THE BALMER DECREMENT IN SOME Be STARS*

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ABSTRACT

Measurements of the Balmer decrement as far as the last visible emission line have been made for four Be stars. These measurements show that the strengths of the lines arising from the high levels ($>H\zeta$) are much greater than would be expected from the theory for a planetary nebula. The effect of self-absorption has been investigated for a simplified transfer problem, and it is shown that this effect could possibly explain the observed decrements, though unknown factors are involved.

I. INTRODUCTION

In a previous paper,[1] which will be referred to as "Paper I," the outer atmospheres of certain Be stars were discussed. In this work measurements of the Balmer emission lines $H\beta$–$H\zeta$ were made, and a tentative model of the outer atmosphere was suggested. Of the six stars studied in Paper I, 11 Cam alone had Balmer emission lines which were single and sharp and could be distinguished as far as $H22$. In the other cases the emission lines were double and appeared to fade out or could not be identified beyond $H\zeta$. In this paper we shall discuss measures of the Balmer emission lines arising from the high levels, in the spectrum of 11 Cam and in the spectra of three other Be stars—12 Aur,[2] HD 37998,[3] and HD 58050[4]—which were not discussed in Paper I.

Calibrated spectra of these latter three stars were obtained in January, 1952, at the McDonald Observatory with the Cassegrain quartz spectrograph (500-mm camera, dispersion 54 A/mm at $H\gamma$), on Eastman Kodak II*a*-O plates. In all three of these stars, as in 11 Cam, emission lines superposed on broad absorption lines were visible from $H\beta$ to beyond the last observable hydrogen absorption line. Brief descriptions of the spectra of 12 Aur, HD 37998, and HD 58050 follow.

12 Aur.—The only description of the spectrum of this star is to be found in the *Catalogue and Bibliography of Be and Ae Stars*,[5] in the *Second Supplement.* Here Miczaika described the hydrogen lines as having sharp dark cores with a suspicion of emission at $H\beta$ in 1948 and 1949. The spectrum has evidently changed radically since then: on our plates no central absorption cores were visible. The emission lines were strong and rather broad (possibly unresolved doubles) and were visible as far as $H22$. A spectrogram obtained at Yerkes in January, 1953, with the one-prism spectrograph and 12-inch camera shows the spectrum of 12 Aur to be similar to its appearance a year previously.

HD 37998.—This star is described in the *Second Supplement.*[5] In 1945 $H\beta$, $H\gamma$, and $H\delta$ consisted of narrow emission components nearly centrally superposed on broad absorption lines, while the absorption lines of *He* I, *Mg* II, and *Si* II resembled those of β Orionis. From our plate this star is seen to be peculiar; a unique spectral classification cannot be assigned. *Si* II $\lambda\lambda$ 4128–4130 does not appear. The hydrogen lines have single, central narrow emission lines visible to $H25$, while the absorption lines are visible to $H17$.

* *Contributions from the McDonald Observatory*, No. 227.

[1] G. R. Burbidge and E. M. Burbidge, *Ap. J.*, **117**, 407, 1953.

[2] MWC 104, $\alpha_{1900} = 5^h9^m.0$; $\delta_{1900} = +46°19'$; $m_v = 6.9$; Sp. B2 V.

[3] MWC 772, $\alpha_{1900} = 5^h37^m.4$; $\delta_{1900} = +25°15'$; $m_v = 8.0$; Sp. B3–5p IV, V.

[4] MWC 176, $\alpha_{1900} = 7^h18^m.8$; $\delta_{1900} = +15°43'$; $m_v = 6.4$; Sp. B2 V.

[5] P. W. Merrill and Cora G. Burwell, *Ap. J.*, **78**, 87, 1933; *Mt. W. Contr.*, No. 471; *Ap. J.*, **98**, 153, 1943; *Mt. W. Contr.*, No. 682; *Ap. J.*, **110**, 387, 1949; *Mt. Wilson and Palomar Reprints*, No. 8.

