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ON THE FRAGMENTATION OF GAS CLOUDS INTO GALAXIES AND STARS

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ABSTRACT

The present paper seeks to trace the steps whereby an extensive cloud of density 10^{-27} gm/cm^3 evolves, first into galaxies and then into stars. The indication is that galaxies are likely to form, not as single objects, but in clusters. Stars apparently form at an early stage in the history of a galaxy. There would seem to be reason why the average mass of such stars should lie somewhat below the solar mass.

I. INTRODUCTION

The arguments of the present paper, it should be stated at the outset, are of a mainly tentative character: the theory is still too qualitative and its comparison with observation not sufficiently strict to justify a stronger statement. Yet, in spite of these shortcomings, there seem to be rather many points where the idea of fragmentation—of gravitational turbulence, as it might be called—holds out hope of explaining important observational data that it seems worth while to describe the process at some length.

As examples of the problems that come within the scope of the present paper the following may be mentioned. Why do galaxies tend to occur in clusters?[1] Why are the masses of the galaxies mainly confined in the range $3 \times 10^9 \odot$ to $3 \times 10^{11} \odot$, with possibly a tendency to fall into two groups at the ends of this range?[2] Why is the typical mass of a type II star of order $\odot$? Why do type II stars apparently form almost simultaneously with the origin of a galaxy?

II. DENSITY AND TEMPERATURE IN AN EXTRAGALACTIC HYDROGEN CLOUD

No attempt is made in the present paper to explain the origin of the extragalactic cloud under consideration. This is a matter that must be treated, not in the present Newtonian theory, but in a theory of the expanding universe.

Reference to the expanding universe allows us, however, to make use of a result that appears in all theories of the expanding universe, namely, that it is necessary that the density of our cloud shall appreciably[3] exceed 10^{-28} gm/cm^3, for otherwise the effects of expansion cannot be neglected; the problem becomes Newtonian only when this condition is satisfied.

Now the density of the cloud may reasonably be bracketed between this lower limit of 10^{-28} gm/cm^3 and the upper limit suggested by Sinclair Smith's determination[4] of the mean density within the great nebular clusters. Again, with Baade's new distance scale, this upper limit turns out to be somewhat greater than 10^{-26} gm/cm^3. In the following work we shall use an intermediate value of 10^{-27} gm/cm^3 for the initial density ρ_0 of our cloud.

To estimate the temperature, it is assumed that, in whatever process originally led to the formation of the cloud, mass motions developed within the gas. It is reasonable to

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[1] J. Neyman, E. L. Scott, and C. D. Shane, *Ap. J.*, **117**, 92, 1953.

[2] T. Page, *Ap. J.*, **116**, 63, 1952.

[3] Using Baade's redetermination of the distance scale.

[4] *Ap. J.*, **83**, 23, 1936.

suppose that the velocities so developed were in excess of 10 km/sec, and perhaps, in the case of very large clouds, in excess of 100 km/sec. Now ordinary turbulence must eventually convert mass motions into heat. Thus it is being assumed, in effect, that in the formation of the cloud a supply of energy, ultimately converted into heat, of certainly more than 10^{12} ergs/gm, and perhaps more than 10^{14} ergs/gm is made available. Assuming, then, an initial supply of energy, the next step is to estimate the temperature to which hydrogen gas will be raised. To make such an estimate it is necessary to collect together the following thermal properties of hydrogen:

a) The thermal energy of atomic hydrogen is $\frac{3}{2}\mathfrak{R}T$/gm, where $\mathfrak{R}$ is the gas constant and T is the temperature in ° K. From this it follows that the energy required to increase the temperature of atomic hydrogen from absolute zero up to 10,000° K is somewhat in excess of 10^{12} ergs/gm.

b) Hydrogen becomes collisionally ionized at temperatures higher than 10,000° K. Thus, even in the absence of radiation, hydrogen changes[5] from being largely un-ionized at 10,000° K to being almost wholly ionized at 25,000° K.

c) The energy required to ionize hydrogen is somewhat greater than 10^{13} ergs/gm.

d) The time of formation of our cloud may be taken as of the same order as Hubble's constant, this being about 10^{17} seconds, with Baade's new distance scale. In this length of time, ionized hydrogen radiates an amount of energy that is much greater than the energies arrived at in *a* and *c*. Excluding line emission, the hydrogen radiation amounts to[6]

$$1.45 \times 10^{-27} T^{1/2} \left(1 + \frac{3.85 \times 10^5}{T}\right) N^2 \text{ ergs/cm}^3\text{/sec}, \tag{1}$$

where N is the number of hydrogen atoms per cubic centimeter, and the kinetic temperature T is in ° K. The first term within parentheses gives the contribution of free-free transitions, and the second term takes account of free-bound transitions. It is particularly to be noticed that equation (1) includes the radiation emitted in the Lyman continuum. This radiation excapes from the cloud only when the hydrogen is effectively wholly ionized, as it is at 25,000° K but not at lower temperatures—say at 15,000° K. Thus the escape of radiation from the cloud at 15,000° K must be calculated, excluding the Lyman continuum. In this case the factor $3.85 \times 10^5/T$ in equation (1) must be replaced by $6.5 \times 10^4/T$.

Compare the two temperatures 15,000° and 25,000° K; in the former case the emission of escaping radiation amounts to 3×10^{13} ergs/gm of ionized hydrogen in 10^{17} seconds, and in the latter case to 1.3×10^{14} ergs/gm in 10^{17} seconds. Both these estimates refer to a hydrogen density of 10^{-27} gm/cm.^{3}

e) As the temperature of hydrogen increases above 25,000° K, the rate of radiation decreases until temperatures above 3.85×10^5 ° K are reached. The rate of radiation then slowly increases. At very high temperatures, however, the main energy requirement comes not from the necessity to make good the loss due to radiation but simply from the energy required to heat the hydrogen. Thus the energy required to heat ionized hydrogen to temperature T is $3\mathfrak{R}T$ per unit mass, which amounts to 2×10^{14} ergs/gm at 10^6 ° K; for comparison, the radiation emitted in 10^{17} seconds by ionized hydrogen at 10^6 ° K is only 7×10^{13} ergs/gm.

If one considers, then, the temperature obtained in the cloud as a function of the energy supply (taken to occupy a time of 10^{17} seconds), one may distinguish the following

[5] Cf., e.g, F. Hoyle, *Some Recent Researches in Solar Physics* (Cambridge: At the University Press, 1949), p. 25.

