# PHOTOELECTRIC STUDIES. IV. COLOR-LUMINOSITY ARRAY FOR STARS IN THE REGION OF THE SUN* 

Olin J. Eggen<br>Lick Observatory, University of California<br>Received March 20, 1950


#### Abstract

S Introduction.-Magnitudes and colors of 180 stars have been determined with the photomultiplier photometer attached to the Lick Observatory 12 -inch refractor; an additional 8 stars have been observed with the Crossley reflector.

Observations of the nearest stars.-The results of 245 observations of 78 stars within 14 parsecs of the sun are listed in Table 3. The color-luminosity array for these stars, derived from trigonometric parallaxes, is shown in Figure 1.

Subgiant extension of the bright-dwarf sequence.-The mean absolute photographic magnitude of 17 subgiants is $+3.60 \pm 0.13$ (A.D.) and is apparently independent of the color $C_{p}$, which ranges from +0.32 to +0.97 mag. It appears that the secondary components of Algol-type eclipsing stars are subgiants.

Dwarf and subdwarf sequences.-The dwarf and subdwarf sequences intersect twice, once near $C_{p}=$ +0 m 55 and again near $C_{p}=+1 \mathrm{~m} 30$.

Blue-dwarf and bright blue-dwarf sequences.-It appears that at least some of the fine structure in that portion of the color-luminosity array populated by the A-type stars probably is reflected in the spectra of these stars.

Rotational velocity.-The bright blue-dwarfs, which include the spectrum variables, appear to rotate more slowly, in the mean, than do the blue-dwarfs.

Concluding remarks.-The stars near the sun populate the same sequences in the color-luminosity array that are found in the four galactic clusters, Hyades, Pleiades, Coma Berenices, and Ursa Major.


## I. INTRODUCTION

The preceding three papers ${ }^{1}$ of this series have been-concerned with the color-luminosity arrays for the stars in four galactic clusters. The present communication is the result of 605 observations of 180 near-by stars made in the last 18 months with the photomultiplier photometer on the Lick Observatory 12-inch refractor; 13 observations of 8 stars made with the Crossley 36 -inch reflector are also included. The color system and magnitude convention used in the 12 -inch work have been described in Paper I; colors and magnitudes observed with the reflector have been converted to the International System by means of the following formulae:

$$
\begin{aligned}
P g_{p} & =P e_{c r}-0^{\mathrm{m}} 19-0.15 C_{c r}, \\
C_{p} & =+0^{\mathrm{m}} 76+0.86 C_{c r} .
\end{aligned}
$$

The effective wave lengths obtained with the photomultiplier and filters used in the Crossley installation will be discussed in more detail in a succeeding paper of this series, but the results obtained for the Harvard Standard Region C 12, given in Table 1, show the close agreement between refractor and reflector observations. As a further check on the Crossley results for the reddest stars, we may compare the observations of Barnard's star and 119 Tau.


* Contributions from the Lick Observatory, Ser. II, No. 30.
${ }^{1}$ Ap.J., 111, 65, 81, 414, 1950. These will be referred to as "Papers I, II, and III."

If we take the observations of the Harvard Standard Region stars as typical for the magnitude range covered, the average deviation of a single determination from the mean magnitude and color, as a function of magnitude, is shown in the last two columns of Table 2, where the individual observations of these stars are listed. In order to extend the magnitude range to that represented by the observations made with the 12 -inch, a few stars selected at random were added to those of the Standard Region in Table 2. Stars A and D in the Region C 12 are used for the determination of the extinction and its dependence upon color whenever this region is observable, so that some 50 observa-

TABLE 1
Colors and Magnitudes of Stars in Harvard Standard Region C 12

| Star | 12-Inch Refractor |  |  | Crossley Reflector |  |  | Mean |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P g_{p}$ | $C_{p}$ | $n$ | $P g_{p}$ | $C_{p}$ | $n$ | $P g_{p}$ | $C_{p}$ |
| A. | 6 m 54 | $-0^{\text {m }} 04$ |  | 6 m 55 | $-0^{\text {m }} 05$ |  | 6 m 54 | $-0^{\text {m }} 045$ |
| C. | 7.34 | +0.06 | 3 | 7.30 | +0.07 | 1 | 7.33 | +0.06 |
| D | 7.60 | +1.12 |  | 7.58 | +1.12 |  | 7.59 | +1.12 |
| E. | 7.88 | +0.865 | 4 | 7.84 | +0.86 | 1 | 7.87 | +0.86 |
| F | 8.26 | +0.24 | 4 | 8.30 | +0.25 | 1 | 8.27 | +0.24 |
| G | 8.33 | +0.23 | 2 |  |  |  | 8.33 | +0.23 |
| H | 8.41 | -0.01 | 4 | 8.44 | 0.00 | 1 | 8.42 | -0.01 |
| I. | 9.08 | +1.16 | 3 | 9.10 | +1.16 | 2 | 9.09 | +1.16 |
| L | 9.38 | +0.36 | 3 | 9.39 | +0.35 | 1 | 9.38 | +0.36 |
| M. | 9.64 | +1.20 | 3 | 9.66 | $+1.21$ | 2 | 9.65 | +1.20 |
| N. | 9.93 | +0.40 | 3 | 9.97 | +0.38 | 2 | 9.95 | +0.39 |
| 0 | 9.97 | +0.955 | 3 |  |  |  | 9.97 | +0.955 |
| P. | 10.10 | +1.10 | 2 |  |  |  | 10.10 | +1.10 |

TABLE 2
Individual Observations of Stars in the Range of Magnitude Observed with the 12-Inch Refractor

| Star | (1) |  | (2) |  | (3) |  | (4) |  | (5) |  | Mean |  | Average Deviation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ |
| Sirius | - 1 m 54 | $-0 \mathrm{~mm} 075$ | - 1 m 53 | $-0^{\text {m }} 08$ | $-1 \mathrm{~m} 53$ | $-0 \stackrel{0}{0} 07$ | $-1 \mathrm{~m} 54$ | $-0 \times 07$ | $-1 \mathrm{~m} 52$ | -0m07 | $-1 \mathrm{~m} 53$ | $-0 \mathrm{~m} 07$ | $\pm 0 \underline{01}$ | $\pm 0 \mathrm{~m} 00$ |
| Vega | - 0.08 | -0.075 | - 0.07 | $-0.08$ | -0.09 | $-0.085$ |  |  |  |  | -0.08 | -0.08 | $\pm .01$ | $\pm .00$ |
| Altair. | + 0.87 | $+0.165$ | +0.89 | $+0.165$ |  |  |  |  |  |  | + 0.88 | $+0.165$ | $\pm .01$ | $\pm .00$ |
| Fomalhaut | +1.11 | +0.02 | +1.11 | +0.005 |  |  |  |  |  |  | + 1.11 | $+0.015$ | $\pm .00$ | $\pm .01$ |
| $\zeta$ Her... | +3.30 +4.25 | +0.53 | + 3.32 | +0.53 +0.40 |  |  |  |  |  |  | + 3.31 | $+0.53$ | $\pm .01$ | $\pm .00$ |
| $\gamma{ }_{61} \mathrm{Ser}$ UMa | $\begin{array}{r}+4.25 \\ +\quad 5.92 \\ \hline\end{array}$ | +0.41 | $+\quad 4.26$ $+\quad 5.91$ | +0.40 +0.63 |  | $+0.63$ |  | +0.63 |  |  | $\begin{array}{r}+4.26 \\ +\quad 5.92 \\ \hline\end{array}$ | +0.405 | $\pm .01$ | $\pm .01$ |
| C 12 A . | + 6.54 | -0.04 | + 6.53 | -0.04 | + 6.54 | -0.05 | +5.93 +6.54 | -0.04 | -6.54 | -0.04 | + 6.54 | -0.04 | $\pm \begin{aligned} & \pm \\ & \pm\end{aligned}$ | $\pm$ 士 . 00 |
| C 12 C . | + 7.34 | +0.06 | + 7.34 | +0.06 | + 7.34 | +0.06 | + 7.35 | +0.06 | -7.34 | +0.06 | + 7.34 | +0.06 | $\pm .00$ | $\pm .00$ |
| C 12 D. | + 7.60 | +1.12 | + 7.60 | +1.12 | + 7.61 | +1.125 | + 7.60 | +1.12 | -7.60 | +1.12 | + 7.60 | +1.12 | $\pm .00$ | $\pm .00$ |
| C 12 E | + 7.88 | +0.86 | + 7.90 | +0.86 | + 7.88 | +0.85 | + 7.89 | +0.87 |  |  | + 7.88 | +0.86 | $\pm .01$ | $\pm .00$ |
| C 12 F . | +8.25 | +0.235 | +8.28 | +0.24 | + 8.25 | +0.235 | + 8.28 | +0.24 |  |  | +8.26 | +0.24 | $\pm .02$ | $\pm .00$ |
| C 12 G | +8.33 | +0.235 | +8.33 | +0.22 |  |  |  |  |  |  | +8.33 | +0.23 | $\pm .00$ | $\pm .01$ |
| C 12 H | + 8.42 | -0.02 | +8.41 | 0.00 | $+8.41$ | $-0.01$ | $+8.39$ | 0.00 |  |  | + 8.41 | $-0.01$ | $\pm .01$ | $\pm .01$ |
| C 12 I . | +9.08 | +1.16 | + 9.10 | +1.16 | + 9.06 | +1.14 |  |  |  |  | + 9.08 | +1.16 | $\pm .01$ |  |
| C 12 L . | +9.38 | +0.36 | + 9.40 | +0.38 | +9.36 | +0.34 |  |  |  |  | +9.38 +9.81 | +0.36 | $\pm .01$ | $\pm .01$ |
| C 12 M . | +9.66 | +1.20 | +9.61 | +1.22 | +9.66 | +1.19 |  |  |  |  | +9.64 | +1.20 | $\pm .02$ | $\pm .01$ |
| C 12 N | +9.93 | +0.42 | +9.95 | +0.40 | +9.91 | +0.395 |  |  |  |  | +9.93 | $+0.40$ | $\pm .01$ | $\pm .01$ |
| C 12 O . | +9.97 | +0.96 | $+10.01$ | +0.955 | +9.93 | $+0.95$ |  |  |  |  | +9.97 $+\quad$ | +0.955 | $\pm .03$ | $\pm .00$ |
| C 12 P .... | +10.11 | +1.11 | +10.09 | $+1.09$ |  |  |  |  |  |  | $+10.10$ | $+1.10$ | $\pm .01$ | $\pm .01$ |
| +680946.. | +10.36 | +1.30 | $+10.34$ | +1.30 | $+10.36$ | +1.29 |  |  |  |  | $+10.36$ | $+1.30$ | $\pm .01$ | $\pm .00$ |
| $+4^{\circ} 3561 .$. | +11.03 | +1.52 |  | +1.53 | +10.99 | +1.54 |  |  |  |  | $+11.03$ | +1.53 | $\pm . .03$ | $\pm .00$ |
| $-4^{\circ} 4226 .$. | +11.25 | +1.21 | +11.23 | +1.23 | +11.33 | +1.23 | +11.23 | +1.23 |  |  | +11.27 | +1.23 | $\pm 0.04$ | $\pm 0.00$ |

tions are available for these two stars; however, only 5 representative determinations are given in Table 2. A screen, absorbing 4.82 mag., was used for Sirius, Vega, and Altair. Although magnitudes and colors have been obtained for stars as faint as $P g_{p}=12 \mathrm{~m} .5$ with the 12 -inch refractor, the number of observations necessary to maintain the accuracy shown in Table 2 increases rapidly for stars fainter than $P g_{p}=11^{\mathrm{m}} 5$, and, except for special objects, observations are usually confined to stars brighter than $P g_{p}=11 \mathrm{~m} .0$ with this instrument.

## II. OBSERVATIONS OF THE NEAREST STARS

Table 3 lists the results for 245 observations of 78 stars within 14 parsecs of the sun. All but 8 of these stars have been observed with the 12 -inch refractor, and each star was observed on at least two different nights. A few red dwarfs were observed as often as six to ten nights to test for variability in these stars. The various columns of Table 3 contain the following information: column 1, the current number; column 2, the Henry Draper Catalogue or Durchmusterung number, when available; column 3, other designations; columns 4 and 5, the 1900 positions; columns 6 and 7, the International photographic magnitude and color derived from the photoelectric observations; column 8, the number of nights each star was observed; column 9, the mean trigonometric parallax (these values are taken from Schlesinger's Catalogue of Stellar Parallaxes," determinations made available since the completion of the catalogue have been incorporated into the means); column 10, the absolute photographic magnitude resulting from columns 6 and 9.

The color-luminosity array for the stars in Table 3 is shown in Figure 1. The filled circles represent stars within 10 parsecs of the sun, while the open circles represent the stars that are $10-14$ parsecs distant. The continuous curves reproduced in Figure 1 have been taken directly from the previous papers of this series concerning the color-luminosity arrays for four galactic clusters: the zero point for all these arrays is determined by the Hyades cluster stars. The dashed lines in the color-luminosity array of Figure 1 represent the extensions added by the material in this paper. The few giant stars in the vicinity of the sun are not included in Table 3 or in Figure 1, but they will be discussed in a later paper of this series.

## III. SUBGIANT EXTENSION OF THE BRIGHT-DWARF SEQUENCE

## LUMINOSITY OF SUBGIANTS

The top section of Table 4 contains those stars in Table 3 which are classified with luminosity class IV in the system of the Atlas. ${ }^{3}$ The middle section of Table 4 contains other parallax stars, more distant than 14 parsecs, for which available spectroscopic luminosity classifications indicate their subgiant nature. The bottom section of Table 4 contains five stars, for four of which no Atlas types are available. From the position of these stars in the color-luminosity array, however, we may assign the subgiant classification to them.

If we omit $\delta \mathrm{Gem}$, which falls on that portion of the bright-dwarf sequence defined by the cluster stars, we find that the straight mean of the luminosities of the subgiants in the first two sections of Table 4 is $\bar{M}=+3 \mathrm{~m} 60 \pm 0 \mathrm{~m} .13$ (A.D.). This result is the basis for the straight-line extension of the bright-dwarf sequence shown as a dashed line in Figure 1. If we assume that all subgiants redder than $C_{p}=+0 \mathrm{~m} 32$ have $M=+3 \mathrm{~m} .60$, we find the "photometric," (ptm), parallaxes given in the table. With but one exception, the photometric parallaxes all lie within the range of the individual trigonometric values;
${ }^{2}$ New Haven: Yale University Press, 1936.
${ }^{3}$ The Yerkes Atlas of Stellar Spectra (Chicago: University of Chicago Press, 1943), by Morgan, Keenan, and Kellman, will be referred to throughout this paper as the "Atlas." The Atlas luminosity classes will also be used: III = giants, $I V=$ subgiants, and $V=$ dwarfs.

TABLE 3
Photoelectric Observations，Luminosities，and Trigonometric Parallaxes for 78 Stars within 14 Parsecs of the Sun

