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THE EVOLUTION OF GALAXIES AND STARS

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ABSTRACT

I. Aims of the theory.—A hydrodynamical scheme of evolution is proposed, confined to events after the time when the average density in the universe was comparable to the density inside a galaxy at our time.

II. Hydrodynamical conditions.—Gas in cosmic space is moving according to hydrodynamics, mostly in a turbulent and compressible manner. Dust is carried with the gas, probably by magnetic coupling. Star systems cannot be described hydrodynamically and hence do not show turbulence and supersonic compressibility.

III. The spectral law of incompressible turbulence.—The relative velocity of two points at a distance l is proportional to $l^{1/3}$. This is deduced from the picture of a hierarchy of eddies.

IV. Compressibility and interstellar clouds.—A hierarchy of clouds is considered.

V. General evolutional scheme for a gaseous body.—A gravitationally stable, turbulent cloud is first flattened into a rotating disk, which then is dissolved into a uniformly rotating central body and a part returning into cosmic space. The time scale of these changes is somewhat larger than the diameter of the cloud divided by the turbulent velocity.

VI. The origin of galaxies.—They seem to have been formed by a competition between expansion and turbulence.

VII. The evolution of galaxies and spiral structure.—Irregular nebulae must be young, spirals intermediate, elliptical nebulae genetically old. Spiral structure is the distortion of turbulent clouds by nonuniform rotation. A bar is more stable than a disk. A two-armed spiral seems to be a distorted bar.

VIII. The origin of the stars.—Three groups of stars are considered instead of Baade's two populations: (*a*) stars belonging to the galactic center dynamically; (*b*) old stars belonging to the disk; (*c*) "young" stars. Stars could be formed as long as there were no stars present, because stellar radiation inhibits the contraction of clouds to form new stars.

IX. "Young stars."—They seem to be, more exactly, rejuvenated stars. The mechanism of the accretion of interstellar matter by a star is discussed hydrodynamically.

X. Rotation, planetary systems, and double stars.—Stars must be formed rotating because of the turbulence of the original clouds. They lose their rotation probably by a combined magnetic-hydrodynamic process. Both formation and loss of rotation provide gaseous disks in which planets and double stars can be formed. Modifications of the author's theory on the origin of planets are discussed.

XI. Giants and white dwarfs.—Giants seem to have highly condensed cores. Their model may be nonstationary, with a slowly expanding atmosphere. Planetary nebulae then would be special types of giants.

I. AIMS OF THE THEORY

The difficulty of cosmogonical theories lies in the interconnection of the facts. The evolution of a single object can be understood only if its temporal and spatial boundary conditions and the external forces acting on it are known. These are defined by the evolution of the larger system of which the object forms a part. So every single problem is likely to lead us back into the problem of the history of the universe. On the other hand, the evolution of large systems can be discussed only if we have some knowledge of the rules governing the evolutionary behavior of their parts. Hence cosmogony has, by necessity, developed as a sequence of general schemes, which, on the one hand, were needed to make the particular problems well defined, while, on the other hand, the very solutions of these particular problems usually disproved some of the fundamental assumptions of the scheme.

This paper again proposes such a general scheme with the purpose of giving a background for more specialized investigations which in the end should, if not supersede, at least correct the scheme. It follows the outline given by some older papers of the author.[1] Since these papers are partly inaccessible to American readers except in a few libraries, their main points are briefly restated and, if necessary, corrected. The results of new calculations, to be published elsewhere, are qualitatively described. The main new point of this paper is a more detailed theory of the origin of the stars.

There are parts of cosmic history for which we have every good reason to assume that they can be understood by applying no other fundamental laws of nature than those known today, while in other, more remote, parts of time and space modified laws may hold. This paper tries to confine the proposed theory strictly to the field of applicability of well-known laws. This excludes all discussions of models of the universe. The only element of general cosmological theories which is used is the assumption that the red shift of the spectral lines of extragalactic nebulae can be interpreted as a Doppler effect and that we must accordingly ascribe an age of not more than a few billion years to the systems which we observe today. The agreement in order of magnitude of this age with the intragalactic and radioactive age determinations seems to render this assumption fairly probable.

The largest system considered in the theory is a vicinity of galaxies. The motion in such a system is described by Newtonian mechanics in a Euclidean space. No cosmological constant and no permanent creation of matter are assumed, but most of the results would not be affected very much by such assumptions.

The earliest time considered is the moment when the average density of the matter from which the galaxies originated was comparable to the average density inside a galaxy today. The problem of the formation of chemical elements which probably belongs to an earlier time is therefore excluded from the theory, while the formation of separate galactic systems is included.

II. HYDRODYNAMICAL CONDITIONS

In cosmic space we know three different "states of aggregation" of matter: gas, dust, and stars. The first question of the theory is what form of mechanics is able to describe the average motions of matter in these states. We will find the following results: (1) The motion of the cosmic gas obeys the equations of hydrodynamics. In most cases it is turbulent and compressible. (2) The dust is built up from and is, in a first approximation, carried with the gas.

(3) The motion of the stars cannot be described by a hydrodynamical approximation. Hence the concepts of turbulence and of dynamical compressibility as used in hydrodynamics cannot be applied to a system of stars. More general methods of statistical mechanics must be used for them.

A system of separate bodies, moving freely in space under a common force (which in our case is gravity) can be statistically described to a first approximation by assigning a

[1] C. F. von Weizsäcker, *Zs.f. Ap.*, **22**, 319, 1943; **24**, 181, 1947; *Zs.f. Naturforschung*, **3a**, 524, 1948; *Fiat Review of German Science, Astronomy, Astrophysics and Cosmogony* ("Naturforschung und Medizin in Deutschland, 1939–1946," Vol. XX) (1948), p. 413; *Naturwiss.*, **35**, 188, 1948; W. Heisenberg and C. F. von Weizsäcker, *Zs.f. Phys.*, **125**, 290, 1948.

density of matter ρ and an average velocity v to every point in space in every moment of time. These quantities will obey the equations of hydrodynamics if the distances l, in which they change appreciably, are large compared with the mean free path λ of the single bodies:

$$\frac{l}{\lambda} \gg 1 . \tag{2.1}$$

The solutions of the hydrodynamical equations are turbulent if the Reynolds number,

$$R = \frac{\rho v l}{\mu} ,$$

is larger than a certain critical value. Here v and l are values of velocity and distance characteristic of the average flow of matter. The molecular viscosity μ is equal to $\rho v_{th} \lambda$, where v_{th} denotes the average thermal velocity of the molecules. The critical value of R at which an average laminar motion becomes unstable is of the order of magnitude 1000. The transition toward a turbulent motion, however, reduces the average velocity, and in a turbulent flow there are eddies whose proper motions and diameters define Reynolds numbers down to about 10.[2] So the condition for turbulence can be roughly written

$$\frac{v}{v_{th}} \frac{l}{\lambda} \gg 1 . \tag{2.2}$$

A gas is compressed by its internal hydrodynamical motion if the velocity of this motion is comparable to, or larger than, the velocity of sound, which is essentially equal to the thermal velocity of the molecules. So the condition for compressibility is

$$\frac{v}{v_{th}} \geqq 1 . \tag{2.3}$$

For *gas* atoms the effective cross-section for a collision, σ, will be the geometrical cross-section, 10^{-16} cm if the atoms are neutral, and considerably larger if they are ionized. The number n of gas particles per cubic centimeter is between 1 and 10 for ordinary interstellar space and higher in dense clouds. So the mean free path, according to the general formula

$$\lambda = \frac{1}{n \sigma} , \tag{2.4}$$

is

$$\lambda_{\text{gas}} < 10^{16} \text{ cm} , \tag{2.5}$$

in which λ_{gas} can be smaller by several orders of magnitude. But even 10^{16} cm is close to the lower limit of observable distances in cosmic space, and observable motions of interstellar gas will consequently be described fairly well by hydrodynamics.

Velocity differences inside a cosmic cloud of the order of magnitude 1 km/sec or more are observed, e.g., in the Orion nebula, and the sharpness of interstellar absorption lines gives an upper limit of only about 6 km/sec for the type of clouds observed by those lines.[3] The velocity differences between different clouds amount to 20 km/sec or in extreme cases to 70 km/sec. If we describe a whole galaxy as a hydrodynamical system, we have to use for v the differences between the rotational velocities of its different parts,

[2] W. Heisenberg, *Zs. f. Phys.*, **124**, 628, 1947.

