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THE ORBIT AND PARALLAX OF PROCYON

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ABSTRACT

The orbital motion and the parallax of Procyon have been determined from plates taken at the Yerkes and McCormick Observatories between 1915 and 1949. The parallax has also been determined from a comparison of the astrometric orbit with the radial-velocity variations derived from the measurements of eleven spectra taken with the coudé spectrograph of the 100-inch telescope between 1926 and 1943.

The photographic observations combined with the visual observations made by Aitken and Barnard between 1897 and 1913 give a period of 40.65 years and a semi-major axis of 4".55. With an absolute parallax of +0".287 and a mass ratio of 0.268, the individual masses of the system are 1.74⊙ and 0.63⊙.

I. INTRODUCTION

Procyon = α Canis Minoris = Schaeberle- = ADS 6251; $7^h36^m.7$, $+5°21'$ (1950), is a visual binary with a period of 40.65 years and a semi-major axis of 4".55, as derived in the present investigation. The combined magnitude on the International photovisual system is 0.45,[1] and the difference between the components is 10.3.[2] The spectrum of the brighter component is F5 IV in the Yerkes system,[3] and the color of the companion has been observed by Kuiper[4] as white.

The most recent investigation of this system was made by Spencer Jones,[5] who obtained the orbit of the bright component relative to the center of mass of the system from a discussion of the meridian observations made between 1755 and 1925 and from a series of high-dispersion spectra taken at the Cape Observatory between 1909 and 1924. Spencer Jones derived a mass for the brighter component which did not agree with the mass-luminosity relation. This deviation was due to the fact that he adopted too large a parallax for the system and found too small a value for the semi-major axis of the relative orbit from a discussion of the visual observations. His orbit did not fully represent the micrometer observations as indicated by the systematic increase with time in the difference between the observed and computed distances after 1902. The results of the present investigation, based mainly on the photographic observations, give a mass of the expected order of value for the brighter component, and the computed motion agrees closely with the micrometer observations of Aitken and Barnard.

[1] G. P. Kuiper, *Ap. J.*, **88**, 477, 1938.

[2] G. P. Kuiper, *ibid.*, **85**, 253, 1937.

[3] W. W. Morgan, unpublished data.

[4] The results kindly communicated in advance of publication.

[5] *M.N.*, **88**, 403, 1926.

II. THE YERKES PHOTOGRAPHIC OBSERVATIONS

Between 1916 and 1949, 104 plates on 66 nights were taken with the 40-inch Yerkes refractor. A special sector with two blades running at different speeds provided a reduction of approximately 10 mag. of the image of Procyon for the plates taken before 1948. This procedure was changed in 1948, after which the plates were taken with a sector giving a reduction of 6 mag., while the additional 4-mag. reduction was provided by the use of an objective grating giving a 4-mag. difference between central image and first-order spectra. As will be seen from the later discussion, the accuracy of the plate solution increased. The exposure time before 1932 was 5 minutes, and after 1939 about 1 minute. The measurements were made by the writer on the Gaertner long-screw measuring machine at the Yerkes Observatory in the direction of right ascension (x) and declination (y) for the approximate equator of the year 2000. Four reference stars of visual magnitudes ranging from 10.3 to 10.8 were used, while the image of Procyon was reduced to 10.2. The standard frame and dependences for the approximate epoch 1947.0 are given in Table 1. The spectra were furnished by Dr. A. N. Vyssotsky.

TABLE 1

REFERENCE STARS FOR YERKES PLATES

No.	AC+5°57	Sp.	Mag.	Diam. (Mm)	x_s (Mm)	y_s (Mm)	Dependences
1..........	130	A2	10.3	0.174	−50.76	− 0.90	0.180+0.00089 (t−1947.0)
2..........	132	A9	10.8	.127	−28.55	+64.16	.177− .00088
3..........	147	B1	10.7	.132	+14.41	−53.74	.298+ .00090
4..........	165	F2	10.5	.144	+64.90	− 9.52	0.345−0.00091
Procyon....	148	.....	10.2	0.192	+12.54	− 8.12	....

Table 2 contains the mean epoch, hour angle, parallax factors in right ascension and declination, elliptical rectangular co-ordinates, the solutions and their residuals in the same order, and the assigned weight, which depended upon the number of exposures and their quality, with allowance for an irreducible night error, when several plates for one night were combined. The elliptical rectangular co-ordinates were derived from the investigation of the orbit discussed in Section V. The scale factor used is 1 mm = 10″.655. The four epochs for 1950, each consisting of four plates, were taken after the completion of the investigation and were not used in the computations.

III. THE MCCORMICK PHOTOGRAPHIC MATERIAL

From a preliminary discussion of the Yerkes material it appeared that a satisfactory solution for the orbital motion could not be obtained, owing to the lack of observations between 1928–1930 and 1933–1938. Fortunately, these time intervals were covered by plates taken with the 26-inch refractor at the McCormick Observatory. Dr. H. L. Alden kindly furnished the measurements of these McCormick series of Procyon. He also took a series of new plates in 1948–1949 to strengthen the conversion of the McCormick solutions to the scale of the Yerkes solutions.

Between 1915 and 1949, ninety-seven plates were taken on fifty nights. The plates obtained in 1915–1916 were taken with a sector, reducing Procyon to a magnitude of 7.5, while on the later plates the star was reduced to a magnitude of 8.9 by means of a neutral filter and a sector. The measurements were made by Drs. C. M. Anderson (the plates taken between 1915–1928 and 1935–1942), P. van de Kamp (1929–1932), and Mr. T. Sully (1948–1949) on the Gaertner long-screw measuring machines at the McCormick

TABLE 2

OBSERVING DATA, MEASURED POSITIONS, AND RESIDUALS

Epoch	H.A. (Min.)	P_α	P_δ	$\bar{X}$	$\bar{Y}$	X (Mm)	Y (Mm)	O−C		Wt.
								x	y	
1916.247....	+79	−0.964	+0.095	−0.9049	−0.7911	+14.4782	−5.1143	+0″025	−0″029	1.0
.274....	+57	− .988	+ .144	−0.9018	− .7927	+14.4788	−5.1181	+ .059	− .059	1.0
.840....	−02	+ .933	− .230	−0.8369	− .8244	+14.4968	−5.1626	+ .130	+ .069	0.5
.883....	+03	+ .812	− .278	−0.8318	− .8267	+14.4794	−5.1816	+ .011	− .081	0.5
.902....	−21	+ .738	− .294	−0.8295	− .8277	+14.4915	−5.1741	+ .176	+ .019	0.3
17.167....	−05	− .737	− .064	−0.7976	− .8409	+14.4094	−5.1914	− .090	+ .009	1.0
.877....	−02	+ .833	− .273	−0.7076	− .8721	+14.4191	−5.2612	+ .088	− .053	0.5
.885....	+20	+ .805	− .280	−0.7066	− .8724	+14.4112	−5.2621	+ .018	− .054	1.2
.943....	−08	+ .549	− .311	−0.6989	− .8746	+14.3926	−5.2665	− .066	− .043	0.8
.962....	+07	+ .445	− .311	−0.6963	− .8753	+14.3840	−5.2456	− .115	+ .197	0.5
18.153....	−08	− .674	− .092	−0.6709	− .8822	+14.3464	−5.2726	− .064	+ .018	1.0
.183....	−23	− .799	− .033	−0.6670	− .8832	+14.3426	−5.2709	− .048	+ .048	0.7
.230....	00	− .933	+ .059	−0.6606	− .8848	+14.3395	−5.2780	− .009	− .012	1.7
.240....	+28	− .953	+ .081	−0.6592	− .8852	+14.3300	−5.2765	− .098	+ .007	1.2
.260....	+19	− .979	+ .117	−0.6566	− .8858	+14.3306	−5.2769	− .069	+ .011	1.2
.806....	−01	+ .978	− .178	−0.5809	− .9014	+14.3480	−5.3321	− .050	− .020	0.7
.868....	+09	+ .862	− .264	−0.5720	− .9028	+14.3602	−5.3301	+ .157	+ .077	0.7
.871....	+15	+ .853	− .267	−0.5716	− .9029	+14.3368	−5.3395	− .088	− .019	1.0
19.103....	−27	− .416	− .181	−0.5383	− .9077	+14.2975	−5.3523	+ .012	+ .025	1.0
.180....	+09	− .786	− .040	−0.5271	− .9091	+14.2879	−5.3601	+ .068	− .030	0.7
.188....	+12	− .816	− .024	−0.5259	− .9092	+14.2754	−5.3612	− .051	− .038	0.7
.865....	−20	+ .872	− .260	−0.4246	− .9162	+14.2751	−5.4235	− .068	− .051	1.0
.887....	−14	+ .800	− .282	−0.4213	− .9163	+14.2849	−5.4181	+ .071	+ .032	1.0
.898....	−21	+ .757	− .290	−0.4196	− .9163	+14.2759	−5.4263	− .005	− .044	1.0
20.050....	−06	− .588	− .125	−0.3961	− .9165	+14.2205	−5.4370	− .114	− .070	1.7
21.151....	−02	− .665	− .096	−0.2204	− .9016	+14.1673	−5.5153	+ .060	+ .042	0.7
.208....	−11	− .881	+ .017	−0.2112	− .9000	+14.1500	−5.5185	− .028	+ .028	1.4
.861....	−06	+ .883	− .256	−0.1028	− .8751	+14.1541	−5.5752	− .077	+ .062	0.5
.915....	−36	+ .682	− .301	−0.0938	− .8724	+14.1568	−5.5787	+ .042	+ .085	1.0
22.169....	−18	− .745	− .060	−0.0511	− .8589	+14.1131	−5.5857	+ .132	+ .167	0.5
.186....	+10	− .810	− .027	−0.0482	− .8579	+14.1045	−5.5930	+ .069	+ .095	1.0
.803....	−37	+ .980	− .174	+0.0559	− .8157	+14.1012	−5.6610	− .110	− .050	0.5
.849....	−18	+ .906	− .242	+0.0635	− .8121	+14.1179	−5.6500	+ .115	+ .126	1.0
23.152....	−01	− .671	− .093	+0.1144	− .7859	+14.0509	−5.6904	+ .017	− .079	1.4
24.160....	−18	− .704	− .079	+0.2781	− .6735	+14.0046	−5.7747	+ .072	− .079	1.4
27.101....	+76	− .416	− .184	+0.5917	− .1173	+13.8789	−6.0404	.000	− .030	1.0
31.191....	+01	− .836	− .012	+0.2543	+ .6930	+13.6792	−6.4726	+ .005	+ .038	1.0
.213....	+08	− .901	+ .031	+0.2507	+ .6959	+13.6720	−6.4751	− .042	+ .028	0.7
.229....	+02	− .938	+ .064	+0.2482	+ .6978	+13.6775	−6.4710	+ .037	+ .082	0.5
32.226....	+22	− .931	+ .058	+0.0845	+ .8017	+13.6135	−6.5962	− .026	− .051	1.7
.248....	+17	− .968	+ .099	+0.0809	+ .8035	+13.6156	−6.5812	+ .022	+ .124	0.5
39.810....	−31	+ .972	− .191	−0.9954	+ .7363	+13.1313	−7.4133	− .004	+ .030	1.0
.959....	+46	+ .450	− .311	−1.0102	+ .7261	+13.1063	−7.4338	− .003	+ .005	1.7
41.182....	−09	− .803	− .031	−1.1212	+ .6349	+12.9829	−7.5499	+ .034	+ .014	1.2
.212....	−30	− .898	+ .029	−1.1237	+ .6324	+12.9765	−7.5525	+ .018	+ .004	1.2
.217....	+05	− .912	+ .040	−1.1241	+ .6321	+12.9786	−7.5529	+ .047	+ .002	1.0
.220....	+14	− .918	+ .046	−1.1244	+ .6318	+12.9786	−7.5484	+ .051	+ .051	1.4
46.883....	−08	+ .806	− .280	−1.3934	+ .1053	+12.5825	−8.1110	+ .032	− .003	0.3
.943....	−31	+ .536	− .311	−1.3941	+ .0993	+12.5625	−8.1182	− .055	− .012	0.7
47.044....	−41	−0.069	−0.260	−1.3953	+0.0890	+12.5452	−8.1274	+0.014	−0.021	0.7