| No． <br> （1） | $\begin{aligned} & \mathrm{HD}(\mathrm{DM}) \\ & (2) \end{aligned}$ | Name <br> （3） | $a(1900)$ <br> （4） | $\underset{(5)}{(1900)}$ | $\begin{aligned} & P_{g_{p}} \\ & (6) \end{aligned}$ | $\begin{gathered} C_{p} \\ (7) \end{gathered}$ | （8） | $\underset{(9)}{(\text { Trig.) }}$ | $\begin{gathered} M_{P g_{p}} \\ (10) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1＊． | $+43^{\circ} 44 \mathrm{~A}$ | ADS 246 A | $0^{\mathrm{h}} 12^{\text {m }} 7$ | $+43^{\circ} 27^{\prime}$ | ＋ $9^{m} 41+$ | $+1 \mathrm{~m} 34$ | 2 | 0 ＂284土 5 | $+11^{\text {m }} 67$ |
| 2＊ | $+43^{\circ} 44 \mathrm{~B}$ | ADS 246 B |  |  | ＋12．65 $\dagger$ | ＋1．55 | 2 | 284土 5 | ＋14．91 |
| 3 ＊ | 4614 | $\eta$ Cas | 043.0 | ＋5717 | ＋ 3.81 | ＋0．46 | 3 | 184土 5 | ＋ 5.13 |
| 4 ＊ |  | Wolf 28 | 043.9 | ＋ 455 | ＋12．50 $\dagger$ | ＋0．41 | 2 | ． $243 \pm 6$ | ＋14．43 |
| 5 | $+61^{\circ} 195 \mathrm{~A}$ |  | 056.3 | ＋6148 | ＋10．81＋ | ＋1．29 | 1 | ．101士 7 | ＋10．83 |
| 6 | ＋61 ${ }^{\circ} 195 \mathrm{~B}$ | Wolf 47 | 057.0 | ＋6150 | ＋14．90† | ＋1．29 | 1 | ．101士 7 | ＋14．92 |
| 7. | 6582 | $\mu \mathrm{Cas}$ | 101.6 | ＋54 26 | ＋ 5.69 | ＋0．58 | 2 | ．133土 5 | ＋6．31 |
| 8 | 10307 | HR 483 | 135.7 | ＋4207 | ＋ 5.36 | ＋0．535 | 3 | ． $087 \pm 5$ | ＋ 5.06 |
| 9 | 10476 | 107 Psc | 137.1 | ＋19 47 | ＋ 5.90 | ＋0．71 | 3 | ．132 $\pm 6$ | ＋ 6.50 |
| 10 | 10700 | $\tau$ Cet | 139.4 | －16 28 | ＋ 4.05 | ＋0．585 | 3 | ．298土 6 | ＋ 6.42 |
| 11 | 10780 | HR 51 | 140.4 | ＋6322 | ＋ 6.22 | ＋0．67 | 2 | 111士 5 | ＋ 6.43 |
| 12＊ | 13974 | $\delta$ Tri | 210.8 | ＋33 46 | ＋ 5.28 | ＋0．49 | 3 | ． $098 \pm 6$ | ＋ 5.24 |
| $13 *$ | 16160 | HR 753 A | 230.6 | ＋ 625 | ＋6．68 | ＋0．89 | 2 | ．147士 4 | ＋ 7.52 |
|  |  | HR 753 B |  |  | ＋12．94 $\dagger$ | ＋1．32 | 1 | ．147士 4 | ＋13．78 |
|  | 16895 | $\theta$ Per | 232.4 | ＋4848 | ＋ 4.41 | ＋0．395 | 3 | ． $078 \pm 5$ | ＋ 3.87 |
| $16$ | 17925 | HR 857 | 247.7 | －13 11 | ＋ 7.34 | ＋0．96 |  | 137士 6 | ＋ 8.02 |
| 17 | 19373 | $\iota$ Per | 301.8 | ＋49 14 | ＋ 4.46 | ＋0．50 | 3 | ．085士 4 | ＋ 4.11 |
| 18 | 20630 | $\chi$ Cet | 314.1 | ＋ 300 | ＋ 5.33 | ＋0．585 | 4 | ． $106 \pm 5$ | ＋ 5.46 |
| 19. | 22049 | $\epsilon$ Eri | 328.2 | － 948 | ＋ 4.87 | ＋0．79 | 3 | ． $303 \pm 6$ | ＋ 6.87 |
| 20 | 26965 | 40 Eri A | 410.7 | － 749 | $+5.12$ | ＋0．725 | 2 | ．202土 3 | ＋6．65 |
| 21 | 30652 | $\pi^{3}$ Ori | 444.4 | ＋ 647 | ＋ 4.02 | ＋0．34 | 3 | ．128士 5 | ＋ 4.02 |
| 22＊ | 32147 | HR 1614 | 455.9 | － 552 | $+7.13$ | ＋0．96 | 3 | ．105士 4 | ＋ 7.24 |
| 23. | 34411 | $\lambda$ Aur | 512.1 | ＋4001 | ＋ 5.12 | ＋0．51 | 3 | ．068土 5 | ＋ 4.28 |
| 24. | 36395 | Cin 705 | 526.4 | － 342 | ＋ 9.12 | ＋1．27 | 3 | ．166士 4 | ＋10．22 |
| 25. | 37394 | HR 1925 | 533.2 | ＋5326 | ＋ 6.88 | ＋0．75 | 3 | ． $097 \pm 5$ | ＋ 6.81 |
| 26＊ | 38393 | $\gamma$ Lep A | 540.3 | －22 28 | ＋ 3.88 | ＋0．36 | 2 | ．122 4 | + +4.31 |
| $27 *$ | 38392 | $\gamma$ Lep B |  |  | ＋ 6.98 | ＋0．87 | 2 | ．122 4 | ＋ 7.41 |
| 28. | 39587 | $\chi^{1} \mathrm{Ori}$ | 548.5 | ＋20 15 | ＋ 4.86 | ＋0．47 | 3 | ．104土 6 | ＋ 4.98 |
| 29. | 48915 | ${ }_{a} \mathrm{CMa}$ | 640.7 | －16 35 | － 1.53 | $-0.07$ | 5 | ． $376 \pm 3$ | ＋1．35 |
| 30 | 76943 | 10 UMa | 854.2 | ＋4211 | ＋4．28 | ＋0．33 | 3 | ． $073 \pm 4$ | ＋ 3.60 |
| 31. | 82885 | 11 LMi | 929.7 | ＋3616 | $+6.00$ | ＋0．66 | 3 | ．113土 6 | ＋ 6.26 |
| 32. | 88230 | Gmb 1618 | 1005.3 | ＋4955 | ＋ 7.96 | ＋1．23 | 3 | ．220土 8 | ＋ 9.67 |
| 33. | $+20^{\circ} 2465$ | Cin 1244 | 1014.2 | ＋20 22 | ＋10．63 | ＋1．32 | 3 | ．202土 6 | ＋12．16 |
| 4＊ | 90839 | 36 UMa A | 1024.2 | ＋5630 | ＋ 5.19 | ＋0．41 | 2 | ．082 $\pm 8$ | ＋ 4.76 |
| 35＊ | ＋56¹458 | 36 UMa B |  |  | ＋ 9.94 | $+1.21$ | 4 | ．082 ${ }^{8}$ | ＋ 9.51 |
| 36＊ | ＋5701266 | 36 UMa C |  |  | ＋8．99 | ＋0．87 | 3 | ．082土 8 | ＋8．56 |
| 37. | 95735 | Lal 21185 | 1057.9 | $+3638$ | ＋8．78 | ＋1．32 | ， | ． $392 \pm 6$ | ＋11．75 |
|  | ＋66 ${ }^{\circ} 717$ | Cin 1383 | 1114.8 | ＋6623 | $+10.52$ | ＋1．28 | 5 | ． $120 \pm 5$ | ＋10．92 |
|  | 101501 | 61 UMa | 1135.8 | ＋34 46 | ＋ 5.92 | ＋0．63 | 4 | ．109士 9 | ＋6．11 |
|  | 102870 | $\beta$ Vir | 1145.5 | ＋220 | ＋ 4.00 | ＋0．49 | 3 | ． $097 \pm 6$ | ＋ 3.93 |
|  | 103095 | HR 4550 | 1147.2 | ＋35 26 | ＋ 7.05 | ＋0．635 | 5 | ． $108 \pm 4$ | +7.92 +7.22 |
|  | 109358 | $\beta \mathrm{CVn}$ | 1229.0 | ＋4154 | ＋ 4.72 | ＋0．48 |  | ．108土 6 | ＋ 4.93 |
|  | 111631 | Cin 1633 | 1245.6 | －013 | ＋ 9.68 | ＋1．26 |  | ．098士 7 | ＋ 9.64 |
|  | 114710 | $\beta$ Com | 1307.2 | ＋28 23 | ＋ 4.63 | ＋0．47 | 2 | ．121士 6 | ＋ 5.05 |
|  | 115617 | 61 Vir | 1313.2 | －1745 | ＋ 5.21 +1.68 | ＋0．60 | 3 | ．116 $\pm 6$ | ＋ 5.77 |
|  | ＋ $11^{\circ} 2576$ | CC 782 | $\begin{array}{lll}13 & 24.9\end{array}$ | ＋1055 | ＋10．37 | ＋1．28 | 3 | ．123 $\pm 10$ | ＋10．82 |
| 47 | 121370 | $\eta$ Boo | 1350.0 | ＋1854 | ＋ 3.22 | ＋0．52 | 3 | ． $116 \pm 11$ | ＋ 3.54 |
| 48 | 141004 | $\lambda$ Ser | 1541.6 | ＋ 740 | ＋+4.87 | ＋0．52 | 2 | ．095 $\pm 5$ | ＋ 4.76 |
| 49 | 142860 | $\gamma$ Ser | 1551.9 | ＋15 59 | ＋ 4.26 | ＋0．405 | 2 | ．078 $\pm$ | ＋ 3.72 |
| 50＊ | 147379 | Cin 2184 A | 1616.5 | ＋6729 | ＋ 9.79 + | ＋1．23 | 4 | ． $093 \pm 4$ | +9.63 |
| 51. | 150680 | $\zeta$ Her | 1637.5 | ＋3147 | ＋ 3.31 | ＋0．53 | 2 | ．113士 3 | ＋ 3.57 |
| 52. | 151288 | Cin 2238 | 1641.4 | ＋33 41 | ＋ 9.34 | ＋1．23 | 4 | ．113士 6 | +9.60 |
| 53＊ | 154363 | CC 1017 | 1659.8 | － 454 | ＋8．94 | $+1.07$ | 4 | ．091士 4 | ＋ 8.74 |
| 54＊ | －404226 | CC 1018 | 1700.0 | －455 | ＋11．27 | ＋1．23 | 4 | ．091士 4 | $+11.07$ |
| 55 | 157214 | 72 Her | 1716.9 | ＋32 36 | ＋ 5.85 | ＋0．49 | 2 | ．076 $\pm$ | ＋ 5.25 |
| 56 | 157881 | Cin 2322 | 1720.8 | ＋ 214 | ＋ 8.74 | ＋1．21 | 9 | ．124士 5 | ＋ 9.31 |
| 57＊ | 160269 | 26 Dra | 17． 33.9 | ＋6158 | ＋ 5.66 | ＋0．48 | 5 | ． $073 \pm 4$ | ＋ 4.98 |
| 58. | ＋68 ${ }^{\circ} 946$ | Cin 2354 | $17^{\circ} 37.0$ | ＋6826 | ＋10．36 | ＋1．30 | 3 | $0.213 \pm 5$ | ＋11．94 |

TABLE 3－Continued

| No． <br> （1） | HD（DM） <br> （2） | Name <br> （3） | $\begin{array}{\|c\|} a(1900) \\ (4) \end{array}$ | $\begin{gathered} \delta(1900) \\ (5) \end{gathered}$ | $\begin{gathered} P g_{p} \\ (6) \end{gathered}$ | $\begin{aligned} & C_{p} \\ & (7) \end{aligned}$ | （8） | $\underset{(9)}{\pi(\text { Trig.) }}$ | $\begin{gathered} M_{P o_{p}} \\ (10) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 59＊ | $+4^{\circ} 3561$ | CC 1069 | $17^{\text {b }} 52 \mathrm{~m} 9$ | $+4^{\circ} 25^{\prime}$ | $+11 \mathrm{~m} 03$ | $+1 \mathrm{~m} 53$ | 9 | $0.543 \pm 3$ | $+14 \mathrm{~m} 71$ |
| 60. | 166620 | HR 6806 | 1806.4 | ＋3828 | $+7.13$ | ＋0．79 | 3 | ．098土 7 | ＋ 7.09 |
| 61＊ | 170153 | $\chi$ Dra | 1822.9 | ＋72 41 | ＋ 3.94 | ＋0．395 | 3 | ．127 $\pm 7$ | ＋ 4.46 |
| 62. | 172167 | $\boldsymbol{a}$ Lyr | 1833.6 | ＋38 41 | $-0.08$ | －0．095 | 3 | ．121士 4 | ＋ 0.34 |
| 63 | 180617 | CC 1143 | 1912.1 | ＋ 503 | ＋10．36 | ＋1．28 | 9 | ．170士 4 | ＋11．51 |
| 64 | 185144 | $\sigma$ Dra | 1932.6 | ＋69 29 | ＋ 5.31 | ＋0．715 | 3 | ．181士 4 | ＋ 6.60 |
| 65 | 187642 | a Aql | 1945.9 | ＋ 836 | ＋ 0.88 | ＋0．165 | 2 | ．208士 5 | ＋ 2.47 |
| 66 | 188512 | $\beta$ Aql | 1950.4 | ＋ 609 | ＋ 4.45 | ＋0．77 | 10 | ．077士 5 | ＋ 3.88 |
| 67. | 199305 | Cin 2707 | 2051.3 | ＋6149 | ＋ 9.78 | ＋1．27 | 4 | ．138士 5 | ＋10．49 |
| 68＊ | 201091 | 61 Cyg A | 2102.4 | ＋38 15 | ＋ $6.20 \dagger$ | ＋1．05 | 2 | ．296士 3 | ＋ 8.56 |
| 69＊ | 201092 | 61 Cyg B |  |  | ＋ $7.17 \dagger$ | ＋1．23 | 2 |  | ＋ 9.53 |
| $70^{*}$ | 210027 | ${ }^{\wedge} \mathrm{Peg}$ | 2202.4 | ＋24 51 | ＋ 4.05 | ＋0．34 | 3 | ． $077 \pm 6$ | ＋ 3.48 |
| 71 | $-15^{\circ} 6290$ | CC 1387 | 2247.9 | －14 47 | ＋11．43† | ＋1．35 | 1 | ． $220 \pm 11$ | ＋13．14 |
| 72 | 216899 | CC 1392 | 2251.8 | ＋1620 | ＋ 9.89 | ＋1．285 | 3 | ．161士 7 | $+10.92$ |
| 73 | 216956 | ${ }_{a}$ Psc A | 2252.1 | －30 09 | $+1.11$ | ＋0．015 | 2 | ．146土 7 | ＋1．93 |
| 74 | 217014 | 51 Peg | 2252.6 | ＋20 14 | ＋ 5.92 | ＋0．54 | 3 | ． $072 \pm 7$ | ＋ 5.21 |
| 75 | 219134 | HR 8832 | 2308.5 | ＋56 37 | ＋ 6.39 | ＋0．875 | 4 | ． $160 \pm 6$ | ＋ 7.41 |
| 76 |  | CC 1445 | 2337.0 | ＋43 39 | ＋13．81 $\dagger$ | ＋1．59 | 1 | ． $316 \pm 9$ | ＋16．31 |
| 77 | $+1^{\circ} 4774$ | Cin 3124 | 2344.0 | ＋ 152 | ＋10．17 | ＋1．24 | 5 | ．167 $\pm 6$ | $+11.30$ |
| 78＊ | 224930 | 85 Peg | 2356.9 | ＋26 33 | ＋ 6.22 | ＋0．55 | 5 | $0.086 \pm 4$ | ＋ 5.89 |

＊The notes for stars marked with an asterisk are as follows：
StAR
1，2．$\rho \sim 40^{\prime \prime}, \theta \sim 60^{\circ} . \quad$ 5，6．Common proper motion，$\rho \sim 300^{\prime \prime}$.
3．ADS $671, \Delta m(P g)>4^{\mathrm{m}}$ ，measured as one star．12．Spectroscopic binary．
4．Van Maanen 2.
$13,14 . \rho \sim 160^{\prime \prime}, \theta \sim 100^{\circ}$ ．
22．The fact that this star lies 0.75 mag．above the dwarf sequence cannot be wholly the result of a poor parallax，for several mod－ ern determinations are in good agreement with one another．The radial velocity is variable in a long period，and the displace－ ment from the dwarf sequence probably is due to a bright companion．
26，27．ADS 4334，$\rho \sim 100^{\prime \prime}, \theta \sim 0^{\circ}$ ．
$34,35,36$ ．Common proper motion and radial velocity．
50．ADS 10157，$\Delta m(P g)>3^{m}$ ，measured as one star．
53,54 ．Common proper motion and radial velocity．
57．ADS 10660，$\Delta m(P g)>3^{m}$ ，measured as one star． 68,69 ．ADS $14636, \rho \sim 30^{\prime \prime}, \theta \sim 140^{\circ}$ ．
59．Barnard＇s proper－motion star．
61．Astrometric and spectroscopic binary．
70．Spectroscopic binary． $\dagger$ Stars observed with the Crossley reflector．
the exception， 0 ＂． 080 for $\gamma$ Cep，exceeds the mean trigonometric value， $0.067 \pm 5$ ，by only two and a half times the probable error．

It is of interest to compare $\epsilon \mathrm{Cep}$ and $\delta \mathrm{Gem}$ with cluster stars of similar luminosity and color，as in the accompanying tabulation．The Mount Wilson spectral classifications

| Star | M | $C_{p}$ | Spectral Type |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Mt．W． | Allas |
| $\epsilon$ Cep． | $+2^{\text {m }} 45$ | $+0^{\text {m．} 22}$ | A6n | F0 V |
| Hyades 75. | $+2.55$ | ＋0．22 | A2 | A1，F5 IV＝F0 II－III（metallic） |
| Hyades 47. | ＋2．62 | $+0.225$ | A6n | A5（K），F2（metallic） |
| $\delta$ Gem． | ＋2．64 | ＋0．225 | A8n | F2 IV |

for $\epsilon$ Cep and $\delta$ Gem are taken from Mount Wilson Contributions，No．511；4 the data for the metallic－line stars are from Paper I．The comparison indicates that the stars $\epsilon$ Cep

[^0]and $\delta$ Gem are quite similar to the two Hyades metallic-line stars; but perhaps the metallic-line tendencies, if present, are not so pronounced as in the Hyades stars.

Four stars in the last section of Table 4 do not have luminosity classifications available on the Atlas system, but spectroscopic absolute magnitudes are given in the Mount Wilson catalogue of spectroscopic parallaxes ${ }^{4}$ (see accompanying table). Since the spectro-

| Star | Sp. | $M$ (Vis) | Star | Sp. | $M$ (Vis) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 74 Ori.... <br> 40 Leo. <br> HR 5691. | $\begin{aligned} & \text { F5 } \\ & \text { F5 } \\ & \text { F6 } \end{aligned}$ | $\begin{aligned} & +3 \mathrm{~m} .4 \\ & +3.3 \\ & +3.1 \end{aligned}$ | $5 \text { Ser. }$ $\qquad$ <br> Mean | F6 | $+3 \mathrm{~m} 3$ |
|  |  |  |  |  |  |
|  |  |  |  | F5. 5 | +3.3 |



Fig. 1.-Color-luminosity array for stars within 14 parsecs of the sun. The filled circles represent stars within 10 parsecs; the open circles stars from 10 to 14 parsecs distant; and the small dots, along the subgiant extension of the bright-dwarf sequence, those more distant stars in Table 4 that were used to define this portion of the sequence. The continuous lines in the color-luminosity array are defined by cluster stars in Papers I, II, and III, while the dashed lines are added by the stars under the present discussion. The cross at $M=7.24, C_{p}=0.96$, represents HR 1614 , which probably has an undiscovered bright companion.

TABLE 4
Photometric, Spectroscopic, and Trigonometric
Data for the Nearer Subgiants

| Star | $P g_{p}$ | $C_{p}$ | $n$ | $\pi$ (Trig.) | $\pi$ (Ptm)* | $M$ (Trig.) | Atlas $\dagger$ | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 UMa . | 4 m 28 | $+0^{\text {m }} 33$ | 3 <br> Mean | $\begin{aligned} & 0 " 070 \pm 9(\mathrm{~A}) \\ & .071 \pm 10(\mathrm{M}) \\ & 0.075 \pm 6(\mathrm{~S}) \end{aligned}$ |  |  |  | Visual binary with $\Delta m=1.8$ mag., so the |
|  |  |  |  | $0.073 \pm 4$ | 0"073 | $+3 \mathrm{~m} 60$ | F4 IV | probably are little affected |
| $\gamma$ Ser.. | 4.26 | $+.405$ |  | $\begin{aligned} & 0.056 \pm 10(\mathrm{~A}) \\ & 0.101 \pm 11(\mathrm{M}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.078 \pm 7$ | . 074 | +3.72 | F6 IV |  |
| $\eta$ Boo. | 3.22 | $+.52$ | 3 | $0.116 \pm 11$ ? | . 119 | +3.54 | G0 IV | Trig. $\pi$ quoted by Kuiper |
| $\zeta$ Her . | 3.31 | $+.53$ | $2$ | $\begin{gathered} 0.117 \pm 9(\mathrm{~A}) \\ .110 \pm 10(\mathrm{M}) \\ .117 \pm 10(\mathrm{~S}) \\ 0.104 \pm 5(\mathrm{y}) \end{gathered}$ |  |  |  | $\begin{aligned} & \text { ADS } 10157, \Delta m \\ & =2.8 \text { (visual) } \end{aligned}$ |
|  |  |  | Mean | $0.112 \pm 3$ | . 115 | +3.57 | G0 IV |  |
| $\beta$ Aql. | 4.45 | $+.77$ | $10$ | $\begin{gathered} 0.081 \pm 9(\mathrm{~A}) \\ .065 \pm 16(\mathrm{~S}) \\ .069 \pm 11(\mathrm{M}) \\ 0.082 \pm 8(\mathrm{Y}) \end{gathered}$ |  |  |  |  |
|  |  |  | Mean | $0.072 \pm 5$ | . 068 | +3.88 | G8 IV |  |
| $\delta$ Gem. | 3.75 | +. 225 | $3$ | $\begin{aligned} & 0.058 \pm 9(\mathrm{~A}) \\ & .063 \pm 7(\mathrm{M}) \\ & 0.059 \pm 8(\mathrm{~W}) \end{aligned}$ |  |  |  | See text |
|  |  |  | Mean | $0.060 \pm 5$ | 063 | $(+2.64)$ | F2 IV |  |
| $\tau$ Boo. | 4.82 | +. 385 |  | $\begin{aligned} & 0.048 \pm 10(\mathrm{~A}) \\ & 0.075 \pm 10(\mathrm{M}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.061 \pm 7$ | . 057 | +3.75 | F6 IV |  |
| $\iota \mathrm{Psc}$. | 4.43 | +. 395 |  | $\begin{aligned} & 0.071 \pm 9(\mathrm{M}) \\ & 0.063 \pm 9(\mathrm{Y}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.067 \pm 6$ | 068 | +3.56 | F8 IV |  |
| $\sigma^{2} \mathrm{UMa}$. | 5.08 | $+.395$ | $3$ | $\begin{gathered} 0.037 \pm 10(\mathrm{M}) \\ .040 \pm 10(\mathrm{y}) \\ 0.078 \pm 10(\mathrm{G}) \end{gathered}$ |  |  |  |  |
|  |  |  | Mean | $0.053 \pm 6$ | . 051 | +3.70 | F6 IV |  |
| $\theta$ Boo. | 4.41 | $+0.41$ |  | $\begin{aligned} & 0.066 \pm 10(\mathrm{~A}) \\ & 0.071 \pm 11(\mathrm{M}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.068 \pm 7$ | 0.069 | +3.57 | F6 IV |  |