[3] W. S. Adams, *Ap. J.*, **97**, 105, 1943; *Mt. W. Contr.*, No. 673; *Ap. J.*, **109**, 354, 1949; *Mt. W. Contr.*, No. 760.

which amount to several hundred km/sec. On the other hand, the temperature of interstellar gas is not higher than 100° K for the larger part of space, and about 10,000° K for *H* II regions.[4] The corresponding thermal velocities for hydrogen are roughly 1 km/sec and 10 km/sec. So, except for small structures, mostly inside *H* II regions, v/v_{th} will never be small compared to unity and in many cases will be much larger. It follows that the motion of the interstellar gas in general must show the characteristics both of turbulence and of compression. Turbulence means irregular fluctuations of velocity. Compression means that these fluctuations of velocity will produce fluctuations of density. Both effects are observed and will be discussed in the next section.

For *dust* we may assume an average particle size of 10^{-5} cm, giving $\sigma \approx 10^{-10}$ cm². Except for especially dense and small dust clouds (globules), one would assume an average space density of the dust not higher than 10^{-25} gm/cm³. This figure follows from the assumption that the abundance ratio of heavy and light elements in stars should not deviate too much from the same ratio in the interstellar matter from which the stars were originally formed. This assumption, it is true, is open to doubt. But, since we know stars composed of heavy elements only (e.g., our earth) and since mechanisms have been proposed initiating the formation of a star by heavy elements only,[5] while, except for the radiation pressure of extremely luminous stars, no effects working in the opposite direction seem to exist, an estimate based on the ratio in our sun may seem conservative.

With these figures we would get roughly $n = 10^{-10}$ cm^{-3} and $\lambda = 10^{20}$ cm. This does not seem to fit well with the observational fact that there are structures of dust which certainly are not gravitationally stable and still show much finer details. The most conspicuous case is probably the thin filamentary structure in the Pleiades nebula, which is dust (since it is a reflection nebula) and whose smallest visible details are not larger than $2 \cdot 10^{15}$ cm. Such facts induce us to suspect that here the dust is carried with the gas. Its structure would then not be due to an interaction of dust particles with other dust particles but to their floating in the moving gas. We would have to assume that there is an uneven distribution of dust in the gas, perhaps due to the radiation pressure of the Pleiades B stars, and that this uneven distribution is then carried around with the eddies of the gas, thus making visible their kinematical pattern. The case would be similar to water clouds or tobacco smoke in our air. This idea seems to be supported by the fact that photographs of interstellar matter tend to show finer details in absorption or reflection than in emission.[6] Details in emission must be real density fluctuations of the gas, while details in absorption or reflection, according to the scheme here proposed, can render visible mere velocity fluctuations, even if they lie below the limit of compressibility.

As to the forces carrying the dust with the gas, only a tentative answer can be given here. The interaction may be caused by collisions of gas atoms with the dust particles or by a magnetic field. The latter coupling mechanism seems to be more efficient.

The coupling by collisions may be estimated as follows: A dust particle moving through the gas with a relative velocity v will change its momentum by its own order of magnitude in a time during which it is hit by a mass of gas comparable to its own mass. If v is small compared with the v_{th} of the gas molecules, this "frictional time scale" will be[7]

$$t_f = \frac{(4/3)\pi r^3 s}{4\pi r^2 \rho v_{th}} = \frac{r s}{3 \rho v_{th}}, \tag{2.6}$$

[4] L. Spitzer, Jr., *Ap. J.*, **93**, 369, 1941; **94**, 232, 1941; **95**, 329, 1942; **107**, 6, 1948; **108**, 276, 1948; **109**, 337, 548, 1949; **111**, 593, 1950; *Centennial Symposia* (Cambridge, Mass.: Harvard College Observatory, 1948), p. 87; *Science*, **109**, 461, 1949; B. Strömgren, *Ap. J.*, **89**, 526, 1939; **108**, 242, 1948.

[5] F. L. Whipple, *Centennial Symposia* (Cambridge, Mass.: Harvard College Observatory, 1948), p. 109; B. J. Bok, *Centennial Symposia*, p. 53.

[6] This was kindly pointed out to the author by Dr. Minkowski.

[7] L. Spitzer, Jr., cf. references n. 4.

where r and s are radius and density, respectively, of a dust particle. Table 1 gives values of t_f for different conditions. Now the gas changes its own velocity by its own order of magnitude during a "turbulent time scale," which, according to the spectral law (cf. next section), is

$$t_t = \frac{l}{v} = \frac{l_0^{1/3} l^{2/3}}{v_0} . \tag{2.7}$$

We may assume $v_0 = 10^5$ cm/sec and $l_0 = 10^{18}$ cm, thereby getting $t_t = 10 l^{2/3}$ sec or $t_t = 10^{13}$ sec for $l = 10^{18}$ cm, and $t_t = 10^{11}$ sec for $l = 10^{15}$ cm. The dust will take part in those motions of the gas for which $t_t > t_f$. This condition will scarcely be fulfilled for ordinary clouds if $l < 10^{18}$ cm. So, for a case like the Pleiades, recourse must probably be had to magnetic fields.

TABLE 1

FRICTIONAL TIME SCALE t_f IN SECONDS

v_{th} (CM/SEC)	ρ (GM/CM^3)		
	10^{-24}	10^{-22}	10^{-20}
10^5	10^{14} sec	10^{12} sec	10^{10} sec
10^6	10^{13} sec	10^{11} sec	10^9 sec

It seems probable today that interstellar magnetic fields of about 10^{-6} gauss exist. For particles of mass m and charge Ze the Larmor period is

$$t_m = \frac{m c}{Z e H} , \tag{2.8}$$

which for $m = 3 \cdot 10^{-15}$ gm and $Z = 10$ would give $t_m = 2 \cdot 10^{10}$ sec. So $t_t > t_m$ would be fulfilled in general. This means a coupling of the dust particles to the lines of force, which, on the other hand, are coupled to the ionized part of the gas.

For a "gas" whose "molecules" are *stars*, even formula (2.1) is not fulfilled, as has been shown by Chandrasekhar.[8] Hence the hydrodynamical concepts cannot be applied to a system of stars. There will be fluctuations of both velocity and density in such a system, but they will correspond to the fluctuations in a gas produced by the random component of thermal motion rather than to turbulence or dynamical compressibility. In a gas, turbulence and dynamical compression (as shock waves and similar phenomena) are of a macroscopic, nonthermal origin. They correspond to solutions of the hydrodynamical equations which would not be substantially altered if, instead of the atomic theory, a continuum theory of matter were true. These fluctuations, if they occur at all, usually are much larger than the thermal fluctuations. But they can occur only if partial volumes of the gas can move as a bulk, transferring momentum only to immediately neighboring volumes. This condition breaks down for star systems. The mean free path of a star in a gravitationally stable star system is, except for extremely dense parts of the system, larger than the diameter of the whole system. So the local conditions in any partial volume of the system can interact directly with the local conditions in any other partial volume. But this "interaction" will seldom lead to a real exchange of momentum or other properties between individual stars. To a good approximation, the motion of a single star

[8] *Principles of Stellar Dynamics* (Chicago: University of Chicago Press, 1942), p. 74.

can be described as an unperturbed orbit in the common field of gravitation produced by the whole system.

A much discussed consequence of this fact is that in our galaxy (and similarly in clusters of galaxies, where the single galaxy plays the role of the single "star" in this analysis) an equipartition of the energies of the single stars cannot be reached within a few billion years. As far as this equipartition does not exist empirically, the theory seems satisfactory. But there are perhaps cases in which it would seem difficult to understand even the approximation toward equipartition which is actually displayed by some systems. Such cases have been considered arguments against the time scale which is now generally adopted.[9] It should, however, be remembered that equipartition is easily reached when the matter is not yet united in stars because then the high turbulent momentum transfer is available. Therefore, the sort or degree of equipartition in a system which is not dense enough to achieve it by stellar interactions may give a hint as to the stage of evolution of the system in which its gas was transformed into stars. We will use such indications in the later sections.

III. THE SPECTRAL LAW OF INCOMPRESSIBLE TURBULENCE

In this section the results of the modern statistical theory of turbulence are described as far as they are needed in this paper.[10] The main purpose is to introduce the concept of the hierarchy of eddies and to formulate Kolmogoroff's law in a way which seems most appropriate for astrophysical use.

We try to describe a turbulent flow. The fundamental concept of the theory is the "element of turbulence" or "eddy." This is a partial volume of the fluid having a rather common motion for an interval of time. This motion will, in general, contain a whirling component—hence the name "eddy." But the whirling is not the defining characteristic, and so the word "eddy" should not be understood as emphasizing the vorticity but rather as an abbreviation for "element of turbulence."

