THE ISOTHERMAL FUNCTION

Many years ago we made an extensive integration of the equation[1]

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\psi}{d\xi}\right)=e^{-\psi},$$

which governs the distribution of matter in hydrostatic equilibrium under its own gravitation and at a constant temperature. This integration has been used in the various investigations of stellar models with isothermal cores which have been carried out at this observatory.[2] In view of the increasing importance of such models for stellar structure,[3] we have thought that it may be useful to publish our integration of the isothermal function.

In addition to the usual quantities, we have also tabulated the homology invariant functions

$$u=\frac{\xi\, e^{-\psi}}{\psi'} \quad \text{and} \quad v=\xi\psi',$$

which are useful in the construction of composite models.

We believe that the tabulated entries after rounding the last figure should be reliable to one or two units.

S. CHANDRASEKHAR
GORDON W. WARES

YERKES OBSERVATORY
March 14, 1949

WAVE LENGTHS OF THE (0, 0) BAND HEADS IN THE INFRARED SYSTEM OF *CN*

Recently Herzberg and Phillips published in this *Journal*[1] a new assignment of vibrational quantum numbers to the infrared *CN* bands near 10930 A. These bands were photographed at the National Bureau of Standards,[2] more than ten years ago, with the spectrograph in which is mounted the Anderson concave grating with 7500 lines per inch. The dispersion in the first-order spectrum is 10 A/mm. The light-source was an arc between graphite electrodes carrying 15 amperes from a 220-volt D.C. circuit. Measurements of three to ten spectrograms gave the following mean values for the wave lengths of the band heads: 10871.71 (2), 10925.74 (10), and 10963.75 (8). The numbers in parentheses are estimated intensities. A strong feature at 11048.42 (10) is probably a blend of several rotation lines.

C. C. KIESS

NATIONAL BUREAU OF STANDARDS
March 31, 1949

[1] For the notation see S. Chandrasekhar, *Introduction to the Study of Stellar Structure* (Chicago: University of Chicago Press, 1939), pp. 155–170.

[2] S. Chandrasekhar and L. R. Henrich, *Ap. J.*, **94**, 525, 1941; S. Chandrasekhar and M. Schönberg, *Ap. J.*, **96**, 161, 1942; M. Harrison, *Ap. J.*, **103**, 193, 1946, and **105**, 322, 1947; and P. Ledoux, *Ap. J.*, **105**, 305, 1947.

[3] A. Reiz, *Ann. d'ap*, **10**, 301, 1947; M. Schwarzschild and R. Richardson, *Ap. J.*, **108**, 373, 1948.

[1] *Ap. J.*, **108**, 163, 1948.

[2] *J. Res. Nat. Bur. Standards*, **20**, 37, 1938.