HD 58050.—This star was suggested by Dr. Morgan, from examination of his plates, as a typical "pole-on" star. In the *First Supplement*[5] there is a note that a spectrum obtained by Swings and Struve in 1943 showed hydrogen emission lines to $H8$ or $H9$, indicating that the emission was stronger then than in 1933. In the *Second Supplement* there is a note that Adams observed $H\beta$, $H\gamma$, $H\delta$, and probably $H\epsilon$ to show broad double emission lines on a coudé spectrogram taken in 1945. On our plates the emission lines are single as far as $H\delta$ or $H\epsilon$, but the higher members are resolved as double, with a mean separation of 145 km/sec, and are visible as far as $H19$.

In the spectra of these three stars and 11 Cam we have found it possible to measure the equivalent widths of the hydrogen emission lines as far as the last visible line. The calibration technique was described in Paper I. The method of obtaining the equivalent widths of those emission lines which are superposed on absorption lines, by drawing in

TABLE 1

EMISSION-LINE EQUIVALENT WIDTHS, IN EQUIVALENT ANGSTROMS

Line	11 Cam	12 Aur	HD 37998	HD 58050
$H\alpha$..........	19.9	....	....	....
$H\beta$..........	1.61	1.63	2.31	4.51
$H\gamma$..........	0.60	....	....	2.07
$H\delta$..........	0.38	....	....	1.02
$H\epsilon$..........	....	....	....	0.92
$H\zeta$..........	0.15	0.61	....	0.71
$H9$..........	0.20	0.76	0.18	0.53
$H10$..........	0.18	0.68	....	0.52
$H11$..........	0.22	0.69	0.15	0.59
$H12$..........	0.15	0.72	0.13	0.50
$H13$..........	0.20	0.86	....	0.53
$H14$..........	0.17	0.63	0.12	0.44
$H15$..........	0.14	0.76	0.18	0.63
$H16$..........	0.20	0.43	0.14	0.44
$H17$..........	0.18	0.53	0.16	0.67
$H18$..........	0.15	0.34	0.15	0.40
$H19$..........	0.19	0.26	0.20	0.22
$H20$..........	0.13	0.25	0.17	....
$H21$..........	0.15	0.21	....	....
$H22$..........	0.07	0.18	....	....
$H23$..........	....	....	0.10	....
$H24$..........	....	....	0.14	....
$H25$..........	....	....	0.06	....

the central parts of the absorption lines, was also described in Paper I. For the reduction of all the lines arising from levels higher than $n = 7$, the calibration-curves for either $H\delta$ or $H\epsilon$ were used. This does not introduce much inaccuracy, since the sensitivity varies only slightly with wave length in the range $\lambda\lambda$ 4000–3600. The equivalent width of the $H\alpha$ line was measured in the spectrum of 11 Cam, from two plates obtained at Yerkes in November, 1952. For these, calibration-curves were obtained with the step-slit sensitometer at the Yerkes Observatory. The dispersion of these spectra was about 250 A/mm at $H\alpha$ and about 90 A/mm at $H\beta$. In spite of this low dispersion, the two plates gave consistent results, and the equivalent width of the emission line at $H\beta$ agreed well with the value obtained a year earlier from the McDonald coudé plate. We think it reasonable to suppose that the spectrum of 11 Cam has not shown significant variations in emission-line intensities in the 10 months' interval.

In Table 1 the equivalent width of $H\alpha$ in 11 Cam is the mean of results from the two

Yerkes plates; the remainder of the lines in 11 Cam were measured from the single McDonald coudé plate. The values for HD 58050 are the means of the results from two plates (the mean difference between the two, from 14 lines for which measures were made for both spectra, was ± 12 per cent). The values for 12 Aur and HD 37998 are from single plates. The lines from $H\gamma$ to $H\epsilon$ in 12 Aur and from $H\gamma$ to $H\zeta$ in HD 37998 were not measured, as the plates were rather too strongly exposed for spectrophotometric work in this region. Other gaps in the table are due to flaws on the plates.