An eddy is characterized by its size, l. The eddy will be destroyed after having traveled a mean free path or "mixing length" which differs from l only by a numerical factor of the order of magnitude unity. Another characteristic of an eddy is its velocity v relative to neighboring eddies. According to a generally accepted similarity hypothesis, for eddies small compared with the largest eddies and large compared with the smallest eddies, all other statistical characteristics can be derived from l and the average value of v. If we want to give a mathematically precise meaning to these concepts, we had best use a Fourier analysis of the velocity field. Instead of the concept "eddy size," we then use the wave length of a Fourier component or its reciprocal value, the wave number k, and instead of v we use the Fourier amplitude corresponding to k.

There are eddies of all sizes from the spatial dimensions of the entire system down to a limit set by the molecular viscosity. The small eddies are inside the large ones. This is what we call the "hierarchy of eddies." It would not be meaningful to describe a particular turbulent flow by telling how many eddies of a given size exist there. The answer would be that they always touch each other, so that, for every eddy size, the number is just the total volume of the fluid divided by l^3. It is meaningful, however, to ask for the "strength" of the eddies of a given size l. This can be defined by the function $v(l)$ indicating the average velocity of an eddy of given size, or the average relative velocity of two points in the fluid at a distance l. The average kinetic-energy density of the eddies of

[9] J. Jeans, *Astronomy and Cosmogony* (Cambridge: At the University Press, 1928).

[10] G. J. Taylor, *Proc. R. Soc. London, A*, **151**, 421, 1935 ff.; T. von Kármán, *J. Aero. Sci.*, **4**, 131, 1937 ff.; A. V. Kolmogoroff, *C.R. Acad. Sci. U.S.S.R.*, **30**, 301, 1941; **32**, 16, 1941; L. Onsager, *Phys. Rev.*, **68**, 286, 1945; C. F. von Weizsäcker, *Zs. f. Phys.*, **124**, 614, 1947; W. Heisenberg, *Zs. f. Phys.*, **124**, 628, 1947; *Proc. R. Soc. London, A*, **195**, 402, 1949; S. Chandrasekhar, *Proc. R. Soc. London, A*, **200**, 20, 1949 ff.; G. K. Batchelor, *Proc. R. Soc. London, A*, **201**, 405, 1950, and previous publications.

size l is $\frac{1}{2}\rho v^2(l)$. In the language of Fourier analysis the same fact is usually represented by a function $F(k)$, defined by

$$\frac{1}{2} v^2 = \int^{\infty} F(k)\, dk \,, \tag{3.1}$$

where $\rho F(k)$ denotes the energy contained in the wave-number interval between k and $k + dk$, and the integral is the energy content of the eddies of the size $l = k^{-1}$. If the lower limit is put equal to zero, the integral is the total energy of the flow.

The spectral function $v(l)$ is determined by a consideration of the dissipation of energy. Under the influence of a viscosity η, the energy dissipated per unit time and volume is

$$S = \eta \,|\, \text{curl}\ \boldsymbol{v} \,|^2 \,. \tag{3.2}$$

The energy of the larger eddies is not immediately transformed into heat, but it is first transferred to the eddies of somewhat smaller size. From these it flows to eddies of even smaller size and so on, until the smallest eddies transfer it to the thermal motion of molecules. The action of the eddies of a size l can be described phenomenologically by an "eddy viscosity,"

$$\eta_l = \rho l v \,. \tag{3.3}$$

This viscosity transfers the energy from the eddies somewhat larger than l to the eddies somewhat smaller than l. For dimensional reasons we must expect, in the average, for eddies of this size,

$$|\, \text{curl}\ \boldsymbol{v} \,|^2 = \text{Const.} \left(\frac{v}{l}\right)^2 . \tag{3.4}$$

Hence the energy transferred through the eddies of size l is

$$S_l = \rho l v \left(\frac{v}{l}\right)^2 = \rho \frac{v^3}{l} \,. \tag{3.5}$$

If the situation is stationary, there must be a constant flow of energy through all eddy sizes, and therefore S_l must be independent of l. It follows that

$$v \propto l^{1/3} \,. \tag{3.6}$$

From equation (3.1) we see that then

$$F(k) \propto k^{-5/3} \,. \tag{3.7}$$

This law, which, in a different mathematical formulation, was first given by Kolmogoroff, can be used to estimate all other statistical properties of the eddies by purely dimensional arguments. The time scale of the dissolution of an eddy will be

$$t = \frac{l}{v} \propto l^{2/3} \,. \tag{3.8}$$

This means that small eddies dissolve (and form) more rapidly than large ones. Therefore, the assumption of a stationary state which was made in deducing formula (3.6) will hold for small eddies, even if the largest eddies are not stationary.

The eddy viscosity is proportional to $lv \propto l^{4/3}$. If we use η_l in the expression for the Reynolds number instead of μ, we find that this $R(\eta_l)$ is of the order of magnitude 1 for eddies somewhat larger than the ones characterized by l. Hence we can also interpret the spectral law by saying that the turbulence below the level l is kept just strong enough to give to eddies above that level an "effective Reynolds number" not much larger than unity. If the turbulence below l were weaker than that, the motion above l would become more turbulent because of a lack of (eddy) viscosity; this increase of turbulence would show up just below l. If, on the other hand, the turbulence below l were stronger than indicated by the spectral law, its eddy viscosity would smooth out the motion above l, and thereby the turbulence below l would diminish.

When the energy flow reaches eddies for which the eddy viscosity is comparable to the molecular viscosity, it will be dissipated into heat; these are the "smallest eddies." According to Heisenberg, the limit is rather $\eta_l \approx 10\mu$; η_l/μ is the Reynolds number for l. This result was used in formula (2.2). Below the limit the spectral law takes the form

$$F(k) \sim k^{-7}. \tag{3.9}$$

Von Hoerner[11] has compared the spectral law with observations of radial velocities in the Orion nebula. Formula (3.6) can also be written

$$\frac{\partial \log v}{\partial \log l} = \frac{1}{3}. \tag{3.10}$$

In an empirical diagram showing $\log v$ as a function of $\log l$, the points can be represented by a straight line with the theoretical inclination 0.33 within the statistical error, while an inclination 0.40 might represent the observed points somewhat more closely. This slight deviation from the theory can perhaps be ascribed to compressibility.

These results will be modified by the consideration of magnetic fields. Turbulence tends to increase existing magnetic fields by lengthening the lines of force that are bound to follow the paths of the ionized particles. On the other hand, strong magnetic fields will prevent turbulent motions of ionized particles. These two effects will affect the spectrum by suppressing the turbulence, beginning from the lower end of the spectrum. The problem is very intricate. It has been treated by Biermann and Schlüter.[12]

IV. COMPRESSIBILITY AND INTERSTELLAR CLOUDS

This section describes the fundamental assumptions and results of a tentative theory of the interaction of turbulence and compressibility proposed by von Hoerner.[11] Supersonic velocities produce compression. In most terrestrial experiments we see single bodies moving with supersonic velocity through a fluid in which no other supersonic velocities are present. This leads to a single, fairly regular shock wave. In cosmic space similar phenomena may occur in the expanding shells of supernovae, as discussed by Oort and Burgers.[13] But this is probably not the average case of cosmic turbulence. In general, the energy source of cosmic turbulence will lie not in a single star but in the largest eddies, which probably should be identified with the rotational motion of the galaxy as a whole. Except for particular regions in their interior, galaxies are not rotating uniformly. The eddy viscosity produced by turbulence on a somewhat smaller scale tends to dissipate the kinetic energy of the relative motion of different parts of the galaxy caused by the nonuniform rotation, and this dissipated energy feeds the turbulence in the way described in the preceding section. The velocity differences involved in this process amount to 100 km/sec or a hundred times the average velocity of sound. Hence we would expect a very complicated statistical mixture of shock waves, if this concept can still be used at all.

Von Hoerner takes not shock waves but density fluctuations of the character of inter-

[11] To be published.

[12] *Zs. f. Naturforsch.*, **5a**, 237, 1950.

[13] J. M. Burgers, *Proc. Kon. Ned. Akad. Wet. Amsterdam*, **49**, 589, 1946; J. H. Oort, *M.N.*, **106**, 159, 1946.

stellar clouds as the fundamental concept of his theory. In analogy to the hierarchy of eddies in incompressible turbulence, he introduces a hierarchy of clouds. Every large cloud is assumed to consist of a certain number of smaller clouds. For the sake of simplicity, the space between the smaller clouds is treated as a vacuum. The smaller clouds, in their turn, consist of even smaller clouds, between which there is vacuum again. This hierarchical structure goes on until clouds are reached with an inner turbulent velocity small compared with the velocity of sound. Again, for simplicity, sharply distinct levels in this hierarchy are assumed and described by an index ν, which can have only integral values, larger ν designating larger clouds. A cloud at the level ν is called a C_ν. Instead of the single function $v(l)$ of the incompressible case, four functions of the index ν must be determined:

- v_ν, the average velocity of the center of gravity of a cloud C_ν with respect to the center of gravity of the next larger cloud $C_{\nu+1}$ in which C_ν is contained;
- ρ_ν, the average density of matter inside C_ν;
- l_ν, the average diameter of a C_ν; and
- N_ν, the average number of C_ν's contained in a $C_{\nu+1}$.

