 3115	2100	9×10^8	9×10^{10}	100
Elliptical Galaxy, M32	460	1.1×10^8	2.5×10^{10}	200
Average S in Double Gal.	—	1.3×10^9	7×10^{10}	50
Average E in Double Gal.	—	8×10^8	2.6×10^{11}	300
Average in Coma Cluster	25000	5×10^8	4×10^{11}	800

The mass-luminosity ratios listed in Table VII indicate the following points:

A. The systems considered of pure population I show a fairly uniform, low mass-luminosity ratio.

B. The Andromeda nebula has a mass-luminosity ratio intermediate between population I galaxies and elliptical galaxies.

C. Systems considered of pure population II do not have a uniform mass-luminosity ratio; rather, the small system of a globular cluster has a much lower mass-luminosity ratio than the elliptical galaxies, as was already emphasized by Oort.

D. The elliptical galaxies have a mass-luminosity ratio greater than that of the population I galaxies by a factor of the order of 40. This big difference has already been pointed out by Oort[11] and Holmberg.[1]

E. The double galaxies give a supporting hint regarding the larger mass-luminosity ratio for elliptical galaxies compared to spirals. In addition, the double galaxies give mass-luminosity ratios approximately three times higher than the preceding data. In view of the great observational uncertainties it seems doubtful whether this fact is significant.

F. The Coma cluster gives a mass-luminosity ratio approximately four to eight times larger than even the elliptical galaxies. Whether this difference is real will only be ascertained when the masses of elliptical galaxies such as NGC 3115 and the total luminosity of the Coma cluster are determined more accurately.

In the discussions of the preceding sections a number of points have become apparent where new observations with instruments now available could greatly strengthen our knowledge in this field. Even though obviously the problems here discussed must compete with many other problems for the available observing time it might seem useful to list here these points:

1. New radial velocity observations of the emission patches in M 33, particularly in the two ranges of central distances from $5'$ to $15'$ and $30'$ to $40'$, to check whether in fact the relation between velocity and distance is as flat as suggested by the curve in Figure 3.

2. In NGC 3115, the determination of the velocity dispersion in the central part of the galaxy to ascertain to what degree the dynamics is there dominated by the random velocities rather than the systematic rotation, and a strengthening and if possible extension to $50''$ of the data on the rotational velocity in an attempt to find the point where the rotational velocity stops rising.

3. The determination of the velocity dispersions in the centers of one or two E0 galaxies, together with the photometry of the luminosity profiles, so as to gain more data on the mass-luminosity ratios of individual elliptical galaxies.

4. An extensive photometry in the Coma cluster for the purpose of obtaining its total luminosity which at present seems less certain than its total mass.

5. An extension of Page's data on double galaxies to about twice as many pairs,[2] equally objectively selected and homogeneously treated, with the same level of accuracy in the velocity determinations so that the results would depend to a somewhat lesser degree than now on a very few cases with large velocity differences. Simultaneously photoelectric photometry of all the pairs measured.

8. *Discussion.* The following discussion will be limited to two points, namely the mass-luminosity ratio of the Andromeda nebula and of the elliptical galaxies.

Formerly the Andromeda nebula was considered as consisting essentially of population II stars since its spiral arms contribute little to the total luminosity and since its color is fairly similar to that of elliptical galaxies and globular clusters. Recently, however, Dr. Baade has pointed out that the main body of the Andromeda nebula might only in part consist of original population II stars and might for the rest consist of stars which were born in the spiral arms but have since diffused out of the arms and are now smoothly distributed throughout the whole body. One might thus represent the Andromeda nebula by a mixture of population II stars and old population I stars. In this formulation, population II can be taken as represented by elliptical galaxies both as regards their color and their mass-luminosity ratio, whereas old population I might be thought of as represented by all the types of stars in the solar neighborhood which have velocity dispersions in the galactic plane of about 25 km/sec. This old population I would not contain any early main sequence stars of spectral types O, B, and A but would contain all the typical red giants and the lower main sequence from spectral type F on. Thus this old population I could be taken to have a color very similar to that of population II but to have, except for the unessential brightest end, a luminosity function very similar to that of the general population I so that its mass-luminosity ratio would agree with that of the pure population I systems.