TABLE 2—*Continued*

Epoch	H.A. (Min.)	P_α	P_δ	$\overline{X}$	$\overline{Y}$	X (Mm)	Y (Mm)	O−C x	O−C y	Wt.
1947.159....	+04	−0.710	−0.076	−1.3964	+0.0774	+12.5131	−8.1332	−0″052	−0″018	2.0
.175....	−22	− .778	− .044	−1.3966	+ .0758	+12.5056	−8.1322	− .098	.000	2.0
.989....	+11	+ .274	− .307	−1.4000	− .0065	+12.4773	−8.2116	− .025	+ .030	2.0
48.129....	+18	− .568	− .135	−1.3997	− .0207	+12.4394	−8.2136	− .076	+ .099	2.0
.139....	−24	− .618	− .117	−1.3997	− .0217	+12.4574	−8.2344	+ .140	− .117	1.7
.199....	−36	− .861	+ .002	−1.3995	− .0278	+12.4398	−8.2316	+ .070	− .061	1.2
.215....	+38	− .907	+ .035	−1.3995	− .0294	+12.4258	−8.2260	− .053	+ .006	1.7
.229....	+56	− .938	+ .062	−1.3994	− .0307	+12.4298	−8.2175	+ .010	+ .103	1.7
.237....	+18	− .952	+ .078	−1.3994	− .0315	+12.4270	−8.2238	− .010	+ .039	2.0
.851....	+61	+ .902	− .248	−1.3948	− .0936	+12.4220	−8.3009	− .080	− .093	0.7
.879....	+46	+ .819	− .279	−1.3944	− .0964	+12.4248	−8.2992	− .003	− .039	2.0
.922....	+65	+ .638	− .308	−1.3939	− .1007	+12.4182	−8.3037	+ .013	− .038	2.9
.999....	+28	+ .212	− .301	−1.3930	− .1085	+12.3994	−8.3063	− .004	+ .009	2.8
49.002....	+28	+ .193	− .300	−1.3929	− .1088	+12.4010	−8.3096	+ .022	− .025	2.0
.135....	+14	− .602	− .122	−1.3911	− .1222	+12.3679	−8.3154	.000	− .004	3.0
.206....	+17	− .884	− .101	−1.3900	− .1293	+12.3565	−8.3186	+ .017	+ .025	2.7
50.017....	+41	+ .096	− .289	−1.3733	− .2105	+12.3208	−8.4030	+ .026	− .040	2.8
.056....	+17	− .144	− .249	−1.3723	− .2142	+12.3044	−8.4013	− .050	+ .004	2.3
.091....	+20	− .360	− .199	−1.3714	− .2176	+12.3061	−8.4062	+ .058	− .028	2.2
.198....	+36	−0.857	−0.001	−1.3685	−0.2283	+12.2813	−8.4120	+0.021	−0.040	2.6

TABLE 3

REFERENCE STARS FOR McCORMICK PLATES

No.	McC-7, 104	Sp.	Mag.	Diam. (Mm)	x_s (Mm)	y_s (Mm)	Dependences
1.........	26	M2	9.6	0.12	−49.48	+19.71	0.231+0.00020 (t−1930)
2.........	32a	M0	8.5	.19	−35.79	−31.01	.243+ .00097
3.........	60	K0	9.3	.14	+20.87	+55.15	.248− .00123
4.........	89	K0	8.1	0.23	+64.40	−43.85	0.278+0.00006
Procyon...	....	....	....	....	+ 2.95	− 1.50	....

Observatory in the direction of right ascension (x) and declination (y) for the approximate equator of the year 2000. Four reference stars were used, of visual magnitudes ranging from 8.1 to 9.6. The standard frame and dependences for the approximate epoch 1930.0 are given in Table 3. The spectra were furnished by Dr. A. N. Vyssotsky.

Table 4 lists the mean epoch, hour angle, parallax factors in right ascension and declination, the elliptical rectangular co-ordinates of the astrometric orbit discussed in Section V, the solutions in x and y, converted to the Yerkes system as explained in Section IV, the final residuals, and the weights which were assigned on the same basis as those for the Yerkes plates.

IV. THE REDUCTION OF THE McCORMICK SOLUTIONS TO THE YERKES SOLUTIONS

A direct reduction of the McCormick and the Yerkes solutions to the same system was not possible because the Yerkes comparison star No. 1 did not appear on the

TABLE 4

OBSERVING DATA, MEASURED POSITIONS, AND RESIDUALS

EPOCH	H.A. (MIN.)	P_α	P_δ	$\overline{X}$	$\overline{Y}$	X' (MM)	Y' (MM)	O−C		WT.
								x	y	
1915.167....	−03	−0.742	−0.064	−1.0175	−0.7209	+14.5817	−5.0150	+0″215	+0″132	1.0
.198....	+20	− .852	− .004	−1.0144	− .7231	+14.5626	−5.0283	+ .067	+ .003	1.0
.200....	+11	− .861	+ .002	−1.0143	− .7232	+14.5805	−5.0493	+ .261	− .219	1.0
.208....	+14	− .885	+ .018	−1.0134	− .7238	+14.5454	−5.0271	− .099	+ .019	1.0
.238....	+44	− .953	+ .078	−1.0105	− .7258	+14.5372	−5.0213	− .145	+ .093	1.0
.831....	+28	+ .947	− .220	−0.9500	− .7654	+14.5488	−5.0887	− .101	− .015	1.5
.853....	−22	+ .901	− .249	−0.9476	− .7669	+14.5547	−5.0909	− .011	− .012	1.7
.858....	−60	+ .887	− .255	−0.9472	− .7672	+14.5574	−5.0784	+ .027	+ .129	1.7
.907....	+10	+ .712	− .300	−0.9419	− .7703	+14.5311	−5.1049	− .167	− .097	1.4
16.194....	+50	− .842	− .010	−0.9107	− .7880	+14.4942	−5.1077	+ .091	+ .051	1.7
.216....	+22	− .905	+ .033	−0.9084	− .7892	+14.4691	−5.1154	− .142	− .022	1.0
.232....	+26	− .941	+ .066	−0.9066	− .7902	+14.4720	−5.1122	− .092	+ .018	1.7
.264....	+28	− .984	+ .129	−0.9030	− .7921	+14.5066	−5.1240	+ .315	− .097	0.7
23.239....	+09	− .953	+ .783	+0.1289	− .7778	+14.0336	−5.6969	− .070	− .087	0.7
.247....	+13	− .965	+ .094	+0.1302	− .7770	+14.0369	−5.6947	− .026	− .059	1.0
.260....	+48	− .980	+ .120	+0.1322	− .7759	+14.0406	−5.6855	+ .025	+ .039	0.7
27.986....	+30	+ .293	− .308	+0.5950	+ .0909	+13.8707	−6.1426	+ .063	− .098	1.7
28.035....	+12	− .012	− .273	+0.5937	+ .1023	+13.8485	−6.1264	− .067	+ .120	1.4
29.217....	+26	− .910	+ .038	+0.5174	+ .3645	+13.7866	−6.2568	+ .068	.000	2.2
.943....	−09	+ .533	− .314	+0.4353	+ .5038	+13.7847	−6.3427	+ .013	+ .036	2.1
.954....	+19	+ .475	− .314	+0.4339	+ .5058	+13.7795	−6.3450	− .021	+ .023	1.7
30.224....	+33	− .927	+ .053	+0.3981	+ .5521	+13.7298	−6.3775	− .012	− .097	1.7
.271....	+42	− .988	+ .142	+0.3917	+ .5598	+13.7244	−6.3716	− .027	− .001	1.7
31.153....	+43	− .684	− .090	+0.2602	+ .6883	+13.6874	−6.4747	− .003	+ .021	1.7
.800....	−14	+ .980	− .173	+0.1554	+ .7621	+13.6978	−6.5508	+ .036	+ .012	2.2
.893....	−20	+ .769	− .290	+0.1401	+ .7713	+13.6860	−6.5670	+ .028	− .017	1.7
32.130....	+09	− .576	− .132	+0.1006	+ .7934	+13.6282	−6.5840	− .062	+ .046	1.7
.204....	+10	− .876	+ .012	+0.0883	+ .7998	+13.6224	−6.5886	+ .009	+ .047	1.7
35.898....	−03	+ .746	− .294	−0.5092	+ .9110	+13.4068	−7.0063	− .119	+ .035	1.4
37.220....	+72	− .918	+ .044	−0.6941	+ .8760	+13.2823	−7.1504	+ .039	− .080	2.2
.924....	+38	+ .629	− .309	−0.7841	+ .8462	+13.2736	−7.2164	+ .072	+ .103	1.3
38.888....	+05	+ .786	− .287	−0.8977	+ .7949	+13.2007	−7.3277	+ .016	− .016	1.7
39.134....	+45	− .594	− .126	−0.9248	+ .7801	+13.1410	−7.3484	− .036	− .010	2.9
.183....	+46	− .808	− .030	−0.9301	+ .7771	+13.1271	−7.3544	− .086	− .045	2.1
.896....	+12	+ .756	− .293	−1.0040	+ .7305	+13.1204	−7.4280	− .018	+ .023	1.7
.909....	−21	+ .700	− .301	−1.0053	+ .7295	+13.1148	−7.4283	− .051	+ .036	1.7
40.193....	+18	− .843	− .009	−1.0329	+ .7096	+13.0478	−7.4586	− .101	− .055	1.3
41.905....	+05	+ .717	− .299	−1.1781	+ .5757	+12.9684	−7.6322	+ .017	+ .011	1.5
.962....	−08	+ .427	− .314	−1.1822	+ .5710	+12.9656	−7.6468	+ .116	− .080	1.7
42.034....	+70	− .005	− .274	−1.1875	+ .5648	+12.9426	−7.6453	+ .050	+ .001	1.7
.036....	−06	− .022	− .271	−1.1877	+ .5647	+12.9443	−7.6554	+ .076	− .104	2.2
.066....	−30	− .210	− .236	−1.1898	+ .5622	+12.9388	−7.6478	+ .094	− .001	1.7
.080....	+03	− .294	− .216	−1.1908	+ .5609	+12.9236	−7.6544	− .033	− .062	1.7
48.182....	−01	− .806	− .032	−1.3996	− .0261	+12.4370	−8.2217	− .020	+ .068	2.0
.229....	+07	− .937	+ .062	−1.3994	− .0307	+12.4310	−8.2255	− .009	+ .049	2.1
.876....	−09	+ .829	− .276	−1.3945	− .0961	+12.4220	−8.2947	− .066	+ .034	1.5
.895....	−20	+ .758	− .292	−1.3943	− .0980	+12.4279	−8.2974	+ .033	+ .029	2.4
49.141....	+10	− .629	− .113	−1.3910	− .1228	+12.3675	−8.3157	− .021	+ .026	1.3
.198....	−36	− .858	.000	−1.3901	− .1285	+12.3639	−8.3264	+ .051	− .064	2.0
.214....	+16	−0.905	+0.033	−1.3898	−0.1302	+12.3488	−8.3192	−0.082	+0.021	1.9