The functions ρ_ν and l_ν can be connected by the equation

$$\frac{\rho_\nu}{\rho_{\nu+1}}=\left(\frac{l_\nu}{l_{\nu+1}}\right)^{-3k_\nu}. \tag{4.1}$$

The exponent k_ν is called the degree of compression at the step ν. If $k_\nu = 0$, there is no compression at this step; k_ν must be smaller than unity.

It can be shown that k_ν also expresses the spectral law. Eliminating the index ν, we can write the law in analogy to formula (3.10)

$$\frac{\partial \log v}{\partial \log l}=\tfrac{1}{3}+k(l). \tag{4.2}$$

So the main task of the theory is the determination of the function k_ν or $k(l)$. This involves considerations of the mechanism of the formation and destruction of the density fluctuations here schematically described as separate clouds. We may consider two limiting cases. If we start from a uniform density throughout the whole volume, the supersonic velocities will soon produce strong density fluctuations by shock waves. If, on the other hand, we start from extreme density fluctuations, e.g., very small and dense clouds separated by distances large compared with their diameters, these clouds (unless gravitation were strong enough to keep them stable) would expand by the pressure of their internal temperature and turbulence; so a more uniform density distribution would be approached. Between the two limiting cases there must be an equilibrium between the compression produced by the collisions and the expansion during the time between the collisions. It is difficult to give an accurate model of these complicated processes, but von Hoerner could show that, with increasing ν and therefore increasing v_ν, the compression k_ν varies from 0 for $v_\nu \ll v_{th}$ to a limiting value k_∞ for $v_\nu \gg v_{th}$. For different assumptions about the mechanism of the collisions, k_∞ varies between 0.09 and 0.23, the value thought to be most probable lying at 0.12. The result about the spectral law of the Orion nebula seems to indicate that there $k_\nu \approx 0.07$, a reasonable value for the observed internal velocities of about 10 km/sec in the nebula.

The result can be expressed in still another way. We ask for the fraction f of the volume inside the largest clouds which is actually occupied by matter. This fraction will depend both on k_∞ and on the ratio between the sizes of the largest and the smallest clouds. If we ascribe a diameter of 10^{22} cm (galaxy) to the largest clouds, and 10^{19} cm (3 parsecs) to the smallest clouds, we get the following values of f as a function of k:

$$k = 0.05\,, \quad 0.10\,, \quad 0.15\,;$$

$$f = 0.36\,, \quad 0.125\,, \quad 0.045\,.$$

The empirical value of f seems to lie somewhere between 0.05 and 0.15.

The analogy of this theory with the theory of the incompressible case is perhaps seen more clearly if we remark that, in addition to the eddy viscosity, we here get a "cloud pressure." Just as the kinetic energy density of molecules appears as gas pressure, the kinetic energy of the clouds below a given level acts as a pressure in the clouds of a higher level; and, since we have supersonic velocities, this cloud pressure is higher than the molecular pressure. And, just as the eddy viscosity serves to define an effective Reynolds number which, for successive levels of eddies in a stationary situation, can never exceed the order of magnitude 10, so we can define an "effective Mach number" for clouds above a level ν with respect to the "velocity of sound" defined by the velocities of clouds below this level. This effective Mach number again will stay small, and so the compression k in a single step can never become very high. This accounts for the existence of a limiting value k_∞. In a less abstract way we can say: Real shock waves in large dimensions cannot exist, since there are always density fluctuations in the next smaller level which immediately divert the wave fronts into statistically distributed directions. Here, again, the action of magnetic fields should be considered. The reader is referred to the paper by Biermann and Schlüter.[12]

V. GENERAL EVOLUTIONARY SCHEME FOR A GASEOUS BODY

In this section we try to describe the evolutionary trend as far as it does not depend on the special conditions by which galaxies, intragalactic clouds, stars, planets, etc., are distinguished.

We assume every gaseous body which we will consider to be formed as a part of a larger mass of gas that existed before the formation of the smaller body. The new body will be stable only if its gravitational energy is larger than its thermal energy:

$$\frac{GM^2}{R} > MkT\,. \qquad (5.1)$$

This is Jeans's[14] condition. It is necessary, but not sufficient, for the actual formation of a separate stable body. If, like Jeans, we consider the original larger mass to have had uniform density and no inner motion, the gravitational instability of a slight density fluctuation might well lead to a first additional contraction. This contraction, however, will be stopped very soon by the conversion of the gravitational energy into heat, unless the heat can be at least partly removed, e.g., by radiation. Another insufficiency of the formula is that it gives only a lower limit to the mass of the body.

The picture becomes somewhat more definite if we assume that there is supersonic turbulent motion in the original mass. Then a hierarchy of clouds will be formed. Instead of Jeans's condition of stability, we will then get an equation in which the hydrodynamical velocity of the inner motion takes the role of the thermal energy:

$$\frac{GM^2}{R} \sim \frac{Mv^2}{2}\,. \qquad (5.2)$$

Here we may write the sign of equality, since the mechanism of compressible turbulence

[14] Jeans, *op. cit.*, chap. xiii; G. Gamow and E. Teller, *Phys. Rev.*, **55**, 654, 1939; G. Gamow, *Phys. Rev.*, **74**, 505, 1948.

will provide for the formation of clouds not much larger than the lower limit. The further contraction of the clouds will again depend on the possibility of getting rid of part of the internal energy. Unlike the adiabatic compression of a quiet cloud, here the contraction is certainly accompanied by one irreversible process—the dissipation of turbulent energy.

If a turbulent cloud is stable and able to contract, the first step of the contraction will be a flattening. The contraction will first be caused by the decay of the inner turbulence and of the corresponding "cloud pressure." Now the part of the turbulent motion which is connected with a resulting angular momentum cannot easily vanish because of the conservation law. Thus a flat rotating disk will remain. Its largest diameter and rotational velocity will not be different by a large factor from the original diameter and turbulent velocity of the cloud. If these quantities are d and v, respectively, the time scale of the flattening will be

$$t = \alpha \frac{d}{v}, \tag{5.3}$$

where α may be put equal to 5, to give an estimate.

The disk will probably already have a central concentration of mass. Its rotation will not be uniform, and the differences in the rotational velocity will go on producing turbulence. The remaining extension of the disk in the direction perpendicular to its plane will be determined by this turbulence as long as its cloud pressure is higher than the molecular pressure.

The remaining turbulence exerts friction and thereby dissipates energy. Therefore, the rotation cannot be stable unless it becomes uniform. This will not be possible for a very extended mass of low density. On the other hand, the angular momentum prevents contraction of the body as a whole. The result will be the contraction of part of the body toward the center, while the gravitational energy set free by the contraction enables the rest of the mass to return to the surrounding cosmic space, carrying with it most of the angular momentum of the body. This evolution has been followed by analytical and numerical solutions of the hydrodynamical equations.[15]

For dimensional reasons the time scale of this loss of angular momentum will again be given by a formula like (5.3), where v can be taken to be the rotational velocity but with a much larger α, since now the turbulence is weaker than before and the process is more complicated. Perhaps we are not quite wrong in assuming an α of 100. But we should rather consider this process in more detail in the special cases.

VI. THE ORIGIN OF GALAXIES

Since the beginnings are the least-known part of history, we offer our hypothesis about the origin of galaxies with more hesitation than the theory about their further development. The actual motions of the galaxies are composed of two parts. One component is the systematic velocity of expansion as deduced from Hubble's law, which is predominant for galaxies far apart. The other component is an irregular proper motion of the single systems. For neighboring systems the relative velocity due to the second component is empirically of just the same order of magnitude as the relative velocity which would follow from Hubble's law alone.

According to the assumptions formulated in Section I, we may follow these motions back to the time t_0 when the galaxies just "touched each other." In the meantime, the velocities should have been essentially preserved by the law of inertia. At t_0 we would describe the first component as a systematic expansion of the part of the universe known to us and the second component as a turbulent motion superimposed on the expansion. It is encouraging for such a picture that the rotational velocities of the actual galaxies, too, are of the same order of magnitude as their irregular proper motions. We try to ex-

[15] R. Lüst and E. Trefftz, to be published (*Zs. f. Naturforsch.*).

plain the origin of the galaxies by the co-operation of these motions and of gravitation, while we do not try to deduce these motions from earlier states of the universe or from more fundamental principles.