It might be thought that systematic errors have been introduced by the process of drawing in the central parts of the underlying absorption lines. However, the last few lines, from $H17$ to $H22$ in 11 Cam, $H15$ to $H22$ in 12 Aur, $H15$ to $H19$ in HD 58050, and $H18$ to $H25$ in HD 37998, had no underlying absorption lines, and there is no appreciable systematic difference between these and the earlier lines.

II. THE BALMER DECREMENT

The energies emitted in the various lines have been obtained by assuming that the stellar continuum in each case is produced by a black body at the temperature given in the table by Keenan and Morgan[6] for the various spectral classes. (There is no evidence from their colors that any of these stars are intrinsically reddened.) The equivalent widths in Table 1 were referred to the apparent stellar continuum, which is depressed below the "true" continuum by the Balmer discontinuity, which occurs near λ 3700 and beyond which the broad absorption lines are no longer resolved. It is also depressed, to a lesser extent, on the long-wave-length side of this point, because of the overlapping of the hydrogen absorption wings. The measured equivalent widths of the higher Balmer emission lines have therefore to be multiplied by a factor giving the black-body intensity at that wave length and be divided by a factor giving the ratio of the "true" to apparent continuum at that wave length.

Barbier and Chalonge[7] give mean values of the Balmer discontinuity for normal stars of the various spectral types. The values for the types B2, B3, and B5 are 0.11, 0.17, and 0.25, respectively, where these figures give the logarithm of the ratio of the intensities on either side of λ 3700. In the four stars considered here, there is some continuous Balmer emission beyond the last resolved emission line (it is especially visible in HD 58050). We therefore took the continuum between the last few Balmer emission lines to represent the level from which the Balmer discontinuity should be measured. We plotted points representing the undepressed continuum on the microphotometer tracings, using the values 0.11 for 12 Aur and HD 58050; 0.17 for 11 Cam; and 0.25 for HD 37998 (since in this star the strength of the hydrogen absorption lines seemed consistent with a type B5 rather than B3). We joined these plotted points smoothly to the continuum between $H\zeta$ and $H9$, where the wings do not overlap; this curve now represents the "true," undepressed, continuum. The ratio of intensities in the "true" to the apparent continuum was obtained for each line, and these ratios were used as correcting factors in obtaining the energies in the various Balmer lines, thus reducing the values for the higher Balmer lines below those which would be obtained if no account were taken of the depression of the continuum. These correcting factors are likely to be over- rather than underestimated, since the hydrogen absorption lines are somewhat weaker in Be stars than in normal B stars.

The resultant energies, referred to $H\beta$ as unity, are given in Table 2 and are the Balmer decrements for the stars. If there is no self-absorption, the energies lead directly, by the method described in Paper I, to the relative populations of the various levels.

Baker and Menzel[8] tabulated the Balmer decrement for various values of T_e, calcu-

[6] *Astrophysics*, ed. J. A. Hynek (New York: McGraw-Hill Book Co., 1951), p. 23.

[7] *Ann. d'ap.*, **4**, 30, 1941.

[8] *Ap. J.*, **88**, 52, 1938.

lated for a gaseous nebula under the assumption that the nebula is excited by the absorption of stellar radiation beyond the Lyman limit. A temperature of 12,600° (T_e) was shown in Paper I to be a reasonable value for the outer atmospheres of Be stars. We have therefore interpolated from Baker and Menzel's tables for $T_e = 12,600°$ for their case B. This is the case in which the nebula is assumed to be opaque to Lyman radiation, and each Lyman transition is assumed to be balanced by an inverse transition. The resulting values of the Balmer decrement are also shown in Table 2.