TABLE 1

ξ	ψ	$e^{-\psi}$	ψ'	u	v	$\xi^2\psi'$	$\rho_c/\bar{\rho}$
0	0.00000000	1.0000000	0.0000000	3.000000	0.000000	0.000000	1.000000
.1	0.00166583	0.9983355	0.0333000	2.998002	0.003330	0.000333	1.001000
.2	0.00665337	0.9933687	0.0664010	2.992028	0.013280	0.002656	1.004001
.3	0.01493288	0.9851780	0.0991076	2.982145	0.029732	0.008920	1.009004
.4	0.02645548	0.9738914	0.1312320	2.968456	0.052493	0.020997	1.016012
.5	0.04115396	0.9596814	0.1625969	2.951106	0.081298	0.040649	1.025030
.6	0.05894408	0.9427595	0.1930388	2.930270	0.115823	0.069494	1.036061
.7	0.07972601	0.9233693	0.2224100	2.906158	0.155687	0.108981	1.049113
.8	0.10338606	0.9017788	0.2505811	2.879000	0.200465	0.160372	1.064193
.9	0.12979853	0.8782724	0.2774418	2.849049	0.249698	0.224728	1.081308
1.0	0.15882769	0.8531434	0.3029014	2.816571	0.302901	0.302902	1.100467
1.1	0.19032975	0.8266865	0.3268897	2.781841	0.359579	0.395537	1.121683
1.2	0.22415488	0.7991913	0.3493560	2.745136	0.419227	0.503073	1.144964
1.3	0.26014913	0.7709366	0.3702684	2.706733	0.481349	0.625754	1.170322
1.4	0.29815626	0.7421854	0.3896126	2.666904	0.545458	0.763641	1.197771
1.5	0.33801943	0.7131814	0.4073907	2.625912	0.611086	0.916629	1.227323
1.6	0.37958273	0.6841468	0.4236191	2.584007	0.677791	1.084465	1.258993
1.7	0.42269255	0.6552801	0.4383270	2.541427	0.745156	1.266765	1.292795
1.8	0.46719876	0.6267555	0.4515543	2.498393	0.812798	1.463036	1.328744
1.9	0.51295569	0.5987233	0.4633503	2.455106	0.880366	1.672695	1.366856
2.0	0.55982298	0.5713102	0.4737714	2.411755	0.947543	1.895086	1.407148
2.1	0.60766620	0.5446204	0.4828796	2.368505	1.014047	2.129499	1.449637
2.2	0.65635733	0.5187375	0.4907410	2.325509	1.079630	2.375186	1.494339
2.3	0.70577514	0.4937257	0.4974247	2.282897	1.144077	2.631377	1.541272
2.4	0.75580536	0.4696322	0.5030011	2.240785	1.207203	2.897286	1.590454
2.5	0.80634082	0.4464889	0.5075413	2.199274	1.268853	3.172133	1.641903
2.6	0.85728143	0.4243140	0.5111157	2.158447	1.328901	3.455142	1.695637
2.7	0.90853409	0.4031147	0.5137938	2.118378	1.387243	3.745557	1.751675
2.8	0.96001257	0.3828881	0.5156432	2.079125	1.443801	4.042643	1.810037
2.9	1.01163730	0.3636231	0.5167295	2.040733	1.498516	4.345695	1.870740
3.0	1.06333512	0.3453023	0.5171153	2.003242	1.551346	4.654038	1.933805
3.1	1.11503902	0.3279025	0.5168608	1.966676	1.602268	4.967032	1.999249
3.2	1.16668782	0.3113966	0.5160225	1.931057	1.651272	5.284070	2.067093
3.3	1.21822587	0.2957544	0.5146545	1.896397	1.698360	5.604588	2.137356