The equality of the velocities of expansion and of turbulence for neighbors is just what we should expect if turbulent compression had determined the formation of the galaxies. In an expanding universe gravitational instability would not be sufficient to form subsystems, while turbulence could do it if its velocity v_t were large enough compared with the velocity of expansion v_{ex}. If r denotes the distance between two points in the universe at the time t_0, their relative velocity consists of the two components, one of which is given by Hubble's law:

$$v_{ex} = a\, r\,, \tag{6.1}$$

while the average value of the other follows from Kolmogoroff's law, which may be modified by considering compressibility and other deviations from stationary incompressible conditions:

$$v_t = b\, r^{1/3+k}\,. \tag{6.2}$$

In these equations a is Hubble's constant at the time t_0 (essentially $a = t_0^{-1}$), b is another constant which may be determined from the irregular motions as seen today, and k is perhaps a constant, perhaps a function of r, but in any case a number which will probably not exceed $\frac{1}{3}$. If $v_t > v_{ex}$, there is a chance that the two points approached each other at the moment t_0 and that they consequently became parts of the same galaxy. This condition defines a maximum value of r which we may identify with the size of the largest galaxy at the time of its formation. So the theory seems to explain that there is a maximum size of galaxies.

Yet the theory has several shortcomings which probably could be removed by closer consideration of the events at and before the time t_0. This transcends the restrictions we have imposed on ourselves in this paper, but we should at least mention the main problems.

First, the connection which probably exists between the two constants a and b is not understood. This may be the most fundamental problem of the theory because it involves the origin of cosmic turbulence, but probably it is the most difficult one, too.

Secondly, one might ask whether our considerations should not be applied to clusters of galaxies rather than to single galaxies. In a theory starting from the idea of a hierarchy of clouds we should probably not be surprised to find the aggregation of matter taking place in different levels at the same time. The competition between turbulence and expansion may lead to the looser aggregation in clusters for very large clouds and to the denser form which we call "galaxies" for somewhat smaller systems. The precise meanings of the quantities t_0, a, and b will not be clear without a more detailed theory of these distinctions.

Thirdly, the theory does not yet include another empirical fact concerning gravitation. Expressed in terms of an energy balance, the condition $v_{ex} = v_t$ means that for the actual galaxies the kinetic energies due to expansion and turbulence are equal. Now, empirically, the inner potential energy of gravitation of a large galaxy, being negative, has roughly the same absolute value as these two kinetic energies. This is seen most easily from the fact that the rotational velocities of galaxies are roughly as large as their "turbulent" translational velocities. The rotational energy of a flat galaxy is connected with its gravitational energy by the virial theorem. Therefore, even in assuming that the origin of the rotation is in the primary turbulence, we would not conclude a priori that the actual velocity of rotation must still be equal to the original velocity of turbulence; it would have had to adjust to the gravitational conditions by a shrinking or an expansion of the system. The empirical coincidence between the velocities of rotation and translation therefore means that the systems did not need to readjust their densities very much after their separation. This may justify our choice of t_0 as the time when the average density of the known part of the universe was comparable to the density inside a galaxy today. But what does this coincidence mean cosmogonically?

At a given time, say, t_0, for every spherical part of the universe of average density, the inner gravitational energy and the kinetic energy of expansion are proportional to each other, no matter how large or how small the total mass of the sphere considered. Let r be the radius of the sphere and M its mass. Then M is proportional to r^3, and

$$E_{\text{grav}} = \text{Const.}\ \frac{M^2}{r} = \text{Const.}\ r^5\ , \tag{6.3}$$

$$E_{\text{exp}} = \text{Const.}\ M\,v^2 = \text{Const.}\ r^5\ . \tag{6.4}$$

Our coincidence now means that at t_0 they were not only proportional but roughly equal, while, later on, for a constant (though expanding) mass M, only E_{exp} remained constant, E_{grav} decreasing proportional to t^{-1}. Before t_0, if we may apply the simple picture of expansion to that time, both energies must have decreased with increasing time. So, from our observational indications, the first formation of galaxies seems to have taken place in the moment when the known part of the universe ceased to be "gravitationally coherent."

A fourth point which would need consideration is the mass distribution of galaxies, especially the fact that there seems to be a lower limit to the possible mass of a galaxy.[16] This is probably connected with the gravitational energy, too. Applying the spectral law (6.2), we find that the turbulent energy decreases more slowly for decreasing r than the potential energy of gravitation:

$$E_t = \text{Const.}\ M\,v_t^2 = \text{Const.}\ r^{11/3+2k}. \tag{6.5}$$

Hence, if condition (5.2) is approximately fulfilled for the largest systems, it cannot be fulfilled for systems much smaller than these. This, however, presupposes that all systems were formed at the same time. In the next section we shall find strong evidence for the assumption that some extragalactic nebulae, especially some of the smaller ones, were formed much later than t_0. So the mass distribution can be understood only in connection with the later evolution of the galaxies.

VII. THE EVOLUTION OF GALAXIES AND SPIRAL STRUCTURE

We assume galaxies to develop according to the scheme proposed in Section V. The time scale for the flattening of a galaxy connected with the decay of its original turbulence may be comparable to its actual period of rotation. The time scale for the loss of rotation can easily be ten or twenty times that period. Thus large galaxies like our own can be as old as the universe, without having yet reached their final stage. For small systems like the Magellanic Clouds or the companions of M 31, however, the time scale must be definitely smaller than the age of the universe. Consequently, they must be either still young in years or already old genetically. Probably systems of both types, including intermediate cases, occur in our neighborhood. Probably not all the original matter was exhausted in the formation of the oldest systems, and, moreover, the old systems must have lost matter in connection with the loss of angular momentum. So there may have been a chance for the formation of young systems from t_0 up to our time.

How to distinguish the stages of evolution? We will consider irregularity of shape and a high content of interstellar matter as indications of youth in a system. Irregularity indicates turbulence and must decay; even independently of our special model, we can conclude that an irregular shape cannot be a stable configuration in any system and there-

[16] Personal communication by Dr. Baade.

fore cannot be old in a time scale defined by the dimensions and inner velocities of the system.[17] Interstellar matter, on the other hand, can be converted into stars, while the opposite process will be rare; but here the time scale depends on details discussed in Section VIII. From these indications we conclude that irregular systems like the Magellanic Clouds are genetically the youngest systems, spirals are older (and probably the older, the "earlier" they are in Hubble's nomenclature), and elliptical nebulae are in a final stage which no longer shows the sort of evolution we consider here. We should keep in mind that the words "old" and "young" here always refer to the proper time scale of the system and that, of two systems of the same absolute age, the larger one will look "younger" today. So there is no objection to assuming that M 31 and its two companions are of the same absolute age, while the Magellanic Clouds must be definitely younger than our galaxy.

The most conspicuous semiregular pattern in galaxies to which we ascribe an intermediate stage in their evolution is spiral structure. We propose to explain it by the theory of Wilczynski and Brown,[18] adapted to the hydrodynamical model.

According to Baade,[19] spiral structure seems to be strictly connected with the appearance of dust. Even in the central part of M 31 some spiral structure is seen, not in the real distribution of the stars but as an absorbing cloud. It is improbable that large dust clouds should exist quite separately from gas (cf. Sec. II). So we take the presence of dust as an indication of the presence of gas. In fact, an increase in the density of dust by a factor of 10 would be much more conspicuous than a similar increase in the density of gas. So we conclude that spiral structure is produced by the interstellar gas. This view is strengthened by the fact that a spiral structure of the star distribution is conspicuous only in "young" stars.

We try to understand spiral structure as a hydrodynamical effect. Spirals are rotating and turbulent. Since they are flat, their turbulence is no longer the original one; it must be produced by nonuniform rotation. Every system, however, which is at the same time turbulent and nonuniformly rotating must, of necessity, show a spiral pattern. Every cloud formed by the turbulence will be distorted by the rotation into a segment of a spiral. This phenomenon is shown by milk put into a cup of coffee after stirring.

The main objection against the older versions of this theory probably was that the spiral arms should be wound many times around the nucleus, while actual spirals never wind more than two or three times. This difficulty no longer exists in the hydrodynamical theory, since turbulence dissolves the clouds as well as creates them, and the time scale for their dissolution is comparable to the period of rotation.