[6] R. Minkowski, *Ap. J.*, **96,** 206, 1942. The formula given by Minkowski is sufficiently accurate for the present purpose. In a more thoroughgoing treatment it would be necessary to include the process of two-quantum emission from the 2s state of the hydrogen atom, discussed by Spitzer and Greenstein in *Ap. J.*, **114,** 407, 1951.

stages. During the early stages (A–B, say), during which atomic hydrogen becomes collisionally ionized, energy is absorbed only in raising the temperature of atomic hydrogen. If all the hydrogen were ionized at 15,000° K (stage C), the energy required for ionization would be 10^{13} ergs/gm, and the energy required to make good the loss through escape of radiation would be 3×10^{13} ergs/gm. But at 15,000° K only about two-thirds of the hydrogen is collisionally ionized. This circumstance reduces the energy required for ionization to $\frac{2}{3} \times 10^{13}$ ergs/gm, and the average radiation rate to $\frac{4}{3} \times 10^{13}$ ergs/gm, thereby giving a total of 2×10^{13} ergs/gm. When the temperature has risen to 25,000° K (stage D), the hydrogen is effectively wholly ionized, and the energy supply must include 10^{13} ergs/gm for ionization, and 1.3×10^{14} ergs/gm for the escape of radiation, giving a total of 1.4×10^{14} ergs/gm. Since radiation then decreases with increasing temperature, the extra energy required to take the hydrogen to 10^{5}° K (stage E) is only a little more than the energy required to reach stage D. The energy at stage C will differ from that at stage B by essentially the difference of thermal energy between 10^{5}° K and 25,000° K. The plot of temperature against energy supply must therefore be very flat between these two stages; it would not steepen again until temperatures exceeding 3×10^{5}° K were reached. This latter steepening arises from the energy required to heat the hydrogen, which becomes the major requirement at temperatures greater than 10^{6}° K. At very high temperatures, the rate of rise of the temperature with energy supply must approach the same slope as during the early stages A–B.

The sequence of events we have described is important in showing that a dichotomy of temperature is to be expected. Excluding temperatures less than 10^{4}° K as corresponding to an improbably small energy supply (mass motions less than 10 km/sec developing during the formation of the cloud), *either the temperature lies in the range 10,000°–25,000° K, or the temperature as a general rule must be high, of the order of* 3×10^{5}° *K or more.*

Intermediate temperatures are not rigorously forbidden, but, owing to the flatness of the temperature rise during stages D–E, it is most unlikely that the energy supply will happen to fall into the narrow range that corresponds to intermediate temperatures.

In concluding the present section, it should be emphasized that although the sequence of effects described would not be affected, its precise form would be altered by changes in the value chosen for the time scale and for the density ρ_0. In this connection it may be noted, however, that the time scale for the formation of the cloud may be expected to be of the order of $(\rho_0 G)^{-1/2}$: for a density of 10^{-27} gm/cm^3 this is precisely of the order of 10^{17} seconds. A reduction of the time scale by a factor y hence implies an increase in density by y^2, and this produces a change in the importance of radiation by y. For $y > 1$ the importance of radiation would be enhanced, and this would have the effect of enhancing to an even greater degree the flatness of the temperature rise between stages D and E. Accordingly, the conclusion of the previous paragraph would be threatened only if $y < 1$, thereby reducing the importance of radiation. But $y < 1$ would imply a time scale $> 10^{17}$ seconds, which seems excluded by data on the expanding universe. Any revision of the values for the time scale and density must, it seems, confirm a fortiori our previous conclusion.

Having now seen physical reasons why a segregation into two temperature classes is to be expected, the further development of a cloud in the low-temperature class will next be investigated in the following three sections; only when we come to Section VII will high-temperature clouds receive further consideration.

It will turn out that, for the low-temperature clouds, radiation by the hydrogen prevents the temperature from ever rising above 25,000° K, while recombination into neutral atoms prevents the temperature from falling much below 10,000° K. Thus fluctuations of temperature are confined to a fairly narrow range. It will be convenient in the following section to ignore any such fluctuations, taking a constant value of 15,000° K as a suitable compromise.

III. THE CONDITION FOR CONTRACTION

It can be rigorously established that when a gas cloud of uniform composition is in hydrostatic equilibrium, with gravitational forces balanced by gas-pressure gradients—radiation pressure being negligible—the total gravitational potential energy Ω (>0) must be equal to twice the total thermal energy of the gas. A spherical cloud contracts if Ω is greater than this and expands if Ω is less than this. As giving an order-of-magnitude estimate, we may adopt a similar criterion for a nonspherical cloud of volume V, the cloud contracting if Ω is greater than twice the total thermal energy. For the total thermal energy we shall write $2.5\,\mathfrak{R}MT$, this being the thermal energy that the cloud would have if all the material were two-thirds ionized at temperature T (hydrogen is two-thirds ionized at $T = 15{,}000^\circ$ K, which is the numerical value used below for T). For Ω we write $GM^2/V^{1/3}$, so that the contraction condition becomes

$$\frac{GM}{V^{1/3}} > 5\,\mathfrak{R}T\,. \tag{2}$$

Combining equation (1) with the condition $M = \rho_0 V$, $\rho_0 = 10^{-27}$ gm/cm^3, and $T = 1.5 \times 10^{4}$ $^\circ$ K gives

$$M > 1.4 \times 10^{10} \odot\,.$$

No cloud with mass appreciably less than this can contract to form a galaxy (this being subject to the value chosen for T, since it is considered improbable that T would initially be much less than $15{,}000^\circ$ K).

IV. FRAGMENTATION OF CLOUD OF VERY LARGE MASS INTO CLUSTER OF GALAXIES

Consider a cloud with mass appreciably larger than the lower limit set for condensation. The time required for contraction to reduce the dimensions to, say, half of the initial size of the cloud is of the order of $(\rho_0 G)^{-1/2}$; that is, of the order of 10^{17} seconds. Now we have seen that in this time interval ionized hydrogen at $T = 15{,}000^\circ$ K emits about 3×10^{13} ergs/gm. This may be compared with the thermal energy made available during contraction, which is only of the order of $2\mathfrak{R}T$/gm in a contraction in which the dimensions of the cloud are halved, which amounts to less than 3×10^{12} ergs/gm at $T = 15{,}000^\circ$ K. Thus the early phases of contraction provide quite insufficient energy to raise the temperature of the hydrogen. Indeed, the hydrogen, if initially largely ionized, will tend toward appreciable recombination.

If we next imagine a further phase of contraction in which the size of the condensation is again halved, the energy made available through the work done by the gas pressure is again only of the order of 10^{12} ergs/gm. The time required for collapse is now reduced, however, since the density ρ was increased in the previous contraction. But, although the time of contraction is thus reduced by $\rho^{-1/2}$, the rise of density has increased the radiation per gram per unit time by ρ, and this more than offsets the reduction in the time scale. Hence the hydrogen tends still more toward recombination. The same argument could be repeated for further steps of contraction.