This table shows that, although our observed values of the Balmer decrement in 11 Cam and HD 58050 are similar to the calculated nebular values for the first few lines,

TABLE 2

THE BALMER DECREMENT

Line	11 Cam	12 Aur	HD 37998	HD 58050	Calculated (Nebula)	γ Cas a) $T=$ 25,000°	γ Cas b) $T=$ 10,000°
$H\alpha$	4.8	....	....	....	2.52	....	....
$H\beta$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$H\gamma$	0.52	....	....	0.65	0.51	0.33	0.28
$H\delta$	0.38	....	....	0.38	0.31	0.15	0.12
$H\epsilon$	....	....	....	0.38	0.20	0.10	0.08
$H\zeta$	0.18	0.72	....	0.31	0.14	0.17*	0.12*
$H9$	0.24	0.92	0.12	0.24	0.10	0.06	0.04
$H10$	0.22	0.80	....	0.23	0.08	0.05	0.04
$H11$	0.28	0.80	0.14	0.27	....	0.06	0.04
$H12$	0.17	0.81	0.11	0.22	....	0.06	0.04
$H13$	0.22	0.96	....	0.23	....	0.08	0.05
$H14$	0.17	0.67	0.08	0.18	....	0.09	0.06
$H15$	0.13	0.83	0.11	0.25	0.024	0.08	0.06
$H16$	0.19	0.48	0.08	0.17	....	0.09	0.06
$H17$	0.18	0.61	0.09	0.27	....	....	....
$H18$	0.15	0.41	0.08	0.17	....	....	....
$H19$	0.19	0.32	0.10	0.09	....	....	....
$H20$	0.13	0.31	0.09	....	0.011	....	....
$H21$	0.15	0.27	....	....	....	....	....
$H22$	0.07	0.24	....	....	....	....	....
$H23$	....	....	0.05	....	....	....	....
$H24$	....	....	0.07	....	....	....	....
$H25$	....	....	0.03	....	0.006	....	....

* Blended with *He* I.

the values for the higher lines all depart very considerably from the calculated values, and differ from star to star. If 11 Cam, for instance, had the calculated Balmer decrement, then the observed equivalent width of $H\beta$ is such that the equivalent width of $H20$ would be 0.01 equivalent angstroms, and would be quite unobservable on our plate. The slowness of the decrement in 12 Aur is remarkable.

Previous measures of the Balmer decrement[9] for $H\beta$–$H\epsilon$ in Be stars have been found not to depart much from those measured for planetary nebulae.[10] This has been interpreted to mean that, in spite of the greatly differing extent and density of the gas in Be

[9] E.g., O. Struve and H. F. Schwede, *Phys. Rev.*, **38,** 1195, 1931, and O. Struve, *Zs. f. Ap.*, **4,** 177, 1932; B. G. Karpov, *Lick Obs. Bull.*, **16,** 159, 1934; O. Mohler, *Pub. Obs. U. Michigan,* **5,** 43, 1934.

[10] See L. H. Aller, *Ap. J.*, **93,** 236, 1941, and references given in that paper; also Aller, *ibid.*, **113,** 125, 1951.

star atmospheres and in planetary nebulae, the conditions for the transmission of radiation are essentially similar. There are very few measures of equivalent widths of the Balmer emission lines of shorter wave length than $H\epsilon$. Walraven and Koelbloed[11] measured the emission lines $H\beta$–$H16$ of γ Cas in 1937 and 1938, when the emission was very strong. We have converted their published equivalent widths to relative energies by assuming that the continuous spectrum was that of a black body at (*a*) 25,000° (corresponding to a B0 star) and (*b*) 10,000° (since the star had appreciable intrinsic reddening at this time). The energies are sensibly constant from $H\epsilon$ to $H16$ in both cases. The decrement is shown in Table 2. The relative energies in the lines of shorter wave length are nearer to the calculated nebular values than in any of the four stars discussed here. Merrill and Wilson[12] give the central emission intensities (not equivalent widths) for some Balmer and Paschen lines in γ Cas; they do not vary appreciably for this star from $H10$ to $H16$. Perhaps the most striking feature shown by Table 2 is the general constancy, for any one star, of the energies emitted in the lines from $H8$ or $H9$ to the last observable line.

III. THE TRANSFER OF RADIATION

If there were no self-absorption, the decrements given here would indicate a considerable overpopulation of the higher levels of the hydrogen atoms. Such a distribution of populations, however, would seem to be very unlikely. We have therefore decided to examine the question of self-absorption of the line radiation as it passes through the outer atmosphere.