3.4	1.26960276	0.2809432	0.5128074	1.862701	1.743545	5.928054	2.210056
3.5	1.32077295	0.2669289	0.5105282	1.829970	1.786849	6.253970	2.285215
3.6	1.37169548	0.2536765	0.5078613	1.798198	1.828301	6.581882	2.362850
3.7	1.42233365	0.2411506	0.5048477	1.767379	1.867937	6.911365	2.442981
3.8	1.47265473	0.2293159	0.5015255	1.737500	1.905797	7.242028	2.525628
3.9	1.52262964	0.2181375	0.4979300	1.708546	1.941927	7.573515	2.610809
4.0	1.57223270	0.2075812	0.4940936	1.680501	1.976374	7.905498	2.698544
4.1	1.62144134	0.1976137	0.4900464	1.653346	2.009190	8.237680	2.788852
4.2	1.67023587	0.1882027	0.4858158	1.627060	2.040426	8.569791	2.881751
4.3	1.71859922	0.1793172	0.4814268	1.601622	2.070135	8.901582	2.977261
4.4	1.76651672	0.1709273	0.4769026	1.577010	2.098371	9.232834	3.075401
4.5	1.81397593	0.1630048	0.4722642	1.553202	2.125189	9.563350	3.176188
4.6	1.86096639	0.1555223	0.4675306	1.530173	2.150641	9.892947	3.279643
4.7	1.90747946	0.1484541	0.4627193	1.507900	2.174781	10.221469	3.385782
4.8	1.95350819	0.1417758	0.4578461	1.486359	2.197661	10.548774	3.494624
4.9	1.99904709	0.1354643	0.4529251	1.465529	2.219333	10.874732	3.606186
5.0	2.04409206	0.1294977	0.4479695	1.445385	2.239847	11.199237	3.720491
5.1	2.08864022	0.1238554	0.4429907	1.425905	2.259253	11.522188	3.837552
5.2	2.13268980	0.1185181	0.4379995	1.407065	2.277598	11.843512	3.957386
5.4	2.21929101	0.10868614	0.4280160	1.371222	2.311287	12.48095	4.205450
5.6	2.30389963	0.09986863	0.4180832	1.337687	2.341266	13.11109	4.464821
5.8	2.38653113	0.09194809	0.4082524	1.306297	2.367864	13.73361	4.735632
6.0	2.46721009	0.08482117	0.3985637	1.276903	2.391382	14.34829	5.018019
6.2	2.54596816	0.07839712	0.3890481	1.249363	2.412098	14.95501	5.312111
6.4	2.62284247	0.07259622	0.3797295	1.223544	2.430269	15.55372	5.618034
6.6	2.69787426	0.06734853	0.3706253	1.199325	2.446127	16.14444	5.935914
6.8	2.77110773	0.06259263	0.3617480	1.176592	2.459886	16.72723	6.265872
7.0	2.84258916	0.05827459	0.3531060	1.155240	2.471742	17.30219	6.608026
7.2	2.91236613	0.05434699	0.3447041	1.135172	2.481870	17.86946	6.962493
7.4	2.98048696	0.05076811	0.3365447	1.116298	2.490431	18.42919	7.329388
7.6	3.04700018	0.04750121	0.3286278	1.098535	2.497571	18.98154	7.708822
7.8	3.11195416	0.04451388	0.3209520	1.081808	2.503425	19.52672	8.100900
8.0	3.17539683	0.04177752	0.3135141	1.066045	2.508113	20.06490	8.505731
8.2	3.23737538	0.03926682	0.3063100	1.051183	2.511742	20.59628	8.923422