The upshot of the argument is that insufficient energy is made available to keep the hydrogen mainly ionized. On the other hand, there is an inverse condition that must be satisfied. This condition requires that the degree of ionization cannot become too low. Thus radiation would effectively cease if the hydrogen were entirely un-ionized. Free electrons would be supplied only by the very small proportion of metal atoms present in the material, and the rate of radiation would then fall to a value lower than the rate of supply of energy by the contraction process (this argument applies not only to radiation by the hydrogen but also to radiation emitted in the forbidden oxygen lines, for example). Hence a balance must be reached in which a proportion—say, a few per cent—of the hydrogen remains ionized. This balance requires the temperature to be maintained near 10,000° K. It follows, therefore, that contraction must be a nearly isothermal process.

Let a uniform isothermal contraction at temperature $T = 10^4$ ° K change the volume of the cloud from V to V'. Then the thermal energy released is given by[7]

$$M\Re T \log_e \frac{V}{V'}. \tag{3}$$

This energy maintains a low degree of ionization in the hydrogen, which adjusts itself so that radiation proceeds at such a rate that the energy leaking away from the system comes eventually into balance with the energy made available by the contraction.

The energy (3), thus lost from the system, may be compared with the gravitational energy released by the compression, which is of the order of

$$\frac{GM^2}{V^{1/3}}\left[\left(\frac{V}{V'}\right)^{1/3} - 1\right]. \tag{4}$$

Now when M is such that $GM^2/V^{1/3}$ is very large compared with $3\,M\Re T$ ($GM^2/V^{1/3} = 3\,M\Re T$ giving the *minimum* mass for contraction when the hydrogen is largely *neutral*), the value of expression (4) is much greater than expression (3), whatever V' may be. Accordingly, the gravitational energy released is not thermally dissipated but must go into mass motions within the condensation, *and these mass motions must be adequate to re-expand the cloud to nearly its original volume.*

It is, of course, possible that before the cloud becomes entirely re-expanded, some of the energy of mass motion so developed will be converted into heat by turbulence and shock waves. This indeed would probably be the ultimate main agency of dissipation if a further process of dissipation were not available. This second process is capable of acting even before substantial mass motions develop, which turbulence and shock waves cannot do. Hence it seems likely that the main dissipation will go through this second process, or, at any rate, that a major proportion of the dissipation so occurs.

The second process arises from the circumstance that since the main cloud mass is, by hypothesis, much larger than the minimum mass that is capable of condensing, the cloud is unstable against the formation of subcondensations. Moreover, since the time of condensation depends only on the inverse square root of the density, it follows that a subregion can condense equally as fast as the main cloud, faster, indeed, if the density in the subregion happens to be higher than the mean density within the cloud.

Next we note that if the contraction condition[8]

$$\frac{GM}{V^{1/3}} > 3\,\Re T \tag{5}$$

is only just satisfied, then formula (4), *applied now with reference to the subregion*, is not much greater than formula (3), provided that V' is not too small compared with V. It will be useful to consider numerical estimates. For $T = 10^4$ ° K, inequality (5) gives a minimum mass of about $3.6 \times 10^9 \odot$, it being assumed in making this estimate that T falls to about 10^4 ° K before the density rises appreciably above 10^{-27} gm/cm^3. Consider

[7] Let v be the specific volume. The thermal energy released by an isothermal contraction dv is $-p dv$ per unit mass. Since $p = \Re\rho T$ for mainly neutral hydrogen, and $\rho = 1/v$, the energy released is $-\Re T\, dv/v$, which, on integrating from v to v', gives $\Re T \log (v/v')$ for the energy released per unit mass. Now for uniform contraction, $v/v' = V/V'$, and hence the energy released is $\Re T \log (V/V')$ per unit mass, and $M\Re T \log_e (V/V')$ for the whole cloud.

[8] This differs slightly from the condition arrived at in Sec. III, the coefficient 3 replacing the previous coefficient 5. The difference is that the temperature has now fallen to about 10,000° K, with the hydrogen mainly neutral, whereas, before, two-thirds of the hydrogen was ionized.

a subunit with approximately this mass. Then for $V' = 10^{-1} V$, expression (3) is comparable with expression (4); for $V' = 10^{-3} V$, expression (3) is still about 30 per cent of (4); but, for $V' = 10^{-6} V$, expression (3) is only about 5 per cent of (4). This means that, for a decrease of linear dimensions by a factor of 2 or 3, there is an almost complete dissipation of the energy of contraction, and hence that a stable bound configuration of about one-third the original dimensions could certainly be formed. It may be that a stable configuration slightly smaller than this could be formed, but in the following discussion we shall accept a permanent shrinkage by a factor of 3. A shrinkage of much more than this would only induce the condition already noted in reference to the main cloud, namely, that the gravitational energy of contraction would go into mass motions that could re-expand the condensation.

Returning, now, to the original cloud of large mass, we have seen that, whereas there is negligible dissipation for a uniform contraction of the whole cloud, the dissipation can be both rapid and considerable if the cloud breaks up into subregions, each subregion possessing a mass that is not greatly in excess of the minimum mass of $3.6 \times 10^9 \odot$, as set by the contraction condition for hydrogen of density 10^{-27} gm/cm^3 that is mainly neutral at a temperature of 10,000° K. We may describe this process by saying that the main cloud fragments into galaxies with masses of the order of $3.6 \times 10^9 \odot$. That the fragments may indeed be described as galaxies is shown by a consideration of their dimensions. Initially, at a density of 10^{-27} gm/cm^3, a sphere of radius 1.2×10^{23} cm contains a mass of $3.6 \times 10^9 \odot$. Permanent contraction by a factor of 3 reduces the radius to about 13,000 parsecs.

Although this is certainly of galactic dimensions, the radius is still considerably greater than the radius of a typical galaxy of mass $3.6 \times 10^9 \odot$. Thus it might appear at first sight as if the degree of permanent contraction were seriously underestimated in the above discussion. It is, however, doubted whether this is so, because there are indications that a considerable further reduction of dimensions is likely to occur at a later phase when stars have formed in these galaxies. This dynamic process is discussed in Section VIII. It may be said that the present calculated radius of 13,000 parsecs should be thought of as a measure of the dimensions of the diffuse extensive halos that surround the galaxies, rather than as an estimate of the final dimensions of the main nucleus.

The results of the present and previous sections might be construed into the conclusion that galaxies never condense singly, that a cluster of galaxies is always formed. Thus if we accept the conclusion of the previous section that no cloud of mass less than $1.4 \times 10^{10} \odot$ can condense in the first place and if we combine this with the galactic mass of $3.6 \times 10^9 \odot$, as deduced above, we see that the smallest condensing cloud should still fragment into about 4 galaxies. The difference between the minimum mass of $1.4 \times 10^{10} \odot$ of Section III and the galactic mass of $3.6 \times 10^9 \odot$, as calculated above, arises from the cooling of the main cloud as it tends to contract. Thus, in Section III an initial temperature of 15,000° K was used, whereas radiative cooling soon reduces this temperature to about 10,000° K. The present argument should not be taken too seriously, however, as it is probably an overstraining of the theory. Even if correct, it does not imply that single galaxies should never be observed; for even if all galaxies do condense into clusters, dynamical encounters must lead to a proportion escaping from their parent-clusters, especially from small clusters.