Miyamoto[13] has discussed self-absorption in Be stars in so far as it affects the strengths of $H\alpha$ and $H\beta$. This involved the solution of coupled equations of transfer for Lyman and Balmer frequencies. If we wish to discuss the strengths of the Balmer lines up to $\sim H20$ by this method, it would involve the solution of a large number of coupled equations, because of the conversion of radiation from one frequency to others by absorption and re-emission. Since it has not yet been possible to evolve a technique to handle such a situation, we shall consider, in order to determine some of the factors involved, the equation of transfer for one particular Balmer frequency ν.

Consider the transfer of radiation through a layer of geometrical thickness x. We will suppose that the layer is illuminated by a source emitting mainly in the ultraviolet. The gas in the layer absorbs radiation beyond the Lyman limit; it is ionized and recombines with the emission of the various line series. That part of this line radiation emitted in the frequency ν, together with that part of the absorbed Lyman radiation which is re-emitted at frequency ν, per unit volume per unit solid angle, will be denoted by J_1. The energy source is situated at the side of the layer, i.e., the line of sight which we are considering does not encounter the source (cf. Paper I, Fig. 13, *b*). J_1 may vary with lateral distance from the source, but this variation will be neglected.

If J_2 is the re-emission per unit volume per unit solid angle, then the equation of transfer in the frequency ν is

$$\cos\theta \frac{dI}{dx} = -\alpha N_2 I + J_1 + J_2 , \tag{1}$$

where α is the coefficient of line absorption per atom for the transition $2 - n$ corresponding to frequency ν, and N_2 is the number of atoms per unit volume in level 2. Then

$$J_2 = \frac{\varpi_0}{4\pi} \int_0^{\pi} \int_0^{2\pi} \alpha N_2 I \sin\theta \, d\theta \, d\phi ,$$

[11] *B.A.N.*, **8**, 299, 1938.

[12] *Ap. J.*, **80**, 19, 1934; *Mt. W. Contr.*, No. 494.

[13] *Jap. J. Astr.*, **1**, 17, 1949; *Pub. Astr. Soc. Japan*, **4**, 1, 28, 1952.

where ϖ_0 is the fraction of all absorbed radiation which is re-emitted at the same frequency. Put $d\tau = -aN_2dx$, $\mu = \cos\theta$, $J_1/aN_2 = D$. Then the equation of transfer becomes

$$\mu \frac{dI}{d\tau} = I - \frac{\varpi_0}{2}\int_{-1}^{+1} I(\mu')\, d'\mu - D. \tag{2}$$

Following the method developed by Chandrasekhar,[14] we replace equation (2) by the $2n$ equations

$$\mu_i \frac{dI_i}{d\tau} = I_i - \frac{\varpi_0}{2}\sum_j a_j I_j - D, \tag{3}$$

where the μ_i's ($i = \pm 1, \ldots, \pm n$) are the zeros of the Legendre polynomials $P_{2n}(\mu)$ and the a_j's ($j = \pm 1, \ldots, \pm n$) are the corresponding Gaussian weights.

The solutions to equation (3) are given by

$$I_i = \frac{D}{1-\varpi_0}\left\{\sum_{a=1}^{n} \frac{L_a e^{-k_a \tau}}{1+\mu_i k_a} + \sum_{a=1}^{n} \frac{L_{-a} e^{+k_a \tau}}{1-\mu_i k_a} + 1\right\}. \tag{4}$$

We shall consider solutions for τ infinite and τ finite.

a) τ INFINITE

In this case we have the boundary conditions that $I_{-i} = 0$ at $\tau = 0$. Also, since none of the I_i's increases more rapidly than $\exp(\tau)$ as $\tau \to \infty$, we can omit the terms in $\exp(+k_a\tau)$. In this case the solution can be obtained in closed form. If we write

$$S(\mu) = \sum_{a=1}^{n} \frac{L_a}{1-\mu k_a} + 1,$$

then, following exactly the procedure devised by Chandrasekhar (*ibid.*, chap. iii), we have

$$S(-\mu) = (1-\varpi_0)^{1/2} H(\mu).$$

Hence

$$I(0, \mu) = \frac{D}{(1-\varpi_0)^{1/2}} H(\mu), \tag{5}$$

and therefore

$$F(\tau = 0) = 2\int_0^1 I(0, \mu)\, \mu\, d\mu = \frac{2D}{(1-\varpi_0)^{1/2}}\, \alpha_1, \tag{6}$$

where α_1 is the first moment of the H function and has been tabulated by Chandrasekhar (*ibid.*, p. 328) for different values of ϖ_0.