TABLE 1-Continued

ξ	ψ	$e^{-\psi}$	ψ'	u	v	$\xi^2\psi'$	$\rho_c/\bar{\rho}$
8.2	3.23737538	0.03926682	0.3063100	1.051183	2.511742	20.59628	8.923422
8.4	3.29793612	0.03695937	0.2993351	1.037161	2.514415	21.12108	9.354065
8.6	3.35712434	0.03483529	0.2925838	1.023924	2.516221	21.63950	9.797763
8.8	3.41498415	0.03287693	0.2860501	1.011421	2.517241	22.15172	10.254614
9.0	3.47155848	0.03106857	0.2797279	0.999604	2.517551	22.65796	10.724708
9.2	3.52688898	0.02939623	0.2736107	0.988431	2.517218	23.15841	11.208136
9.4	3.58101599	0.02784739	0.2676920	0.977861	2.516305	23.65327	11.704994
9.6	3.63397856	0.02641090	0.2619652	0.967856	2.514866	24.14271	12.215363
9.8	3.68581441	0.02507674	0.2564237	0.958383	2.512952	24.62693	12.739332
10.0	3.73655997	0.02383596	0.2510612	0.949408	2.510612	25.10612	13.276975

ξ	ψ	$e^{-\psi}$	ψ'	u	v
10	3.73655997	0.02383596	0.2510612	0.949408	2.510612
11	3.97511952	0.018777057	0.2267131	0.911053	2.493844
12	4.19117842	0.015128447	0.2059457	0.881501	2.471348
13	4.38799408	0.012425629	0.1881317	0.858618	2.445712
14	4.56825346	0.010376066	0.1727558	0.840869	2.418581
15	4.73417833	0.008789668	0.1594003	0.827132	2.391005
16	4.88761436	0.007539388	0.1477278	0.816571	2.363645
17	5.03010344	0.006538134	0.1374657	0.808553	2.336917
18	5.16294168	0.005724834	0.1283929	0.802591	2.311072
19	5.28722535	0.005055769	0.1203294	0.798305	2.286259
20	5.40388741	0.004499057	0.1131274	0.795397	2.262548
21	5.51372659	0.004031057	0.1066652	0.793625	2.239969
22	5.61743057	0.003633966	0.1008417	0.792800	2.218517
23	5.71559468	0.003294191	0.0955726	0.792763	2.198170
24	5.80873691	0.003001218	0.0907872	0.793385	2.178893
25	5.89731020	0.002746823	0.0864256	0.794563	2.160640
26	5.98171237	0.002524500	0.0824371	0.796207	2.143365
27	6.06229439	0.002329051	0.0787786	0.798242	2.127022
28	6.13936715	0.002156288	0.0754129	0.800607	2.111561
29	6.21320709	0.002002804	0.0723080	0.803249	2.096932
30	6.28406094	0.001865808	0.06943642	0.806122	2.083093
32	6.41767169	0.0016324527	0.06429989	0.812420	2.057596
34	6.54171438	0.0014420143	0.05984538	0.819253	2.034743
36	6.65742606	0.0012844482	0.05595102	0.826440	2.014237
38	6.76582795	0.0011524929	0.05252153	0.833843	1.995818
40	6.86777147	0.0010407939	0.04948146	0.841361	1.979258
42	6.96397286	0.0009453334	0.04677039	0.848913	1.964356
44	7.05503961	0.0008630486	0.04433949	0.856441	1.950938
46	7.14149083	0.0007915711	0.04214884	0.863897	1.938847
48	7.22377312	0.0007290465	0.04016560	0.871249	1.927949
50	7.30227320	0.0006740049	0.03836248	0.878469	1.918124
52	7.37732783	0.0006252695			
54	7.44923187	0.0005818884			
55	7.4840848	0.00056195727	0.03450171	0.895830	1.897594
60	7.6484895	0.00047676374	0.03136331	0.912079	1.881799
65	7.7986151	0.00041030281	0.02876508	0.927155	1.869730
70	7.9368274	0.00035733838	0.02658032	0.941060	1.860622
75	8.0649553	0.00031436516	0.02471855	0.953834	1.853891
80	8.1844398	0.00027896067	0.02311346	0.965535	1.849077
85	8.2964345	0.00024940449	0.02171552	0.976232	1.845819
90	8.4018769	0.00022444568	0.02048701	0.985996	1.843831
95	8.5015380	0.00020315568	0.01939874	0.994899	1.842880
100	8.5960597	0.00018483267	0.01842785	1.003007	1.842785
105	8.6859815	0.00016893754	0.01755609	1.010387	1.843389
110	8.7717612	0.00015505026	0.01676883	1.017097	1.844571
115	8.8537906	0.00014283927	0.01605414	1.023195	1.846226
120	8.9324072	0.00013203980	0.01540223	1.028733	1.848268
125	9.0079040	0.00012243822	0.01480503	1.033755	1.850629
130	9.0805373	0.00011386042	0.01425576	1.038307	1.853249
135	9.1505319	0.00010616332	0.01374870	1.042429	1.856075
140	9.2180866	0.00009922837	0.01327906	1.046156	1.859068