V. THE HIERARCHY STRUCTURE

The argument presented in the previous section shows that, even when the mass of a galactic condensation is close to the minimum mass set by the contraction condition, effective dissipation of the gravitational energy of contraction is limited to decrease in dimensions by a factor of about 3. Additional contraction simply leads to the development of dynamical motions, with the tendency to re-expand the condensation. Evolution can proceed alternatively, however, through the formation of a hierarchy structure. *Thus an exactly similar development can be applied to subunits developing within the galaxy itself.* Consider the following step-by-step model, in which the approximation of a constant temperature of 10^4 ° K will be used throughout. This simplification omits any slight cooling that may occur in the earliest phases of contraction.

MODEL FOR THE HIERARCHY STRUCTURE

Step 1.—A spherical galactic condensaton, of mass $3.6 \times 10^9 \odot = M_0$ say, temperature 10^4 ° K, initial density $\rho_0 = 10^{-27}$ gm/cm^3, initial radius $R_0 = 1.2 \times 10^{23}$ cm, contracts by a factor $k^{2/3}$ and then divides into k equal masses, each of radius R_0/k, density $\rho_0 k^2$, and temperature 10^4 ° K.

It will be noticed that the obvious requirements are satisfied by these assumptions. Thus each of the subunits satisfies the contraction condition. Moreover, with their assumed radii the subunits can be fitted into the contracted volume of the galaxy itself. Remembering that in the first shrinkage the galaxy decreases in dimensions by a factor of about 3, it follows that we must put $k^{2/3} \doteqdot 3$, which gives $k \doteqdot 5$.

Step 2.—The subunits of step 1 themselves contract by a factor of $k^{2/3}$ and then divide into k equal fragments, each of radius R_0/k^2, density $\rho_0 k^4$, and temperature 10,000° K. Once again these smaller fragments satisfy the contraction condition.

Step 3.—The subunits of step 2 themselves contract by a factor of $k^{2/3}$ and then divide into k equal still smaller fragments, each of radius R_0/k^2, density $\rho_0 k^6$, and temperature 10,000° K. Once again these still smaller fragments satisfy the contraction condition.

And so on for further steps.—Now, since the time scale for condensation at each step may be expected to follow the inverse square root of the density, it follows that the ratio of the time required for an infinity of such steps to the time required for just the first step is given by

$$1+\frac{1}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\ldots=\frac{k}{k-1},$$

which for $k = 5$ gives a ratio close to unity. Thus we see that the main expenditure of time lies in the first step and that further steps require comparatively little time. It is indeed this rapid convergence of the steps that provides the strongest reason for the opinion that dissipation arises from the development of the sort of hierarchy structure postulated above. An alternative scheme of dissipation would lie in the development of dynamical motions in a more or less uniform contraction of the main mass and of the turbulent dissipation of these motions. This may occur in some degree but is considered less likely in view of the time-scale estimates for fragmentation.

Of course, the hierarchy picture suggested above has many analogies with the phenomenon of turbulence. It may be thought of as turbulence in a gravitational field rather than in a pressure field. From the turbulence point of view the present suggestions would seem plausible, since the fact that the gravitational field becomes stronger than the pressure field supplies the very basis of the fragmentation process.

Although it may be useful to think of fragmentation in a hierarchy structure as representing a sort of gravitational turbulence, it is important to realize that a crucial difference exists between the formation of eddies in a pressure field and in the present process. The rise of pressure due to compression in ordinary turbulence tends to disrupt the denser elements of material. It is for this reason that the density of a particular sample of material fluctuates in ordinary turbulence, sometimes being denser than the general average and sometimes being less dense. The formation of a denser element in gravitational turbulence, on the other hand, only increases the ability of the element to hold itself together. Thus there is a rooted difference between the two cases: denser elements are evanescent in ordinary turbulence, denser elements tend to become permanent condensations in gravitational turbulence.

The description of the hierarchy model is still incomplete, in that an explanation of why the temperature is maintained constant at 10,000° K must still be given. It can readily be shown that the dissipation of energy is the same in each step of the hierarchy structure, the dissipation per step being of the order of

$$\frac{GM_0^2}{R_0}(k^{2/3}-1). \tag{6}$$

Now, since the time scale for each successive step decreases as $1/k$, it follows that the rate at which thermal energy is made available per unit mass increases by a factor k at each step. On the other hand, the rate of radiation per unit mass increases as ρ, which increases by k^2 at each step. Accordingly, radiation is favored by the changes that take place as fragmentation proceeds. There is, therefore, a tendency for the temperature to fall, *but the temperature cannot fall much below 10,000° K, since otherwise the hydrogen would become insufficiently ionized for effective radiation to occur*. This conclusion is subject to the implicit assumption that the fragments remain optically thin to radiation. The entire nature of the condensation process is altered if the fragments become optically thick to the whole of the radiation spectrum of the hydrogen. A discussion of this issue forms the main topic of the following section.

It is desirable to add that the model laid down above is an obvious idealization. We may expect deviations from the model to occur in a number of respects: The formation of subunits would be continuous, not a succession of discrete steps. Even with discrete steps, however, one step need not be exactly like another. Also we have assumed that at each step all the material condenses into subunits. The possible consequences of this assumption's not being strictly satisfied are discussed in the last section of the paper.

VI. FRAGMENTATION OF A GALAXY INTO TYPE II STARS

It is now necessary to investigate whether step-by-step fragmentation continues indefinitely or whether there is some reason why it should end after a finite number of steps. Looking back over the previous argument, it will be seen that fragmentation owes its origin to the maintenance of a near-isothermal condition, that a near-isothermal condition is maintained throughout the condensation on account of radiation by the hydrogen. It is this that leads to the development of dynamical motions, which forms the basis of fragmentation.

If, in contrast, an adiabatic condition was maintained, then the work done by the gas pressure in a uniform change of volume from V to V' releases thermal energy[9]

$$1.5M\mathfrak{R}T_0\left[\left(\frac{V}{V'}\right)^{2/3}-1\right], \tag{6a}$$

where M is the total mass and T_0 is the temperature at volume V. Comparison with expression (4) shows that the dependence on V/V' is now stronger in expression (6a). Hence under adiabatic conditions a condensation does not necessarily tend toward dynamical motions as it shrinks. Indeed, a mass of gas that is just unstable against contraction shinks only a little before the thermal energy becomes adequate to maintain the gas in hydrostatic equilibrium. It is true that a cloud with a mass that is very large compared with the minimum mass for contraction can shrink appreciably and that during the shrinkage dynamical motions that produce fragmentation can develop; but such fragments cannot themselves contract significantly, since they tend to possess masses that are only slightly above the minimum for contraction.