b) τ FINITE ($= \tau_1$)

In this case we have the boundary conditions that $I_{-i} = 0$ at $\tau = 0$, and $I_{+i} = 0$ at $\tau = \tau_1$. In the second approximation $n = 2$ ($i = \pm 1, \pm 2$; $j = \pm 1, \pm 2$) we find, by

[14] *Radiative Transfer* (Oxford: Clarendon Press, 1950).

evaluating the constants L_α and $L_{-\alpha}$ from the boundary conditions and using $F = 2\Sigma a_i \mu_i I_i$, that

$$F(\tau = 0) = \frac{-4D}{\varpi_0 A_5}\left\{\frac{1}{k_1}(A_1 - A_3) + \frac{1}{k_2}(A_4 - A_2)\right\}. \tag{7}$$

In equation (7)

$$\begin{aligned} A_1 - A_3 = (e^{k_1\tau_1} - 1)\{ & e^{k_2\tau_1}(a_2 - b_2)(a_1 b_2 - a_2 b_1) + e^{-k_1\tau_1}(a_3 - b_3)(a_2 b_4 - a_4 b_2) \\ & + e^{-k_2\tau_1}(a_4 - b_4)(a_4 b_1 - a_1 b_4) + e^{(k_2 - k_1)\tau_1}(a_2 - b_2)(a_2 b_3 - a_3 b_2) \\ & + (a_1 - b_1)(a_4 b_2 - a_2 b_4) + e^{-(k_1 + k_2)\tau_1}(a_4 - b_4)(a_3 b_4 - a_4 b_3)\}; \end{aligned}$$

$$\begin{aligned} A_4 - A_2 = (e^{k_2\tau_1} - 1)\{ & e^{k_1\tau_1}(a_1 - b_1)(a_2 b_1 - a_1 b_2) \\ & + e^{-k_2\tau_1}(a_4 - b_4)(a_1 b_3 - a_3 b_1) + e^{-k_1\tau_1}(a_3 - b_3)(a_3 b_2 - a_2 b_3) \\ & + e^{(k_1 - k_2)\tau_1}(a_1 - b_1)(a_1 b_4 - a_4 b_1) + (a_2 - b_2)(a_3 b_1 - a_1 b_3) \\ & + e^{-(k_1 + k_2)\tau_1}(a_3 - b_3)(a_4 b_3 - a_3 b_4)\}; \end{aligned} \tag{8}$$

$$\begin{aligned} A_5 = {} & e^{(k_1 + k_2)\tau_1}(a_1 b_2 - a_2 b_1)^2 + e^{-(k_1 + k_2)\tau_1}(a_3 b_4 - a_4 b_3)^2 - e^{(k_1 - k_2)\tau_1}(a_1 b_4 - a_4 b_1)^2 \\ & - e^{(k_2 - k_1)\tau_1}(a_2 b_3 - a_3 b_2)^2 + 2(a_3 b_1 - a_1 b_3)(a_4 b_2 - a_2 b_4) \end{aligned}$$

where

$$\begin{aligned} a_1 &= (1 - \mu_1 k_1)^{-1}, & a_2 &= (1 - \mu_1 k_2)^{-1}, \\ a_3 &= (1 + \mu_1 k_1)^{-1}, & a_4 &= (1 + \mu_1 k_2)^{-1}, \\ b_1 &= (1 - \mu_2 k_1)^{-1}, & b_2 &= (1 - \mu_2 k_2)^{-1}, \\ b_3 &= (1 + \mu_2 k_1)^{-1}, & b_4 &= (1 + \mu_2 k_2)^{-1}, \end{aligned} \tag{9}$$

and k_1^2 and k_2^2 are the roots of the characteristic equation

$$\frac{1}{\varpi_0} = \frac{a_1}{(1 - \mu_1^2 k^2)} + \frac{a_2}{(1 - \mu_2^2 k^2)}.$$