[^1]TABLE 4-Continued

| Star | $P_{g}$ | $C_{p}$ | $n$ | $\pi$ (Trig.) | $\pi(\mathrm{Ptm})^{*}$ | $M$ (Trig.) | Atlas $\dagger$ | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v$ And | 4 m 43 | $+0^{\mathrm{m}} 41$ | 3 <br> Mean | $\begin{aligned} & 0.064 \pm 9 \text { (A) } \\ & 0.055 \pm 12 \text { (M) } \end{aligned}$ |  |  |  |  |
|  |  |  |  | $0.061 \pm 7$ | 0"068 | +3m36 | F8 IV |  |
| $\chi$ Her . | 5.03 | $+.47$ | 2 <br> Mean | $\begin{aligned} & 0.064 \pm 7(\mathrm{~A}) \\ & 0.049 \pm 9(\mathrm{M}) \end{aligned}$ |  |  |  |  |
|  |  |  |  | $0.057 \pm 6$ | . 052 | +3.81 | G0 IV |  |
| $10 \text { Tau. }$ | 4.66 | $+.48$ | 3 <br> Mean | $\begin{aligned} & 0.056 \pm 8(\mathrm{~A}) \\ & .047 \pm 8(\mathrm{M}) \\ & 0.060 \pm 10(\mathrm{Y}) \end{aligned}$ |  |  |  |  |
|  |  |  |  | $0.056 \pm 5$ | . 061 | +3.40 | F7 IV |  |
| 70 Vir | 5.50 | $+.635$ | Mean | $\begin{aligned} & 0.037 \pm 8(\mathrm{~A}) \\ & 0.056 \pm 10(\mathrm{M}) \end{aligned}$ |  |  |  |  |
|  |  |  |  | $0.042 \pm 6$ | . 042 | +3.62 | G5 IV-V |  |
| $\eta$ Cep. | 4.17 | $+.81$ | $5$ | $\begin{aligned} & 0.069 \pm 10(\mathrm{~A}) \\ & .067 \pm 13(\mathrm{M}) \\ & 0.069 \pm 12(\mathrm{y}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.069 \pm 7$ | . 076 | +3.36 | K0 IV |  |
| $\gamma$ Cep. | 4.07 | $+.92$ | $2$ | $\begin{aligned} & 0.066 \pm 10(\mathrm{~A}) \\ & .065 \pm 8(\mathrm{M}) \\ & 0.070 \pm 10(\mathrm{G}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.067 \pm 5$, | . 080 | +3.20 | K1 IV |  |
| $v^{2} \mathrm{CMa} .$. | 4.88 | + . 97 | 2 <br> Mean | $\begin{aligned} & 0.059 \pm 9(\mathrm{M}) \\ & 0.051 \pm 8(\mathrm{Y}) \end{aligned}$ |  |  |  |  |
|  |  |  |  | $0.055 \pm 6$ | 0.055 | +3.58 | K2 III-IV |  |
|  |  |  |  |  | Mean | $+3.60 \pm$ | 0.13(A.D.) |  |
| $\epsilon$ Cep | 4.28 | $+.22$ | 3 <br> Mean | $\begin{gathered} 0.030 \pm 9(\mathrm{~A}) \\ .056 \pm 10(\mathrm{M}) \\ .063 \pm 14(\mathrm{y}) \\ 0.029 \pm 13(\mathrm{~S}) \end{gathered}$ |  |  |  | Discussed in text |
|  |  |  |  | 0.043士 5 | 0.045 | $(+2.45)$ | F0 V |  |
| 74 Ori. | 5.30 | $+.33$ | 3 | $0.042 \pm 7$ (A) | . 046 | +3.42 |  | $\ddagger$ |
| 40 Leo. | 5.07 | +. 37 | 3 | $0.051 \pm 11$ (A) | . 051 | +3.60 |  | $\ddagger$ |
| HR 5691 | 5.51 | $+.44$ |  | $\begin{aligned} & 0.046 \pm 6(\mathrm{~A}) \\ & .045 \pm 10(\mathrm{M}) \\ & 0.051 \pm 9(\mathrm{G}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.047 \pm 5$ | . 042 | +3.87 |  |  |
| 5 Ser. | 5.39 | +0.47 | 3 | $\begin{aligned} & 0.033 \pm 8(\mathrm{~A}) \\ & .035 \pm 12(\mathrm{M}) \\ & 0.048 \pm 10(\mathrm{Y}) \end{aligned}$ |  |  |  |  |
|  |  |  | Mean | $0.038 \pm 6$ | 0.044 | +3.29 |  |  |

$\ddagger$ Further trigonometric parallazes are necessary before 74 Ori and 40 Leo can be definitely assigned to the bright-dwarf or dwarf sequences. The fact that the luminosity difference between the two sequences near the color of these stars is very small will make the assignment difficult unless spectroscopic criteria are applicable.
scopic luminosity is in terms of visual magnitudes, we must add +0.4 mag., the color of F5.5 taken from Figure 2, to the mean, which yields $\bar{M}=+3$ m. 7 , in excellent agreement with the value of $\bar{M}=+3 \mathrm{~m} .60 \pm 0 \mathrm{~m} .13$ (A.D.) for the subgiants in Table 4.

The relationship between color and spectral types of the subgiants is given in Figure 2, from Atlas spectral types only. It is a curious fact that subgiants of type later than K2 must be quite rare, since none have been found in the following three independent surveys:

1. "Atlas."-No subgiants of type later than K2 are included in this monograph.
2. Keenan. ${ }^{5}$-In determining the spectroscopic luminosities and spectral types for stars brighter than the eighth visual magnitude and of Henry Lraper Catalogue types G5-K5 in two fields-one in Cassiopeia and the other centered at the north galactic pole-Keenan found that ". . . among the stars later than K2, none was classified as a subgiant; this was true also of the calibration stars."
3. McCuskey. ${ }^{6}$-Observations of spectra of all stars to photographic magnitude 10.5 in a field in Aquila of 14.3 square degrees revealed no subgiants later than K0.


Fig. 2.-The color-spectral-type relationship for subgiants (luminosity class IV) in the Atlas. The open circles represent individual stars; the filled circles, means of two or more stars of the same spectral type.

Since the subgiants redder than $C_{p}=+0^{\mathrm{m}} 32$ appear to be of constant photographic luminosity, surveys Nos. 2 and 3 above may indicate the luminosity function of subgiants in the regions covered. The lack of subgiants later than K2, however, might mean only that the spectroscopic-luminosity criteria are not valid for later types, since the brighter visual magnitude may place these stars, spectroscopically, in the category of normal giants.

Since the relationship between color and magnitude of the subgiants is dependent upon the region of the spectrum in which we observe these quantities, it is accidental that the photographic absolute magnitude is essentially constant for all subgiants redder than

[^2]$C_{p}=+0^{\mathrm{m}} .32$ (regardless of color). ${ }^{7}$ It is, however, a fortunate accident, and it may have many astrophysical applications. A good example of such an application is to the study of those eclipsing systems possessing either one or two subgiant components. A program to observe these systems is now in progress at this observatory, and a few preliminary results, relevant to the subject under discussion, follow.

## SUBGIANTS IN ECLIPSING BINARIES

Algols.-An Algol system usually contains a small, bright, late B- or early A-type component and a fainter, larger subgiant companion. Three classical examples of this type of system are U Cep, U Sge, and the prototype star itself. All three of these systems have been observed at maximum light, and $U C e p$ and $U$ Sge have also been observed during the total phase of the primary eclipse. The results are given in the accompanying tabulation. The depths of the total eclipses immediately give the luminosities of the brighter

stars, in terms of the total light of the system; hence we can compute the individual magnitudes and colors. The small reflection and ellipticity effects, deduced from existing light-curves, may also be allowed for. In the case of Algol, the results of a previous sixcolor study ${ }^{8}$ may be used to determine the difference in magnitude between each component and the combined light of the system. The resulting magnitudes and colors for both components of the three systems are given in the accompanying tabulation. The

| Star | Component 1 |  | Component 2 |  | $\begin{gathered} M(2) \\ \text { (Assumed) } \end{gathered}$ | $M-m$ | $M(1)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P g_{p}$ | $C_{p}$ | $P g_{p}$ | $C_{p}$ |  |  |  |
| U Cep. | 6 m .65 | $-0 \mathrm{~m} 14$ | 9 m 65 | +0. ${ }^{\text {m }} 75$ | $+3 \mathrm{~m} 60$ | $-6.05$ | $+0^{\mathrm{m}} .60$ |
| U Sge. | 6.32 | $-.15$ | 9.92 | +.80 | +3.60 | $-6.32$ | . 00 |
| Algol. | 1.96 | $-0.16$ | 5.42 | +0.59 | $+3.60$ | $-1.82$ | +0.14 |

absolute magnitudes of the secondary components, $M$ (2), are derived from the assumption that they are subgiants. The reason for this assumption is that the resulting moduli for the systems give luminosities for the primary components, $M(1)$, that are consistent with the colors and spectral types. We see that these stars are very similar to some of the Pleiades stars that populate the blue-dwarf sequence discussed in Paper II; examples of such stars are Pleiades No. 67, with $M=+0^{\mathrm{m}} 20$ and $C_{p}=-0 \mathrm{~m} .15$, and No. 28, with $M=+0^{\mathrm{m}} 53, C_{p}=-0^{\mathrm{m}} 13$.

These results, if the basic assumption of subgiant character for the secondary components is correct, show that negligible reddening or selective absorption, either circum-

[^3]stellar or interstellar, affects the brighter stars of these three systems. ${ }^{9}$ The eclipsing binaries U Cep and U Sge are too distant to have reliable trigonometric parallaxes, but the photometric parallax of Algol, 0 ".043, agrees well with two trigonometric values, $0.044 \pm 4(\mathrm{~A})$ and $0.034 \pm 5(\mathrm{~S}),{ }^{10}$ both of which are based upon many plates.

AR Lacertae-type systems.-Another type of eclipsing system known to contain subgiant components is represented by the prototype, AR Lac. Photoelectric observations of this system by G. E. Kron ${ }^{11}$ present the first photometric evidence for possible largescale surface activity, such as dark and bright patches, on stars other than the sun. W. E. Harper ${ }^{12}$ determined velocity-curves for both components of the system, and A. B. Wyse, ${ }^{13}$ who classified the stars as G5 and K0, noted sharp emission features at H and K of $C a$ II. Observations of magnitude and color made at maximum light and at primary total eclipse give

$$
\begin{aligned}
\text { Maximum light: } P g_{p}=6 . \mathrm{m} .87, & C_{p}=+0^{\mathrm{m}} 66 \\
\text { Total eclipse: } P g_{p}=7 . \mathrm{m} 69, & C_{p}=+0 \mathrm{~m} .83
\end{aligned}
$$

F. B. Wood ${ }^{14}$ was unable to separate the reflection and ellipticity effects in the lightcurve from the intrinsic variation of the light of the system. Kron, however, obtained light-curves during intervals when the source of the intrinsic variation was relatively quiescent; and from these curves we find that the depth of primary eclipse given above must be corrected by -0.07 mag., in the photographic region. Although the ellipticity and reflections effects are wave-length dependent, we have arbitrarily corrected the visual depth by the same amount, since the error introduced can hardly amount to more than 0.01 mag . We see, therefore, that at the total eclipse half the photographic light of the system is lost; this result we might have anticipated from our previous conclusion that, photographically, the subgiants redder than $C_{p}=+0^{\mathrm{m}} 32$ have the same absolute magnitude, regardless of their color. We find, then, the colors and magnitudes of the individual components to be as follows:

$$
\begin{array}{ll}
\text { Blue component: } P g=7 \mathrm{~m} 65, & C_{p}=+0 \mathrm{~m} 55 ; \\
\text { Red component: } P g_{p}=7 \mathrm{~m} 65, & C_{p}=+0 \mathrm{~m} .82 .
\end{array}
$$

The modulus of the system, from the assumption that both components are subgiants with $M=+3^{m} .60$, is 4.05 mag., or a photometric parallax of 0.10155 . From the colors of the components and from the color-spectral-type relation of Figure 2, we can estimate that the spectral type of the primary-eclipsed star, which is also known from the photometric data to be the smaller, is G2 and that the larger component is K0.

There are a few other stars mentioned in the literature that may be similar to AR Lac in that both components are subgiants. When approximate spectral classes are known for the components and both are later than, say, F5 but are separated by several subclasses and when the light-curve exhibits a total eclipse during which half the total photographic light is lost, the system is quite likely to be composed of two subgiants. If the

[^4]elements are known, we should expect the star of later spectral type to be the larger. A good example of such a system is RS Ari. Wyse ${ }^{13}$ called the spectra F9 and G5, and stated: "The primary spectrum [F9] is intermediate between giant and dwarf." Many visual observations give the primary eclipse, which is total, a depth of 1.0 mag . From Figure 2 we find that a G5 star is 0.2 mag . redder than one of class F9, so that the photographic depth of the total eclipse, during which the F9 star is occulted, is 0.8 mag.; when allowance is made for reflection and ellipticity effects, we may conclude that, photographically, the stars are of about the same magnitude. The elements quoted by the Gaposchkins ${ }^{15}$ indicate that the G5 star is the larger of the two.

A similar system is RW UMa, the components of which Wyse ${ }^{13}$ has classified as F9 and G9. The primary eclipse is total, with a visual depth of 1.10 mag., or 0.75 mag . photographically if we correct for the difference in color between the two stars. Struve ${ }^{16}$ determined the velocity-curve of the F9 star, which is the star occulted at the total eclipse. He also measured ". . . two fairly well-marked and narrow. emission lines of $C a$ II K and H . These lines belong to the eclipsing star [G9] at primary eclipse and have about the same amplitude as the absorption lines of the [visually] brighter component [F9]. ${ }^{17}$

## IV. DWARF AND SUBDWARF SEQUENCES

DWARFS
The dwarf sequence has been defined in Papers I, II, and III by the cluster stars that are gathered together in Table 5; they range in color from $C_{p}=+0$ m. 02 to $+1^{\text {m. }} 05$. The main sequence defined by these stars is given in Table 6, but the individual deviations from this main sequence are tabulated as $\mathrm{O}-\mathrm{S}$ (observed minus sequence) in the last column of Table 5. The sequence between $C_{p}=+0 \mathrm{~m} .06$ and $+0^{\mathrm{m}} .16$ is not well defined, owing to a real lack of cluster stars in this interval, and the main sequence is represented by only one point in this range. Table 7 contains the dwarf stars from Table 3, with the individual modern determinations of the trigonometric parallaxes and their means. The photometric parallaxes given in the penultimate column of Table 7 are derived from the colors of the stars and from the dwarf sequence of Table 6. In nearly every instance the photometric parallax falls within the range of the trigonometric determinations. There is some selection of the stars in Table 7, in that, generally, only stars with several modern determinations of parallax were included on the observing program. The values of $\mathrm{O}-\mathrm{S}$ in Table 7 are in the sense of mean trigonometric parallax minus the photometric parallax. When we recall that the zero point of the dwarf sequence given in Table 6 is defined by the Hyades stars, the mean systematic difference in $\mathrm{O}-\mathrm{S}$ of $-0^{\prime \prime} .001$ indicates the sameness of cluster and noncluster dwarfs.

A few dwarfs, more distant than 14 parsecs, are included in Table 8; the arrangement of the material is the same as in the previous table. Although it is true that for these stars the trigonometric parallaxes are, on the whole, rather discordant, the photometric parallax usually falls within the range of the individual determinations. Only those dwarfs redder than $C_{p}=+0$ m. 20 are discussed here; the bluer stars, that is to say, those earlier than about F0, are discussed in connection with the blue-dwarf and bright bluedwarf sequences in Section V.

[^5]TABLE 5
Luminosities and Colors of Cluster Dwarfs

| Name* | $M_{P g_{p}}$ | $C_{p}$ | O-S | Name | $M_{P O_{p}}$ | $C_{p}$ | O-S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C 130. | +0m. 42 | $+0 \mathrm{~m} 03$ | $-0 \mathrm{~m} 08$ | C 86. | $+4^{\text {m }} 32$ | $+0^{m} 35$ | $+0.02$ |
| C 107. | +0.58 | +. 03 | + . 08 | C 162. | +4.32 | +0.36 | -. 06 |
| H 60 | +0.96 | +. 08 | - . 06 | C 114. | +4.37 | +0.36 | -. 01 |
| H 117 | +1.28 | +. 09 | + . 04 | HR 4867 | +4.41 | +0.36 | $+.03$ |
| H 139. | +1.27 | +. 095 | - . 03 | P 23. | +4.43 | +0.38 | -. 08 |
| H 119 | +1.30 | +. 10 | -. 03 | P 49. | +4.55 | +0.395 | -. 05 |
| H 84. | +2.10 | $+.15$ | - . 04 | C 53. | +4.55 | +0.395 | -. 05 |
| H 34 | $+2.20$ | +. 15 | $+.04$ | H 19 | +4.52 | +0.40 | -. 11 |
| H 141 | +2.37 | +. 16 | +. 07 | P9 | +4.57 | +0.40 | -. 06 |
| H 96. | +2.50 | +. 17 | $+.05$ | C 58 | +4.64 | +0.40 | $+.01$ |
| P 58. | +2.60 | +. 18 | $\pm .00$ | P 55. | +4.72 | +0.41 | +. 03 |
| H 39 | +2.82 | +. 20 | $\pm .00$ | H 10. | +4.87 | +0.43 | +. 06 |
| P 15. | +2.86 | +. 20 | +. 04 | P 53. | +4.87 | +0.45 | -. 08 |
| P 20. | +2.96 | +. 21 | +. 04 | C 76. | +4.98 | +0.45 | +. 02 |
| H 8. | +3.08 | +. 22 | $+.06$ | C 97. | +5.02 | +0.45 | +. 06 |
| H 102 | +3.14 | + . 23 | $\pm .00$ | C 85. | +5.22 | +0.475 | +. 11 |
| H 14. | +3.25 | + . 24 | -. 02 | P 57 | +5.13 | +0.48 | - . 01 |
| P 71 | +3.26 | + . 24 | -. 01 | H20. | +5.15 | +0.48 | + . 01 |
| H 4 | +3.28 | + . 24 | $+.01$ | H 57 | +5.15 | +0.485 | - . 02 |
| C 49 | +3.52 | +. 26 | +. 02 | P 32. | +5.18 | +0.49 | -. 02 |
| 37 UMa | +3.58 | +. 27 | - . 02 | H 15. | +5.38 | +0.50 | +. 11 |
| H 21. | +3.61 | +. 28 | -. 09 | P 38. | +5.28 | +0.51 | - . 06 |
| P 72 | +3.66 | +. 28 | - . 04 | H 18 | +5.44 | +0.53 | -. 01 |
| H 13 | +3.76 | +. 29 | -. 03 | HD 115043 | +5.63 | +0.54 | +. 11 |
| C 19 | +3.82 | + . 29 | +. 03 | H 24. | +5.79 | +0.56 | +. 13 |
| C 36 | +3.82 | + . 29 | +. 03 | C 132 | +5.86 | +0.60 | - . 01 |
| H 38. | +3.88 | + . 29 | + . 09 | CH 14. | +6.10 | +0.64 | -. 01 |
| P 24. | +3.97 | +. 30 | +. 08 | CH 21 | +6.53 | +0.70 | +. 06 |
| C 118. | +4.10 | +. 33 | -. 05 | H 89 | +6.65 | +0.72 | +. 06 |
| C 101 | +4.14 | +. 33 | -. 01 | H 91 | +6.72 | +0.75 | -. 05 |
| H 6. | +4.15 | +. 33 | $\pm .00$ | H 106 | +7.02 | +0.80 | +. 05 |
| P 42. | +4.15 | +. 33 | $\pm .00$ | H 53 | +7.49 | +0.87 | $\pm .00$ |
| H 138. | +4.23 | +. 34 | $\pm .00$ | H 90 | +8.37 | +1.03 | -. 08 |
| C 90. | +4.34 | +0.345 | +0.07 | H 72 | +8.54 | +1.05 | -0.03 |

* $\mathrm{C}=$ Coma Berenices; $\mathrm{P}=$ Pleiades $; \mathrm{H}=$ Hyades .