If the theory is correct, the arms should trail behind (move in the direction of their convex side), except if there are regions where the angular velocity increases with increasing distance from the center. Babcock's[20] measurements seem to indicate that such a region exists in M 31, while the spiral structure certainly does not change the direction of winding in that region. But Babcock's measurements certainly do not refer to the interstellar matter in that region, and additional observations may be needed.[21]

The abundance of systems with just two spiral arms is probably caused not by turbulence but by gravitation. In fact, for "late" spirals, which contain most interstellar matter and look most turbulent, the spiral structure, though conspicuous in all large clouds, cannot, in general, be described by the concept of two long, coherent arms. The number 2 is most evident in the least turbulent-looking systems like barred spirals. We may understand the bars as elongated equilibrium figures of rotation similar to Jacobi's

[17] H. Shapley, *Galaxies* (Philadelphia: Blakiston Co., 1943), pp. 216 ff.

[18] E. Z. Wilczynski, *Ap. J.*, **4**, 97, 1896; *A.J.*, **20**, 67, 1899; E. W. Brown, *Observatory*, **51**, 277, 1928.

[19] Private communication.

[20] *Lick Obs. Bull.*, **19**, 41, 1939.

[21] I am grateful to Dr. N. U. Mayall for an interesting discussion on this point.

liquid ellipsoids. In a rotating disk of circular symmetry, no gravitational energy can be gained by concentrating the mass toward the center, since the conservation of angular momentum prevents such a dislocation; but all the matter moving on the same circular orbit around the center can be moved toward two opposing single points on the circle without changing the angular momentum, but with a gain of gravitational energy. Hence a bar is a more stable structure than a disk. But the bar can be kinematically stable only if the system rotates uniformly. Uniform rotation presupposes a gravitational potential quadratic in the distance from the center. Hence it will be possible in the neighborhood of a potential minimum. This may explain the fact that barred spirals have bars near the center and spiral structure in the outer parts.

Regular spirals with two arms, according to this explanation, would be close enough to uniform rotation not to destroy the "bar" entirely but to distort it strongly.[22] Both in M 31 and in M 33 the easily visible spiral arms lie in regions where the rotation does not deviate strongly from uniformity. It is remarkable in M 31 that outside the nucleus (which has its own, higher angular velocity) there is another region of nearly uniform rotation. The fact that some barred spirals show a small spiral inside the nucleus (a "wheel inside the bar") raises the same problem even more clearly. A very special law of mass distribution is needed to achieve this effect, and it is not to be expected a priori that the system during its formation should assume this special mass distribution. The only explanation I can offer is that turbulent friction tends to establish a uniform rotation and that it may set up hydrodynamical currents achieving the necessary rearrangement of the masses.

Spiral structure as a turbulent pattern cannot last forever. Two final developments can be imagined. The interstellar matter can either be absorbed by the stars or be lost, together with an amount of angular momentum. According to the following sections, both events will happen, but probably with a preference for the loss into cosmic space. In any case a system consisting of stars alone will be the final stage. Such a system can preserve its internal motion for a time long compared with the age of the universe.

VIII. THE ORIGIN OF THE STARS

The distribution of stars and of interstellar matter gives us some indications about the possible processes of formation of the stars. We may use Baade's[23] two populations as a starting point of an empirical discussion. These populations were first distinguished by three criteria:

Physical parameters of the stars: Their H-R diagrams are different.

Spatial distribution: Population I is found in the disk of spirals, especially in the arms; population II in the nuclei of spirals, in elliptical nebulae, and in globular clusters.

Kinematical properties: In the neighborhood of our sun, population I prefers circular orbits, population II elliptical orbits.

There has been some discussion whether the two populations are sharply distinguished or whether they indicate extreme points of a continuous range of types. Baade maintains that they are sharply distinguished, but he partly separates the criteria, e.g., by saying that a large percentage of the stars in spiral disks belong to population II.[24] We cannot try to discuss these difficult observational questions here. To have a nomenclature that would not involve too many special assumptions, we propose to characterize every type of star by three numbers, x, y, and z. The first number should classify it according to its physical parameters, the second one according to its spatial position, and the third number should describe its kinematical properties. In this paper we shall only use integers for

[22] Dr. A. Schlüter is considering the possibility that spiral arms are stabilized by magnetic fields (private communication).

[23] *Ap. J.*, **100**, 137, 1944; *Mt. W. Contr.*, No. 696.

[24] Private communication.

x, y, z, leaving it to further discussion if some of them should rather be considered to vary within a continuous range of values.

With respect to physical parameters, we accept the distinction of two groups as defined by their color-luminosity diagrams. We write $1yz$ for Baade's population I, and $2yz$ for Baade's population II, as far as populations I and II are defined by the H-R diagram. But we want to give some emphasis to a subgroup of I by calling it $0yz$. These are the stars that can be seen by merely physical arguments to be young (or "rejuvenated"; cf. next section). For most of them several independent criteria of youth apply. The early types of the main sequence must be young because they exhaust their hydrogen rapidly. This criterion may even include the A stars in group $0yz$, if there is no mixing process of matter inside the stars.[25] The O and B stars of the main sequence and the supergiants are mostly connected with dust.[26] Just as in the preceding chapter, we shall assume the dust to be the tracer for the presence of gas. Then we may conclude that these stars, if any, have a chance of still acquiring large amounts of interstellar matter. Finally (cf. Sec. X), rotation seems to be a sign of youth in a star, and this property is confined to early main-sequence stars down to the early F stars.

We might ask if all $1yz$ stars are young, so that we might identify $0yz$ and $1yz$. But if the sun is a typical population I star, as Baade maintains, this population also contains old stars, since the sun cannot plausibly be assumed to be younger than the earth. So, if the sun is to be retained in group $1yz$, this group must be defined by other parameters than age, and $0yz$ is only a subgroup of population I. We do not try to solve this problem in this paper.

With respect to spatial distribution, the three areas of which Baade's group II is the only population seem to have something in common. According to our picture, elliptical nebulae are old spirals that have lost their spiral arms; so, in general, we will expect them to behave similarly to the nuclei. Globular clusters (and the few stars in the space between them) may be considered to form an extended "atmosphere" of the galactic center. Such an atmosphere may be a remnant of an original cloud, but, even if the original cloud had completely disappeared, a new atmosphere would probably have been formed. For the virial theorem postulates a high average velocity for the stars in the center, and in every reasonable statistical distribution this means that there must be some stars present which have enough energy to move very far away from the center.

We shall call all stars belonging to the "center plus atmosphere" or to an elliptical system, group $x2z$. Baade's empirical result is expressed by saying that for them always $x = z$. As long as we do not know exactly what the distinction between $1yz$ and $2yz$ means, we cannot deduce this result theoretically. But it seems understandable that no young stars of this type, i.e., no $02z$ stars, exist, since interstellar matter is very scarce in those regions of space today.

Besides the center plus atmosphere, spirals possess a flat disk of stars. We call the stars contained in this disk group $x1z$. According to Baade, there exist stars both of type $11z$ (e.g., the sun) and $21z$.

Inside the disk we can again distinguish as a subgroup the stars contained only in spiral arms; we shall call them group $x0z$. Empirically,[27] arms seem to be whiter than the rest of the disk. It seems possible that in this group always $x = 0$. That means that only young stars are confined to arms, as we should suppose if the arms are the concentrations of interstellar gas. Stars that accidentally occur within an arm but belong to a type which is also present in the rest of the disk should not be included in group $x0z$.

Kinematically, we may distinguish stars in circular orbits and stars in eccentric orbits: $xy1$ and $xy2$. It is probable that circular orbits are typical of disk stars, so in $xy1$

[25] This possibility was pointed out to the author by Dr. Chandrasekhar.

[26] Both Drs. Baade and Nassau kindly discussed the evidence with the author.

[27] C. K. Seyfert and J. J. Nassau, *Ap. J.*, **101**, 179, 1945.

we expect $y = 1$ or 0, while x may be 0, 1, or 2. For $xy2$ stars, $x = 0$, and probably even $x = 1$, seem to be excluded.

The concentration of disk stars toward the galactic plane is a criterion that is in some way coupled with kinematical properties. The more strictly circular the orbit is, the more concentrated toward the plane the stars seem to be. $00z$ stars are certainly the most concentrated ones, as O and B stars. These kinematical criteria are the ones which show the most continuous variation of the parameters.

We may summarize this analysis by mentioning the groups of stars we have found:

000: Young stars, moving in spiral arms and in circular orbits.
111: Population I, including old stars like our sun.
212: Population II in the disk. It is doubtful if also 211 stars exist.
222: Population II in the center plus atmosphere.