[9] Let v again be the specific volume. The work done in a small change dv is again $-p dv$ per unit mass. Also $-p dv = -\mathfrak{R}T\, dv/v$ as before, but T now varies as $v^{-2/3}$, so that the work done for a change from v to v' is $1.5\mathfrak{R}T_0\,[(v/v')^{2/3}-1]$ per unit mass. Thus for uniform contraction with $v/v' = V/V'$, the total energy released for mass M is $1.5M\mathfrak{R}T_0\,[(V/V')^{2/3}-1]$.

We can readily see why fragmentation should ultimately result in an approximation to the adiabatic condition. Thus after n steps in the hierarchy model the density is $\rho_0 k^{2n}$, and the radius of a fragment is R_0/k^n. Accordingly, the total opacity across the fragments increases with n according to k^n, so that for sufficiently large n the fragments must become highly opaque. This causes a transition to the adiabatic condition. It is now proposed to investigate the nature of this transition.

Opacity may be thought of as arising partly from scattering and partly from absorption. In the present problem the scattering, arising from free electrons, can be shown to be unimportant compared with absorption. Now absorption can become important in impeding the escape of radiation from a fragment only when the absorption by the gas of energy from the radiation field becomes comparable with the energy radiated by the gas. But this is just the condition that the radiation field, in the frequencies under consideration, shall approximate to the black-body distribution. It follows, therefore, that a transition to the adiabatic condition implies that the black-body distribution (over the part of the spectrum that is of interest) has been built up inside the fragment under consideration. The properties of the fragment then become similar to those of a star; and, just as with a star, the fragment radiates at a rate L that can be estimated in terms of temperature, density, opacity, and radius.

For a transition to the adiabatic condition, we require L to become less than the rate at which gravitational energy is being released in our problem. The gravitational energy released per fragment in the nth step of the hierarchy process is of the order of

$$\frac{GM_0^2}{R_0 k^n}\left(k^{2/3}-1\right),$$

the release occupying a time of order $k^{-n}(G\rho_0)^{-1/2}$. In this time the energy radiated is $Lk^{-n}(G\rho_0)^{-1/2}$. Hence a transition from the isothermal to the adiabatic condition begins to occur at the stage where

$$L=\frac{G^{3/2}M_0^2\rho_0^{1/2}}{R_0}. \tag{7}$$

Now the outward flux at distance r from the center of the fragment is given by

$$\frac{16\pi a c r^2 T^3}{3\kappa\rho}\frac{dT}{dr}, \tag{8}$$

where a is Stefan's constant, c the velocity of light, and κ the coefficient of opacity.

If the fragment were composed of entirely pure hydrogen, κ would become extremely small for temperatures below about 5000° K, since even in the presence of black-body radiation the hydrogen would then be effectively un-ionized (having regard to the important densities turning out to be of the order of 10^{-9} gm/cm^3). Hence expression (8) would become increasingly large as T fell below 5000° K, until ultimately a value equaling $4\pi acR^2T^4$ would be attained. This would imply that the surface temperature of the fragment had been reached, the value so reached being in the neighborhood of 4000° K. At such a surface temperature the fragment would radiate at a much greater rate than the luminosity that obtains when only a very small proportion of the mass consists of nonhydrogenic materials. It is therefore of importance to take account of the presence of a small admixture of elements other than hydrogen.

Metals have the important effect of increasing the opacity for values of T in the range 3000°–5000° K. Free electrons are supplied by the metals, approximately one electron per metal atom. The attachment of free electrons to the neutral hydrogen atoms then causes negative hydrogen ions to be produced, and it is these ions (as was pointed out by Wildt) that produce the opacity in the temperature range in question. At temperatures below 3000° K molecules begin to be formed, with the probable consequence of a steep rise in the opacity. Indeed, so great is the absorptive power of the molecules likely to be that the smallest value of κ would seem to lie in the range 3000°–5000° K. According to the tables of Chandrasekhar and Münch,[10] the smallest value of κ is likely to occur toward the upper end of this range, where its value is close to $10^{-1} P_e$, the electron pressure being denoted by P_e. For the purpose of obtaining a numerical estimate, we shall use $T = 4500°$ K, which is safely below the temperature at which the ionization of hydrogen would have to be taken into account. Such a temperature is sufficiently small compared with the central temperature of 10,000° K for the radius R of the fragment to be used as a suitable approximation for the radial co-ordinate r in expression (8). Next we note that the temperature gradient dT/dr will tend to be least for those values of T where κ is least. But the least value of dT/dr must be of the order of T/R in the present problem.[11] Hence expression (8) must be of the order of

$$\frac{160\, a\, cRT^4}{3\,\rho P_e}. \tag{9}$$

Although expression (9) has not been obtained from a consideration of the extreme outer layers, it must represent the luminosity of the fragment, since, if the surface radiates less energy than is received from below, the surface simply heats up, whereas if the surface radiates more, the surface temperature declines accordingly. Where the opacity is higher toward the center, the same flux must be maintained by a marked steepening of the temperature gradient.

The electron pressure is given by

$$P_e = x \mathfrak{R} \rho T ,$$

where x is the number ratio of metals to hydrogen. Inserting this value for P_e in expression (9) shows that the luminosity is proportional to T^3/ρ^2, which may be taken as not very different from the central value $T_0^3/\rho_0^2 k^{4n}$ (the central density after n fragmentations being $\rho_0 k^{2n}$, ρ_0 being the original density of the cloud). With this further approximation and putting $R = R_0 k^{-n}$, the luminosity becomes

$$\frac{160\,\pi\, a\, cR_0 T_0^3}{3\, x \mathfrak{R}\, \rho_0^2}\, k^{-5n}. \tag{10}$$

We are now in a position to return to our original objective. The luminosity of the fragment is given by expression (10) when the adiabatic condition is satisfied, since it is only then that the black-body distribution is set up. The transition to the adiabatic condition occurs at the stage of the hierarchy system where equation (7) is satisfied. If we insert expression (10) for L in equation (7), we obtain (writing $GM_0/V^{1/3} = 3\mathfrak{R}T_0$)

$$k^{5n} = \frac{160\,\pi\, a\, cG^{1/2}}{27\, x \mathfrak{R}^3}\, \frac{T_0}{\rho_0^{5/2}}. \tag{11}$$

This procedure constitutes an approximation in the sense that the substitution of expression (10) in equation (7) means that we are using formula (10) at the onset of the

[10] *Ap. J.*, **104**, 446, 1946.