TABLE 6
The Dwarf Sequence Defined by Cluster Stars
IN THE RANGE $C_{p}=+0$ m. 02 TO +1 m .22

| $C_{p}$ | $M_{P_{g_{p}}}$ | $C_{p}$ | $M_{P_{g_{p}}}$ | $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+0^{m} 02$ | $+0^{m} 40$ | +0 ${ }^{\text {m }} 38$ | $+4^{\mathrm{m}} 51$ | $+0^{m} 66$. | $+6 \mathrm{~m} 23$ | $+0^{\text {m }} 94$ | +7m91 |
| +. 04 | +0.60 | +. 40 | +4.63 | + . 68. | +6.35 | +0.96. | +8.03 |
| + . 06. | +0.80 | + . 42 | +4.75 | +. 70 | +6.47 | +0.98. | +8.15 |
| +. 10 | +1.33 | + . 44 | +4.90 | + . 72 . | +6.59 | +1.00 | +8.27 |
| +. 16 | +2.30 | +. $46^{*}$ | +5.01 | + . 74. | +6.71 | +1.02 | +8.39 |
| + . 18 | +2.60 | +. 48 | +5.14 | + . 76. | +6.83 | +1.04 | +8.51 |
| +. 20 | +2.82 | + . 50. | +5.27 | + . 78. | +6.95 | +1.06. | +8.63 |
| + . 22 . | +3.02 | + . 52. | +5.39 | + . 80 . | $+7.07$ |  |  |
| + . 24 . | +3.27 | + . 54. | +5.51 +5.51 | + . 82. | +7.19 | +1.08 $\dagger$. | +8.75 |
| +.26. | +3.50 | + . 56. | +5.63 | + . 84. | +7.31 | +1.10. | +8.87 |
| + . 28. | +3.70 | + . 58. | +5.75 | + . 86 . | +7.43 | +1.12. | +8.99 |
| + . 30 . | +3.89 | +. 60 . | +5.87 | + . 88. | $+7.55$ | +1.14. | +9.11 |
| +. 32 . | +4.07 | +.62. | +5.99 | +. 90 | +7.67 | +1.16 | +9.23 |
| + . 34. | +4.23 | +0.64. | +6.11 | +0.92. | +7.79 | +1.18. | +9.35 |
| +0.36. | +4.38 |  |  |  |  | +1.20 | +9.47 |
|  |  |  |  |  |  | +1.22. | +9.59 |

[^6]SUBDWARFS
In 1922 Adams and Joy ${ }^{18}$ discovered＂．．．three stars of large proper motion，which，in absolute magnitude，seem to form a group midway between the faint＇white dwarfs＇．．． and the stars of early type belonging to the main sequence＂；they called these stars＂in－ termediate white dwarfs．＂In 1938 Kuiper ${ }^{19}$ pointed out that these stars existed all along the main sequence，and he stated that＂．．．since these stars ．．．are much more similar

TABLE 7
Comparison of the Trigonometric and Photometric
Parallaxes of the Dwarfs in Table 3

| Star | Trigonometric Parallaxes |  |  |  |  |  | $\underset{\text { Ptm．}}{\text { ¢ }}$ | O－S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | M | S | Y | Others | Mean |  |  |
| $\eta$ Cas A | $0 " 179 \pm 9$ | $0 " 182 \pm 80$ | 0＂194土 8 |  |  | $0 " 184 \pm 5$ | 0＂175 | $+0.009$ |
| 107 Psc． | ．128土 7 | ． $146 \pm 11$ |  |  |  | ． $132 \pm 6$ | ． 134 | $-.002$ |
| HR 511 |  | ．111 $\pm 9$ |  |  | 0＂116 $\pm 10$（W） | ． $111 \pm 5$ | ． 105 | $+.006$ |
| $\delta$ Tri． | ． $065 \pm 9$ | ． $135 \pm 9$ | ． $087 \pm 11$ |  |  | ． $098 \pm 6$ | ． 096 | ＋． 002 |
| HR 753 | ． $149 \pm 9$ | ．141 7 |  | $0 \prime 153 \pm 8$ |  | ． $147 \pm 4$ | ． 153 | －． 006 |
| HR 857 |  | ．139士 9 |  |  | ． $134 \pm 10$（C） | ． $137 \pm 6$ | 137 | ． 000 |
| $\kappa$ Cet | $115 \pm 7$ | ． $092 \pm 10$ |  | $104 \pm 10$ |  | ． $106 \pm 5$ | 125 | －． 019 |
| $\epsilon$ Eri |  | ． $303 \pm 9$ |  | $298 \pm 10$ | $.301 \pm 6$（C） | ． $303 \pm 6$ | 320 | －． 017 |
| 40 Eri A | $201 \pm 6$ | $210 \pm 9$ |  | $200 \pm 11$ | $.190 \pm 9(\mathrm{C})$ | 202 $\pm 3$ | 200 | $+.002$ |
| $\pi^{3}$ Ori． | ．128土 8 | ． $156 \pm 11$ |  | $110 \pm 9$ |  | $128 \pm 5$ | 110 | $+.018$ |
| HR 1925 A | ．086 $\pm 8$ | $105 \pm 10$ |  |  |  | ． $097 \pm 5$ | ． 095 | $+.002$ |
| $\gamma$ Lep A | ．147 $\pm 9$ | ． $104 \pm 6$ |  |  | $.136 \pm 8(\mathrm{C})$ | $122 \pm 4$ | ． 125 | $-.003$ |
| $\gamma$ Lep B | $147 \pm 9$ | ． $104 \pm 6$ |  |  | ． $136 \pm 8$（C） | ． $122 \pm 4$ | 125 | －． 003 |
| $\chi^{1}$ Ori． | 098 $\pm 8$ | ．110 $\pm 8$ |  |  |  | ． $104 \pm 6$ | 110 | －． 006 |
| $11 \mathrm{LMi} \mathrm{A}$. | $127 \pm 8$ | ． $110 \pm 11$ | $.113 \pm 19$ |  | $(.065) \pm 8(\mathrm{~W})$ | ． $113 \pm 6$ | 112 | $+.001$ |
| 36 UMa A． | ． $074 \pm 9$ | ．085 $\pm 9$ | $.102 \pm 14$ |  |  | ． $082 \pm 8$ | 080 | ＋． 002 |
| 61 UMa | ．109 9 |  |  |  |  | ． $109 \pm 9$ | 107 | ＋． 002 |
| $\beta \mathrm{CVn}$ ． | ．114士 9 | ． $107 \pm 10$ | ． $093 \pm 18$ |  |  | ． $108 \pm 6$ | 110 | －． 002 |
| $\beta$ Com |  | $.124 \pm 10$ | $133 \pm 18$ |  |  | $.121 \pm 6$ | 122 | $-.001$ |
| 61 Vir． |  | $.131 \pm 11$ |  | ． $127 \pm 9$ |  | ． $129 \pm 7$ | 128 | $+.001$ |
| 72 Her | $071 \pm 11$ | ． $074 \pm 11$ | $.062 \pm 13$ |  |  | ． $076 \pm 6$ | 074 | $+.005$ |
| 26 Dra |  | $.073 \pm 10$ | $.078 \pm 16$ | $0.078 \pm 6$ |  | ． $073 \pm 4$ | 078 | $-.002$ |
| HR 6806 | ．098土 9 | ． $109 \pm 13$ |  |  |  | ． $098 \pm 7$ | 095 | $+.003$ |
| $\chi$ Dra |  | $.124 \pm 10$ | ． $120 \pm 9$ |  | $136 \pm 10$（G） | $.127 \pm 7$ | 135 | －． 008 |
| $\sigma$ Dra | ． $180 \pm 7$ | ． $184 \pm 9$ |  |  | ． $168 \pm 12$（G） | $181 \pm 4$ | 178 | $+.003$ |
| 61 Cyg A | ． $288 \pm 6$ | ． $310 \pm 6$ | $.271 \pm 14$ |  | $0.320 \pm 10$（W） | ． $296 \pm 3$ | 296 | ． 000 |
| 51 Peg． | ． $072 \pm 9$ | ． $076 \pm 10$ |  |  |  | ． $072 \pm 7$ | ． 083 | $-.011$ |
| HR 8832 | $0.140 \pm 8$ | $0.161 \pm 10$ | $0.179 \pm 13$ |  |  | $0.160 \pm 6$ | 0.168 | －0．008 |
|  |  |  |  |  |  | Av．de |  | $\left\{\begin{array}{l}-0.001 \\ \pm 0.005\end{array}\right.$ |

to main－sequence stars than to white dwarfs，the name＇subdwarfs＇is suggested for this class of stars，in analogy with＇subgiant．＇＂This group of objects contains＂stars not over 2 or 3 mag．below the main sequence．＂In 1947 he further stated that＂．．．subdwarfs are all or nearly all，high velocity stars．${ }^{\prime 20}$ Joy ${ }^{21}$ has summarized the spectroscopic fea－ tures that distinguish subdwarfs from ordinary dwarfs．We shall apply the term＂sub－ dwarf＂to the stars populating that sequence which，in Figure 1，runs below and parallel

[^7]TABLE 8
Photometric and Trigonometric Data for Dwarfs
More Distant than 14 Parsecs

| Star | $P g_{p}$ | $C_{p}$ | $n$ | $\pi$ (Trig.) | $\pi$ (Ptm) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 6 \text { Cet. . . } \\ -4^{\circ} 3049 . \end{gathered}$ | 5 m 16 | $-0^{\text {m }} 36$ | 3 | $0.062 \pm 10$ (Y) | 0.070 |
|  | 8.02 | $+.62$ | 2 <br> Mean | $\begin{aligned} & 0.047 \pm 12 \text { (M) } \\ & .065 \pm 11 \text { (Y) } \\ & 0.043 \pm 11 \text { (C) } \end{aligned}$ |  |
|  |  |  |  | $0.049 \pm 7$ | . 040 |
| HR 4867. | 6.13 | $+.36$ | 5 <br> Mean | $\begin{aligned} & 0.038 \pm 10(\mathrm{~A}) \\ & 0.041 \pm 11(\mathrm{~W}) \end{aligned}$ |  |
|  |  |  |  | $0.039 \pm 7$ | . 045 |
| HR 5455. | 6.46 | $+.35$ | 3 <br> Mean | $\begin{aligned} & 0.035 \pm 6(\mathrm{~A}) \\ & .051 \pm 10(\mathrm{M}) \\ & 0.034 \pm 8(\mathrm{Y}) \end{aligned}$ |  |
|  |  |  |  | $0.036 \pm 4$ | . 037 |
| 45 Boo.. | 5.20 | $+.34$ | 2 <br> Mean | $\begin{aligned} & 0.058 \pm 10(\mathrm{~A}) \\ & 0.071 \pm 12(\mathrm{M}) \end{aligned}$ |  |
|  |  |  |  | $0.063 \pm 8$ | 064 |
| $+39^{\circ} 2950$. | 9.27 : | $+.84$ | 3 <br> Mean | $\begin{aligned} & 0.054 \pm 7(\mathrm{~A}) \\ & 0.062 \pm 6(\mathrm{y}) \end{aligned}$ |  |
|  |  |  |  | $0.052 \pm 6$ | . 041 |
| $+0^{\circ} 3593$. | 7.26 | $+.68$ | 3 | $\begin{gathered} 0.082 \pm 10(\mathrm{~A}) \\ .033 \pm 12(\mathrm{M}) \\ .073 \pm 8(\mathrm{Y}) \\ 0.047 \pm 8(\mathrm{~W}) \end{gathered}$ |  |
|  |  |  | Mean | $0.061 \pm 5$ | . 066 |
| $+1^{\circ} 3421$. | 7.32 | $+.43$ | 3 <br> Mean | $\begin{aligned} & 0.040 \pm 10(\mathrm{~A}) \\ & 0.032 \pm 8(\mathrm{Y}) \end{aligned}$ |  |
|  |  |  |  | $0.036 \pm 6$ | . 032 |
| $+4^{\circ} 3599$. | 7.19 | +. 475 | 6 | $\begin{aligned} & 0.051 \pm 8(\mathrm{~A}) \\ & .041 \pm 9(\mathrm{M}) \\ & 0.033 \pm 8(\mathrm{Y}) \end{aligned}$ |  |
|  |  |  | Mean | $0.041 \pm 5$ | . 038 |
| $\theta$ Cyg | 4.70 | +0.30 | 2 <br> Mean | $\begin{aligned} & 0.060 \pm 9(\mathrm{~A}) \\ & 0.083 \pm 11(\mathrm{M}) \end{aligned}$ |  |
|  |  |  |  | $0.067 \pm 7$ | 0.069 |

to the dwarf sequence from $C_{p}=+0^{\mathrm{m}} .60$ to +1 m .23 and crosses the dwarf sequence at $C_{p}=+0 \mathrm{~m} .55$ and +1 m 35 . Although this nomenclature presents a situation in which subdwarfs may be brighter than dwarfs, it is analogous to the fact that some "main-sequence dwarfs" are brighter than giants. As stated in Paper II, all these designations are unsatisfactory, and they should be replaced by less misleading ones as soon as the colorluminosity array is more completely mapped.

The stars populating the subdwarf sequence in Figure 1 are listed in Table 9, which also includes three probable subdwarfs found in the Hyades cluster. Two stars-Cin 1244, with $M=+12^{\mathrm{m}} 16$ and $C_{p}=+1 \mathrm{~m} 32$, and Lal 21185, with $M=+11^{\mathrm{m}} 75$ and $C_{p}=$ +1 m 32 -occur near the junction of the two sequences, so that it is difficult to say to which group they belong; both stars are omitted from Table 9.

TABLE 9
Luminosities and Colors of Subdwarfs

| Star | $M_{P o_{p}}$ | $C_{p}$ | Star | $M_{P g_{p}}$ | $C_{p}$ | Star | $M_{P g_{p}}$ | $C_{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta$ Vir* | 3 m .93 | $+0^{\text {m }} 49$ | 85 Peg A | 5.89 | $+0^{\text {m }} 55$ | CC 1018 | 11 m 07 | $+1 \mathrm{~m} 23$ |
| $\iota$ Per | 4.11 | $+.50$ | Hyades 17 $\ddagger$ | 6.01 | + . 565 | Cin 3124 | 11.30 | +1.24 |
| $\lambda$ Aur | 4.28 | + . 51 | $\mu \mathrm{Cas}$ | 6.31 | $+.58$ | CC 1143 | 11.51 | +1.28 |
| $\lambda$ Ser. | 4.76 | + . 52 | Hyades 7 $\ddagger$ | 6.39 | +. 58 | Cin 2354 | 11.94 | +1.30 |
| HR 483 | 5.06 | $+.535$ | $\tau$ Cet. | 6.42 | + . 585 | CC 1069. | 14.71 | $+1.53$ |
| 51 Peg $\dagger$ | 5.21 | $+. .54$ | HR 4550 | 7.22 | $+.635$ | ADS 246 B | 14.96 | +1.55 |
| Hyades $40 \ddagger$ | 5.80 | $+0.55$ | 36 UMa. | 8.56 | +0.87 | CC 1445.... | 16.31 | $+1.59$ |

[^8]Table 10 contains some of the stars in Table 3 that populate the dwarf sequence from $C_{p}=+0^{\mathrm{m}} 30$ to +1 m 30 , together with the spectral classes assigned these stars by Kuiper, ${ }^{22}$ the Mount Wilson observers, ${ }^{4}$ and the Atlas. The Atlas spectral types have been extended to stars later than K 5 by Morgan. ${ }^{23}$ The resulting color-spectral-type relationships for these stars are plotted in Figure 3, where the open circles represent individual stars and the filled circles are the means of two or more stars of the same spectral class. The crosses in all three cases represent the two components of 61 Cygni. It appears that, to $C_{p} \sim+0^{\mathrm{m}} 70$, the three systems of classification are quite similar, in that they give a linear relationship between color and spectral type; but, whereas the Yerkes systems show a sudden discontinuity at $C_{p}=+0^{\mathrm{m}} 70$ to +0 m 75 , spectral type K0-K1, the Mount Wilson system preserves the linear relationship to $C_{p} \sim+0$ m. 90 , spectral type K5, where it then shows a somewhat less abrupt discontinuity. Since the color-luminosity array is linear to $C_{p} \sim+1 \mathrm{~m} 23$, Figure 3 indicates that the Yerkes spectral types corresponding to $C_{p}=+0 \mathrm{~m} 75$ to +1 m .23 are compressed between K0 and K5, while the Mount Wilson system, for the same color range, contains types K0-M0. The second discontinuity, apparent at $C_{p}=+1 \mathrm{~m} 23$ in all three parts of Figure 3, results from the sharp break in the color-luminosity array at this point.

The mean color-spectral-type relationships for the Mount Wilson and Yerkes systems are reproduced as the lines in Figure 4, while the points plotted in this figure represent the colors and spectral types of the subdwarfs given in Table 11 on these same sys-

[^9]tems. There appears to be a systematic difference in the relationship for the two kinds of stars, which amounts to three subclasses at F8 and disappears at G5. It is interesting that the two relationships merge at $C_{p}=+0 \mathrm{~m} .55$ to +0 m 60 , which is near the point of intersection of the dwarf and subdwarf sequences in the color-luminosity array of Figure 1.

The systematic difference noted above may be due to certain spectroscopic peculiarities in subdwarfs. For example, a recent spectrophotometric study of bright stars in the range F8-M0 by T. Setterberg ${ }^{24}$ indicates that the $C N$ and $C H$ molecular absorptions, as well as the atomic absorptions in the region $\lambda \lambda 4190-4340 \mathrm{~A}$ are weaker in the subdwarfs $\iota$ Per, $\lambda$ Aur, and $\mu$ Cas than they are in dwarfs of the same color; this result is in agreement with Joy's ${ }^{21}$ remark that "the spectroscopic features which distinguish subdwarfs from ordinary dwarfs are: for types A-M, a general weakening of the absorption spectrum. . . ."

TABLE 10
Spectral Types for Dwarfs in the Color Range from $\mathrm{C}_{p}=+0 \mathrm{~m} 34 \mathrm{TO}+1 \mathrm{~m} .30$

| Star | $C_{p}$ | Spectral Type |  |  | Star | $C_{p}$ | Spectral Type |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Kuiper | Mt. W. | Atlas |  |  | Kuiper | Mt. W. | Allas |
| $\pi^{3}$ Ori. | $+0^{\text {m }} 34$ | F6 | F5 | F6 V | HR 6806 | $+0 \mathrm{~m} 79$ | K2 | K2 |  |
| $\gamma$ Lep A | $+.36$ | F6 | F6 |  | $\gamma$ Lep B. | +0.87 | K3 | K6 |  |
| $\chi$ Dra. . | +. 395 | F8 | F5 | F6 V | HR 8832 | +0.875 | K3 | K5 | K3 V |
| $\eta$ Cas | + . 46 | G0 | F9 | G0 V | HR 753 A | +0.89 | K3 | K4 | K3 |
| $\chi^{1}$ Ori. | + . 47 | G0 | F9 | G0 V | 61 Cyg A. | +1.05 | K3 | K6 | K5 V |
| $\beta$ Com | + . 47 | G0 | G0 | G0 V | Cin 2322. | +1.21 | K7 | M0 | K6 |
| $\beta \mathrm{CVn}$ | +. 48 | G0 | G0 | G0 V | Cin 2238 | +1.23 | K6 | M0 | K6 |
| $\kappa$ Cet. | +. 585 | G4 | G5 | G5 V | 61 Cyg B. | +1.23 | K5 | M0 | K6 |
| 61 Vir. | + . 60 | G5 | G6 |  | Gmb 1618 | +1.23 | K8 | M0 | K6 |
| 61 UMa. | + . 63 | G4 | G6 | G8 V | Cin 1633. | +1.24 | K8 | M0 |  |
| 11 LMi . | +. 66 | G8 | K0 |  | Cin 705 | +1.27 | M1 | M3 |  |
| HR 511 | + . 67 | K1 | K0 |  | Cin 2707 | +1.27 | M0+ | M2 |  |
| 107 Psc. | + . 71 | K0 | G9 | K0 V | Cin 1383 | +1.28 | M1 | M1 | M1 |
| $\sigma$ Dra | + . 715 | G9 | K0 |  | CC 782. | +1.28 | M1 | M1 |  |
| 40 Eri A. | + . 725 | K1 | K0 |  | CC 1392. | +1.285 | M2 | M2.5 |  |
| HR 1925 | $+.75$ | K2 | K1 |  | +61 195 . | +1.29 | M1+ |  |  |
| $\epsilon$ Eri | +0.79 | K2 | K2 |  |  |  |  |  |  |

The most obvious characteristic of the dwarf sequence is the abrupt break at $C_{p} \sim$ $+1 \mathrm{~m} 23 .{ }^{25}$ From Figure 3 we see that this color corresponds to K 6 on the Yerkes systems, or to M 0 on the Mount Wilson system; in both cases, this is the point at which the TiO bands begin to show strongly in the spectra. A simple explanation of the break, then, would be that the $T i O$ absorption band heads at $\lambda \lambda 4762,4954,5168$, and 5445 A are affecting the observed colors, since the color defines only a small part of the shorter-wave-length side of the spectral energy-curve. There is some evidence, however, that the explanation is not so simple. For example, it is difficult to explain the fact that, although

[^10]

Fig. 3.-The color-spectral-type relationship for dwarfs classified by (a) Kuiper, (b) Mount Wilson, and (c) the authors of the Atlas. The open circles represent individual stars and the filled circles the means of two or more stars of the same spectral type; the crosses in all three sections of the figure represent the two components of 61 Cyg.


Fig. 4.-The color-spectral-type relationship for subdwarfs. The straight lines are reproduced from Fig. 3.
in the Mount Wilson system "the types of the M-dwarfs are based strictly upon the strength of the titanium oxide bands in the blue region of the spectrum,'"21 there exists a large color difference between dwarfs and subdwarfs of the same spectral subclass, as is the case with Wolf 47, an M6 dwarf with $C_{p}=+1 \mathrm{~m} 31$, and CC 1445, an M5.5 subdwarf with $C_{p}=+1^{\mathrm{m}} 59$. There is evidently some other feature of the spectral energycurve, besides TiO absorption, that differentiates dwarfs from subdwarfs in the M stars.