McCormick plates, owing to its position under the sector, and the McCormick comparison stars Nos. 3 and 4 were outside the usable field of the Yerkes plates. Since the two series of plates were exposed to different limiting magnitudes, there appeared no possibility of choosing a common set of comparison stars.

In order to obtain the necessary scale and orientation factors for converting the McCormick solutions to the Yerkes solutions, the co-ordinates of the Yerkes comparison stars Nos. 2, 3, and 4 were measured on the McCormick standard plate No. 26687 (1929.943), and the co-ordinates for the McCormick comparison stars Nos. 1 and 2 were measured on the Yerkes standard plates Nos. 12673–12674 (1947.175).

From these measurements the following equations were obtained:

$$X' = +1.9531X - 0.0014Y\,, \qquad Y' = +0.0028X + 1.9498Y\,,$$

where X' and Y' are the values of the X, Y solutions on the McCormick plates reduced to the orientation and the scale of the Yerkes plates. The above values for the scale factors agree well enough with the value 1.947 obtained from the ratio of the scale values normally used for the McCormick (1 mm = 20″.748) and the Yerkes plates (1 mm = 10″.655).

In order to check the factors derived above and to find the zero-point corrections as well as the correction for differential proper motion between the two reference systems, normal places were formed from plates taken at both observatories at nearly the same epoch. Both series were corrected for a parallax of 0″.293 (Yerkes 0.0275 mm; McCormick 0.0141 mm). An additional correction for proper motion was applied to the Yerkes results to bring them to the epoch of the McCormick plates. These corrections were: -0.0628 in x and -0.0941 in y in units of millimeters per year. Additional small corrections were applied for the orbital motion as derived from the provisional orbit.

Table 5 lists the results for the Yerkes normal places in the following order: mean epoch, number of plates, X and Y in millimeters corrected for a parallax of 0″.293 (0.0275 mm), X and Y in the fifth and sixth columns corrected for proper motion, and orbital motion to the epoch given in the last column.

Table 6 lists the results for the McCormick normal places in the following order: mean epoch, number of plates, X and Y in millimeters corrected for a parallax of 0″.293 (0.0141 mm), X' and Y' in millimeters, using the above conversion factor for orientation and scale. The next two columns give the difference between the Yerkes and the McCormick solutions, while the following two columns give the differences in x and y for the individual normal place as compared with the mean. The agreement in the x co-ordinate is extremely close, indicating no differential proper motion between the two systems and giving a p.e. of the individual normal place of ± 0.0011 mm. The differences in the Y column show the presence of differential proper motion between the systems. A further study of these residuals shows that a term linear with time fails to bring the necessary agreement between the two series of observations and that the introduction of an additional term quadratic with time is necessary to bring the two systems into a close agreement in the y co-ordinate. The residuals from this conversion are given in the last column of Table 6 and indicate a p.e. of the individual normal place equal to ± 0.0011 mm. The factors for reducing the McCormick solutions (X, Y) to those of the Yerkes (X', Y') are thus

$$X' = +1.9531X - 0.0014Y + 13.8542\,,$$

$$Y' = +0.0028X + 1.9498Y + 0.00100\,(E - 1930) + 0.000061\,(E - 1930)^2 - 6.3943\,,$$

where E is the epoch of the observations.

The acceleration term at present cannot definitely be attributed to the McCormick series. However, the computations which follow for the orbital motion show preference for the application of this term to the McCormick series. To explain this term solely as a spurious acceleration term which depends upon the product of yearly change of the dependences of the reference stars due to the proper motion of the central star and upon the proper motion of the reference stars will necessitate a proper motion of 0.05 mm per year for either McCormick reference stars No. 2 and No. 3, both having the largest yearly changes in their dependences ($\Delta D_2 = +0.0010$; $\Delta D_3 = -0.0012$), or a proper motion of half this amount for each of the stars in opposite directions. It seems unlikely that the proper motions are as large as indicated by this effect.

TABLE 5

YERKES NORMAL PLACES FOR SCALE

Epoch	No. of Plates	X (Mm)	Y (Mm)	X (Mm)	Y (Mm)	Epoch
1916.260...	2	+14.5053	−5.1195	+14.5069	−5.1175	1916.236
23.152...	2	+14.0693	−5.6878	+14.0650	−5.6956	23.249
32.233...	3	+13.6402	−6.5932	+13.6438	−6.5859	32.167
39.884...	2	+13.0992	−7.4166	+13.0979	−7.4184	39.902
48.191...	12	+12.4605	−8.2240	+12.4594	−8.2254	48.206
48.895...	9	+12.4012	−8.2933	+12.4016	−8.2927	48.889
49.170...	8	+12.3826	−8.3166	+12.3809	−8.3187	49.193

TABLE 6

MCCORMICK NORMAL PLACES FOR SCALE

Epoch	No. of Plates	X (Mm)	Y (Mm)	X′ (Mm)	Y′ (Mm)	Yerkes−McCormick		O−C		
						x (Mm)	y (Mm)	x (Mm)	y (Mm)	y (Mm)
1916.236	4	+0.3342	+0.6553	+0.6515	+1.2786	+13.8554	−6.3961	+0.0012	−0.0207	+0.0004
23.249	3	+ .1075	+0.3605	+0.2092	+0.7032	+13.8558	−6.3988	+ .0016	− .0232	− .0003
32.167	4	− .1070	−0.0988	−0.2091	−0.1929	+13.8529	−6.3930	− .0013	− .0176	− .0012
39.902	4	− .3878	−0.5337	−0.7566	−1.0417	+13.8545	−6.3767	+ .0003	− .0013	+ .0017
48.206	4	− .7157	−0.9572	−1.3966	−1.8683	+13.8560	−6.3571	+ .0018	+ .0183	− .0012
48.889	3	− .7430	−0.9912	−1.4496	−1.9371	+13.8512	−6.3556	− .0030	+ .0198	− .0020
49.193	5	−0.7547	−1.0085	−1.4729	−1.9685	+13.8538	−6.3502	−0.0004	+0.0252	+0.0024

The acceleration can also be attributed to an error in the orientation of the plates relative to the optical axis of the telescope. This would give rise to a similar effect, owing to the change with time in the position of Procyon relative to the comparison stars and the center of the plate.