TABLE 1-Continued

ξ	ψ	$e^{-\psi}$	ψ'	u	v
140	9.2180866	0.00009922837	0.01327906	1.046156	1.859068
145	9.2833780	0.00009295659	0.01284271	1.049522	1.862193
150	9.3465633	0.00008726481	0.01243613	1.052556	1.865420
155	9.4077838	0.00008208266	0.01205627	1.055286	1.868722
160	9.4671661	0.00007735030	0.01170049	1.057738	1.872078
165	9.5248250	0.00007301651	0.01136650	1.059933	1.875473
170	9.5808640	0.00006903728	0.01105226	1.061895	1.878884
175	9.6353776	0.00006537454	0.01075605	1.063638	1.882309
180	9.6884519	0.00006199531			
200	9.8878344	0.000050788815	0.00949605	1.069683	1.899210
225	10.1124639	0.000040570722	0.00851205	1.072411	1.915211
250	10.3150266	0.000033131485	0.00771984	1.072933	1.929960
275	10.4996104	0.000027547180	0.00706674	1.071990	1.943354
300	10.6692309	0.000023249407	0.00651806	1.070076	1.955418
325	10.8261826	0.000019872324	0.00604992	1.067536	1.966224
350	10.9722557	0.000017171566	0.00564538	1.064596	1.975883
375	11.1088768	0.000014978769	0.00529198	1.061425	1.984493
400	11.2372028	0.000013174824	0.00498042	1.058130	1.992168
425	11.3581863	0.000011673534	0.00470353	1.054793	1.999000
450	11.4726215	0.000010411272	0.00445574	1.051469	2.005083
475	11.5811785	0.000009340241	0.00423262	1.048196	2.010495
500	11.6844281	0.000008423981	0.00403061	1.045001	2.015305
525	11.7828607	0.000007634290	0.00384682	1.041900	2.019581
550	11.8769011	0.000006949081	0.00367888	1.038902	2.023384
575	11.9669199	0.000006350863	0.00352479	1.036018	2.026754
600	12.0532422	0.000005825637	0.00338290	1.033250	2.029740
625	12.1361551	0.000005362099	0.00325182	1.030596	2.032388
650	12.2159134	0.000004951038	0.00313035	1.028056	2.034728
675	12.2927442	0.000004584891	0.00301747	1.025628	2.036792
700	12.3668510	0.000004257404	0.00291229	1.023313	2.038603
725	12.4384166	0.000003963367			
750	12.5076063	0.000003698415	0.00272209	1.019001	2.041568
800	12.6394420	0.0000032416052	0.00255478	1.015071	2.043824
850	12.7634002	0.0000028636881	0.00240644	1.011509	2.045474
900	12.8803509	0.0000025476202	0.00227403	1.008280	2.046627
950	12.9910281	0.0000022807001	0.00215514	1.005348	2.047383
1000	13.0960569	0.0000020533110	0.00204780	1.002691	2.047800
1050	13.1959740	0.0000018580667	0.00195043	1.000277	2.047952
1100	13.2912436	0.0000016892202	0.00186170	0.998089	2.047870
1150	13.3822697	0.0000015422475	0.00178052	0.996105	2.047598
1200	13.4694066	0.0000014135495	0.00170599	0.994296	2.047188
1250	13.5529661	0.0000013002342	0.00163731	0.992660	2.046638
1300	13.6332245	0.0000011999574	0.00157384	0.991171	2.045992
1350	13.7104273	0.0000011108031	0.00151500	0.989825	2.045250
1400	13.7847941	0.0000010311931	0.00146032	0.988599	2.044448
1450	13.8565218	0.0000009598183	0.00140938	0.987481	2.043601
1500	13.9257878	0.0000008955858	0.00136180	0.986473	2.042700
1550	13.9927521	0.0000008375774	0.00131726	0.985565	2.041753
1600	14.0575601	0.0000007850173	0.00127550	0.984734	2.040800
1650	14.1203437	0.0000007372464	0.00123625	0.983989	2.039813
1700	14.1812233	0.0000006937020	0.00119930	0.983318	2.038810
1750	14.2403091	0.0000006539015	0.00116446	0.982711	2.037805
1800	14.2977019	0.0000006174289	0.00113156	0.982159	2.036808
1850	14.3534945	0.0000005839243	0.00110043	0.981671	2.035796
1900	14.4077722	0.0000005530752	0.00107094	0.981234	2.034786
1950	14.4606138	0.0000005246085	0.00104297	0.980840	2.033792
2000	14.5120922	0.0000004982857	0.00101639	0.980501	2.032780
2050	14.5622750	0.0000004738974	0.00099112	0.980194	2.031796
2100	14.6112247	0.0000004512589	0.00096706	0.979922	2.030826
2150	14.6589998	0.0000004302069			
2200	14.7056544	0.0000004105967			