For $\tau_1 \gg 1$, equation (7) becomes

$$F(\tau = 0) = \frac{4D}{\varpi_0}\left\{\frac{1}{k_1} + \frac{1}{k_2} - \mu_1 - \mu_2\right\}. \tag{7a}$$

The flux computed from equation (7*a*) will not exactly agree with that computed from equation (6), since (7*a*) is the solution in the second approximation only.

We have evaluated $F(\tau = 0)$ for several values of ϖ_0 and τ_1. These resulting fluxes from the atmosphere are shown in Table 3. In this table, the values of F for $\tau_1 = 1$ and 5 have been derived from equation (7), while the values for $\tau_1 > 5$ are from equation (6).

If there is no self-absorption, the equation of transfer becomes

$$\mu \frac{dI}{d\tau} = -D'. \tag{10}$$

Equation (10) has been derived from equation (2) by putting the first and second terms on the right-hand side equal to zero and by replacing D by D'. Now D' represents the original Balmer emission only, since it is now assumed that no Lyman radiation is absorbed and hence none is re-emitted at Balmer frequencies. The relative values $D'(Hn)/D'(H\beta)$ are equivalent to the Balmer decrement given by Baker and Menzel.[8]

From equation (10) we find that the outward flux

$$F(\tau=0)=2\int_0^1 I(\mu)\,\mu d\mu=2D'\tau_1 . \qquad (11)$$

The equivalent width of the emission line whose central frequency is ν can be written as

$$w=2K\int_0^{\Delta\nu} F_\nu(\tau=0)\,d\nu ,$$

TABLE 3

VALUES OF THE FLUX AND f/τ_1

ϖ_0	τ_1	$F(\tau=0)$	f/τ_1
0.9	1	$2D\times0.87$	0.87
	5	2.38	.48
	10	2.61	.26
	10^3	2.61	.0026
0.7	1	$2D\times0.70$	.70
	5	1.17	.23
	10	1.24	.12
	10^3	1.24	.0012
0.5	1	$2D\times0.58$	.58
	5	0.87	.17
	10	0.85	.085
	10^3	0.85	0.0009

where $\Delta\nu$ is half the total extent of the line, which is considered symmetrical, and K is a factor of proportionality. Thus

$$w=2K\int_0^{\Delta\nu} 2Df\,d\nu , \qquad (12)$$

where

$$f=\frac{\alpha_1}{(1-\varpi_0)^{1/2}} \quad (\text{for case } a, \text{ from eq. [6]})$$

and

$$f=\frac{-2}{\varpi_0 A_5}\left\{\frac{1}{k_1}(A_1-A_3)+\frac{1}{k_2}(A_4-A_2)\right\} \quad (\text{for case } b, \text{ from eq. [7]}) .$$

Similarly, the equivalent width, if there were no self-absorption, would be

$$w=2K\int_0^{\Delta\nu} 2D'\tau_1 d\nu \quad (\text{from eq. [11]}) .$$

Thus the cut-down factor due to self-absorption is given by

$$\frac{\int_0^{\Delta\nu} D f \, d\nu}{\int_0^{\Delta\nu} D' \tau_1 \, d\nu}. \tag{13}$$

The optical depth $\tau_1 = N_2 a x$. The shape of the outer atmosphere is known only very approximately, and N_2 is one of the unknown quantities of the problem. From Paper I (Sec. X) we may put $x \approx 2R_* \approx 10R_\odot$. As a first approximation, a value of N_2 can be obtained from the work of Baker and Menzel,[8]

$$N_2 = b_2 N_2^{(0)} ,$$

where $N_2^{(0)}$ is the population of level 2 in thermodynamic equilibrium at $T_e = 12{,}600°$. From the tables of Baker and Menzel we estimate that $b_2 \approx 0.009$, so that $N_2 = 5.66 \times 10^4$/cc.