TABLE 11
Spectral Types for Subdwarfs in the Color Range from $\mathrm{C}_{p}=+0 \mathrm{~m} .49$ to +1 m 30

| Star | Spectral Type |  | Star | Spectral Type |  | Star | Spectral Type |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mt. W. | Atlas |  | Mt. W. | Allas |  | Mt. W. | A tlas |
| $\beta$ Vir. | F8 | F8 V | Hyades 40.. | G3 |  | HR 4550. | G5 | G8 V |
| $\iota$ Per. | G1 | G0 V | $85 \mathrm{Peg} \mathrm{A..}$. | G1 | G3 V | 36 UMa C. |  | K0* |
| $\lambda$ Aur | G0 | G2 IV-V | Hyades 17.. | G5 |  | CC 1018... | M3 | M2 |
| $\lambda$ Ser | G0 | G2 V | $\mu \mathrm{Cas}$. | G4 | G5 V | Cin 3124. | M2 | M2 |
| HR 483. | G0 |  | Hyades 7 | G4 |  | CC 1143. | M3 | M3 |
| 51 Peg. | G0 |  | $\tau$ Cet. | G4 | G8 V | Cin 2354 | M3 | M4 |

* 36 UMa C: Classified by Popper on the Atlas system.

It is interesting to note that the three stars that represent the extension of the subdwarf sequence to the red of $C_{p}=+1 \mathrm{~m} 32$ in Figure 1 are all classified as subdwarfs by Joy. ${ }^{21}$ These stars are listed in the accompanying tabulation. The second and third stars

*Ap. J., 105, 96, 1947; Mt. W. Contr., No. 726.
in this group show emission lines of both hydrogen and H and K of $C a \mathrm{II}$, while the first, Barnard's star, shows neither. The star L 726-8, which W. J. Luyten ${ }^{26}$ has announced as an irregular variable, was observed on two nights. The individual results are $P g_{p}=$ 13 m .92 and 13 m .94 , and $C_{p}=+1 \mathrm{~m} .55$ and +1 m .55 . The star is a close double, and Luyten gives 0.50 mag . as both the photographic and the photovisual magnitude difference between the components; he also gives the value 13.69 mag . as the total photographic magnitude of the system. If we adopt the value 0.50 mag . for the photographic $\Delta m$, we obtain from the photoelectric observations, $P g_{p}(\mathrm{~A})=14.45$ and $P g_{p}(\mathrm{~B})=14.95$; Joy and Humason ${ }^{27}$ have classified this star as dM5.5e. Since the color of the system is similar to that of ADS 246 B , the stars probably are subdwarfs, although we cannot eliminate the possibility that one star is a dwarf and the other a subdwarf, of different colors, with a combined color of +1 m . 55 . If a value for $C_{p}=+1 \mathrm{~m} 55$ approximately represents the color of both components, then, by comparison with ADS 246 B and CC 1445, the parallax probably lies between 0 ". 1 and 0.2 ; a more accurate value is not possible in view of the uncertainty introduced by the presence of a companion and the steepness of

[^11]the subdwarf sequence in this color region; Luyten estimates the parallax to be 0 ". $56 \pm 0$ ". 07 .

It should, perhaps, be noted, although not stressed, that there is some suggestion in Figure 1 that the turning-back of the dwarf sequence, evident in the last observed point, Wolf 47, continues on through such stars as van Maanen No. 2 to the white dwarfs. Some support for this suggestion is offered by such stars as Wolf 489, which Kuiper ${ }^{28}$ classifies as of the type $\mathrm{K} 5 \pm$. From Figure 3 we see that this spectral type corresponds to $C_{p}=+1 \mathrm{~m} 1 \pm$ and, since $m_{v}=14 \mathrm{~m} 3$ and $\pi=0$ ". $130 \pm 9$, as given by Kuiper, we find $M(P g)=16^{m} 0 \pm$, which would place this star on the dwarf sequence between Wolf 47 and van Maanen No. 2. A search of the literature for parallax stars that populate this portion of the color-luminosity array yielded those given in the accompanying tabulation. These are stars contained in a recent catalogue of white dwarfs by Luyten ${ }^{29}$ for

| Star | Pg | I.C. | $\pi$ (Trig.) | $M(\mathrm{pg})$ | Star | Pg | I.C. | $\pi$ (Trig.) | $M(\mathrm{pg})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vr 7 | $14{ }^{\text {m.0 }}$ | $-0 \mathrm{~m} 3$ | Hyades | +11 ${ }^{\text {m }} 0$ | van Maanen 2. | $12^{\text {m }} 6$ | $+0^{\text {m }} 4$ | $0 \mathrm{~m} 243 \pm 6$ | $+14^{\mathrm{m}} 5$ |
| Vr 16 | 13.7 | $-.3$ | Hyades | +10.7 | Wolf 457 | 16.0 | +0.5 | $078 \pm 8$ | $+15.5$ |
| Wolf 219. | 15.2 | +0.3 | $0 \prime 067 \pm 8$ | +14.3 | Wolf 489 | 15.4 | +1.1 | $0.130 \pm 9$ | +16.0 |

which available parallaxes indicate a distance of 14 parsecs or less; in addition, two white dwarfs in the Hyades cluster are included, and the mean cluster parallax given in Paper I, 0 ". 0245 , is used to obtain the luminosity of these stars. The photographic magnitudes and colors are those given by Luyten. The magnitude and color obtained by Luyten for van Maanen No. 2, $P g=12^{\mathrm{m}} 6$ and I.C. $=+0^{\mathrm{m}} 4$, agrees well with the values in Table 3, $P g_{p}=12 \mathrm{~m} .50$ and $C_{p}=+0^{\mathrm{m}} 41$. Also, the color of Wolf 489 is the same as that obtained above from the spectral type given by Kuiper. The colors and luminosities of these stars have been plotted in Figure 5. Taking into account the uncertainty in the photographic colors and the smaller parallaxes, Figure 5 is highly suggestive of a connection between the red and the white dwarfs. All the stars for which Luyten's magnitudes and colors have been used above will be observed on the present photoelectric program.

## V. BLUE-DWARF AND BRIGHT BLUE-DWARF SEQUENCES

CLUSTER STARS
The stars in the Pleiades, Coma Berenices, and Ursa Major clusters, which were found in Papers II and III to populate the blue-dwarf sequence, are gathered together in Table 12. The mean sequence defined by these stars is given in Table 13. The individual deviations from this mean are tabulated as $\Delta M(\mathrm{O}-\mathrm{S})$ in Table 12 for stars redder than $C_{p}=-0 \mathrm{~m} .12$; the sequence is poorly defined between $C_{p}=-0 \mathrm{~m} .12$ and -0 m .17 ; and at $C_{p}=-0^{\mathrm{m}} .17, M=-0^{\mathrm{m}} 50$, the sequence seems to rise vertically to $M=-3$ mag. with no further change in color. The blue-dwarf sequence joins the dwarf sequence at $C_{p}=$ +0 m. 18 .

The stars in Papers I, II, and III which were found to populate the bright bluedwarf sequence are listed in Table 14. The mean sequence is in Table 15.
"ATLAS" STARS

In Paper II it was pointed out that, in the Pleiades, the spectra of the bright bluedwarfs indicate a smaller amount of $H \gamma$ and $H \delta$ absorption than do the spectra of bluedwarfs of corresponding color. This effect probably is due to less pronounced wings of
${ }^{28}$ Ap. J., 87, 595, 1938.

$$
{ }^{29} \text { Ap.J., 109, 528, } 1949 .
$$



Fig. 5.-The position of the "white dwarfs" in the color-luminosity array. The large open circles represent stars within 14 parsecs for which photographic magnitudes and colors (Luyten) are available. The continuous curves are taken from Fig. 1.

TABLE 12
Luminosities and Colors of Blue-Dwarf Stars in Galactic Clusters

| Star | $M_{P g_{p}}$ | $C_{p}$ | $\Delta m(0-S)$ | Star | $M_{P g_{p}}$ | $C_{p}$ | $\Delta m(0-S)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P 17. | $-1 \mathrm{~m} 70$ | $-0 \mathrm{~m} 18$ |  | P 69 | $+1 \mathrm{~m} 70$ | $-0^{\mathrm{m}} 01$ | $-0^{\text {m. }} 07$ |
| P 48. | -3.10 | -. 175 |  | C 183. | +1.78 | . 00 | -. 05 |
| P 59. | $-2.33$ | -. 17 |  | P 46. | +2.04 | +. 05 | - . 04 |
| P 54. | -0.55 | - 1.17 |  | P 44. | +2.11 | +. 07 | $-.05$ |
| P 16. | -0.35 | -. 17 |  | P 64. | +2.27 | $+.08$ | $+.07$ |
| P 25. | $-2.09$ | -. 16 |  | C 139 | +2.22 | +. 09 | $\pm .00$ |
| P 67. | +0.20 | -. 15 |  | C 168 | +2.28 | $+.105$ | -. 01 |
| P 27. | -0.16 | -. 145 |  | C 145. | +2.22 | +. 11 | -. 09 |
| P 28. | +0.53 | -. 12 | -0m. 02 | P 5 | +2.38 | +. 12 | +. 03 |
| P 63. | +0.75 | $-.115$ | + . 12 | P 66. | +2.44 | +. 125 | +. 07 |
| P 40. | +0.94 | - . 09 | - . 03 | C 104 | $+2.36$ | +. 13 | -. 03 |
| P 43. | +1.10 | -. 08 | $\pm .00$ | P 73. | +2.41 | $+.13$ | +. 02 |
| P 70. | +1.11 | -. 07 | -. 11 | P 61 | +2.44 | +. 13 | +. 05 |
| P 45 | +1.35 | -. 06 | $\pm .00$ | 80 UMa. | +2.45 | +. 13 | +. 06 |
| P 33. | +1.57 | -. 03 | - . 05 | P 41. | +2.65 | $+.165$ | +. 12 |
| P 68. | +1.75 | $-.02$ |  | P 3 . | +2.61 | +0.17 | +0.07 |
| P 62. | +1.79 | -0.02 | +0.09 |  |  |  |  |

the hydrogen lines in the bright blue-dwarfs-an indication, according to the Atlas, of higher luminosity. In an attempt to separate the brighter A stars into the sequences discussed above, as well as to find additional spectroscopic evidence of differences in the spectra of stars populating the various sequences, the stars classified with luminosity classes IV and V and those of Atlas spectral types A0-A7 are discussed in the following paragraphs. The stars C Hya, $\lambda \mathrm{UMa}, 38 \mathrm{Lyn}$, and $\zeta \mathrm{Vir}$, are omitted for lack of reliable trigonometric parallaxes.

TABLE 13
The Blue-Dwarf Sequence Defined by Cluster Stars

| $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-0^{\mathrm{m}} 12$. | $+0^{\text {m }} 55$ | -0m04 | +1m55 | +0.04. | $+2^{\text {m }} 05$ | $+0^{\mathrm{m}} 12$. | $+2 \mathrm{~m} 35$ |
| - . 10. | +0.85 | -. 02. | +1.70 | $+.06$ | +2.12 | + . 14. | +2.43 |
| -.08 . | +1.10 | $\pm .00$ | +1.83 | +.08 . | +2.20 | + .16. | +2.50 |
| -0.06. | +1.35 | +0.02. | +1.95 | +0.10. | +2.27 | +0.18. | +2.58 |

TABLE 14
Luninosities and Colors of Bright Blue-Dwarfs in Clusters

| Star | $M_{P g_{p}}$ | $C_{p}$ | $\Delta M(\mathrm{O}-\mathrm{S})$ | Star | $M_{P g_{p}}$ | $C_{p}$ | $\Delta M(\mathrm{O}-\mathrm{S})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\epsilon \mathrm{UMa}$. | $+0 \mathrm{~m} 01$ | -0m. 11 | $+0^{\text {m }} 03$ | P 22. | +19.43 | $+0 \mathrm{~m} 03$ | $-0 \mathrm{~m} .02$ |
| C 146*. | +0.55 | -. . 08 | + . 15 | C 10 | +1.50 | +. 05 | -. 10 |
| $\gamma$ UMa. | +0.59 | -. 06 | - . 01 | $\delta$ UMa. | +1.70 | + . 05 | +. 10 |
| $\beta$ UMa. | +0.66 | -. 05 | -. . 04 | P 77 | +1.82 | +. 08 | $\pm .00$ |
| C 160*. | +0.88 | -. 03 | -. 02 | C 62 | +1.82 | + . 09 | -. 06 |
| P 47. | +1.01 | -. 03 | + . 11 | P 75 | +1.93 | + . 09 | $+.05$ |
| H 61. | +1.18 | $\pm .00$ | $-.01$ | H 50. | +1.84 | $+.095$ | $-.08$ |
| P 56. | +1.19 | $\pm 0.00$ | $\pm 0.00$ | C 60 | +1.98 | +0.10 | +0.03 |

* Slightly and erratically variable in magnitude ( $\pm 0 \mathrm{~m} 05$, A.D.), and in color ( $\pm 0 \mathrm{~m} 02$, A.D.).

TABLE 15
The Bright Blue-Dwarf Sequence Defined by Cluster Stars

| $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ | $C_{p}$ | $M_{P g_{p}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-0^{\text {m }} 12$. | $-0 \mathrm{~m} 15$ | -0m.06. | $+0^{m} 60$ | $\pm 0 \mathrm{~m} 00$. | +1m19 | +0m06. | $+1{ }^{\text {m }} 68$ |
| -. 10 . | +. 12 | -. 04 . | +0.80 | $\pm .02$. | +1.37 | +. 08. | +1.82 |
| -0.08. | +0.40 | -0.02. | +1.00 | +0.04. | +1.53 | +0.10. | +1.95 |

SPECTRUM A0
$\gamma$ UMa.-We have already shown, in Paper III, that this member of the Ursa Major cluster is a bright blue-dwarf.
a $\operatorname{Lyr}(V e g a)$.-If we assume that Vega is a bright blue-dwarf, then from Table 15 we find $M=+0^{m} 45$, or a photometric parallax of 0 ". 127 , which falls well within the range of the trigonometric values given in Table 16. Since a parallax of 0 ". 173 would be required to make Vega a blue-dwarf, the assumption that it is a bright blue-dwarf seems to be correct.

TABLE 16
Standard A-Type Stars in the atlas

| Star | $P_{g}$ | $C_{p}$ | $n$ | Allas | $\pi$ (Trig.) | $\pi$ ( Ptm ) | Sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \gamma \mathrm{UMa} . \\ & \underset{\text { (Vyr.. }}{\text { (Vega) }} \end{aligned}$ | 2 m 31 | -0m06 | 3 | A0 V | 0.046* | 0"046 | Bright blue-dwarf |
|  | -0.08 | $-.075$ | 3 | $\mathrm{A} 0 \mathrm{~V}$ <br> Mean | $\begin{aligned} & 0.120 \pm 6(\mathrm{~A}) \\ & .134 \pm 8(\mathrm{M}) \\ & 0.117 \pm 13(\mathrm{Y}) \end{aligned}$ |  | Bright blue-dwarf |
|  |  |  |  |  | $0.121 \pm 4$ | . 127 |  |
| $a \mathrm{CrB}$. | +2.12 | $-.05$ | 3 | A0 V <br> Mean | $\begin{aligned} & 0.049 \pm 9(\mathrm{~A}) \\ & 0.053 \pm 10(\mathrm{M}) \end{aligned}$ |  | Bright blue-dwarf |
|  |  |  |  |  | $0.051 \pm 7$ | . 049 |  |
| ${ }_{a}$ CMa. (Sirius) | $-1.53$ | $-.07$ | 5 | A1 V <br> Mean | $\begin{gathered} 0.370 \pm 5(\mathrm{~A}) \\ .360 \pm 8(\mathrm{M}) \\ .387 \pm 9(\mathrm{Y}) \\ 0.370 \pm 11(\mathrm{y}) \end{gathered}$ |  | Blue-dwarf |
|  |  |  |  |  | $0.373 \pm 3$ | . 367 |  |
| $\gamma$ Gem. | +1.88 | $-.065$ | 2 | A1 V <br> Mean | $\begin{aligned} & 0.031 \pm 8(\mathrm{M}) \\ & 0.047 \pm 11(\mathrm{y}) \end{aligned}$ |  | Bright blue-dwarf |
|  |  |  |  |  | $0.040 \pm 6$ | . 056 |  |
| $\beta$ UMa...... <br> $\beta$ Aur. | +2.23 | $-.05$ | 5 | A1 V | 0.049* | . 049 | Bright blue-dwarf |
|  | +1.84 | $+.01$ | 5 | A2 IV <br> Mean | $\begin{aligned} & 0.039 \pm 4(\mathrm{~A}) \\ & 0.029 \pm 12(\mathrm{M}) \end{aligned}$ | . 035 | Dwarf |
|  |  |  |  |  | $0.034 \pm 4$ |  |  |
| $a$ PsA (Fomalhaut) | +1.11 | $+.015$ | 2 | A3 V <br> Mean | $0.134 \pm 9(\mathrm{Y})$ $0.157 \pm 8(\mathrm{C})$ | . 142 | Blue-dwarf |
|  |  |  |  |  | $0.146 \pm 6$ |  |  |
| $\delta$ UMa. | +3.28 | $+.05$ | 3 | A3 V | 0.049* | . 048 | Bright blue-dwarf |
| $\beta$ Leo. | +2.11 | $+.08$ | 2 | A3 V <br> Mean | $\begin{gathered} 0.066 \pm 6(\mathrm{~A}) \\ .075 \pm 11(\mathrm{M}) \\ .103 \pm 15(\mathrm{y}) \\ 0.121 \pm 15(\mathrm{~S}) \end{gathered}$ | . 089 | Bright blue-dwarf |
|  |  |  |  |  | 0.083 $\pm 7$ |  |  |
| $\delta$ Her. | +3.08 | $+.04$ | 3 | A3 IV <br> Mean | $\begin{aligned} & 0.032 \pm 8(\mathrm{~A}) \\ & 0.019 \pm 10(\mathrm{M}) \end{aligned}$ | . 032 | Dwarf |
|  |  |  |  |  | $0.027 \pm 6$ |  |  |
| $\beta$ Ari | +2.65 | $+0.09$ | 3 | A5 V | $\begin{gathered} 0.075 \pm 10(\mathrm{~A}) \\ .094 \pm 13(\mathrm{M}) \\ .040 \pm 13(\mathrm{y}) \\ 0.035 \pm 12(\mathrm{~S}) \end{gathered}$ | 0.067 | Bright blue-dwarf |
|  |  |  |  | Mean | $0.063 \pm 6$ |  |  |

[^12]TABLE 16-Continued

| Star | $P g_{p}$ | $C_{p}$ | $n$ | Atlas | $\pi$ (Trig.) | $\pi$ ( Ptm ) | Sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta$ Cas | $+2^{m} 64$ | $+0^{\mathrm{m}} 10$ | 3 | A5 V <br> Mean | $\begin{aligned} & 0 " .062 \pm 10(\mathrm{M}) \\ & .025 \pm 9(\mathrm{y}) \\ & 0.009 \pm 9(\mathrm{~W}) \end{aligned}$ | 0 " 055 | Dwarf (?) |
|  |  |  |  |  | $0.033 \pm 7$ |  |  |
| 80 UMa. | +4.05 | +. 13 | 4 | A5 V | 0.048* | 047 | Blue-dwarf |
| a Aql. (Altair) | +0.88 | $+.165$ | 2 | A7 V <br> Mean | $\begin{aligned} & 0.213 \pm 8(\mathrm{M}) \\ & .217 \pm 10(\mathrm{Y}) \\ & 0.187 \pm 10(\mathrm{y}) \end{aligned}$ |  | Blue-dwarf |
|  |  |  |  |  | $0.208 \pm 5$ | 213 |  |
| $a$ Cep.. | +2.42 | +0.165 | 4 | A7 V | $\begin{aligned} & 0.069 \pm 10 \text { (A) } \\ & .062 \pm 7 \text { (y) } \\ & 0.064 \pm 10 \text { (G) } \end{aligned}$ | 0.068 | Bright-dwarf |
|  |  |  |  | Mean | $0.068 \pm 4$ |  |  |

* Cluster parallax from Paper III.
a CrB .-The photometric parallax derived for this star on the assumption that it is a bright blue-dwarf, 0 ". 049 , is in excellent agreement with the trigonometric values in Table 16; a parallax of 0 ". 070 would be required to place the star on the blue-dwarf sequence. It is noteworthy that if we assume membership in the Ursa Major cluster for a Coronae and compute the cluster parallax with the constants derived in Paper III, we find $\pi_{c}=0$ " 047 .