If one correspondingly represents the mass and the luminosity of the Andromeda nebula by

$$\mathrm{M_A} = \mathrm{M_I} + \mathrm{M_{II}}, \quad L_\mathrm{A} = L_\mathrm{I} + L_\mathrm{II}, \qquad (16)$$

and if one takes from Table VII the corresponding mass-luminosity ratios

$$\frac{\mathrm{M_A}}{L_\mathrm{A}} = 16, \quad \frac{\mathrm{M_I}}{L_\mathrm{I}} = 4, \quad \frac{\mathrm{M_{II}}}{L_\mathrm{II}} = 150, \qquad (17)$$

then one obtains by solving (16) and (17)

$$\mathrm{M_I} = 0.23\,\mathrm{M_A}, \quad \mathrm{M_{II}} = 0.77\,\mathrm{M_A},$$
$$L_\mathrm{I} = 0.92\,L_\mathrm{A}, \quad L_\mathrm{II} = 0.08\,L_\mathrm{A}.$$

Thus one finds in this somewhat formal representation that most of the mass, 77 per cent, of the Andromeda nebula is provided by the original population II while most of the light, 92 per cent, is provided by the old population I. Whether this representation is useful can only be decided when observations of further characteristics are available in terms of which the Andromeda nebula can be compared with other systems of pure populations.

Regarding the elliptical galaxies the question arises what form must the luminosity function in these systems have so as to give a mass-luminosity ratio 40 times larger than that of the solar neighborhood. To investigate the luminosity function in elliptical galaxies only two data appear available at present, the color index and the mass-luminosity ratio. The color indicates that the bulk of the total luminosity is produced either by K giants or by K dwarfs. The mass-luminosity ratio indicates that the bulk of the total mass must be contributed either by faint dwarfs or by stars which were formerly rather bright but are now exhausted and faint. Combining these two data one may consider the following three possibilities.

In the first possibility, the light is to be produced mainly by K dwarfs and the mass is to be contributed mainly by very faint dwarfs. This would suggest a luminosity function quite similar in form to that of the solar neighborhood, however shifted as a whole to fainter luminosities by about five magnitudes. Such a luminosity function would suggest an average stellar mass approximately four times smaller than that in the solar neighborhood.

In the second possibility, the luminosity is mainly to be produced by K giants and the mass is mainly to be contributed by faint dwarfs. The corresponding luminosity function would have to have a rather particular form. It would have to rise relatively slowly from the K giants at about 0 absolute magnitude to the F stars around +3 since otherwise these earlier types would contribute to the total light and hence affect the color. On the other hand, at the fainter absolute magnitudes the luminosity function would have to rise very steeply so that at an absolute magnitude of +15 or fainter the numbers of stars were sufficiently high to produce the large total mass.

In the third possibility, the luminosity is again mainly to be produced by K giants, but the mass is to be contributed mainly by exhausted faint stars. According to the recent analysis by Sandage of the globular cluster M 3,[19] population II stars which were born with an absolute magnitude brighter than +3 should by now be exhausted and faint. This suggests that in the early phases of the life of an elliptical galaxy the luminosity function had a form similar to that in the solar neighborhood but as a whole shifted to brighter stars by about 7 magnitudes; which would correspond to an average stellar mass about 6 times larger than that in the solar neighborhood. The bulk of the mass would then be contributed by stars which originally had an absolute magnitude of around +1 which, however, by now would be exhausted and faint. The bulk of the present luminosity on the other hand would be produced by the brightest still unexhausted stars which should be quite similar to the brightest stars now found in globular clusters. This third possibility would suggest a rather high frequency of star deaths in the early phases of a population II system. Other indications for the possible occurrence of this phenomenon have been discussed previously.[20]

It would seem much worth-while to try to find additional observable characteristics of elliptical galaxies which might help in discriminating between the above three possibilities.

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