V. THE DETERMINATION OF THE ASTROMETRIC ORBIT

The X and Y values for the Yerkes (Table 2, seventh and eighth cols.) and the McCormick nightly means (Table 4, seventh and eighth cols.) were corrected for a parallax of 0″.293 (0.0275 mm) and 0″.291 (0.0273 mm), respectively, assuming an absolute parallax of 0″.295 for both systems. In addition, the nightly means were reduced to the epoch of 1930.0, adopting the provisional proper motions:

$$\mu_x = -0.06630 \text{ mm}, \qquad \mu_y = -0.09529 \text{ mm}.$$

The nightly means combined into seasonal means in X and Y were plotted against time in order to derive the time of the periastron passage and the eccentricity from the projected displacement-curves, following the same procedure as that first outlined by K. Schwarzschild for spectroscopic binaries.[6] Normally, the displacement-curves in the two co-ordinates serve as an independent check on the derived values. In this case, however, it was found that the points on the $\Delta\delta$-curve for periastron and apastron fell close to the maximum and minimum displacements, thus making the time of periastron almost indeterminate from this co-ordinate. The periastron was determined as $T = 1927.5$ from the displacement-curve in $\Delta\alpha \cos\delta$. The value of the eccentricity determined from the ratio of the projected distances, center focus, and center periastron in the two displacement-curves was derived to be: $e = 0.39$. Prior to the determination of these values for T and e, the period 40.75 years was found by comparing the position angles observed between 1897 and 1909 with those derived from the displacement-curves.

In order to improve the dynamical elements derived from the graphical method, a series of seventeen orbits was computed. For each orbit a set of dynamical elements was adopted, and the orientational elements were computed from a least-squares solution, using the sum of the squares of the residuals as a criterion for the fitness of the orbit. In this investigation the period was varied between 39.5 and 41.0 years, the eccentricity between 0.32 and 0.44, and the time of periastron between 1927.0 and 1927.7.

This investigation indicated that the following orbital elements gave the closest agreement with the photographic observations, as well as with the visual position angles observed by Aitken and Barnard between 1897 and 1909:

$$P = 40.65\,, \qquad T = 1927.60\,, \qquad e = 0.40\,.$$

The corresponding values for the orientational elements are:

$$A = +0.0901 \text{ mm}\,, \qquad B = +0.0232 \text{ mm}\,,$$

$$F = -0.0287 \text{ mm}\,, \qquad G = +0.1109 \text{ mm}\,.$$

The co-ordinates for the center of gravity were: $c_x = +13.6998$ mm, $c_y = -6.3669$ mm. A change of 0.1 year in P and T and a change of 0.01 in the value of e from the above value showed definite increases in the residuals of the photographic observations.

Since the above elements were derived after assuming the parallax and the proper motion of the system and with equal weights assigned to each normal place regardless of the number of observations it contained, a definitive solution was made in both co-ordinates in which the final corrections were derived for the adopted parallax and proper motion and for the orientation elements, together with the probable errors. Because the photographic observations did not cover a complete period, a correction for this value could not be obtained, and no attempt was made to correct either the eccentricity or the time of periastron passage.

The residuals Δx and Δy between the measured co-ordinates and the provisional values for the parallax, proper motion, and orbital motion give the following equations of condition for the zero epoch 1930.0:

$$\Delta x = \Delta c_x + \Delta c_x' + \Delta B\overline{X} + \Delta G\overline{Y} + \Delta\pi P_\alpha + \Delta\mu_x t\,,$$

$$\Delta y = \Delta c_y + \Delta c_y' + \Delta A\overline{X} + \Delta F\overline{Y} + \Delta\pi P_\delta + \Delta\mu_y t\,,$$

where Δc_x, Δc_y, and $\Delta c_x'$, and $\Delta c_y'$ are the corrections to the zero point for the Yerkes and the McCormick plates; $\Delta\mu_x$ and $\Delta\mu_y$ the component corrections in proper motion; t the time of observations relative to 1930 in units of 10 years; $\Delta\pi$ the correction to the pro-

[6] *A.N.*, **152**, 65, 1900.

visional parallax, P_α; P_δ the parallax factor; ΔA, ΔB, ΔF, and ΔG the corrections to the orientational elements; and $\bar{X}$ and $\bar{Y}$ the elliptical rectangular co-ordinates as derived from the dynamical elements given above.

The normal equations are

$$\begin{array}{rrrrrrl}
+79.00\Delta c_x + & 0.00\Delta c'_x - & 68.62\Delta B - & 21.08\Delta G - & 21.07\Delta\pi + & 37.94\Delta\mu_x = & -0.1228\,\text{mm} \\
 & +80.30 & -53.07 & +15.17 & -12.16 & +27.62 & +0.0699 \\
 & & +156.76 & +8.13 & +25.87 & -119.61 & +0.0269 \\
 & & & +62.49 & +1.56 & +68.04 & +0.0322 \\
 & & & & +92.33 & -17.96 & -0.0687 \\
 & & & & & +269.77 & -0.0212\,,
\end{array}$$

$$\begin{array}{rrrrrrl}
+79.00\Delta c_y + & 0.00\Delta c'_y - & 68.63\Delta A - & 21.08\Delta F - & 9.57\Delta\pi + & 37.94\Delta\mu_y = & +0.0743\,\text{mm} \\
 & +80.30 & -53.07 & +15.17 & -10.64 & +27.62 & -0.1119 \\
 & & +156.76 & +8.13 & +16.71 & -119.62 & +0.0281 \\
 & & & +62.49 & -0.78 & +68.04 & -0.0302 \\
 & & & & +5.99 & -12.49 & -0.0015 \\
 & & & & & +269.77 & -0.0385\,.
\end{array}$$

The separate solutions in Δx and Δy for the combined Yerkes and McCormick plates lead to the following values expressed in millimeters:

$$\begin{array}{llll}
\Delta c_x = -0.0024\,, & & \Delta A = -0.0002\,, \\
\Delta c_y = +0.0010\,, & & \Delta B = -0.0007\,, \\
\Delta c'_x = +0.0003\,, & & \Delta F = +0.0008\,, \\
\Delta c'_y = -0.0018\,, & & \Delta G = -0.0002\,, \\
\Delta\pi_x = -0.0011\,, & & \Delta\mu_x = -0.00009\,, \\
\Delta\pi_y = -0.0023\,, & & \Delta\mu_y = -0.00046\,.
\end{array}$$

After computing their probable errors, these corrections were applied to the provisional values, leading to the following results:

$$\begin{array}{lll}
c_x = +13.6974 \pm 0.0010\text{ mm}\,, & A = +0''958 \pm 0''009\,, & \\
c_y = -\ 6.3659 \pm 0.0008\text{ mm}\,, & B = +0''240 \pm 0''011\,, & \\
c'_x = +13.7001 \pm 0.0009\text{ mm}\,, & F = -0''298 \pm 0''010\,, & \\
c'_y = -\ 6.3687 \pm 0.0008\text{ mm}\,, & G = +1''180 \pm 0''013\,, & \\
\pi_x = +\ 0''282 \pm 0''007\,, & \mu_x = -0''7065 \pm 0''0007 & \multirow{2}{*}{\}\text{ Yerkes}\,,} \\
\pi_y = +\ 0''268 \pm 0''026\,, & \mu_y = -1''0158 \pm 0''0006 & \\
\text{p.e. } 1_x = \pm 0''062\,, & \mu_x = -0''7065 \pm 0''0007 & \multirow{2}{*}{\}\text{ McCormick}\,.} \\
\text{p.e. } 1_y = \pm 0''047\,, & \mu_y = -1''0265 \pm 0''0006 &
\end{array}$$

The proper motion of Procyon according to the FK_3 is: $\mu\alpha \cos\delta = -0\overset{''}{.}712$, $\mu\delta = -1\overset{''}{.}021$, which shows the following dependence–mean proper motion for the reference stars:

$$\mu_x = -0\overset{''}{.}005\ ; \qquad \mu_y \text{ (Yerkes)} = -0\overset{''}{.}005\ ; \qquad \mu_y \text{ (McCormick)} = +0\overset{''}{.}006\ .$$

The FK_3 proper motions for the center of mass of the system were derived from the observations of the bright component reduced by means of the orbit published by Spencer Jones.

The probable error of unit weight is especially large in the x co-ordinate as compared with the average value of $\pm 0\overset{''}{.}035$ normally obtained for other series at both observatories; one likely explanation would be the large reduction of the bright star by means of a rotation sector or a neutral filter. The last eleven nightly means obtained at Yerkes, where the reduction of the bright star was obtained by means of a sector and an objective grating, reduced the probable errors of unit weight to $\pm 0\overset{''}{.}031$ and $\pm 0\overset{''}{.}028$ for $\Delta\alpha \cos\delta$ and $\Delta\delta$, respectively, thus showing the superior accuracy obtained by this method.

The yearly normal places derived from the Yerkes and the McCormick plates, corrected for the final parallax and proper motions, have been plotted against the orbital motion in Figures 1 and 2.

VI. SPECTROSCOPIC OBSERVATIONS

Dr. W. S. Adams kindly called the author's attention to a series of spectrograms taken at the coudé focus of the 100-inch Mount Wilson telescope. These plates were lent to the author by Dr. Bowen, and a series of seven prism and four grating spectrograms taken between 1926 and 1943 were found suitable for measurement. While the material was insufficient to establish the orbital motion in the line of sight, it was used for the derivation of the parallax of the system, by adopting the orbital motion derived from the astrometric orbit. The plates prior to 1937 were prism spectrograms, while the subsequent ones were taken with Wood diffraction gratings, as described in detail by Adams.[7]

The spectrograms were measured on the small Gaertner machine of the Dearborn Observatory by Dr. (Mrs.) Helen Steel Lillibridge, assisted by Mr. Harry Rymer. The region $\lambda\lambda$ 4175–4272 was chosen for the radial velocities. On each plate thirty-three stellar lines were measured, primarily *Fe* I, chosen for their freedom of blends, medium intensity, and lack of diffuseness, while thirty-one lines in the iron comparison spectrum were measured. The lines were identified by Mrs. Lillibridge. The values for the wave lengths of the lines were the International secondary standards of iron[8] and those published by Burns and Walters,[9] increased by 0.002 A to adjust the vacuum-arc values to the wave lengths observed in air. Each plate was measured in direct and reverse position, with three settings on each line.

In the reduction of readings to wave lengths, the region was divided into two parts at λ 4219, and a linear relation was set up between readings and wave lengths at the ends of the region. The departures of the computed intermediate wave lengths of iron lines from the standard values were then plotted against computed wave length. From the resulting curve, corrections were obtained to the computed wave lengths of the stellar lines. Then $\Delta\lambda/\lambda$, averaged for the thirty-three lines and multiplied by the velocity of light, gave the observed radial velocity.