[11] The temperature T falls from 5000° to 3000° K in a distance of the order of, but less than, R. Hence the mean temperature gradient for this range is of the order of T/R, $T = 4500°$ K. Now the mean temperature gradient gives an adequate approximation for dT/dr, since κ does not vary much between 5000° and 3000° K; if there were marked variations from the mean temperature gradient, there would be corresponding variations in the outward flux, and such variations would eventually be damped out. The present argument is, of course, dependent on the surface temperature being lower than 3000° K, a condition that at a later stage will be shown to be satisfied.

transition to the adiabatic condition, whereas, properly, formula (10) should be used only *after* the adiabatic condition is satisfied. Accordingly, we cannot be sure that the numerical value of n given by equation (11) represents the complete end of the fragmentation process. Fragmentation might well go one step further than this. Nevertheless equation (11) is important in giving the end of fragmentation within an uncertainty of about one step. That the uncertainty is not worse than this is shown by the strong dependence of expression (10) on n: an increase of n by 1 decreases formula (10) by a factor of about 5^5.

Now, since $T_0 = 10{,}000°$ K and $\rho_0 = 10^{-27}$ gm/cm^3, the right-hand side of equation (11) is known numerically as soon as a value of x is specified. In stars of type I the value of x is about 5×10^{-5}. This value is now believed, however, to be considerably higher than the value of x occurring in the early history of a galaxy: a value of 5×10^{-6} would perhaps be a more suitable value. Using this for x gives

$$k^n = 2.3 \times 10^9 \,. \tag{12}$$

Thus for $k = 5$, as indicated in the definition of our model, this gives a value of n of about 13.

The feature of greatest interest, however, is not the number of steps in the hierarchy structure but the mass of the final fragments; this is M_0/k^n. Remembering that M_0 is $3.6 \times 10^9 \odot$, we obtain with equation (12) a fragment mass of about $1.5\odot$. There is an uncertainty, however, in n, as was indicated in the previous paragraph. Thus equation (12) gives k^n only at the onset of the transition to the adiabatic condition, so that one further fragmentation might well occur, which for $k = 5$ would give a final fragment mass of about $0.3\odot$.

The present uncertainty over the precise end of the fragmentation process is, of course, a result of a serious imperfection in the model employed; for it is the assumed discrete nature of the steps that lies at the root of this uncertainty. In a proper theory the whole process would have to be described in continuous terms, and the uncertainty of the discrete model would be replaced by a continuous dispersion of the final masses. It is not possible to say whether or not this dispersion would give a variation between the same values as those indicated above, namely, between $0.3\odot$ and $1.5\odot$. One is encouraged in the belief that it would by the circumstance that the final fragment masses arrived at in the present treatment are entirely independent of the parameter k. In this connection it may be noted that the approximations involved in deriving expression (10) are of a less serious nature than the discrete character of the model, since the approximations affect only the masses of the final fragments in the fifth root.

The radius of the stellar fragment at the onset of the transition to the adiabatic condition is given by $R_0 k^{-n}$, where $R_0 = 1.2 \times 10^{23}$ cm and k^n is given by equation (12): a radius of about 5×10^{13} cm. The luminosity at this stage can be calculated from expression (10) or more simply from the consideration that the dissipation in each step of the hierarchy model is given by expression (6). Thus with $M_0 = 3.6 \times 10^9 \odot$, $R_0 = 1.2 \times 10^{23}$ cm, and $k = 5$, expression (6) amounts to 2.8×10^{55} ergs. This represents the total energy available for the formation of about 5×10^9 fragments, giving about 5.6×10^{45} ergs/fragment. To obtain the luminosity per fragment, this value of 4.6×10^{45} ergs must be divided by the time sacle of the final fragmentation, this being less than the time scale—10^{17} seconds—for the first fragmentation by just the factor k^n. Accordingly, the luminosity during the final fragmentation is about 1.5×10^{38} ergs/sec, corresponding to a bolometric luminosity of about -6.5. The effective surface temperature for a radius of 5×10^{13} cm must be close to 2000° K.

It should now be emphasized that because the first step in the hierarchy system takes longer than all the other steps together, the formation of the stars must be spread over the whole time scale of 10^{17} seconds. This amounts to an average rate of star formation of about one per year. There will, of course, be variations from this average rate, as, for instance, when any stellar fragment is formed, there must be three or four other stellar fragments produced at the same time. The total bolometric luminosity of such a group would be some five times greater than that of a single fragment, that is, about -8, although the visual magnitude would be substantially less than this, owing to the red color that must accompany the low surface temperatures of the fragments. It might also be added that such an emission would last only for a few years and that fragmentation would induce variability.

After the adiabatic condition becomes satisfied, the star at first shrinks quickly. As the shrinkage proceeds, it slows down, and the luminosity declines. Eventually, in the last stages of the shrinkage, occupying a time of 10^7 years or more, the usual mass-luminosity relation for hydrogen stars comes into force. Shrinkage ceases when the central temperature has risen high enough for the luminosity to be balanced by energy production in the interior.

VII. THE FORMATION OF GALAXIES OF VERY LARGE MASSES

Throughout the last four sections it has been assumed that the initial heating was insufficient to lift the temperature to very high values. In Section II we saw that two temperature classes are to be expected, one in the range 10,000°–25,000° K and the other at temperatures in excess of 10^5 ° K. In the present section we shall consider the latter case.

Once again, if the mass of a cloud is much larger than the minimum mass defined by (now for wholly ionized hydrogen)

$$\frac{GM_0}{V^{1/3}} = 6\mathfrak{R}T_0\,, \tag{13}$$

division into a large number of fragments with masses approximately satisfying equation (13) is to be expected.

On the other hand, there is an important difference when T_0 rises above 3×10^5 °K, because radiation then becomes inadequate to dispose of the thermal energy of the hydrogen. Thus at 3×10^5 ° K, hydrogen of density 10^{-27} gm/cm^3 radiates 8×10^{13} ergs/gm in 10^{17} seconds, as compared with the thermal energy content of slightly more than 7×10^{13} ergs/gm. At $T_0 = 10^6$ ° K, the radiation rate is 7×10^{13} ergs/gm in 10^{17} seconds, while the thermal energy is 2×10^{14}/gm.

Now when radiation is unable to dispose of the thermal energy of the hydrogen,we have an approximation to the adiabatic condition, and there is no substantial contraction of clouds with masses slightly in excess of equation (13). Accordingly, it would seem that the formation of galaxies is precluded when T_0 is above 3×10^5 ° K.

But T_0 is not likely to be much below 3×10^5 ° K on account of the flatness of the theoretical relation from 25,000 to 10^5 ° K; so that galaxies of the high-temperature class would seem to fall mainly into the fairly narrow range for T_0 of 10^5 ° to 3×10^5 ° K. Consider the intermediate case $T_0 = 1.5 \times 10^5$ ° K. Then the mass defined by equation (13)—using $M_0 = V\rho_0$ and $\rho_0 = 10^{-27}$ gm/cm^3—is $5 \times 10^{11}\odot$. Galaxies forming in high-temperature clouds should have masses of this order.