## SPECTRUM A1

a CMa (Sirius).-The photometric parallax in Table 16 is derived from data in Table 12, with the assumption that Sirius is a blue-dwarf; the agreement with the trigonometric values is good.
a Gem (Castor).-The visual components of Castor are too close to be resolved with the 12 -inch photometer, and so they are not included in Table 16; but, for the present purpose, we shall adopt the Harvard revised visual magnitudes ${ }^{29 \mathrm{a}}$ of $m(\mathrm{~A})=1 \mathrm{~m} .99$ and $m(\mathrm{~B})=2^{\mathrm{m}} 85$. If we assume that component A is a blue-dwarf and, as is indicated by the spectral type, similar to Sirius in color, we find a photometric parallax of 0 ". 071 , compared with the mean trigonometric value of $0 \prime \prime 073 \pm 3$. The agreement is satisfactory, and we may assume that component A is a blue-dwarf. If we now apply the same parallax to component B , we find $M(\mathrm{~B})=+2^{\mathrm{m}} 2$, or $+2^{\mathrm{m}} \cdot 5$ if we allow for the binary nature of this star. If the $\mathbf{B}$ component fell on the dwarf sequence, it would then be similar to such stars as Hyades Nos. 96 and 141 (Paper I), which have a mean $M=+2 \mathrm{~m} 45$ and $C_{p}=+0 \mathrm{~m} 16$. Both Hyades stars are metallic-line objects like $a$ Gem B, a fact which supports our assumptions concerning component A .
$\gamma$ Gem.-The trigonometric parallax of this star is small and uncertain, but the assumption that it is a bright blue-dwarf gives a photometric parallax that falls within the range of the trigonometric determinations; since the star is a long-period spectroscopic binary, the companion may have some influence on the magnitude and color.
$\beta U M a$.-We have already shown, in Paper III, that this member of the Ursa Major cluster is a bright blue-dwarf. The cluster parallax is given in place of the trigonometric values in Table 16.

[^13]On the basis of the foregoing discussions, the Atlas standards at A1 V appear to consist of two blue-dwarfs, Sirius and $a$ Gem A, and of two bright blue-dwarfs, $\gamma$ UMa and probably $\gamma$ Gem. In the Atlas, the authors state: "It is possible that the [hydrogen] wings are slightly less pronounced in the spectrum of $\gamma$ Gem than in the other [A1 V] stars listed." We might expect a similar effect in $\beta \mathrm{UMa}$ as compared with Sirius.

## SPECTRUM A2

$\beta$ Aur.-The observed color and magnitude are $P g_{p}=1 \mathrm{~m} .84$ and $C_{p}=+0 \mathrm{~m} 01$, and, although the star is a spectroscopic and eclipsing binary, ${ }^{30}$ the two stars are equally bright and probably of the same color, so that, correcting for the duplicity, we have $P g_{p}=2 \mathrm{~m} .59$ and $C_{p}=+0^{\mathrm{m}} .01$ for each component. The dwarf sequence of Table 6 stops at $C_{p}=+0^{\mathrm{m}} 02$; but, if we extrapolate to $C_{p}=+0^{\mathrm{m}} 01$, we find $M=+0 \mathrm{~m} 30$, or a photometric parallax of 0 ". 035 if the components are dwarfs; the mean trigonometric parallax in Table 16 is $0 \prime 034 \pm 4$. This star, like $a$ CrB discussed above, is often assigned membership in the extended Ursa Major group. If we compute the cluster parallax with the constants derived for the cluster in Paper III, we find 0 ". 034 in good agreement with the photometric value. The components of $\beta$ Aur, therefore, probably are dwarfs similar to the brightest dwarfs thus far found in galactic clusters.
$\beta$ Ser.-This star will be discussed in the following section concerning binary stars.

## SPECTRUM A3

a PsA (Fomalhaut).-If Fomalhaut is a blue-dwarf, then, from Table 13, $M=$ +1 m 89 , and the photometric parallax is 0.142 , very near the mean of the two trigonometric values in Table 16. The authors of the Atlas state: " $a$ Pisces Australis gives spectroscopic evidence of having the lowest luminosity of any star in the table [of A3 stars]." Fomalhaut probably is the only A3 blue-dwarf in the Atlas.
$\delta U M a$.-We have already shown in Paper III that this member of the Ursa Major cluster is a dwarf. The cluster parallax is given in place of the trigonometric values in Table 16.
$\beta$ Leo.-The assumption that this star is a bright blue-dwarf leads to a photometric parallax of 0 ".089, which is in excellent agreement with the mean of the rather scattered trigonometric values. The authors of the Atlas state: "The hydrogen lines are weaker in the spectrum of $\beta$ Leonis than in the other [A3] dwarfs. . . ."
$\delta$ Her.-The trigonometric parallax of this star is small and relatively uncertain, but the mean value gives $M=+0$ m. 24 , compared to $M=+0^{\mathrm{m}} .60$ obtained from Table 6 under the assumption that the star is a dwarf; a parallax of 0 ". 049 would be required tc place it on the bright blue-dwarf sequence.

## SPECTRUM A5

$\beta$ Ari.-Although the individual values of the trigonometric parallax are rather dis. cordant, this spectroscopic binary, with a period of 107 days, probably is a bright blue. dwarf, since the photometric parallax in Table 16, derive irom this assumption, is ir good agreement with the mean trigonometric value.
$\delta$ Cas.-This star is rather similar to $\beta$ Ari in both magnitude and color, but thi trigonometric determinations of the parallax are even more discordant than those fo $\beta$ Ari. From data in Tables 6,15 , and 13 we find $M=+1 \mathrm{~m} 33,+1 \mathrm{~m} 95$, or +2 m 27 fo the assumptions that the star is a dwarf, bright blue-dwarf, or blue-dwarf, respectively The corresponding photometric parallaxes are $0 " 055,0.072$, or $0^{\prime \prime} 084$, of which 0.055 from the assumption that the star is a dwarf, is the only value that falls within th rather large range of the trigonometric values.

80 UMa.-We have already shown in Paper III that this member of the Ursa Majo

[^14]cluster is a blue-dwarf. The cluster parallax is given in place of the trigonometric values in Table 16.

## SPECTRUM A7

a Aql (Altair).-This star is situated near the junction of the blue-dwarf and dwarf sequences, so that it is difficult to decide to which it belongs; but the large rotational velocity, discussed later in this paper, supports its membership in the blue-dwarf sequence. It is interesting to note here that in Papers I and III it was found that several of the stars lying near the intersection of the blue-dwarf and dwarf sequences were metallic-line stars, and we might therefore expect the spectrum of Altair to show "metallic-line" tendencies.
a Cep.-From Tables 6 and 13 we find $M=+2^{m} 45$ and $2^{m} 54$, respectively, from the assumptions that this star is a dwarf or a blue-dwarf. Either of these values for $M$ would require a photometric parallax in excess of $0 \prime 11$, as compared with the three accordant trigonometric values given in Table 16, 0 ". $069 \pm 10(\mathrm{~A}), 0$ ". $062 \pm 7$ (y), and 0 ".064 $\pm$


Fig. 6.-The color-spectral-type relationship for the A-type stars in the Atlas. The crosses indicate dwarfs; the open circles, blue-dwarfs; and the filled circles, bright blue-dwarfs.
$10(\mathrm{G})$, and with one discordant value, $0.0094 \pm 9(\mathrm{M})$. The mean trigonometric parallax yields $M=+1 \mathrm{~m} 58$, which makes the star similar to $v$ and 71 Tau, Nos. 66 and 70 in Paper I, which have mean $M=+1^{\mathrm{m}} 45$ and $C_{p}=+0^{\mathrm{m}} 18$. Both these Hyades stars fall on a portion of the bright-dwarf sequence that was shown in Paper I to contain rapidly rotating A5-A7 stars of luminosity class III; the mean rotational velocity of the two Hyades stars was noted in Paper I as $200 \mathrm{~km} / \mathrm{sec}$, which is the same as that observed for $a$ Cep by Miss Westgate. ${ }^{31}$ The fact that luminosity class V, instead of IV or III, is assigned to this star in the Atlas may be caused by the haziness of the spectral lines resulting from the lar ${ }_{5}$, rotational velocity.

From the foregoing examples, it is apparent that at least some of the fine structure in that portion of the color-luminosity array populated by the A-type stars probably is reflected in the spectra of these stars. Close scrutiny of high-dispersion spectra of the brighter stars populating the dwarf, blue-dwarf, and bright blue-dwarf sequences may reveal further criteria for differentiating these stars purely from their spectra. If such a separation can be made, the spectroscopic method of deriving individual parallaxes would become precise and powerful. Figure 6 shows the rather ill-defined color-spectral-type relationship for the Atlas stars, discussed above.

[^15]
## PARALLAX STARS NOT IN THE "ATLAS"

A few additional parallax stars not included in the Atlas but with $C_{p} \leqq+0$ m. 17 have been observed; data for these stars are given in Table 17 and are discussed below.
$\lambda$ Gem.-The photometric parallax derived from Table 15 on the assumption that the star is a bright blue-dwarf is in excellent agreement with the mean trigonometric value given in Table 17; but, since a slightly larger parallax of 0 ". 049 would place the star on the blue-dwarf sequence, we cannot confidently eliminate the possibility that it falls on the latter sequence.
$\delta$ Leo.-The individual determinations of the trigonometric parallax of this star are quite discordant. The assumption that the star is a dwarf leads to a photometric parallax of 0 " 057 , which is close to the mean of the trigonometric values; but, if the star is a bright blue-dwarf, the photometric parallax is 0 ". 074 , which is still within the range of

TABLE 17
Stars With $C_{p} \leqq+0 \mathrm{~m} 17$ but Not Included in the Atlas

| Star | $P g_{p}$ | $C_{p}$ | $n$ | $\pi$ (Trig.) | $\pi$ ( Ptm ) | Sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\lambda$ Gem . | 3m59 | $+0^{\mathrm{m}} 04$ | 4 <br> Mean | $\begin{aligned} & 0.045 \pm 7(\mathrm{~A}) \\ & .034 \pm 11(\mathrm{M}) \\ & 0.041 \pm 12(\mathrm{~S}) \end{aligned}$ |  | Bright blue-dwarf |
|  |  |  |  | $0.041 \pm 6$ | 0".039 |  |
| $\delta$ Leo . | 2.55 | $+.10$ | 3 <br> Mean | $\begin{aligned} & 0.040 \pm 6(\mathrm{~A}) \\ & 0.078 \pm 10(\mathrm{M}) \end{aligned}$ |  | Dwarf (?) |
|  |  |  |  | $0.050 \pm 5$ | . 057 |  |
| ^UMa. | 3.30 | $+.165$ | 3 <br> Mean | $\begin{aligned} & 0.066 \pm 7(\mathrm{~A}) \\ & 0.066 \pm 14(\mathrm{M}) \end{aligned}$ |  | Blue-dwarf (?) |
|  |  |  |  | $0.066 \pm 8$ | . 069 |  |
| 72 Oph. | 3.72 | +0.08 | 4 | $\begin{aligned} & 0.035 \pm 9(\mathrm{~A}) \\ & .033 \pm 11(\mathrm{M}) \\ & 0.047 \pm 8(\mathrm{Y}) \end{aligned}$ |  | Bright blue-dwarf |
|  |  |  | Mean | $0.040 \pm 5$ | 0.042 |  |

the parallax determinations. We may conclude, with some uncertainty, that the star probably is not a blue-dwarf.

८UMa.-This star is quite similar to Altair, in that it has the same color and a large rotational velocity. Also, like Altair, it occurs near the junction of the dwarf and bluedwarf sequences, a fact that prohibits a definite assignment to either sequence. From Tables 6 and 13 we find, for $C_{p}=+0^{\mathrm{m}} 165, M=+2^{\mathrm{m}} .38$ and $+2^{\mathrm{m}} \cdot 52$, respectively, or photometric parallaxes of 0.069 , dwarf, or 0 " 073 , blue-dwarf. Either value for the photometric parallax is consistent with the trigonometric determinations given in Table 16.
$720 p h$.-The trigonometric parallaxes are quite consistent with the assumption that this star is a bright blue-dwarf.

## VI. VISUAL BINARIES

Although the trigonometric parallaxes of stars more distant than those listed in Table 3 are not trustworthy enough to use in the construction of accurate color-lumi-
nosity arrays, we are able to test the slopes of the various sequences and the relative position of one sequence to another, by the use of visual binaries and widely separated stars known to possess common proper motion and radial velocity. These systems may be regarded as small clusters; ${ }^{32}$ and, when one component is properly fitted to a sequence of the color-luminosity array, the other component should fall into place on another, or the same, sequence. The systems observed in the present program are discussed below and are listed in Table 18. The six wide pairs included in Table 3 are not entered in Table 18. For purposes of comparison with visual binaries, the dwarf sequence defined by the cluster stars and given in Table 6 is extrapolated from $C_{p}=+1$ m 06 to $+1 \mathrm{~m} \cdot 22$; the extrapolated portion is given as the second section of Table 6.

111 Tau.-If the brighter star is a dwarf, we find, from Table $6, M=+4.82$, or a photometric parallax of 0.078 , which, when applied to the fainter component, gives $M=+8^{\mathrm{m}} 30$ as compared to $+8^{\mathrm{m}} 39$, given in Table 6 for $C_{p}=+1 \mathrm{~m} 02$.
$\rho$ Gem.-If both stars are dwarfs, Table 6 gives $M=+3 \mathrm{~m} .15$ and +7 m 25 , or moduli of $m-M=+1^{\mathrm{m}} 26$ and $+1^{\mathrm{m}} 16$. The mean photometric parallax is then $0 \prime 057$, which is identical with the mean trigonometric value.
a Leo (Regulus).-If the companion is a dwarf, then $M=+6 \mathrm{~m} 89$ from Table 6, and hence $M=-0 \mathrm{~m} .83$ for Regulus. The companion is itself a binary, but the magnitude difference is great enough, 5 mag., to insure that the observed magnitude and color is that of the brighter component alone. Regulus is, then, a blue-dwarf similar to Pleiades No. 54 (Paper II) with $M=-0^{\mathrm{m}} 55$ and $C_{p}=-0^{\mathrm{m}} 17$. The Pleiades star is classified B8 by Morgan, and it has a rotational velocity of $300 \mathrm{~km} / \mathrm{sec}$, compared to B 8 V (Atlas) and $200 \mathrm{~km} / \mathrm{sec}$ for Regulus. The photometric parallax of the system, $0^{\prime \prime} 041$, is the same as the mean trigonometric value.
$\gamma$ Vir.-This double is too close to be separated with the 12 -inch photometer, but the components differ by only 0.04 mag. ${ }^{33}$ and are of the same spectral type. From the observed magnitude and color we find $P g_{p}=3 \mathrm{~m} 75$ for each component. If the stars are dwarfs, Table 6 gives $M=+3$ m 38, or a photometric parallax of 0 " 084 , which agrees well with the mean trigonometric value, $0^{\prime \prime} 089 \pm 7$.
$+7^{\circ} 2690$ and $+7^{\circ} 2692$. -These two stars, separated by $500^{\prime \prime}$, have a common proper motion. The Mount Wilson observers assign to the brighter star a spectroscopic absolute magnitude of +3.8 and spectral type G2. If the fainter star is a dwarf, then, from Table 6 , we find $M=+8^{\mathrm{m}} 75$, or $m-M=+2^{\mathrm{m}} 31$, and a photometric parallax of $0 " 034$. If we apply this photometric parallax to the brighter component, we find $M=+4^{\mathrm{m}} 66$. The brighter component is then a subdwarf similar to such stars in Table 9 as $\lambda$ Ser, with $M=+4^{\mathrm{m}} 78$ and $C_{p}=+0^{\mathrm{m}} \cdot 52$, and $\lambda$ Aur, with $M=+4^{\mathrm{m}} .28$ and $C_{p}=+0^{\mathrm{m}} .51$. We can eliminate the possibility that the fainter component lies on the subdwarf sequence, as defined in Table 9 , because, if such were the case, $M$ would be near +10.0 mag., which would give a photometric parallax of 0.06 -a value ruled out by the trigonometric determinations given in Table 18. Combining $M=+4 \mathrm{~m} 66$ and $C_{p}=+0^{\mathrm{m}} 52$, we find the visual absolute magnitude of the brighter star to be +4.1 mag., which is in reasonably good agreement with the value +3.8 mag. found spectroscopically.
a Lib.-The two components of this system are separated by $230^{\prime \prime}$. The brighter component is given in the Atlas as a metallic-line star, but the separation between the K-line

[^16]${ }^{33}$ J. Stebbins, U. Illinois Bull., Vol. 4, No. 25, 1907.

TABLE 18
Photometric and Trigonometric Data for Visual Binaries
(Wide Pairs in Table 3 Excluded)

| Star | $P g_{p}$ | $C_{p}$ | $n$ | $\pi$ (Trig.) | $\pi$ (Ptm) | Sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 111 Tau A.B. | $\begin{aligned} & 5 \mathrm{~m} 36 \\ & 8.84 \end{aligned}$ | $\begin{aligned} & +0 \mathrm{~m} .43 \\ & +1.02 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ <br> Mean | $\begin{aligned} & 0 " .065 \pm 8(\mathrm{~A}) \\ & 0.059 \pm 9(\mathrm{M}) \end{aligned}$ |  | Dwarf Dwarf |
|  |  |  |  | $0.062 \pm 7$ | 0".078 |  |
| $\rho \text { Gem A. } \mathrm{B} .$ | $\begin{aligned} & 4.39 \\ & 8.41 \end{aligned}$ | $\begin{aligned} & +0.23 \\ & +0.83 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ <br> Mean | $\begin{aligned} & 0.062 \pm 10(\mathrm{~A}) \\ & 0.055 \pm 8(\mathrm{M}) \end{aligned}$ |  | Dwarf Dwarf |
|  |  |  |  | $0.057 \pm 6$ | . 057 |  |
| $a$ Leo A. B. | $\begin{aligned} & 1.09 \\ & 8.81 \end{aligned}$ | $\begin{aligned} & -0.16 \\ & +0.77 \end{aligned}$ | $\begin{aligned} & 2 \\ & 3 \end{aligned}$ <br> Mean | $\begin{aligned} & 0.067 \pm 10(\mathrm{M}) \\ & 0.026 \pm 8(\mathrm{~V}) \end{aligned}$ |  | Blue-dwarf Dwarf |
|  |  |  |  | $0.041 \pm 6$ | . 041 |  |
| $\gamma \operatorname{Vir} A B$. | 3.00 | +0.25 | 2 <br> Mean | $\begin{aligned} & 0.101 \pm 15(\mathrm{M}) \\ & 0.088 \pm 8(\mathrm{Y}) \end{aligned}$ |  | Dwarfs |
|  |  |  |  | $0.089 \pm 7$ | . 084 |  |
| $\begin{aligned} & +7^{\circ} 2690 \ldots \ldots \\ & +7^{\circ} 2692 \ldots \ldots \end{aligned}$ | $\begin{array}{r} 6.97 \\ 11.06 \end{array}$ | $\begin{aligned} & +0.52 \\ & +1.08 \end{aligned}$ | $\begin{aligned} & 2 \\ & 7 \end{aligned}$ <br> Mean | $\begin{array}{r} 0.025 \pm 14(\mathrm{~W}) \\ .028 \pm 6(\mathrm{~A}) \\ .040 \pm 10(\mathrm{M}) \\ 0.010 \pm 9(\mathrm{Y}) \end{array}$ |  | Subdwarf Dwarf |
|  |  |  |  | $0.026 \pm 4$ | . 034 |  |
| $a \underset{\mathrm{~B}}{\mathrm{Lib}} \mathrm{~A} .$ | $\begin{aligned} & 2.72 \\ & 5.33 \end{aligned}$ | $\begin{array}{r} +0.08 \\ 0.29 \end{array}$ | 5 <br> 4 <br> Mean | $\begin{aligned} & 0.069 \pm 13 \text { (M) } \\ & .047 \pm 7 \text { (Y) } \\ & 0.041 \pm 17 \text { (y) } \end{aligned}$ |  | Dwarf Dwarf |
|  |  |  |  | $0.051 \pm 5$ | . 049 |  |
| $\begin{aligned} & -15^{\circ} 4041 \ldots \\ & -15^{\circ} 4042 \ldots \end{aligned}$ | $\begin{array}{r} 9.67 \\ 10.09 \end{array}$ | $\begin{aligned} & +0.65 \\ & +0.74 \end{aligned}$ | 5 6 <br> 6 <br> Mean | $\begin{gathered} 0.051 \pm 10(\mathrm{M}) \\ .050 \pm 9(\mathrm{C}) \\ .040 \pm 14(\mathrm{D}) \\ .046 \pm 12(\mathrm{y}) \\ 0.029 \pm 7(\mathrm{Y}) \end{gathered}$ |  | Subdwarf Subdwarf |
|  |  |  |  | $0.040 \pm 4$ | . 035 |  |
| $-0^{\circ} 2944 \mathrm{AB}$. | 7.18 | $+0.67$ | 5 <br> Mean | $\begin{gathered} 0.059 \pm 6(\mathrm{~A}) \\ .049 \pm 8(\mathrm{y}) \\ .046 \pm 10(\mathrm{M}) \\ .060 \pm 12(\mathrm{Y}) \\ 0.094 \pm 11 \text { (W) } \end{gathered}$ |  | Dwarfs (?) |
|  |  |  |  | $0.057 \pm 4$ | . 047 |  |
| $\beta \text { Ser } \underset{\mathrm{BC}}{\mathrm{~A}} .$ | $\begin{aligned} & 3.61 \\ & 8.51 \end{aligned}$ | $\begin{array}{r} +0.03 \\ +0.56 \end{array}$ | $\begin{aligned} & 5 \\ & 3 \end{aligned}$ <br> Mean | $\begin{aligned} & 0.035 \pm 10(\mathrm{~A}) \\ & 0.033 \pm 10(\mathrm{M}) \end{aligned}$ |  | Dwarf <br> Dwarfs |
|  |  |  |  | $0.034 \pm 7$ | . 024 |  |
| $\begin{array}{r} 36 \text { Oph } \begin{array}{r} \mathrm{AB} \ldots \\ \text { C. ... } \end{array} . \end{array}$ | $\begin{aligned} & 5.01 \\ & 7.39 \end{aligned}$ | $\begin{aligned} & +0.75 \\ & +1.02 \end{aligned}$ | $\begin{gathered} 5 \\ 3 \\ \text { Mean } \end{gathered}$ | $\begin{aligned} & 0.165 \pm 8 \text { (Y) } \\ & 0.190 \pm 9(\mathrm{C}) \end{aligned}$ |  | Dwarfs <br> Dwarf |
|  |  |  |  | $0.175 \pm 5$ | 0.160 |  |