Table 7 lists the plate number; the epoch; the observed radial velocity, v_o, with its probable error; the average dispersion in the region of measurements; the combined corrections for the heliocentric, diurnal, and barycentric motions of the earth, v_e; and the velocity, v, of Procyon A relative to the sun.

[7] *Ap. J.*, **93**, 11, 1941.

[8] *Trans. I.A.U.*, **3**, 86, 1929.

[9] *Pub. Allegheny Obs.*, **6**, 159, 1929.

We derive the motion in the line of sight in kilometers per second from the orbital elements of the final orbit after adopting a provisional parallax for the system. A comparison of these velocities with the observed velocities will then give the corrections to the assumed parallax and the velocity of the center of the system (v_c). A least-squares solution, based on each observed velocity having unit weight, gave the following values:

$$\pi = +0\overset{''}{.}316 \pm 0\overset{''}{.}013 \text{ (p.e.)}, \qquad v_c = -4.11 \pm 0.05 \text{ km/sec (p.e.)}.$$

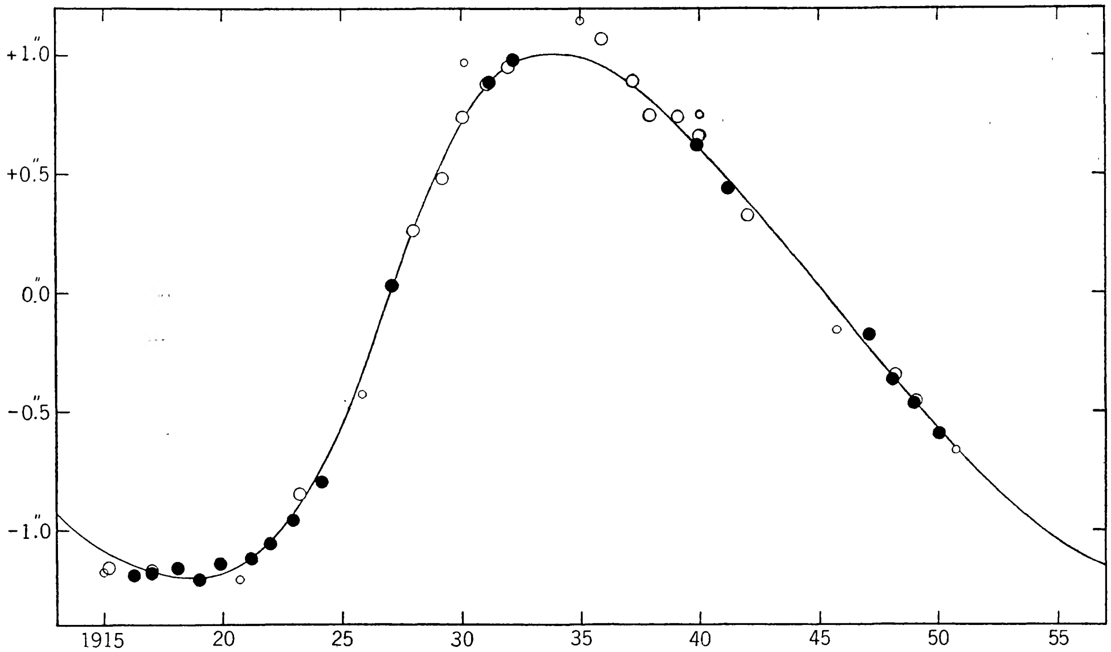

FIG. 1.—The orbital motion in $\Delta\alpha \cos \delta$. The yearly normal places are marked with a dot for the Yerkes plates and with a circle for the McCormick plates. The small circles are the normal places derived from the meridian observations.

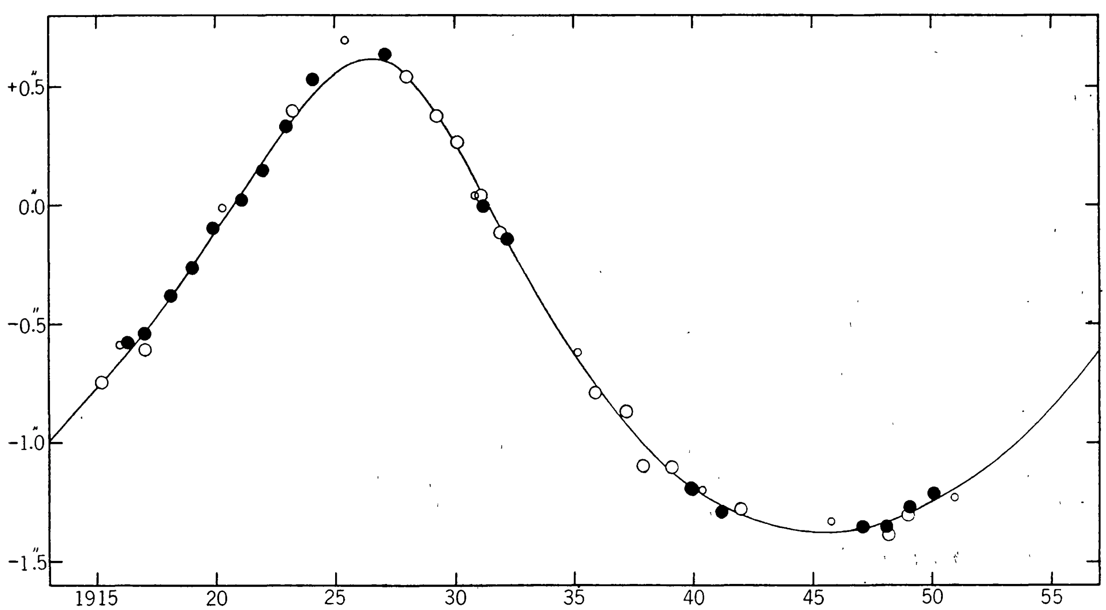

FIG. 2.—The orbital motion in $\Delta\delta$. The symbols are identical with those in Figure 1

The residuals between the observed and computed radial velocities are listed in the last column of Table 7. The probable error for one plate is ±0.09 km/sec.

A similar solution for the radial velocities obtained by Spencer Jones[10] from high-dispersion plates (8.4 A/mm), taken at the Cape Observatory, gave the following results:

$$\pi = +0\overset{''}{.}294 \pm 0\overset{''}{.}008 \text{ (p.e.)}, \qquad v_c = -4.17 \pm 0.14 \text{ km/sec (p.e.)}.$$

The smaller probable error, ±0.05 km/sec, for each Cape normal place as compared with the Mount Wilson plates results from the large number of plates (119) used by Spencer Jones for his seven normal places. However, the probable error of the Cape velocity of the system is larger than that derived from the Mount Wilson plates. This is due to Spencer Jones's method of measuring all plates differentially against the same standard plate, thus making v_c dependent upon a single plate.

The combined results from the radial velocities are: $\pi = +0\overset{''}{.}300 \pm 0\overset{''}{.}007$ (p.e.) and $v_c = -4.12 \pm 0.05$ km/sec (p.e.). The latter value is at variance with the value −3.69 km/sec derived by Spencer Jones[10] and −3.2 km/sec listed as the result of 114 spectra taken at the Lick Observatory between 1897 and 1926.[11, 12] The last series, reduced for the orbital motion and final parallax derived in the present investigation, gives $v_c = -3.50 \pm 0.08$ km/sec (p.e.).

TABLE 7

RADIAL VELOCITIES FROM MOUNT WILSON PLATES

Plate No.	Epoch	v_o p.e. (Km/Sec)	Disp. (A/Mm)	v_e (Km/Sec)	v (Km/Sec)	O−C (Km/Sec)
51.....	1926.878	−28.56±0.18	2.6	+24.95	−3.61	0.00
136.....	29.002	−12.16 .10	2.6	+ 7.04	−5.12	− .09
221.....	29.870	−30.99 .19	4.3	+25.53	−5.46	− .03
720.....	33.761	−33.98 .08	2.6	+28.17	−5.81	+ .06
739.....	33.840	−33.63 .12	2.6	+27.76	−5.87	.00
740.....	33.840	−33.71 .07	2.6	+27.64	−6.07	− .20
741.....	33.840	−33.31 .10	2.6	+27.56	−5.75	+ .12
1595.....	37.974	−17.04 .06	3.1	+11.84	−5.20	+ .22
1626.....	38.201	+19.30 .08	3.1	−24.68	−5.38	+ .01
1824.....	38.853	−32.21 .06	3.1	+26.73	−5.48	− .17
3309.....	43.934	−22.68±0.08	2.9	+18.12	−4.56	+0.07

VII. THE MERIDIAN OBSERVATIONS

In 1873 Auwers[13] published his second investigation of the orbital motion of Procyon based upon the extended series of meridian observations up to the year 1870 which he had collected and discussed. Using the same method as that first adopted by Bessel[14] for showing the variable proper motion of this star, Auwers compared the right ascension of Procyon, derived from a certain series of observations with the tabular right ascension as given by Wolfers' *Tabulae reductionum*. Next, the right ascensions of six stars—α Tauri, β Orionis, α Orionis, β Geminorum, α Hydrae, and α Leonis—were compared with their corresponding tabular positions in Wolfers' tables. The excess of the first difference over the mean of the second difference is the quantity q, showing the orbital motion of Procyon in right ascension. The mean position of the six reference stars differs by 28 minutes

[10] *Op. cit.*, pp. 424–427.

[11] *Pub. Lick Obs.*, **16**, 116, 1928.

[12] *Pub. Lick Obs.*, **18**, 66, 1932.

[13] *Monatsberichte d. Akad. Wiss. Berlin*, May, 1873.

[14] *A.N.*, **22**, 151, 1844.

in right ascension and 30′ in declination from the co-ordinates of Procyon. It can therefore be expected that any systematic errors in right ascension depending upon right ascension and declination have practically been eliminated.