The line-absorption coefficients for the centers of some Balmer lines were computed from the well-known formula. These values, together with the values of N_2 and x, give the following representative values:

$$\tau_1(H\alpha) = 1.7 \times 10^4 , \qquad \tau_1(H\zeta) = 1.3 \times 10^2 ,$$

$$\tau_1(H\beta) = 2.4 \times 10^3 , \qquad \tau_1(H22) = 5.0 .$$

For all except the very high members of the series, the optical depths can be considered infinite, as far as the fluxes determined from the transfer equation are concerned.

The line-absorption coefficient varies rapidly over the line width because of damping and kinetic Doppler broadening. However, the observed line width of the emission line is governed by the Doppler width, owing to mass motions, and this is much greater than that due to the kinetic Doppler motion. Consequently, we can use the value of a_{ν_0} for the center of the line to determine the effective optical depth.

From equation (13), the cut-down factor now becomes

$$\frac{\int_0^{\Delta\nu} D f(a_{\nu_0}) \, d\nu}{\int_0^{\Delta\nu} D' \tau_1(a_{\nu_0}) \, d\nu} = \frac{f(a_{\nu_0}) \int_0^{\Delta\nu} D \, d\nu}{\tau_1(a_{\nu_0}) \int_0^{\Delta\nu} D' \, d\nu} = \frac{\overline{D} f(a_{\nu_0})}{\overline{D'} \tau_1(a_{\nu_0})}. \tag{14}$$

Thus the cut-down factor is directly proportional to $1/\tau_1$ for large values of τ_1. Some values of $f(a_{\nu_0})/\tau_1(a_{\nu_0})$ are given in Table 3.

Although the values of τ_1 are uncertain, the ratios from line to line are known. The dependence of the cut-down factor on $1/\tau_1$ acts in the right sense to explain the large intensities of the high-level lines; but if this were the only term involved, then the lines from the lower levels would be cut down by much too large factors. If one assumes that $a_1\overline{D}/\overline{D'}(1 - \varpi_0)^{1/2}$ decreases on going to higher levels, then the observed Balmer decrements could be explained.

IV. CONCLUSION

We have shown that the effect of self-absorption will be to produce an apparent Balmer decrement in Be stars, which can depart considerably from that given in the ideal case of a planetary nebula and may approach that actually measured for the stars studied here. However, it is not possible to calculate the Balmer decrement exactly without setting up and simultaneously solving equations of transfer for radiation in all the frequencies. Lack of precise knowledge of the geometry of the outer atmosphere would lead to difficulties, even if the equations could be set up and solved. We have not considered the variation of the effective radiation field with distance from the source, which can be expected to change the ratio $\alpha_1\bar{D}/\overline{D'}(1 - \varpi_0)^{1/2}$. This will correspond to a varying dilution factor. Miyamoto[13] has shown that the $H\alpha/H\beta$ ratio is very sensitive to variations in the assumed dilution factor.

It is important to realize that, of the "pole-on" stars studied in Paper I, 48 Per, 105 Tau, 56 Eri, ω CMa, and β Psc did not show this anomalous Balmer decrement. The complete decrement is not measurable in these stars, since the emission lines are too faint to measure beyond $H\zeta$; but it would appear that if these lines are so faint as to be unobservable, then the Balmer decrement is nearer to the ideal nebular case. Although, for the purpose of estimating the self-absorption effects, the physical characteristics of the atmospheres of the four stars studied here have been considered to be similar, we do not know what physical conditions they have in common which cause them to emit relatively so much more energy in the high-level Balmer lines than do the other five stars mentioned above. They are not a very homogeneous group: 11 Cam is a typical "pole-on" star with single emission lines, HD 58050 has double emission lines with a fairly large separation, 12 Aur has shown considerable changes in a three-year interval, and HD 37998 probably has a variable spectrum, to which a unique type cannot be assigned.

Since many of the Be stars show atmospheric variations with time, as measured by total emission strengths, for example, it will be important to look for possible changes in the Balmer decrement with time.

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