At this stage the general line of the argument will be interrupted. It would be natural to go on now to discuss further properties of these specially massive galaxies. This will not be done, however, because this section of the present investigation is so frankly speculative that it has been thought better to include first a more definitive section on the dynamical development of the less massive type of galaxy.

VIII. THE FINAL RADIUS OF A GALAXY OF MASS $3.6 \times 10^9\odot$

At first sight it might seem as if the theory presented above is seriously deficient, in that it predicts far too large sizes—13,000 parsecs (for galaxies of mass 3.6×10^9.)

instead of radii of 1000 or 2000 parsecs. This objection would seem to be entirely overcome, however, by the argument of the present section, which indicates that subsequent dynamical developments greatly reduce the initial radius.

For simplicity, suppose that the angular momentum of the system can be neglected. Then the various groups of condensed stars fall toward the center, where frequent dynamical encounters take place. Now dynamical encounters must introduce a tendency toward an equipartition of energy, in which the small members of the hierarchy structure receive the same energy as the large members. But when a particular member of the hierarchy receives a sufficiently large supply of energy, it is thereby enabled to break loose from the larger member of which it was originally a part. Accordingly, equipartition breaks up the hierarchy structure.

Let us examine the extreme case in which dynamical encounters are adequate to break up the entire detailed structure of the system, so that the stars come to belong to a general amorphous mass. Energy is required for this process. Thus we saw at the end of Section V that the radiative dissipation is the same in all steps of the hierarchy. This dissipation shows itself initially in the binding of the various members of the hierarchy sequence. To disrupt this binding, so that an amorphous group of stars is formed, requires about thirteen times (there are about thirteen steps in the hierarchy) the energy that was released by gravitation in just the first step. This energy has to be supplied by a shrinkage of the main mass, which must accordingly contract down to about one-thirteenth of the main radius at the end of the first step: *that is, the galaxy must shrink to a radius of about 1000 parsecs.*

It may be useful to mention the mechanism by which this shrinkage takes place. The largest members of the hierarchy acquire dynamical energy through falling toward the center of the system. The dynamical energy so acquired is then transferred in encounters to smaller members, thereby rendering the large members unable to expand out to their original distances from the center. It may also be remarked that the process is probably cumulative—the more the system shrinks, the more frequently do dynamical encounters take place.

The estimate of 1000 parsecs is indeed an upper limit to the final radius (the angular momentum still supposed zero); for the above considerations assume that none of the smaller members of the system acquire velocities large enough for them to escape entirely from the galaxy. If any appreciable fraction of the mass does manage to escape, the shrinkage will be greater than that just estimated.

Now a radius of 1000 parsecs is somewhat larger than, but comparable with, the observed radii of galaxies of type E_0 (allowing for Baade's new distance scale). Accordingly, the theory seems in satisfactory agreement with observation in the case where the system possesses negligible angular momentum. When the angular momentum is appreciable, the situation is somewhat altered. Then the final amorphous distribution of stars will possess an ellipticity that depends on the angular momentum. The estimate of 1000 parsecs can then be applied only to the polar radius of the distribution. It must indeed be an upper limit to the polar radius, for the reason that rotation restricts contraction toward the axis of rotation, so that the gravitational energy required to break up the hierarchy structure must be supplied by an extra contraction toward the plane of symmetry of the distribution.

IX. SPECULATIONS ON THE DEVELOPMENT OF A GALAXY OF MASS $5 \times 10^{11}\odot$

We have seen that the temperature is likely to be closely determined in this case. The trend of the theoretical relation above 25,000° K requires the temperature in general to be high, but the necessity for the hydrogen radiation to be capable of radiating the thermal energy places an upper limit to the temperature of about 3×10^5 ° K.

Now, assuming that radiation is adequate for the initial contraction to be nearly isothermal, the energy supplied per gram of material varies with the density according to $\log \rho$. Thus, since the time scale for contraction varies as $\rho^{-1/2}$, the rate of release of thermal energy per gram varies as $\rho^{1/2} \log \rho$. In contrast, the rate of radiation per gram varies as ρ. Accordingly, radiation slowly becomes dominant as contraction proceeds. This causes a departure from the isothermal condition, and the temperature must fall. But as the temperature falls, radiation becomes even more dominant, since the rate of radiation increases as $T^{-1/2}$, as T falls below 3×10^5 ° K. It seems probable that a temperature instability must develop in this way. A temperature instability implies that the temperature falls low—to about 10,000° K—in a time that is too short for any effective contraction to occur.

The importance of a temperature instability is that the contracting cloud becomes unstable against the formation of subunits of very much smaller mass than that of the galaxy itself. Consider the following model of a large galaxy. A galaxy of mass $M_0 = 5 \times 10^{11} \odot$, initial density $\rho_0 = 10^{-27}$ gm/cm³, temperature $T_0 = 1.5 \times 10^5$ K,° and initial radius $R_0 = 5 \times 10^{23}$ cm condenses to radius 10^{23} cm, maintaining a high temperature throughout this contraction. A temperature instability then develops that swiftly reduces the temperature down to 10,000° K. Some fragmentation may occur during the high-temperature phase, but the main fragmentation occurs after the steep fall of temperature. The gas, then having a density of about 10^{-25} gm/cm³, is unstable against the formation of masses of the order of $3 \times 10^8 \odot$. Each of these masses then fragments along a hierarchy structure of the pattern already discussed in previous sections. Thus it would seem that the characteristic first stage in the evolution of a massive galaxy is not a closely knit series of steps but a widespread breakup into about a thousand or so dwarf galaxies, the whole distribution possessing a radius of some 30,000 parsecs.

Subsequent dynamical evolution is then likely to follow somewhat different lines from those described in the previous section, where the main effect was simply the breakup of the general hierarchy system. In the present case dynamical encounters among the multitude of more or less separate dwarf sytsems must lead to some of the systems acquiring sufficient energy to escape altogether from the galaxy. This causes a general compacting of the whole structure. In the case where such systems acquire only the limiting velocity of escape, the radius of the galaxy varies as the square of the remaining mass: thus if loss of material were to reduce the mass to $2.5 \times 10^{11} \odot$, the radius would shrink from 30,000 to 7500 parsecs. In the case where the escaping material acquires more than the limiting velocity, the shrinkage of the galaxy would be even greater than this.

Accompanying such a shrinkage there would also be a general disintegration of the multitude of dwarf systems, which would again produce an evolution into a general amorphous mass of stars. It can readily be shown that the energy required for such a disintegration does not appreciably add to the shrinkage of the galaxy, in marked contrast with the case discussed in Section VIII. Once most of the dwarf subsystems become disrupted, the effects of dynamical evolution are likely to be much reduced, and the galaxy reaches a more or less stable state.

It is, of course, possible that a subsystem might itself become compacted as a result of the escape of certain of its components caused by internal dynamical encounters. If such were to occur in a sufficient degree, it would be possible for a stable subsystem to arise, capable of resisting the disruptive effects of external dynamical encounters. It is tempting to identify the globular clusters with such cases.