For the comparison in declination, Auwers did not find it possible to choose a series of fundamental stars whose mean declination differed little from that of Procyon without having their mean right ascension differ considerably from that of Procyon. The seven comparison stars had a mean position differing 9′ in declination from Procyon's position, while the difference in right ascension equaled 6^h46^m. The observed variation, r, in the declination of Procyon is therefore affected by the systematic error in declination depending upon right ascension in the individual series but is practically independent of errors in the adopted latitude and refraction.

Spencer Jones[5] added to Auwers' lists the observations made since 1870, incorporated earlier observations which had not already been used by Auwers, and revised such values published by him for series of early observations where new reductions had led to improved values. As a basis throughout, Spencer Jones adopted the positions and proper motions from the *P.G.C.*[15] The values for q and r taken from Auwers' investigations were accordingly corrected for the differences in positions and proper motions of the stars between Wolfers' *Tabulae reductionum* and the *P.G.C.*

Spencer Jones also pointed out that the derived variations, q and r, were affected by errors due to the use of different values of the constants of nutation and aberration at the different times in the reductions of the apparent places of the stars to their mean places. While the effect of aberration could safely be neglected by taking the means of observations at different times of the year, Spencer Jones found it necessary to apply a correction to the mean value of r resulting from a series of observations to reduce the effect of nutation to the basis of the value $9\overset{''}{.}210$.

The values of q and r as found in Tables I and II of Spencer Jones's article can be expressed as follows:

$$q = c_x + \mu_x (t - 1850) + B'\overline{X} + G'\overline{Y} ,$$

$$r = c_y + \mu_y (t - 1850) + A\overline{X} + F\overline{Y} ,$$

where c_x and c_y are the corrections to the positions in the *P.G.C.*; μ_x and μ_y the component corrections to the proper motion (counted from the epoch 1850); and A, B', F, and G' the orientational elements ($B' = B\,[\sec\delta/15]$; $G' = G\,[\sec\delta/15]$) of the final orbit, reduced to the equator of 1850.0; X and Y are the elliptical rectangular co-ordinates as derived from the Johannesburg tables from the mean anomaly and the eccentricity.

In view of the expected accuracy of the meridian observations, these would add little to the solutions derived from the photographic observations; therefore, no attempt was made to obtain the orbital elements from the above equations. Instead, the terms depending upon the orbital motion were taken from the orbit derived in Section V, and least-squares solutions for c_x, c_y, μ_x, and μ_y were made to find the general agreement of the adopted orbit with the meridian observations.

The obtained solutions were:

$$c_x = +0\overset{s}{.}0014 \pm 0\overset{s}{.}0011 \text{ (p.e.)} , \qquad c_y = +0\overset{''}{.}010 \pm 0\overset{''}{.}026 \text{ (p.e.)} ,$$

$$\mu_x = -0\overset{s}{.}00022 \pm 0\overset{s}{.}00003 \text{ (p.e.)} , \qquad \mu_y = -0\overset{''}{.}0021 \pm 0\overset{''}{.}0007 \text{ (p.e.)} ,$$

$$\text{p.e. } 1 = \pm 0\overset{s}{.}023\ (0\overset{''}{.}34) , \qquad \text{p.e. } 1 = \pm 0\overset{''}{.}39 .$$

These corrections to the proper motion applied to the *P.G.C.* give the proper motion $\mu\alpha \cos\delta = -0\overset{s}{.}0471$ and $\mu\delta = -1\overset{''}{.}029$ against $\mu\alpha \cos\delta = -0\overset{s}{.}0473$ and $\mu\delta = -1\overset{''}{.}032$

[15] Lewis Boss, *Preliminary General Catalogue of 6188 stars for the Epoch 1900* (Washington: Carnegie Institution of Washington, 1910).

from the *G.C.*;[16] and $\mu\alpha \cos \delta = -0\overset{s}{.}0474$, $\mu\delta = -1\overset{''}{.}029$ from the FK_3 (all computed for the equinox 1950).

Since the corresponding probable errors of unit weight as derived from the residuals listed by Spencer Jones are $\pm 0\overset{s}{.}021$ ($0\overset{''}{.}32$) and $\pm 0\overset{''}{.}41$, it is seen that both orbits fit the meridian observations equally well. The derived probable errors of unit weight may be compared with the value $\pm 0\overset{''}{.}30$ adopted by the *P.G.C.* as the probable error of unit weight for meridian observations in general, since Spencer Jones used this system for weighing the observations. Since the observations of very bright stars are usually the least accurate, neither one of the probable errors can be considered as substantially larger than normal; this shows that the agreement between the meridian observations and the adopted orbit is satisfactory.

The total weights of the meridian observations in right ascension and declination are 599.5 and 411.5, respectively. If these weights are expressed in the same units as for the photographic observations, they become 19.9 and 5.8 as compared with the weights 159.3 for each of the two co-ordinates of the photographic observations. The meridian observations would therefore have contributed little if a combined solution had been attempted. The meridian observations reduced to one period and combined into eight normal places have been plotted in Figures 1 and 2.

VIII. MICROMETRIC OBSERVATIONS OF ADJACENT STARS

Several series of micrometer measures are available of the bright components of Procyon relative to adjacent stars. The most complete observations of this kind are those made by O. Struve[17] between 1851 and 1890 and listed by Spencer Jones (*op. cit.*, pp. 436–437) in his Table IX, together with an early observation by Bessel (1822) and observations by Auwers and Quetelet. The two reference stars were BD$+5°$1738 (mag. 9.0) and BD$+5°$1741 (mag. 8.5) preceding and following Procyon at intervals of 24 and 22 seconds of time, respectively, and differing from Procyon in declination by less than 2 minutes of arc at maximum. In the third column of the table Spencer Jones has listed the difference between the declination of Procyon and the mean declination of the two comparison stars.

Adopting the final orbit, we find from a least-squares solution the following values for the zero point and proper motion from the differences in declination: $-70\overset{''}{.}66 - 1\overset{''}{.}022\ (t - 1870)$. The probable error of unit weight as derived from the residuals is $\pm 0\overset{''}{.}20$ as compared with the value $\pm 0\overset{''}{.}19$ derived from Spencer Jones's orbit. These errors agree with the probable error of unit weight equal to $\pm 0\overset{''}{.}16$ as estimated by Struve himself for the accuracy of his observations. No attempt was made to compare the orbital motion with his less accurate observations of right-ascension differences.

IX. OBSERVATIONS OF THE FAINT COMPANION

The large difference in magnitude between the two components makes the observations of their relative positions extremely difficult even at the time when their angular separations are near maximum. For this reason the complete list of micrometer measures in Table 8 should be used with extreme caution in the evaluation of the semi-major axis of the relative orbit. The fact that neither Barnard nor Aitken saw the companion after 1913 makes it doubtful that observations after that time are real, except for the observations by van den Bos and Finsen, made at the Union Observatory under excellent seeing conditions. The observations of van den Bos and Finsen in 1927–1929 are, however, still quite uncertain, owing to the proximity of the two components, which were in the same relative position as when Burnham looked in vain for the companion in 1888 and 1890 with the 36-inch Lick refractor.

[16] *General Catalogue of 33342 Stars for the Epoch 1950* (Washington: Carnegie Institution of Washington, 1937).

[17] *Obs. Poulkovo*, **10**, 72–76, 1893.

TABLE 8

VISUAL OBSERVATIONS OF PROCYON AND THEIR RESIDUALS (EQUINOX 2000)

EPOCH	OBSERVED p	OBSERVED d	NO. OF NIGHTS	OBSERVER	O−C p	O−C p (Arc)	O−C d
1896.93	320°9	4″63	4	Schaeberle	+ 1°8	+0″15	−0″15
97.00	321.6	4.83	4	Aitken	+ 2.2	+0.18	+0.04
97.16	320.3	4.65	3	Hussey	− 0.1	−0.01	−0.16
97.82	324.9	4.66	6	Schaeberle	+ 0.6	+0.05	−0.21
97.83	327.9	4.80	1	See	+ 3.6	+0.31	−0.07
97.83	329.1	4.82	1	Boothroyd	+ 4.8	+0.41	−0.05
97.88	324.3	4.70	3	Aitken	− 0.3	−0.03	−0.18
98.21	325.8	4.82	3, 2	Aitken	− 0.7	−0.06	−0.09
98.21	326.5	4.83	6	Barnard	0.0	0.00	−0.08
98.24	326.5	4.26	1	Lewis	− 0.1	−0.01	−0.65
98.28	325.5	4.50	1	Hussey	− 1.3	−0.11	−0.41
98.88	331.1	4.97	4	Aitken	+ 0.9	+0.08	+0.01
99.07	331.1	4.91	3	Barnard	− 0.2	−0.02	−0.06
99.96	335.0	4.88	3	Aitken	− 1.2	−0.10	−0.15
1900.05	336.5	5.09	6	Barnard	− 0.1	−0.01	+0.06
00.23	338.8	4.83	1	Lewis	+ 1.2	+0.11	−0.21
00.29	332.9	4.60	4	See	− 5.0	−0.44	−0.44
01.20	339.00	5.13	3, 2	Aitken	− 3.8	−0.34	+0.05
01.30	338.4	5.00	5	See	− 4.9	−0.43	−0.08
01.88	344.0	5.06	2	Barnard	− 2.4	−0.21	−0.04
02.21	345.4	5.39	2	Lewis	− 2.6	−0.23	+0.28
02.24	347.0	5.11	1	Hussey	− 1.2	−0.11	0.00
02.25	338.9	5.35	3	See	− 9.4	−0.84	+0.24
02.72	351.1	5.33	3	Aitken	+ 0.4	+0.04	+0.20
03.15	351.5	5.16	3	Barnard	− 1.4	−0.13	+0.02
04.29	355.7	4.93	2	Bowyer	− 3.1	−0.28	−0.22
04.80	357.9	5.36	1	Barnard	− 3.5	−0.32	+0.20
05.14	6.7	5.14	1	Aitken	+ 3.6	+0.32	−0.02
05.17	5.8	4.46	1	Lewis	+ 2.4	+0.22	−0.70
05.99	10.7	....	1	Aitken	+ 3.2	+0.29	....
06.77	28.7	....	1	Aitken	+17.2	+1.53	....
09.16	23.0	5.26	4, 3	Barnard	− 0.7	−0.06	+0.10
09.30	23.0	5.04	1	Bowyer	− 0.4	−0.04	−0.12
10.10	24.8	5.21	2, 1	Barnard	− 3.8	−0.34	+0.06
11.07	29.3	4.69	2	Jonckheere	− 4.2	−0.38	−0.45
13.14	34.8	....	1	Bowyer	− 9.5	−0.84	....
13.16	43.1	5.09	3	Barnard	− 1.4	−0.12	+0.01
14.30	29.5	6.14	1	Bowyer	−21.0	−1.80	+1.11
14.94	30.5	5.25	1	Jonckheere	−23.5	−1.99	+0.25
17.24	47.9	4.12	1	Jonckheere	−19.1	−1.58	−0.70
18.22	59.3	4.63	1	Jonckheere	−13.6	−1.11	−0.07
21.21	94.6	(5. ±)	1	Bernewitz	+ 1.3	+0.10	+0.8 ±
24.19	106. ±	(5.5±)	1	Dick	−17. ±	−0.94	+2.3 ±
27.11	199.0	3.06	1	van den Bos	+19.5	+0.75	+0.81
28.98	237.9	2.68	3	van den Bos	+ 7.9	+0.34	+0.23
29.06	252.0	3.99	2, 1	Finsen	+20.4	+0.86	+1.52
32.27	278.5	3.57	6	van den Bos	− 1.2	−0.08	−0.10
32.28	276.0	3.96	4	Finsen	− 3.8	−0.24	+0.28