TABLE 18-Continued

| Star | $P g_{p}$ | $C_{p}$ | $n$ | $\pi$ (Trig.) | $\pi$ (Ptm) | Sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & +8^{\circ} 3689 \ldots \ldots \\ & +8^{\circ} 3692 \ldots \end{aligned}$ | $\begin{aligned} & 8^{\mathrm{m} .25} \\ & 8.85 \end{aligned}$ | $\begin{aligned} & +0 \mathrm{~m} .59 \\ & +0.69 \end{aligned}$ | $\begin{aligned} & 3 \\ & 3 \end{aligned}$ | $\begin{aligned} & 0.033 \pm 8(\mathrm{~A}) \\ & .038 \pm 9(\mathrm{M}) \\ & 0.036 \pm 7(\mathrm{Y}) \end{aligned}$ |  | Dwarf <br> Dwarf |
|  |  |  | Mean | 0.035 $\pm 4$ | 0".033 |  |
| $\begin{aligned} & 17 \mathrm{Cyg} \quad \mathrm{AB} \\ & \text { ADS } 12889 \mathrm{AB} \end{aligned}$ | $\begin{aligned} & 5.25 \\ & 8.45 \end{aligned}$ | $\begin{aligned} & +0.38 \\ & +0.89 \end{aligned}$ | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ | $\begin{aligned} & 0.045 \pm 5(\mathrm{~A}) \\ & .046 \pm 7(\mathrm{M}) \\ & 0.049 \pm 7(\mathrm{y}) \end{aligned}$ |  | Bright-dwarf Dwarfs |
|  |  |  | Mean | $0.047 \pm 4$ | 0.047 |  |

and the metallic-line type is only between A3 and A7, and the star is not included in A. Slettebak ${ }^{3} s^{34}$ catalogue of metallic-line stars brighter than fifth magnitude and north of declination $-20^{\circ}$. If the fainter star is a dwarf, $M=+3^{m} 80$, and the photometric parallax is $0^{\prime \prime} 049$, compared with the mean trigonometric value, 0 ". $051 \pm 5$. If we apply the photometric parallax to the brighter component, we find $M=+1 \mathrm{~m} 17$. The bright star is then also a dwarf, for Table 6 gives, at $C_{p}=+0^{\mathrm{m}} 08, M=+1 \mathrm{~m} 07$. If this star is a metallic-line star, it is the brightest one yet discussed; from its position in the dwarf sequence we would expect the Atlas classification to be A3-A5 V.
$-15^{\circ} 4041$ and $-15^{\circ} 4042$. - These two stars, separated by $300^{\prime \prime}$, have a common proper motion and radial velocity. The large radial velocities of both components, $+306 \pm$ $2 \mathrm{~km} / \mathrm{sec}$ for $-15^{\circ} 4041$ and $+300 \pm 3 \mathrm{~km} / \mathrm{sec}$ for $-15^{\circ} 4042$, obtained by D. M. Popper, ${ }^{35}$ prove that the components are physically connected. Popper classifies $-15^{\circ} 4041$ as sdG6. If we assume the stars to be subdwarfs and interpolate for $C_{p}=+0.6 .5$ and $+0^{\mathrm{m}} 74$ in Figure 1 or Table 9, we find $M=+7{ }^{\mathrm{m}} 40$ and $+7 . \mathrm{m}^{\mathrm{m}} 80$ for the brighter and fainter stars, respectively. The photometric parallaxes, then, are both 0 ". 035 , as compared with the mean trigonometric value, $0 " 040 \pm 4$. Since a parallax of 0.019 would be required to put these stars on the dwarf sequence, we may conclude that both are subdwarfs and, as such, they fill the gap, in Table 9, between HR 4550 and 36 UMa C.
$-0^{\circ} 2944$.-This star, ADS 9544, is a close double, with a separation of about 0 " 1 , with an orbital period probably less than 5 years. R. G. Aitken, ${ }^{36}$ the only observer from whom measures are available, gives the components equal magnitude, but this observation cannot be conclusive in so difficult a pair. If we correct the observed magnitude for the duplicity of the star and assume that the components are of equal magnitude, we find $P g_{p}=7^{\mathrm{m}} 93$ for each star. From Table 6 we find $M=+6^{\mathrm{m}} 30$ if the stars are dwarfs, or a photometric parallax of 0.047 , as compared with the mean trigonometric value 0 ". $057 \pm 4$, which is formed from several discordant values. The system has a fairly high radial velocity of $-70 \mathrm{~km} / \mathrm{sec}$ and a large proper motion of 1 ". 38 . Since a parallax of 0 ". 078 would place the stars on the subdwarf sequence, we cannot altogether eliminate the possibility that the stars are subdwarfs. If, however, we disregard the discordant Mount Wilson value for the trigonometric parallax, given in Table 18, and allow for a small difference in magnitude between the components, we can conclude that the stars are probably dwarfs.
$\beta$ Ser.-The companion to $\beta$ Ser is $+15^{\circ} 2906$, which is $1600^{\prime \prime}$ distant from the brighter star but shares its proper motion. The brighter star is classified A2 IV in the Atlas, which

[^17]the same. ${ }^{41}$ In any case, the presence of two sequences, the blue-dwarf and bright bluedwarf, probably is not an effect of rotation on the observed colors; for, if that were true, we would not expect the sharp demarcation between the sequences, as is observed, but, instead, a rather ill-defined region in which the position of each star was determined by its rotational speed and by the orientation of its rotational axis.

TABLE 20
Estimated Rotational Velocities of Dwarfs and Bright Blue-Dwarfs

| Dwarfs |  |  | Bright Blue-Dwarfs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Star | $M$ ( Ptm ) | $\begin{gathered} V_{r} \sin i \\ (\mathrm{Km} / \mathrm{Sec}) \end{gathered}$ | Star | $M(\mathrm{Ptm})$ | $\begin{gathered} V_{r} \sin i \\ (\mathrm{Km} / \mathrm{Sec}) \end{gathered}$ |
| $\delta \mathrm{UMa}$. | $+0 \mathrm{~m} 70$ | 175 | $\epsilon \mathrm{UMa}$. | $\pm 0 \mathrm{~m} 00$ | 25 |
| H 60. | +1.07 | 150 | $a \mathrm{Lyr}$. | +0.40 | 0 |
| H 117. | $+1.20$ | 0 | $\gamma$ UMa | +0.60 | 125 |
| H 139. | +1.25 | 100 | $\boldsymbol{\beta}$ UMa. | +0.70 | 0 |
| H 119. | +1.33 | 50 | P 47. | $+0.90$ | 0 |
| H 84. | +2.15 | 25 | H 61. | +1.19 | 0 |
| H 34. | +2.15 | 0 | P 56. | $+1.19$ | 50 |
| H 141 | +2.30 | 0 | P 22. | +1.45 | 0 |
| H 96. | +2.45 | 0 | $\lambda$ Gem | +1.53 | 150 |
| P 58. | $+2.60$ | 50 | P 77. | +1.82 | 75 |
| H 39. | +2.82 | 0 | $\beta$ Leo | +1.82 | 125 |
| P 15. | +2.82 | 75 | 72 Oph. | +1.82 | 75 |
| P 20. | +2.92 | 50 | P 75. | +1.88 | 75 |
| H 8. | +3.02 | 100 | H 50. | +1.92 | 0 |
| H 102 | +3.14 | 75 |  |  |  |
| H 14. | +3.27 | 0 |  |  |  |
| P 71. | +3.27 | 75 |  |  |  |
| H 4. | +3.27 | 25 |  |  |  |
| H 21 | +3.70 | 25 |  |  |  |
| H 13. | +3.79 | 0 |  |  |  |
| H 38. | +3.79 | 0 |  |  |  |
| H 6. | +4.15 | 25 |  |  |  |

## MAGNETIC VARIABLES

The foregoing conclusion, that the bright blue-dwarfs rotate more slowly, in the mean, than do the blue-dwarfs, may have some bearing on the presence of stellar magnetic fields recently measured by H. W. Babcock. All the stars for which Babcock has announced magnetic fields are "peculiar" 42 A-type stars, the majority of which are also known to be spectrum variables. He concludes that it is "not impossible that future observations will show that all spectrum variables of type A have variable fields and, indeed, that the spectrum variability results from the magnetic variations." ${ }^{43}$ Furthermore, con-
${ }^{41}$ The role of $H^{-}$in the opacity of the A-type stars has been recognized since the discussions by Struve and by Hynek were published. Chandrasekhar and Münch (Ap.J., 104, 446, 1946) have shown that, for those stars in which the contribution of both $H^{-}$and $H$ are important, for a fixed effective temperature the color temperature is sensitive to small changes in pressure. If $g_{\text {grav }}$ is diminished by rotation, then the pressure will decrease in such a way as to make the star appear bluer. An accurate computation of the magnitude of this effect would be difficult, but qualitatively we can see that it would operate to make a rapidly rotating star bluer than a nonrotating star.
${ }^{42}$ The term "peculiar" is used here to indicate those A-type stars showing strong spectral features attributed to the elements manganese, europium, chromium, and strontium. Such stars are to be differentiated from the other peculiar A-type stars usually designated as "metallic-line" stars.
${ }^{43}$ Ap. J., 108, 200, 1948.
sequence" stars. In Paper II the color-luminosity array for the Pleiades stars added the blue-dwarf and bright blue-dwarf sequences to the fine structure in the region populated by the A-type stars. A discussion of the rotational velocities of these stars revealed that, at least for the Pleiades, the broad-lined, rapidly rotating $n$ stars populate the bluedwarf sequence, whereas the five stars that fall on the bright blue-dwarf sequence are sharp-lined objects; the rapidly rotating A-type giants, which populate the upper end of the Hyades' bright-dwarf sequence, are absent from the Pleiades. Table 19 reproduces the results obtained in Paper II for the run of the mean rotational velocity with luminosity of the blue-dwarfs in the Pleiades. Only four blue-dwarfs with available estimates of rotational velocity, which are not spectroscopic binaries, are added by the present discussion. These stars are Sirius, Fomalhaut, 80 Ursae Majoris, Altair, and $\iota$ Ursae Majoris, with rotational velocities of $0,85,200,200$, and $125 \mathrm{~km} / \mathrm{sec},{ }^{38}$ respectively. For comparison, the rotational velocities for the bright blue-dwarfs and the dwarfs are listed in Table 20, from which it appears that, on the average, both these sequences are populated by more slowly rotating stars than is the blue-dwarf sequence. The relatively large number of sharp-lined stars on the bright blue-dwarf sequence, as compared to the blue-dwarf

TABLE 19
Run of Mean Observed Rotational Velocities with Luminosity of the Blue-Dwarfs in the Pleiades

| $\overline{V_{r} \sin i}$ <br> $(\mathrm{Km} / \mathrm{Sec})$ | $M$ | No. of <br> Stars | $\overline{V_{r} \sin i}$ <br> $(\mathrm{Km} / \mathrm{Sec})$ | $M$ | No. of <br> Stars |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $170 \ldots \ldots \ldots$ | $-3^{\mathrm{m}}$ to $-1^{\mathrm{m}}$ | 4 | $100 \ldots \ldots \ldots$ | $+2^{\mathrm{m}}$ to $+2^{\mathrm{m} 5}$ | 4 |
| $180 \ldots \ldots \ldots$ | -1 to +1 | 5 | $60 \ldots \ldots \ldots$ | +2.5 to +3 | 4 |
| $130 \ldots \ldots \ldots$ | +1 to +2 | 5 |  |  |  |

sequence, suggests that the luminosity difference between the $n$ and $s$ stars noted by Adams and Joy was caused by the presence of both blue-dwarfs, predominately n stars, and bright blue-dwarfs, predominately s stars, in their material. From a compilation of the A-type stars in the Mount Wilson catalogue of spectroscopic parallaxes, ${ }^{4}$ J. A. Hynek ${ }^{39}$ found that, in the mean, the s stars are brighter than the n stars by 0.8 mag . for spectral types A0-A4; stars classified later than A4 are not included here for fear of vitiating the mean luminosities by including A-type stars populating the dwarf sequence. In approximate agreement with this result, Tables 13 and 15 show a difference of about 0.75 mag. between blue-dwarfs and bright blue-dwarfs for $C_{p}=-0 \mathrm{~m} .12$ to +0 m 02 . Struve has suggested that the luminosity differences between n and s stars may be caused by (1) ". . . systematic errors in the spectral classification used by Adams and Joy... ," (2) ". . . the more luminous stars actually rotate more slowly than the less luminous stars. . .," or (3) ". . the decrease in surface gravity due to rapid rotation simulates [an] absolute magnitude effect. . . ." ${ }^{40}$ Struve suggests that the absolute-magnitude effect might arise from the fact that, if a star of a given spectral class could be given rapid rotation, the higher level of ionization resulting from the lowered surface gravity would lead to an earlier spectral class, while the color of the star would remain

[^18]is the same classification given $\beta$ Aur. If, like $\beta$ Aur, $\beta$ Ser is a dwarf, then, from Table 6, $M=+0^{m} 50$, and the photometric parallax is 0 ". $024 .{ }^{37}$ If we apply this parallax to the faint star, we find $M=+5^{m} 40$, compared to $M=+5^{\mathrm{m}} 63$ given in Table 6 for $C_{p}=$ $+0^{\mathrm{m}} 56$. The faint star is itself a visual binary, ADS 9766 , the components of which are separated by $6^{\prime \prime}$ and were observed in this program as one star. Since we find that the combined light of the system falls 0.23 mag. above the dwarf sequence, we would expect the magnitudes of the components to differ by about 1.5 mag. photographically, or approximately 1.2 mag. visually, which may be compared with 1.0 mag . given as the rough, visual estimate by double-star observers. We conclude that $\beta$ Ser and both components of the fainter star are dwarfs.

36 Oph.-The close pair, AB, of this triple system was measured as one star, while the C component was measured separately. The components of the close pair are of equal magnitude; hence $P g_{p}=5 \mathrm{~m} 76$ for each component. If the bright stars are dwarfs, we find $M=+6^{m} 78$ from Table 6 and a photometric parallax of $0^{\prime \prime} .160$, which falls just outside the range of the two discordant trigonometric values given in Table 18. If we apply the photometric parallax to component C, we find $M=+8.41$, compared to $M=+8$ m 39 from Table 6 for $C_{p}=+1 \mathrm{~m} 02$. All three stars, therefore, are dwarfs.
$+8^{\circ} 3689$ and $+8^{\circ} 3692$. These two stars, separated by $600^{\prime \prime}$, have common proper motion and radial velocity. If $+8^{\circ} 3689$ is a dwarf, then from Table $6, M=+5^{m} .81$, and the photometric parallax is $0 " 032$, which agrees with the accordant trigonometric values. If we apply the photometric parallax to $+8^{\circ} 3692$, we find $M=+6^{m} 41$, which is the same value given in Table 6 for $C_{p}=+0 \mathrm{~m} 69$. We conclude that both stars are dwarfs.

17 Cyg and ADS 12889.-The star 17 Cyg is a visual binary, ADS 12913, with components that differ more than 3 mag ; in brightness and that are separated by $25^{\prime \prime}$. The pair ADS 12889, $52^{\mathrm{s}}$ preceding and $7^{\prime}$ south of 17 Cyg, has the same proper motion and radial velocity. The components of ADS 12889 are of equal magnitude and, because of their small separation, were measured as one star; the large magnitude difference between the components of 17 Cyg makes it certain that the observations of this pair are uncontaminated by the closer companion. If we correct the observations of ADS 12889 for equal luminosity of the components, we find $P g_{p}=+9$ m 20 for each star. From the assumption that 17 Cyg is a subgiant, as is indicated by the spectroscopic absolute magnitude, we have $M=+3^{m} .60$ and a photometric parallax of 0 ". 047 , which is the same as the mean of the trigonometric values; the latter in each case are the mean of the observed values for 17 Cyg and ADS 12889. If we apply the photometric parallax to the components of ADS 12889, we find $M=+7 \mathrm{~m} 55$, whereas Table 6 gives, for $C_{p}=$ $+0^{\mathrm{m}} .89, M=+7^{\mathrm{m}} 61$. Both components of ADS 12889 are probably dwarfs, but the magnitudes may be slightly unequal.