A further tentative identification may be put forward. In recent years it has become increasingly clear that the average space density of faint dwarf galaxies must be rather high. These systems may owe their origin to escape from larger galaxies.

X. SPECULATIONS ON THE FORMATION OF TYPE I STARS

The stars so far discussed are all of type II: stars that form during the actual condensation of the galaxies themselves. Now the above discussion has proceeded on the basis that all the initial gas passes through the hierarchy system. This is probably valid as a zero-order approximation, but not in higher approximation. Thus at any stage of the condensation process, contraction occurs only where the gas density is higher than average. Where the density is lower than average, condensation is not likely to occur, if only because the self-gravitation of an element of abnormally low density is unable to withstand the tidal shearing of the main field. In step 1 of the hierarchy system the main field should be understood as arising from the whole galaxy; in step 2 as arising from the first set of fragments; and so on.

For this reason it is to be expected that a residuum of gas will remain uncondensed into stars, constituting perhaps 20 or 30 per cent of the original mass. This gas cloud will shrink with the galaxy. Even after the star distribution attains an effective equilibrium size, the gas cloud will continue to shrink; in systems without angular momentum the cloud will shrink toward the center of the system, while in systems with appreciable angular momentum the cloud will shrink to a disk. In either case the density within the cloud eventually rises to such a value that a second phase of condensation can occur: that is, the self-gravitation of an element of material is eventually able to overcome the tidal shearing of the main field.

Two cases may then be distinguished. If the radiative properties of the gas are of the type discussed above, the gas cloud will fragment into stars in essentially the manner already discussed. Such stars would be of type II in the sense of the present paper. In the case where such a condensation occurs in a gaseous disk, we may speak of disk–type II stars or of late–type II stars. A second and radically different case arises, however, if dust has been produced (in some way outside the scope of this paper) in the condensing gas. Dust allows molecules to be formed, and molecules are able to radiate at low temperatures, in contrast to hydrogen, which cannot radiate, once the electrons and protons recombine to form atomic hydrogen. In the dust case, on the other hand, the temperature may well fall to 50° K, as compared to the value of 10,000° K in the type II case.

The whole problem of star formation requires a rediscussion for the low-temperature case, the opacity question being particularly troublesome on account of the effects of molecular absorption. Stars formed at low temperatures may be tentatively defined as the type I stars.

It is of interest to note that the condition for contraction, applied to the case $T_0 = 50°$ K and $\rho_0 = 10^{-23}$ gm/cm^3 (a likely value of the density after contraction), gives a minimum mass of about $10^4\odot$, comparable with the masses of the larger interstellar clouds in the galaxy. The fragmentation time for such a cloud would be less than 4×10^7 years, using the arguments of previous sections. On this basis we should at first sight expect the residuum of gas to become entirely condensed into stars in only 4×10^7 years, reckoned from the time that its density rises high enough for tidal shearing to be unimportant. But such an exhaustion of the remaining gas certainly has not occurred in the galaxy or in the other spirals. The question is: Why? Three tentative suggestions may be made:

a) That in the type I case the fragments become opaque to infrared radiation through absorption by molecules at an early stage of the hierarchy development. If the opacity were sufficiently great, a good approximation to the adiabatic condition could arise, in which case fragmentation would cease. Not only this but contraction could (under suitable conditions) be slowed up almost indefinitely. The dark globules observed in certain interstellar clouds might possibly be identified with fragments in this state.

b) Fragmentation into stars may occur, but into stars that eventually gain (either by direct condensation or by subsequent accretion in the low-temperature gas) masses considerably greater than the masses of the type II stars. If very luminous stars of high temperature are so formed—and there is, of course, observational evidence that this is so—a considerable heating must occur in the gas.[12] This arises from the absorption of

[12] For the details of this process the reader is referred to a forthcoming paper by Oort and Spitzer. The writer is much indebted to Dr. Spitzer for a private communication on this matter and also for general discussions on the whole of the present paper.

ultraviolet light from the stars. It is readily possible for a few highly luminous O stars to heat a gas cloud of mass $10^4\odot$ to such a degree that the cloud is forced to expand under the action of gas-pressure forces—not of radiation pressure. Thus the following situation may well arise: When there are no luminous high-temperature stars, condensation occurs, and O stars are formed. But as soon as O stars are formed, the clouds are expanded and condensation ceases. After the bright stars become exhausted, the clouds then re-form and more stars condense. On this picture a sort of cyclic process is set up—clouds condense; stars form, including O stars; clouds blow up, and star formation ceases; O stars die; clouds re-form; more stars condense, including O stars; etc. The interstellar clouds of small mass would represent bits of clouds left over from the blowing-up process.

c) It has in recent years been thought that the galaxy may possess a substantial magnetic field. It has been suggested that the magnetic field owes its origin to the general motions of ionized interstellar clouds. But this suggestion has the disadvantage, from the point of view of explaining the polarization properties of the interstellar medium, that the magnetic vector cannot apparently maintain its direction over sufficiently great distances. This difficulty is much alleviated if it be supposed that the magnetic field is built up, not in the motions of the interstellar clouds, but by the motions of the original gas cloud out of which the galaxy originally formed. In this case the magnetic vector would also alter in direction from point to point, but alignment might be expected over far greater distances than on the earlier view. Moreover, the concentration of the residuum of gas to a disklike shape would tend to align such a magnetic field parallel to the galactic plane, in accordance with the requirement of Davis and Greenstein in their theory of the polarization properties of the medium.[13]

Now it may occur,[14] after a sufficient fraction of the gas in the disk has become condensed into stars, that the self-gravitation of the remaining gas is insufficient to overcome the magnetic pressure, in which case there is no tendency for large-scale condensation into stars. In exceptional regions, self-gravitation may indeed overcome the magnetic pressure—for example, in the Orion nebula—and extensive condensation may occur there. In this way, it might be possible to explain why a gaseous disk has in the main managed to survive in the galaxy and in other spirals for several billion years.

It remains to mention a final point concerning the residuum of gas. It has been remarked that this might amount to 20 per cent of the original mass of the galaxy. In the case of a galaxy of mass $5 \times 10^{11}\odot$ this would give $10^{11}\odot$ in the form of gas. Now, although the residuum of gas thus amounts to no more than 20 per cent originally, if the mass condensed into a type II hierarchy is reduced by the dynamical escape of various subunits, the type II mass comes nearer to equality with the gaseous residuum. Thus for a loss of half the type II mass, the gas would have mass $10^{11}\odot$, and the mass condensed into the type II hierarchy would be $2 \times 10^{11}\odot$, giving a total mass of $3 \times 10^{11}\odot$.

[13] L. Davis and J. L. Greenstein, *Ap. J.*, **114**, 206, 1951.

[14] I am indebted to Dr. S. Chandrasekhar for this remark.