The series of distance measures by Barnard and Aitken were used for the purpose of determining the semi-major axis of the relative orbit. For each of the observed distances, the corresponding distance, d_a, was computed from the final orbit. The factor k, with which d_a has to be multiplied to correspond to the observed relative distances, d_o, provides the semi-major axis of the relative orbit, ka, and the total mass in units of the fainter component, $(M_A + M_B)/M_B = k$.

The values of k are listed in Table 9 for Aitken's and Barnard's observations separately. The fact that k does not show a systematic run with time shows the general agreement

TABLE 9

THE RELATIVE DISTANCES AND THEIR RESIDUALS

EPOCH	d_o	NO. OF NIGHTS	d_a	k	d_c	$d_o - d_c$
			Aitken			
1897.00.....	4″83	4	1″282	3.77	4″79	+0″04
97.88.....	4.70	3	1.306	3.60	4.88	− .18
98.21.....	4.82	2	1.313	3.67	4.91	− .09
98.88.....	4.97	4	1.326	3.75	4.96	+ .01
99.96.....	4.88	3	1.345	3.63	5.03	− .15
1901.20.....	5.13	2	1.358	3.78	5.08	+ .05
02.72.....	5.33	3	1.371	3.89	5.13	+ .20
05.14.....	5.14	1	1.381	3.72	5.16	− .02
			Barnard			
1898.21.....	4.83	6	1.313	3.68	4.91	− .08
99.07.....	4.91	3	1.330	3.69	4.97	− .06
1900.05.....	5.09	6	1.346	3.78	5.03	+ .06
01.88.....	5.06	2	1.365	3.71	5.10	− .04
03.15.....	5.16	3	1.374	3.76	5.14	+ .02
04.80.....	5.36	1	1.380	3.88	5.16	+ .20
09.16.....	5.26	3	1.381	3.81	5.16	+ .10
10.10.....	5.21	1	1.378	3.78	5.15	+ .06
13.16.....	5.09	3	1.358	3.75	5.08	+0.01

between the final orbit and the observations of these two excellent observers. For Aitken's observations we find $k = 3.73 \pm 0.02$ (p.e.) and for Barnard's 3.75 ± 0.01 (p.e.), showing no marked systematic difference between the two observers. The mean value, $k = 3.738 \pm 0.011$ (p.e.) was adopted, and we find, accordingly, that the semi-major axis of the relative orbit is 4″548 ± 0″015 (p.e.) and $M_B/(M_A + M_B) = 0.268 \pm 0.001$ (p.e.).

The last column of Table 9 lists the differences between the distances observed by Aitken and Barnard and those computed from the astrometric orbit multiplied by the adopted value k. Assigning a weight equal to 1 for a single night's observation and a weight of 2 for the mean of two or more nights' observations, we find that the residuals indicate a probable error of ±0″10 for a distance measure of unit weight. The corresponding error for the same observers for the position angles (tangential deviations $d \sin [v_o - v_c]$) was found to be ±0″16, if we omit the single night's observation by Aitken in 1906, which is obviously erroneous.

These values for the errors are remarkably small as compared with a probable error of $\pm 0\overset{''}{.}12$ found by the writer[18] as the average expected error for visual observations of pairs with separations between 2″ and 9″ and with relatively small differences in magnitude.

It has already been mentioned that the position angles observed between 1897 and 1909 were effective in determining the period of the orbit. The same observations proved the correctness of the adopted eccentricity, as can be seen from Figures 3 and 4, where the cosine and sine of the observed position angles are plotted against the final orbit and against one of the provisional orbits with an eccentricity of 0.35.

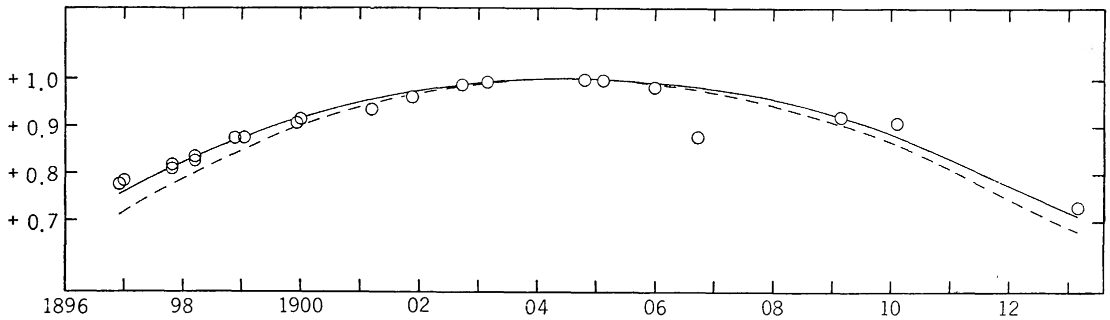

FIG. 3.—Cosines of the visually observed position angles (*circles*) plotted against *a*, a solid curve representing the cosines of the position angles for the final orbit ($e = 0.40$), and *b*, a dotted curve representing the same quantities for a provisional orbit ($e = 0.35$; same period and periastron passage as the final orbit).

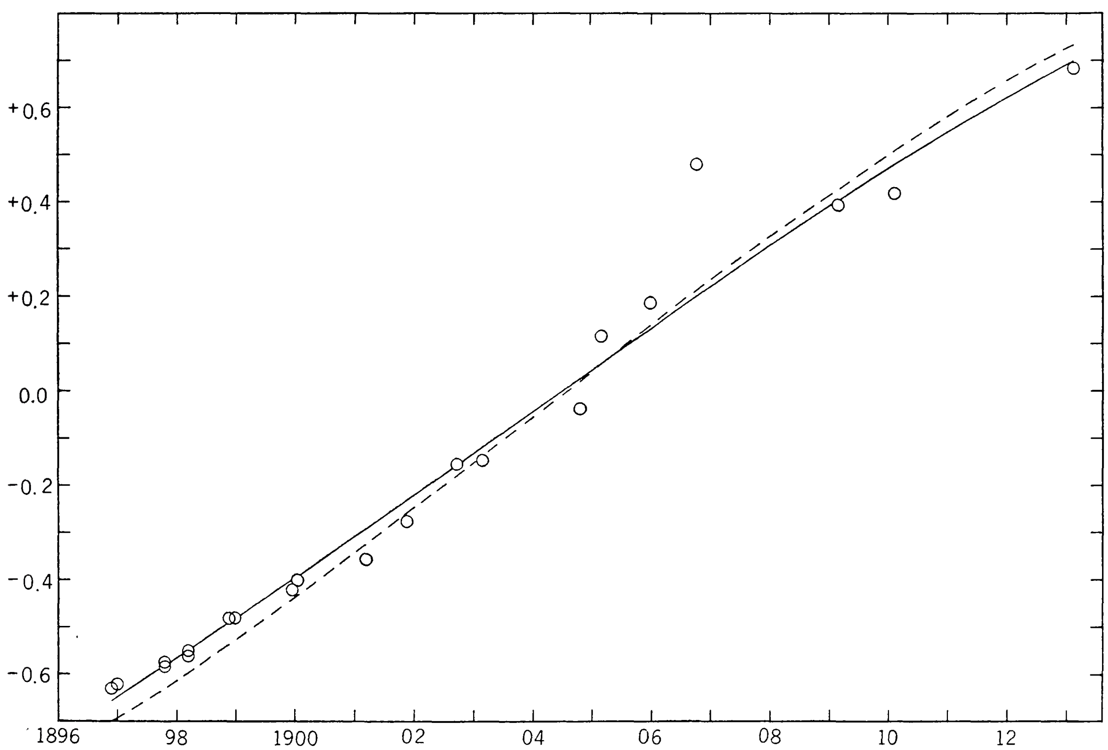

FIG. 4.—Sines of the visually observed position angles (*circles*) plotted against *a*, a solid curve representing the sines of the position angles for the final orbit ($e = 0.40$), and *b*, a dotted curve representing the same quantities for a provisional orbit ($e = 0.35$; same period and periastron passage as the final orbit).

[18] *A.J.*, **49**, 166, 1942.

The last three columns of Table 8 contain the residuals between the observed and computed values for all published observations. Some of the large discrepancies either are contradicted by the observations of Aitken and Barnard made at nearly the same time or were made after Barnard considered the pair too difficult to measure. The observations prior to 1914 give probable errors of $\pm 0\overset{''}{.}25$ and $\pm 0\overset{''}{.}19$ for the position angles and distances, respectively, for observations of unit weight.