## VII. ROTATIONAL VELOCITY

## An AND AS STARS

We may recall from Paper I that the bright-dwarf sequence of the Hyades cluster appears to be populated by stars that, on the average, possess higher speeds of rotation than do stars populating the dwarf sequence. It was concluded that the separation between these two sequences was not an effect of rapid rotation on either the colors or the luminosities of the bright-dwarfs; the effect is just opposite to that noted by Adams and Joy, ${ }^{18}$ who first pointed out that the rapidly rotating A-type stars, designated as An stars, are, in the mean, of lower luminosity than the sharp-lined, or As, stars. If we convert the colors into spectral classes, the separation between the dwarf and the brightdwarf sequences occurs between the late A-type giants and the early A-type "main-

[^19]cerning the connection between the star's velocity of rotation and its magnetic field, he adds: "The spectra of some normal main-sequence stars of early type such as Sirius (A0) and $\gamma$ Geminorum (A3), show little or no evidence of the Zeeman effect on plates taken with the analyzer. As there is a great range in equatorial velocity among the A-type stars, from practically zero up to $250 \mathrm{~km} / \mathrm{sec}$, it is possible that Sirius and $\gamma$ Geminorum have a low rate of axial rotation; and as a working hypothesis we may retain the modified proposition that the most intense magnetic activity occurs in the stars that are rotating most rapidly." ${ }_{44}$

## SPECTRUM VARIABLES

There are three recognized spectrum variables included with the bright blue-dwarfs in Table 14: C $146=17$ Coma, C $160=21$ Coma, and $\epsilon \mathrm{UMa}$. The star 17 Com is described in the Atlas as being very similar to 78 Vir , which was the first star shown by Babcock to possess a magnetic field. ${ }^{44}$ Also, 17 Com is very nearly matched in color and luminosity by $a \mathrm{CVn}, 45$ which Babcock has recently announced to possess a magnetic field. ${ }^{46}$ Unfortunately, the three spectrum variables in Table 14, with the addition of 78 Vir and a CVn, represent only about 20 per cent of the total known (1947). A. J. Deutsch, ${ }^{47}$ however, has shown, from a consideration of the trigonometric parallaxes for more than half the total number of known spectrum variables, that as a group they lie between 0.5 and 1.2 mag. above the main sequence. Since the main sequence in this case probably refers to the blue-dwarf sequence, it appears reasonable to assume that at least the majority of spectrum variables populate the bright blue-dwarf sequence; some support is given this assumption by Deutsch's observation that "the peculiar A-stars generally exhibit less strongly winged hydrogen lines than do the main sequence stars of the same spectral type." ${ }^{21}$
To summarize, we may state the following conclusions concerning rotational velocities:

1. The bright blue-dwarfs rotate more slowly, in the mean, than do the blue-dwarfs. The separation between the two sequences in this respect, however, is not complete; for, although the zero rotational velocity of, say, Sirius, a blue-dwarf, may result from the fact that we are viewing this star nearly "pole-on," there are stars, such as $\gamma \mathrm{UMa}$ or $\lambda$ Gem, which populate the bright blue-dwarf sequence and have rapid velocities of rotation.
2. If we associate magnetic fields with rapid rotation, then it is necessary to assume that the spectra of stars such as $\epsilon \mathrm{UMa}, 17 \mathrm{Com}, 21 \mathrm{Com}, 78 \mathrm{Vir}$, and $a \mathrm{CVn}$ show negligible rotational disturbances because their axes of rotation are directed toward the sun. Since, according to Babcock, all spectrum variables of type A may possess variable

[^20]magnetic fields and since we have found that at least the majority of the spectrum variables populate the bright blue-dwarf sequence, then it appears that intense magnetic fields are a feature of bright blue-dwarfs, or, more precisely, of those bright blue-dwarfs which have their poles of rotation directed toward the sun. The requirement that the pole of rotation be directed toward the sun is probably only an observational one, since the Zeeman effect would be lost in rotationally broadened lines.
3. There appears to be no correlation between color and spectrum, or magnetic variability among the bright blue-dwarfs. For example, Vega, with $C_{p}=-0$ m.075, is of nearly the same color as $17 \mathrm{Com}, C_{p}=-0$ m. 08 , and yet Vega is not a recognized spectrum variable. The same situation exists for 78 Vir, with $C_{p}=-0^{m} 01$, which is similar to Hyades No. 61 or to Pleiades No. 56, both with $C_{p}=0$ m 00 in Table 14, but the two cluster stars are not recognized spectrum variables. It may be that spectrum variability, like magnetic variability, also requires that the stellar poles of rotation be directed toward the sun and that Vega, $\beta$ UMa, and the other bright blue-dwarfs in Table 19, which show no rotational line-broadening, fail to do so because of a real lack of rapid rotation. It is possible, also, that the spectrum variables possess shells that are rotating much more slowly than are the photospheres of the stars; the association of rapid rotation with the presence of magnetic fields could then be preserved without the necessity of assuming that all spectrum variables are viewed pole-on.

## VIII. CONCLUDING REMARKS

There are several obvious questions which present themselves in the interpretation of the foregoing data. The answers to some will require more observations, which are now being obtained at the Lick Observatory.

An important question pertains to what extent the various sequences represent real differences in the stars. The bright-dwarf and subdwarf sequences have been discussed at some length. We have seen that the separation between the blue-dwarf, bright bluedwarf, and dwarf sequences is reflected in the spectra of these stars; our knowledge of Sirius and Vega alone, two early A-type stars with the well-determined difference in luminosity amounting to 1 mag ., is convincing evidence for the reality of this separation. There is strong evidence that the spectrum and magnetic variables of class A occur among the bright blue-dwarfs and that the brighter components of at least three Algoltype eclipsing stars-U Cep, U Sge, and Algol-populate the blue-dwarf sequence. Unfortunately, it is in just such eclipsing systems that Struve has found spectroscopic evidence that the velocity-curves are not a true representation of the motions of the stars, so that we are not able to make use of the spectroscopic data to estimate the size and masses of the components. Among the brighter stars on the bright blue-dwarf sequence, only three are known spectroscopic binaries, and one of these, a CrB , is also an eclipsing star. D. B. McLaughlin ${ }^{48}$ has studied $a \mathrm{CrB}$ spectroscopically and has found that its rotational period is much shorter than the period of revolution. In this respect $a \mathrm{CrB}$ is similar to the brighter stars of the U Cep and U Sge systems, for which Struve has also found a lack of equality between rotational and orbital velocities. The similarity, however, ends with this feature, for the Algol systems have periods less than 5 days and the components are subgiants, of larger size than the brighter stars by at least a factor of 2, compared to the well-separated components of $a \mathrm{CrB}$, the fainter of which has only 30 per cent of the radius of the brighter. We also suspect that YZ Cas is a bright bluedwarf. This system was not observed on the present program because the wide visual pair is not resolvable with the 12 -inch photometer. However, two-color photometry by Kron ${ }^{49}$ provides enough information to derive the approximate International magnitudes and colors of the components. From Kron's data we find $\operatorname{Pg}(A)=5 . \mathrm{m} 76, C=-0 \mathrm{~m} .01$, and
${ }^{48}$ Pub. Obs. U. Michigan, 5, 91, 1933.
${ }^{40}$ Ap. J., 96, 87, 1942; Contr. Lick Obs., Ser. II, No. 5, and Lick Obs. Bull., 19, 59, 1939.
$P g(B)=8^{m} 71, C=+0^{m} 33$. If we assume that the fainter component is a dwarf, then, from Table $6, M(B)=+4.14$ and, therefore, $M(A)=+1$ m. 19 . From Table 15 we find that a bright blue-dwarf with $C=-0$ m 01 has $M=+1$ m. 10 . From the color of the dwarf component, $C=+0^{\mathrm{m}} 33$, which corresponds to dF 5 , we estimate the mass to be $1.5 \odot$, which, when combined with the mass functions, gives $3 \odot$ for the mass of the brighter star. From unpublished work Kron estimates the fainter components of $a \mathrm{CrB}$ to be of approximately solar type; hence, if, by analogy with the YZ Cas system, we assume it to be a dwarf of solar mass, we derive, from the mass function, $3 \odot$ as the mass of the brighter star. The resulting radii are then $2.7 \odot$ and $2.8 \odot$ for the bright blue-dwarf components of YZ Cas and a CrB , respectively. The radii derived for the dwarf components, $1.4 \odot$ and $1.0 \odot$, respectively, are consistent with the assumed masses for these stars. Although the values quoted above are necessarily rough, they provide us with an estimate that the masses and radii of bright blue-dwarfs exceed the solar values by a factor of $2-3$. The distribution of the A-type eclipsing systems in the various sequences is further marked by the presence of $\beta$ Aur on the dwarf sequence. The system of $\beta$ Aur contains two very nearly equal dwarfs with masses of $\sim 2.2 \odot$ and radii of $\sim 2.4 \odot$; these values are somewhat smaller than those of the bright blue-dwarfs. To summarize, then: the somewhat limited data appear to separate the A-type eclipsing stars into the three sequences populated by the A-type stars, as follows:

1. Blue-dwarfs: Algol systems with large subgiants as the secondary components
2. Bright blue-dwarfs: 50 a CrB systems with relatively smaller dwarfs as secondary components
3. Dwarfs: ${ }^{51} \beta$ Aur systems with both components very nearly equal in mass, luminosity, and temperature

The occurrence of the components of eclipsing stars on the various sequences will be discussed in more detail in a succeeding paper of this series.

Another question that might be mentioned here pertains to the distribution of the so-called "high-" and "low-velocity" stars among the various sequences. The most striking sequence in this respect is the subdwarf sequence, where all but two stars have peculiar space motions exceeding $70 \mathrm{~km} / \mathrm{sec}$. The two exceptions are 36 UMa C and ADS 246 B ; the space motion of 36 UMa C is less than $30 \mathrm{~km} / \mathrm{sec}$, while that of ADS 246 is about $60 \mathrm{~km} / \mathrm{sec}$. It may be significant that both these stars have dwarf companions, although ADS 246 may escape the net for high-velocity stars because of the arbitrary limit of $70 \mathrm{~km} / \mathrm{sec}$ and the high dispersion for the velocities of such stars. In contrast to the fact that all the single stars populating the subdwarf sequence have peculiar space motions in excess of $70 \mathrm{~km} / \mathrm{sec}$, the stars on the dwarf sequence contain a mixture of both high- and low-velocity stars. For example, the motions of the dwarfs 72 Her , CC 1017, and both components of 61 Cyg exceed $100 \mathrm{~km} / \mathrm{sec}$. It is interesting that, in contrast to the low-velocity system of 36 UMa , which contains a subdwarf and two dwarfs, we have the system of CC 1017 and 1018, with a peculiar space motion of over $100 \mathrm{~km} / \mathrm{sec}$, which also contains members of both sequences. A large number of highvelocity stars is found among the subgiants; for about 20 per cent of the stars in Table 4 are of this type, the most notable examples being $\zeta$ Her, $\gamma$ Ser, and $\eta$ Cep, all with peculiar space motions exceeding $100 \mathrm{~km} / \mathrm{sec}$.

I am indebted to my colleagues, Messrs. Herbig, Kron, Mayall, and Weaver, for discussions on some of the questions raised here.

[^21]
[^0]:    ${ }^{4}$ Ap．J．，81，189， 1934.

[^1]:    * It should be noted that the differences between the $\pi$ (Trig.) and $\pi$ (ptm) are less than the probable errors of the trigonometric parallaxes in sixteen cases, equal in five cases, and greater in one case. A study of the probable errors given by Schlesinger on the basis of agreements between different determinations for each star shows that his probable errors should be reduced by an average factor of 0.7 . With the recomputed values of the probable errors, it is found that $\pi$ (Trig.) $-\pi$ (ptm) is less in twelve cases and greater in ten. Since Schlesinger had, in general, increased the published probable errors in a manner described in the General Catalogue of Stellar Parallaxes, it appears that for these stars, at least, he was overpessimistic.
    $\dagger$ The luminosity classes for $\iota$ Psc and $\chi$ Her are by Nassau and van Albada ( $A p_{.} J_{., 10} 107,16,1948$ ); those for 10 UMa, 10 Tau, and $v^{2}$ CMa are unpublished values determined here by Mrs. Belserene (Emilia Pisani) in connection with Moore and Paddock's radial-velocity program of faint stars (Ap.J., 112, 48, 1950).

[^2]:    ${ }^{5}$ Ap. J., 91, 506, 1940.
    ${ }^{6}$ Ap. J., 109, 426, 1949.

[^3]:    ${ }^{7}$ It might be pointed out that, even in terms of visual magnitude, the absolute magnitudes of the subgiants between $C_{p}=+0.97$ and $C_{p}=+0^{\mathrm{m}} .32$ will vary only by 0.65 mag .
    ${ }^{8}$ Ap. J., 108, 1, 1948.

[^4]:    ${ }^{9}$ Since the blue-dwarf sequence begins its nearly vertical rise at $C_{p} \sim-0^{\mathrm{m} 16}$, the effects of selective absorption on the bright components would be conspicuous; this is one reason that the use of some B-type stars in detecting interstellar absorption is so successful.
    ${ }^{10}$ The following convention will be followed throughout in regard to the source of individual trigonometric parallaxes: $(\mathrm{A})=$ Allegheny, $(\mathrm{C})=$ Cape of Good Hope, $(\mathrm{G})=$ Greenwich, $(\mathrm{M})=\mathrm{McCor}-$ $\operatorname{mick},(S)=$ Sproul, $(\mathrm{V})=$ Van Vleck, $(\mathrm{W})=$ Mount Wilson, $(\mathrm{Y})=$ Yale, and $(\mathrm{y})=$ Yerkes. The probable errors are given in terms of 0 ". 001 .
    ${ }^{11}$ Pub. A.S.P., 59, 261, 1947.
    ${ }^{12}$ Pub. R.A.S. Canada, 27, 146, 1933.
    ${ }^{13}$ Lick Obs. Bull., 17, No. 464, 37, 1934.
    ${ }^{14}$ Princeton U. Obs. Contr., No. 21, 1946.

[^5]:    ${ }^{15}$ Variable Stars ("Harvard College Observatory Monographs," No. 5 [Cambridge, 1938]).
    ${ }^{16}$ Ap. J., 102, 74, 1945.
    ${ }^{17}$ It is strange that, although the primary eclipse of RW UMa indicates that the components are of nearly equal photographic brightness, only the emission lines of the redder component are observed, while in WW Dra, for example, where the total eclipse exceeds 1.0 mag . photographically, Joy observed both absorption and emission lines of the fainter star. The emission lines in WW Dra, like those in RW UMa, give a velocity-curve which is a mirror image of that of the primary star, but the amplitudes of the absorption-line velocity-curves are quite unequal (Ap.J., 94, 407, 1941; Mt. W. Contr., No. 654).

[^6]:    * For $C_{p}=+0 \mathrm{~m} 46$ to +1 m 22 the values of $M$ may be computed from the relation $M=+2 \mathrm{~m} 27+6.0 C_{p}$.
    $\dagger$ From $C_{p}=+1 \mathrm{~m} 06$ to +1 m 22 the main sequence is derived from the stars in Table 3.

[^7]:    ${ }^{18}$ Ap．J．，56，262，1922；Mt．W．Contr．，No． 244.
    ${ }^{19}$ Ap．J．，89，548， 1938.
    ${ }^{20}$ A．J．，53，149， $1948 . \quad{ }^{21}$ Ap．J．，105，96，1947；Mt．W．Contr．，No． 726.

[^8]:    *Three consistent determinations of the trigonometric parallax-0.100 $\pm 10(\mathrm{~A}), 0!102 \pm 9(\mathrm{Y})$, and $0.093 \pm 9(\mathrm{M})-$ indicate that this star falls below the bright-dwarf sequence; however, the possibility that the star is a subgiant, with photometric parallax of 0.083 , cannot be entirely eliminated.
    $\dagger$ Two determinations of the trigonometric parallax, $0.072 \pm 9(\mathrm{~A})$ and $0.076 \pm 10(\mathrm{M})$, place this star on the subdwarf sequence; but, since a parallax of 0.083 would place it on the dwarf sequence, the point is of low weight.
    $\ddagger$ The data for these stars are taken from Paper I.

[^9]:    ${ }_{22}$ Ap. J., 95, 201, 1942.
    ${ }^{23}$ Ap. J., 87, 589, 1938.

[^10]:    ${ }^{24}$ Stockholm Obs. Ann., Vol. 15, No. 1, 1947.
    ${ }^{25}$ The existence of such a discontinuity was first pointed out by E. Hertzsprung in 1915, when he found a similar effect in his measurements of the effective wave lengths of absolutely faint stars. He concluded: "It is an obvious suggestion for the explanation of the fact noted above that the absolute magnitude +3 (sun $=-0.33$ ), corresponding to the temperature $3400^{\circ} \mathrm{Abs}$. of a black body the size of the sun, represents the stage of a cooling star at which the formation of relatively dark solid matter on its surface begins, the remaining fluid giving practically all the radiation." Perhaps it is near this point in the color-luminosity array that the dwarfs reach a minimum effective temperature and become fainter as a result of decreasing size alone.

[^11]:    ${ }^{26} A$ p. J., 109, 532, 1949.
    ${ }^{27}$ Pub. A.S.P., 61, 134, 1949.

[^12]:    * Cluster parallax from Paper III.

[^13]:    ${ }^{29 \mathrm{a}}$ Harvard Mimeograms, Ser. II, No. 2, 1938.

[^14]:    ${ }^{30}$ The eclipses are only about 0.1 mag. deep, and the observed magnitude refers to maximum light.

[^15]:    ${ }^{31}$ Ap. J., 78, 46, 1933.

[^16]:    ${ }^{32}$ It is an attractive hypothesis to regard visual binaries as the remains of galactic clusters. For example, stars Nos. 145 and 146 in the Coma Berenices cluster (Paper III) are close enough together to be linked in ADS 8568. One of these stars is a spectrum variable, the other a metallic-line star, and they form a pair very similar to $\pi$ Boo, ADS 9338, one component of which also is a spectrum variable and the other a metallic-line star. When disintegration of the Coma cluster is completed, perhaps by galactic rotation or by some other disrupting force, it is possible that the two Coma stars may hold each other by mutual attraction and become another binary system like $\pi$ Boo. The systems of $\epsilon$ Lyr, Castor, and many others are highly suggestive of such an origin.

[^17]:    ${ }^{34}$ Ap. J., 109, 547, 1949.
    ${ }^{35}$ Ap. J., 95, 307, 1942.
    ${ }^{36}$ New General Catalogue of Double Stars (Washington: Carnegie Institution of Washington, 1932).

[^18]:    ${ }^{38}$ The sources of the rotational velocities quoted in this paper are Miss Westgate (Ap.J., 78, 48, 1933) and C. T. Elvey ( $A p . J ., 71,228,1930$ ); the rotational velocities of the Hyades and Pleiades stars, the work of Struve and of Struve and Smith, are taken from Papers I and II. Estimates of the rotational velocities of the Coma Berenices cluster stars are, unfortunately, not available.
    ${ }^{39}$ Ap. J., 83, 476, 1936.
    ${ }^{40}$ Observatory, 54, 84, 1931.

[^19]:    ${ }^{37} \beta$ Ser is often included as a member of the extended Ursa Major cluster. If we adopt the constants derived in Paper III for this cluster, we find a cluster parallax of 0.024 , which is identical with the photometric parallax.

[^20]:    ${ }^{44} 78$ Vir was not included in the discussion of the bright blue-dwarf sequence because of its relatively small and uncertain trigonometric parallax, $0.018 \pm 5$. The observations of magnitude and color give $P g_{p}=4 \mathrm{~m} 83, C_{p}=-0 \mathrm{~m} 01$. If we assume the star to be a bright blue-dwarf, then, from Table $15, M=$ +1 m 10 , and the photometric parallax is 0.018 . The assumption is usually made that this star belongs to the Ursa Major cluster, and the constants derived for this cluster in Paper III lead to the cluster parallax of 0 ". 017 for 78 Vir.
    ${ }^{45} a \mathrm{CVn}$ was not observed on the present program because of its small and uncertain trigonometric parallax and also because it is not possible to separate the visual components of the wide double with the 12 -inch photometer. The individual visual magnitudes are given in Harvard Mimeograms, Ser. III, No. 2, as 2.74 and 5.26 mag. A. Deutsch ( $A p . J ., 105,283,1947$ ) has classified the fainter component as F5 IV, which, from Fig. 2, corresponds to $C_{p}=+0 \mathrm{~m} 37$ or $P g(B)=5 \mathrm{~m} .63$. The brighter component is contained in Hertzsprung's catalogue of effective wave lengths (B.A.N., 9, 101, 1940); so, by converting $c_{2} / T$ to $C_{p}$, we find $C_{p}=-0$ m 07 or $P g(A)=2 \frac{m}{6}$. Since the $B$ component is a subgiant, we take $M=+3$ m 60 , and the modulus of the system is +2.03 mag., which gives a photometric parallax of 0.039 ; the mean trigonometric parallax is $0.033 \pm 5$. Applying the distance modulus, +2.03 mag., to the brighter star, we find $M=+0 \mathrm{~m} 71$ at $C_{p}=-0 \mathrm{~m} .07$, compared with $M=+0 \mathrm{~m} .50$ for $C_{p}=-0 \mathrm{~m} .07$ in Table 15 , with the assumption that the star is a bright blue-dwarf.
    ${ }^{46}$ Pub. A.S.P., 61, 226, 1949.
    ${ }^{47}$ Ap. J., 105, 283, 1947.

[^21]:    ${ }^{50}$ As a result of the large differences in size and luminosity of the components of these systems, both eclipses are very shallow and have been discovered by photoelectric methods only.
    ${ }^{51} \mathrm{WW}$ and AR Aur probably are similar systems.