If we now try to apply the ideas of Section V to the formation of stars, we seem to encounter a paradox. The time scale for the formation of a cloud of stellar mass as defined by formula (5.3) is much shorter than the age of the galaxy. Choosing $l = 3 \cdot 10^{18}$ cm and $v = 10^5$ cm/sec, we get about $t = 5 \cdot 10^6$ years. If the formation of stars is possible at all today, it is not to be understood why there still should be as much interstellar matter present as we find in the surroundings of our sun. We are led to the suspicion that no formation of stars is possible today, except perhaps under conditions which generally are not fulfilled. This view is strengthened by the analysis of Section IX, which seems to show that the stars of group 000 are not, in general, newly formed but rather rejuvenated and that even this process of rejuvenation needs peculiar external conditions.

On the other hand, the stars are there, so the conditions for their formation must once have been better. What was the difference between then and now? We propose that it consisted in the fact that when the stars were formed there were no stars present. In other words, the presence of stars inhibits the formation of new stars. Consider a cloud of stellar mass before it has started to contract appreciably. It must be an equilibrium between gravitation and gas pressure if it is stable at all. It can contract further only if it can radiate away part of its thermal energy. This is probably possible if no stars are in the neighborhood. If stars are present, however, they will maintain the temperature of the cloud by their radiation as long as it is still transparent to their light, and so the contraction cannot start. In fact, we know that the interstellar matter in our surroundings is kept at a temperature of at least 50° K by the stars,[4] and at these temperatures clouds of stellar mass and a density smaller than about 10^{-18} gm/cm^3 are not dynamically stable at all but will expand.

What, then, do we know about the conditions under which the formation of stars began? We have empirical indications that at least the stars of group $x1z$ (stars in the disk) were formed after the decay of the initial turbulence or in its last phase. Else they would not now be concentrated in the galactic plane. For stars, once formed, no longer suffer turbulent friction, and therefore a system of stars should approximately retain its kinematic structure and hence the general characteristics of its shape as they were at the time of the formation of the stars. Thus globular clusters seem to have formed their stars before much of their turbulence decayed, while the opposite conclusion applies to the galaxy.

If clouds are formed inside a galaxy by a compressible turbulence caused by rotation, they cannot be expected to be gravitationally stable from the outset. The galaxy as a whole is just in such an equilibrium between gravitational and centrifugal forces. For smaller clouds the gravitational energy decreases more rapidly with decreasing size than the kinetic energy of their internal turbulence (cf. Sec. VI). But the smallest clouds can dissipate energy by their internal incompressible turbulence. The time scales of this dissipation and of the expansion of the cloud caused by the same internal turbulence and by temperature are of the same order of magnitude, both being determined by the same values of v and l. The dissipation will be especially high in the moment of the collision of two such clouds under a still supersonic relative velocity. Therefore, the smallest clouds have a good chance of becoming gravitationally stable if they are able to radiate away the heat produced. If they can cool down to the temperature T, we would expect their total mass to be given by Jeans's condition, which can be written:

$$M = \left(\frac{KT}{G}\right)^{3/2} \rho^{-1/2} . \quad (8.1)$$

As a function of ρ and T, Table 2 gives M, roughly estimated. The real star masses lie in the region between 0°2 and 5° for $\rho = 10^{-23}$ and between 1° and 10° for $\rho = 10^{-21}$.

We do not attempt to estimate the temperature to which the clouds really can be cooled down. Hence two possibilities must be considered: either the clouds will reach temperatures as required—say, a few degrees—or they stay at a higher temperature.

TABLE 2

MASS OF SMALLEST CLOUDS

ρ (GM/CM^3)	T			
	100° K	10° K	1° K	0°1 K
10^{-23}......	10^{37} gm	$10^{35.5}$ gm	10^{34} gm	$10^{32.5}$ gm
10^{-21}......	10^{36} gm	$10^{34.5}$ gm	10^{33} gm	$10^{31.5}$ gm

In the first case the temperature reached seems to determine the average mass of the stars. In the second case a secondary effect must have been active, reducing the masses of the stars to the values observed today. Radiation pressure may have been such an effect, since we know that for the highest known star masses the radiation pressure at the surface is strong enough to blow away the upper layers of the atmosphere. This, however, would lead to the unpleasant result that the larger part of the cloud's mass would not be kept in the star but would become interstellar matter again. Since we assume that existing stars prohibit the formation of new stars, we would be left with more interstellar matter than is actually observed.

Observational facts seem to indicate a compromise. Empirically, we do not find appreciable amounts of interstellar matter in the regions of pure group 222, which always are regions of high density. If we assume that in regions both of high and of low density the same temperature of a few degrees was reached, this would lead to the formation of stars of about solar mass in the regions of high density but to stars of about ten times that mass in the regions of low density. Therefore, only in the regions of low density did radiation pressure have a good chance to return a part of the mass into interstellar space.

It should not be forgotten, however, that the hydrodynamical process connected with the loss of angular momentum also carries matter back into cosmic space. A more detailed theory will be needed to describe these processes.

IX. "YOUNG" STARS

The stars of group 000 either can have been formed as entirely new individuals or can be older stars which were rejuvenated by the accretion of interstellar matter.

The only possible mechanism which could lead to the recent formation of new stars seems to be the one proposed by Spitzer and Whipple.[4, 5] They assume the formation of clouds consisting mainly of dust which is brought together by a "quasi-gravitational" action of radiation pressure. We shall not try here to find out whether such conditions can exist. Every star, be it formed recently or at an earlier time, may have a chance of growing by the accretion process. We shall discuss this problem without deciding how the original star was formed.

Hoyle and Lyttleton[28] have proposed a mechanism for the accretion of interstellar matter by stars. We shall rediscuss the question under hydrodynamical aspects. Consider a star of mass M and a small volume of interstellar matter of density ρ, which, if following a straight path of inertial motion, would pass the star at a smallest distance r with the velocity v. The total duration of the encounter is roughly

$$\tau = \frac{r}{v}. \tag{9.1}$$

The force exerted on the gas by the star is

$$f = \frac{GM\rho}{r^2}. \tag{9.2}$$

The total momentum transferred by this force is approximately

$$p = f\tau = \frac{GM\rho}{rv}. \tag{9.3}$$

This momentum is equal to the original momentum ρv at a distance

$$r_0 = \frac{GM}{v^2}. \tag{9.4}$$

Particles hitting an area of the order of magnitude r_0^2 are therefore deflected by a large angle.

Now we introduce the ideas of hydrodynamics. In general, r_0 is larger than the mean free path of gas particles. Accordingly, we assume that all matter entering a sphere of radius r_0 will be maintained as a turbulent cloud surrounding the star. The turbulent friction causes a loss of kinetic energy, by which at least a part of the matter in this sphere is forced to stay in the gravitational field of the star, and so in the end to be united with the star. The total mass inside the sphere is roughly $r_0^3\rho$, and it is replenished in the time $\tau(r_0)$. Thus the increase of the star's mass per unit time is

$$\frac{\partial M}{\partial t} = \gamma r_0^3 \rho \frac{v}{r_0} = \gamma \rho r_0^2 v = \gamma \frac{G^2 M^2 \rho}{v^3}, \tag{9.5}$$

where γ is a numerical constant probably not much smaller than unity. This is the formula of Hoyle and Lyttleton. A high initial mass of the star and a small relative velocity between the star and the cloud favor a rapid increase of the mass. But both quantities must be close to the acceptable limits, in order to give an appreciable effect in "historic" times. The solution of equation (9.4) is

$$M = \frac{v^3}{\gamma G^2 \rho (t_1 - t)}. \tag{9.6}$$

Thus, for the finite time $t = t_1$, M would diverge. Before that moment several of our approximations (e.g., $\rho =$ Const., $v =$ Const.) would break down, but t_1 may be used to measure the time scale of the increase of the star's mass. If M_0 is the initial mass,

$$t_1 = \frac{v^3}{\gamma G^2 \rho M_0}. \tag{9.7}$$

[28] *Proc. Cambridge Phil. Soc.*, **35**, 405, 592, 1939.

The value of t_1 is given in Table 3 for $\rho = 10^{-23}$ and different M_0 and v. We see that only very small velocities, together with a mass about ten times that of the sun, give reasonably short time scales. In very dense clouds the times are somewhat shorter. But, in general, we would conclude that only a star which starts with a mass larger than that of the sun and which stays inside a very quiet cloud for more than 10^7 years has a chance of increasing its mass considerably. For $M = 2 \cdot 10^{34}$ gm, $\rho = 10^{-23}$ gm/cm^3, and $v = 10^4$ cm/sec, we get roughly $r_0 = 10^{19}$ cm = 3psc, a rather large feeding volume.