X. THE PARALLAX OF THE SYSTEM

A combination of the results listed in Section V for the relative parallax derived from the x and y co-ordinates of the combined Yerkes and McCormick material gives the following weighted mean value for the relative parallax: $+0\overset{''}{.}281 \pm 0\overset{''}{.}007$ (p.e.). The correction $+0\overset{''}{.}002$ to absolute parallax was determined from the table by Vyssotsky and Williams,[19] using an average photovisual magnitude of 10.5 mag. and a galactic latitude of $15\overset{\circ}{.}0$ for the comparison stars and a solar velocity of 20 km/sec.

The derived parallax supersedes the previously published value by the Yerkes Observatory, $+0\overset{''}{.}307 \pm 0\overset{''}{.}009$ (15 plates)[20] and those published by the McCormick Observatory, $+0\overset{''}{.}309 \pm 0\overset{''}{.}007$ (16 plates)[21] and $+0\overset{''}{.}293 \pm 0\overset{''}{.}010$ (23 plates).[22] All three parallaxes were based on part of the present material.

The radial velocities discussed in Section VI led to a parallax of $+0\overset{''}{.}300 \pm 0\overset{''}{.}007$ (p.e.). Combined with the absolute trigonometric parallax, we obtain a final value of $+0\overset{''}{.}291 \pm 0\overset{''}{.}005$ (p.e.). Other adjusted absolute parallaxes derived from photographic series are: Sproul,[23] $+ 0\overset{''}{.}288 \pm 0\overset{''}{.}014$ (11 plates); Yale,[24] $+ 0\overset{''}{.}288 \pm 0\overset{''}{.}009$ (22 plates); and Van Vleck,[25] $+ 0\overset{''}{.}274 \pm 0\overset{''}{.}008$ (48 plates). These three series give a mean absolute parallax of $+0\overset{''}{.}281 \pm 0\overset{''}{.}005$, thus showing a close agreement with the parallax obtained from the Yerkes and McCormick plates. Combining, according to weight, the trigonometric parallaxes with the parallax obtained from the radial velocities, we obtain the absolute parallax of $+0\overset{''}{.}287 \pm 0\overset{''}{.}004$ (p.e.).

XI. THE FINAL ELEMENTS

Summarizing the results obtained in the previous sections, we derive the following orbital elements for the bright component relative to the center of gravity (Equinox 2000):

$$\begin{aligned}
T &= 1927.60\,, & A &= +0\overset{''}{.}958 \pm 0\overset{''}{.}009 \text{ (p.e.)}\,,\\
P &= 40.65\,, & B &= +0\overset{''}{.}240 \pm 0\overset{''}{.}011 \text{ (p.e.)}\,,\\
e &= 0.40\,, & F &= -0\overset{''}{.}298 \pm 0\overset{''}{.}010 \text{ (p.e.)}\,,\\
n &= 8\overset{\circ}{.}8561\,, & G &= +1\overset{''}{.}180 \pm 0\overset{''}{.}013 \text{ (p.e.)}\,,\\
a &= 1\overset{''}{.}217 \pm 0\overset{''}{.}002 \text{ (p.e.)}\,, & C &= +0\overset{''}{.}710 \pm 0\overset{''}{.}004 \text{ (p.e.)}\,,\\
i &= 35\overset{\circ}{.}7 \pm 0\overset{\circ}{.}2 \text{ (p.e.)}\,, & H &= +0\overset{''}{.}002 \pm 0\overset{''}{.}004 \text{ (p.e.)}\,,\\
\omega &= 89\overset{\circ}{.}8 \pm 0\overset{\circ}{.}3 \text{ (p.e.)}\,, & \pi L &= +29.79\,,\\
\Omega &= 104\overset{\circ}{.}3 \pm 0\overset{\circ}{.}3 \text{ (p.e.)}\,, & \pi N &= +0.08\,.
\end{aligned}$$

The probable errors for the elements a, i, ω, and Ω were derived from those obtained for the Thiele-Innes orientational elements.

[19] *Pub. McCormick Obs.*, **10**, 34, 1948.
[20] *Pub. Yerkes Obs.*, **4**, Part III, 11, 1920.
[21] *Pub. McCormick Obs.*, **3**, 229, 1920.
[22] *Pub. McCormick Obs.*, **8**, 246, 1940.
[23] *Pub. Sproul Obs.*, **4**, 48, 1919.
[24] *A.J.*, **44**, 39, 1934.
[25] *Pub. Van Vleck Obs.*, **1**, 142–145, 1938.

If the above elements are advanced to the equinox 2100, the following corrections for precession and space motions are obtained:

$$\Delta A = -0''.0017\,, \quad \Delta F = -0''.0105\,, \quad \Delta a = 0''.000\,, \quad \omega = -0^\circ.02\,,$$

$$\Delta B = +0''.0084\,, \quad \Delta G = -0''.0026\,, \quad i = +0^\circ.03\,, \quad \Omega = +0^\circ.52\,.$$

The orbit of the companion about the primary has the same dynamical elements as those given above, while the remaining elements are as follows (Equinox 2000):

$$A = -3''.582\,, \qquad a = 4''.548\,,$$

$$B = -0''.898\,, \qquad i = 35^\circ.7\,,$$

$$F = +1''.112\,, \qquad \omega = 89^\circ.8\,,$$

$$G = -4''.410\,, \qquad \Omega = 284^\circ.3\,.$$

Adopting the absolute parallax $+0''.287$ derived in Section X, we obtain a total mass of the system equal to 2.37⊙, while the individual masses are $M_A = 1.74$⊙, $M =$ 0.63⊙. The derived mass for the brighter component is the expected for an F5 IV star.

As there is no observed spectrum of the companion, its effective temperature is not known. From Kuiper's observation of its color, it is probable that its effective temperature does not exceed 10,000° and is not less than 6500°. With the absolute visual magnitude of 13.0 and the appropriate bolometric corrections, the two values of the computed diameter are 0.008 and 0.016. The corresponding densities are 1.8×10^6 and 0.22×10^6 c.g.s.

XII. EPHEMERIS OF PROCYON

Table 10 gives an ephemeris covering a complete period. The chosen dates are those listed by Spencer Jones for his ephemeris, so that the differences from the old orbit may be found from simple inspection. The second and third columns are the values required to be added to the right ascension and declination of the center of gravity of the system to obtain the position of the bright star. The remaining two columns give the position angle and the distance of the faint component relative to the bright component. The positions are computed for the equinox of date. For any other equinox differing by n orbital periods, the values change, owing to proper motion and precession, by the following amounts:

$$\Delta\alpha\colon +0^s.00025\ \Delta\delta n\ ; \qquad \Delta\delta\colon -0''.53\ \Delta\alpha n\ ; \qquad \Delta v = +0^\circ.21 n\,.$$

It is a pleasure to acknowledge the support of the Research Corporation, the American Philosophical Society, the Watson Fund of the National Academy of Sciences, and the Graduate School of Northwestern University. Their grants made it possible to carry out the measurements of the spectra and the extensive computations. I wish to express my sincere thanks to Dr. (Mrs.) Helen Steel Lillibridge for her part in measuring and analyzing the spectra; to Mr. R. Spong and Mr. T. Littlejohn, who carried out most of the tedious computational work. I also wish to express my appreciation to Directors H. T. Alden and Ira S. Bowen for allowing me to use their material in the present investigation. It is impossible to acknowledge my indebtedness to each of the observers at the Yerkes and the McCormick Observatories, whose observations over the span of more than thirty-five years have made this investigation possible. I am especially grateful to Dr. G. Van Biesbroeck, who has always emphasized the importance of including continuous photographic series of binaries in the parallax program.

TABLE 10

EPHEMERIS OF PROCYON

Date	Δα	Δδ	v	d
1908.0......	−0ˢ.028	−1″.32	17°.3	5″.17
09.0......	− .035	−1.28	22.4	5.16
10.0......	− .043	−1.22	27.5	5.15
11.0......	− .050	−1.16	32.7	5.14
12.0......	− .056	−1.08	37.9	5.11
13.0......	− .062	−0.99	43.2	5.08
14.0......	− .068	−0.89	48.5	5.04
15.0......	− .072	−0.79	53.9	4.99
16.0......	− .076	−0.67	59.4	4.93
17.0......	− .079	−0.54	65.2	4.84
18.0......	− .080	−0.41	71.1	4.73
19.0......	− .080	−0.27	77.4	4.59
20.0......	− .079	−0.12	84.0	4.43
21.0......	− .076	+0.03	91.4	4.21
22.0......	− .070	+0.18	99.5	3.96
23.0......	− .062	+0.32	108.9	3.65
24.0......	− .051	+0.44	120.2	3.29
25.0......	− .037	+0.54	134.5	2.90
26.0......	− .020	+0.60	153.2	2.53
27.0......	− .002	+0.61	177.3	2.26
28.0......	+ .017	+0.54	204.7	2.24
29.0......	+ .034	+0.42	229.9	2.46
30.0......	+ .047	+0.26	249.7	2.82
31.0......	+ .057	+0.08	264.8	3.21
32.0......	+ .064	−0.11	276.6	3.58
33.0......	+ .067	−0.29	286.3	3.90
34.0......	+ .068	−0.47	294.7	4.17
35.0......	+ .067	−0.62	302.1	4.39
36.0......	+ .064	−0.77	308.9	4.57
37.0......	+ .060	−0.89	315.2	4.71
38.0......	+ .054	−1.01	321.2	4.83
39.0......	+ .048	−1.10	327.0	4.92
40.0......	+ .041	−1.18	332.5	4.99
41.0......	+ .034	−1.25	337.9	5.04
42.0......	+ .026	−1.30	343.2	5.08
43.0......	+ .018	−1.34	348.5	5.11
44.0......	+ .010	−1.37	353.7	5.14
45.0......	+ .002	−1.38	358.8	5.15
46.0......	− .006	−1.38	4.0	5.16
47.0......	− .014	−1.36	9.1	5.17
48.0......	− .023	−1.34	14.2	5.17
49.0......	−0.031	−1.30	19.3	5.17