TABLE 3

TIME SCALE t_1 FOR THE INCREASE OF A STAR MASS FROM THE INITIAL VALUE M_0, ASSUMING $\rho = 10^{-23}$ GM/CM^3 AND DIFFERENT VALUES FOR v

v (CM/SEC)	M_0 (GM)		
	$2 \cdot 10^{32}$	$2 \cdot 10^{33}$	$2 \cdot 10^{34}$
10^4........	$3 \cdot 10^9$ years	$3 \cdot 10^8$ years	$3 \cdot 10^7$ years
$3 \cdot 10^4$......	10^{11} years	10^{10} years	10^9 years

X. ROTATION, PLANETARY SYSTEMS, AND DOUBLE STARS

If a star originates or is rejuvenated in one of the ways described above, it must rotate. If we assume $r_0 = 10^{18}$ cm and $v = 10^4$ cm/sec, a resultant average rotational velocity of the original cloud of 10^3 cm/sec may be a conservative estimate. By contraction to a final radius of 10^{11} cm, the conservation of angular momentum would lead to a rotational velocity of the star of 10^{10} cm/sec = 10^5 km/sec, while $5 \cdot 10^2$ km/sec is the limit of stability. Even if most of the angular momentum is carried away, the stars must originate rotating.

This mechanism provides a rotating disk of gas around an originating star as was supposed in the author's work on the origin of planets.[29] Yet still another mechanism for the formation of such a gaseous disk can have existed, connected with the loss of rotation by the stars. We shall try to describe this second mechanism without deciding in which phase of the evolution of the sun our solar system was built.

In discussing the rotation, we will consider only main-sequence stars. Giants, except for very early supergiants,[30] do not show any rotation. This is not very surprising. Their radii are so large that any large rotational velocity would destroy the star by its centrifugal force. White dwarfs, on the other hand, can uphold a high rotational velocity with a small angular momentum, comparable to that of our sun, and so we should not be surprised if we should, as a rule, find them rotating. For main-sequence stars, however, the distinction between rotating and nonrotating stars must be genetically significant. Visible rotation for them is empirically limited to the types which also for other reasons must be considered to be "young" stars. Not all stars of those types rotate, it is true. On the other hand, no stars of other types (except binaries) rotate with a measurable velocity.

We conclude that there must be a mechanism by which stars can lose their angular momentum. Ter Haar[31] has pointed out that the mechanism connected with the dissolu-

[29] *Zs. f. Ap.*, **22**, 319, 1943.

[30] This exception was mentioned to the author by Dr. Nassau.

[31] *Ap. J.*, **110**, 321, 1949.

tion of an original gas cloud around the sun will not suffice. Probably the magnetic coupling first considered by Alfvén[32] will be a working mechanism. Such a coupling would carry an ionized cloud around the sun like a rigid body. This can be true only up to a distance from the sun at which the period of a planetary orbit around the sun is equal to the sun's rotational period. Probably beyond that distance hydrodynamical transport mechanism must take its place. Calculations about this problem are now being carried out at the Max Planck Institut für Physik.

Lüst[15] has shown that there exist solutions of the hydrodynamical equations with turbulent friction in which a rotating disk of finite mass carries an infinite angular momentum from the center toward infinity. These solutions presuppose a boundary condition expressing a stationary transfer of angular momentum from the central body to the innermost part of the disk. This boundary condition may schematically describe the effect of Alfvén's mechanism. If the total angular momentum transferred is finite, the disk will not directly disappear into infinity but will be extended into a distance from the central body corresponding to the angular momentum transferred. From there on, the disk will develop and in the end dissolve, according to solutions without a momentum transfer at the center. The distance to which a disk is carried can be estimated as follows: The angular momentum of a body revolving around the sun at a distance r, according to Kepler's third law, is proportional to $r^{1/2}$. If the sun originally rotated with a velocity close to the stability limit and if the fraction f of the sun's mass had to carry this angular momentum from the sun's surface to a distance r, then r will be f^{-2} times the sun's radius. For $f = \frac{1}{10}$, this would be approximately the radius of the orbit of the earth. So the disk required by the theory of the origin of the planets can be formed in this way too.

Here I want to point out what other parts of my paper on the origin of the planets ought to be changed or modified in the light of new information. The regular system of vortices drawn there now seems to me to be too particular a description of a more general consequence of the theory of turbulence. Tuominen[33] has pointed out that Bode's law can formally be derived from Prandtl's rule that the average eddy size is proportional to the distance from the wall, the sun in this case being the wall. This, however, would in itself give only an average rule for the location of eddies and not a well-defined sequence of distances at which planets must originate. Such a sequence is given by Kuiper's new theory.[34] In this theory the gravitational action of the disk on itself is considered, and it is assumed that it forms "protoplanets," within which the later planet is built up while the protoplanet in the end is dissolved again.

The growth of planets out of small bodies which are themselves formed by chemical forces must take place inside the protoplanets as well, as was considered in the original picture. The absence of the rare gases on the earth is a clear indication of this "cold" formation of the earth. This fact and the later heating-up of the earth have been considered in detail by Urey and H. Brown.[35]

For the formation of double stars, two ways seem still to be open, and perhaps both of them are used: the immediate formation of two centers in an original rotating cloud or the growth of a planet or a protoplanet to the size of a full, stable companion. Whether double stars could ever be formed by the breakup of a single star seems very doubtful now, and I want explicitly to revoke my tentative hypothesis connecting supernovae with such an event.

[32] *Ark. f. mat., astr. och fysik*, Vol. **28A**, No. 6, 1942.

[33] *Ann. d'ap.*, **10**, 179, 1947.

[34] I am extremely grateful to Dr. Kuiper for the opportunity to see his paper before publication and for many discussions.

[35] H. Urey, to be published; H. Brown, *Ap. J.*, **111**, 641, 1950. I am very grateful to Drs. Urey and Brown for many interesting discussions.

XI. GIANTS AND WHITE DWARFS

That white dwarfs are stars which have exhausted their hydrogen now seems generally accepted. No satisfactory model, however, seems yet to have been given for giants in general. We shall use the name "giants" here for all stars right on and above the main sequence. There are giants in group 222. Hence giants can be old stars. It is therefore necessary to find a model for at least some giants that will provide an energy source which will outlast nuclear energy sources. The only sufficient source known in the physics of today is gravitation. Enough gravitational energy can be available only if the giant contains a highly condensed core.[36]

This idea puts the giants in one family with the white dwarfs. It would not exclude the possibility that some giants (especially subgiants) still produce energy by nuclear reactions outside an isothermal core.[37] But this certainly cannot be the theory of highly luminous old giants, because the nuclear energy would not suffice for them. The difference between giants and white dwarfs must then be that giants are above Chandrasekhar's[38] limiting mass and therefore have ample energy resources in their contraction toward a density comparable to the density in the atomic nucleus. The theory proposed here would assume that luminous giants are not in a final state but that their core is still contracting and thus liberating energy.

The main objection to this picture is that stationary models of such stars are always very small and dense, lacking the large atmosphere which would be seen as a giant star from outside. Yet such small stars cannot be stable at their surface if they are highly luminous. The radiation pressure must drive the atmosphere outward. If there are no stationary solutions for stars with such extended atmospheres, the real solutions must be nonstationary, with an atmosphere moving slowly but steadily outward (or containing other systematic motions). A planetary nebula would then be a special type of giant in which the atmosphere happens to be transparent to visible light. As far as I know, no calculations for such models have as yet been made.

It can be asked what determines the rate of release of gravitational energy from the contracting core. Perhaps it is given by the rate of loss of the angular momentum remaining in the core. It is not sure a priori how far a mass-luminosity relation can be expected for such models.

For the stars of group 111 this would lead to the conclusion that the B-type supergiants are a later stage of evolution than the B-type main-sequence stars. But, since evolution at such luminosities is rapid, they can still be young stars on an absolute-time scale.

[36] P. Jordan, *Die Herkunft der Sterne* (1947), pp. 38 ff.

[37] R. S. Richardson and M. Schwarzschild, *Ap. J.*, **108**, 373, 1948; G. Gamow and E. Teller, *Phys. Rev.*, **55**, 791, 1939; G. Gamow and G. Keller, *Rev. Mod. Phys.*, **17**, 125, 1945; G. Gamow, *Phys. Rev.*, **67**, 120, 1945; A. Reiz, *Ann d'ap.*, **10**, 301, 1947.

[38] *M.N.*, **95**, 207, 226, 676, 1935; *An Introduction to the Study of Stellar Structure* (Chicago: University of Chicago Press, 1939).