1948ApJ...108..462A

THE ABSORPTION-LINE SPECTRA OF THE CENTRAL STARS OF THE PLANETARY NEBULAE

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Received May 13, 1948

ABSTRACT

Under the terms of a co-operative arrangement with the University of Chicago and the University of Texas, a systematic spectroscopic study of the brighter central stars of the planetary nebulae has been undertaken at the McDonald Observatory. The intensities of all available absorption lines have been measured. From the Balmer lines it is possible to estimate the number of hydrogen atoms above the photosphere and the electron pressure, which is equal to half the gas pressure if hydrogen is the most abundant element. If the atmosphere is in hydrostatic equilibrium, the gas plus radiation pressure must just balance the weight of the overlying layers. Hence we may estimate the effective surface gravity of the star; and, if we can allow for the radiation pressure and find the stellar radius, we may obtain a lower limit to the mass of the star.

To within the limits imposed by observational uncertainties the spectra of these stars appear consistent with the suggestion that they are submain-sequence dwarfs with masses appropriate to their luminosities in accordance with the mass-luminosity law. There is no evidence from their spectra that they are white dwarfs or dense, extremely massive stars. The hotter planetary nuclei may resemble the old novae, although the absorption-line objects seem cooler and larger.

Attention is called to the high-excitation character of the nucleus of NGC 246.

I. INTRODUCTION

Further progress in the interpretation of the planetary nebulae would appear to depend, to some extent, on better information concerning their central stars. Thanks to the work of W. H. Wright,[1] Plaskett and his co-workers,[2] Struve and Swings,[3] and others, we have good qualitative information on the spectra of these objects. The planetary nuclei show spectra of four general kinds: (1) Wolf-Rayet type, e.g., NGC 40, BD +30°3639; (2) combined absorption and emission features, e.g., NGC 6543 and IC 418; (3) spectra predominantly of the absorption type with but few emission lines, e.g., NGC 2392, NGC 6891; and (4) continuous spectra with no trace of emission lines with the dispersion available, e.g., NGC 7662 or NGC 7009.

Under the terms of a co-operative arrangement between the University of Chicago, the University of Texas, and Indiana University, I have undertaken a spectroscopic and spectrophotometric study of the brighter nuclei of the planetary nebulae, with the aim of applying techniques worked out for other hot stars to an interpretation of these objects. The McDonald plates have been supplemented by observations taken with the quartz slitless spectrograph at the Crossley reflector of the Lick Observatory. The latter spectra are to be used primarily for measuring the energy distribution in the emission lines and in the continuous spectrum. In this paper I shall confine my attention to the absorption lines; the emission features will provide material for later articles.

II. THE OBSERVATIONS

The McDonald survey covers planetary nuclei brighter than apparent magnitude 12.0 and a few fainter objects as well. For the brighter nuclei, I have employed the

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[1] *Pub. Lick Obs.*, **13**, 193, 1918.

[2] *Pub. Dom. Ap. Obs., Victoria*, **1**, 366, 1922.

[3] *Proc. Nat. Acad. Sci.*, **26**, 548, 454, 1940; **27**, 225, 1941; *Ap. J.*, **92**, 289, 1940.

Cassegrain spectrograph with quartz prisms and 500-mm camera. For fainter objects it has been necessary to use the *f*/2 Schmidt camera. To make a photometric study of the absorption lines it is necessary to use as high a dispersion as possible; hence only plates made with the 500-mm camera have been employed for this purpose. The *f*/2 films are of great assistance for a qualitative study of the spectra of the fainter stars.

The usual observational procedure has been to lengthen the slit to the diameter of the nebular image and to let the latter drift slightly, so as to obtain a widened spectrum of the central star. The distinction between nebular and nuclear emission lines then may be easily made.

Table 1 lists the McDonald observations of interest in the present connection. Successive columns give the NGC number of the object, the α and δ for 1900, the magnitude

TABLE 1

LIST OF PLANETARY NUCLEI STUDIED FOR ABSORPTION-LINE INTENSITIES

Object	α	δ	Mag.	Spectral Class	Plate	Camera	Quality
NGC 246......	$0^h42^m.0$	$-12°25'$	11.2	O7	CQ4610	500-mm	p
					4618	500-mm	g
					4624	500-mm	g
					6277	Q *f*/2	g
NGC 1535......	4 9.6	−13 0	11.8	Cont.	6362	Q *f*/2	g
					6363	Q *f*/2	f
IC 418......	5 22.8	−12 45	10.8	WC7+O5	4653	500-mm	g
					6364	Q *f*/2	p
NGC 2392......	7 23.3	+21 7	10.5	O8e	4652	500-mm	f
					5222	500-mm	g
NGC 3242......	10 19.9	−18 8	11.9	Cont.	7944	Q *f*/2	g
IC 3568......	12 30.4	+83 7	12.0	Cont.	8045	Q *f*/2	g
					8074	Q *f*/2	f
IC 4593......	16 7.0	+12 20	10.2	Cont.	4593	500-mm	g
					4603	500-mm	f
					5959	500-mm	g
NGC 6210......	16 40.3	+23 59	11.7	O6+WC7	4599	500-mm	g
					4628	500-mm	g
NGC 6543......	17 58.6	+66 38	11.2	WN6	4600	500-mm	g
					4637	500-mm	g
NGC 6826......	19 42.1	+50 17	10.7	WN6, O5	4590	500-mm	f
					4596	500-mm	f
					5960	500-mm	g
NGC 6891......	20 10.4	+12 24	11.6	Cont.	4613	500-mm	f
					4622	500-mm	g
NGC 7009......	20 58.7	−11 46	11.9	Cont.	4609	500-mm	p
					4617	500-mm	p
					4623	500-mm	f
					6360	Q *f*/2	f
NGC 7662......	23 21.1	+41 59	12.7	Cont.	6281	Q *f*/2	g

of the central star as given by Berman,[4] the spectral classes assigned by Verontsov-Velyaminov,[5] the plate number, the camera used, and the quality.

Satisfactory plates with the 500-mm camera, which gives a dispersion of 75 A/mm at $H\gamma$, have been obtained for the central stars of NGC 246, 2392, 6210, 6543, 6891, and IC 418 and 4593. In addition, I have paid special attention to the central stars of NGC 1535, NGC 3242, IC 3568, NGC 7009, and NGC 7662, whose spectra have been described as continuous. Unfortunately, the 500-mm camera (CQ) plates of NGC 7009 and NGC 7662 were somewhat underexposed. The strong nebular recombination lines of *He* II, *H*, etc., would tend to obliterate the corresponding presumably weak absorption lines in the central star. Plates taken with the $f/2$ camera give insufficient dispersion to detect more than the strongest absorption lines. Thus strong λ 4452 *He* II absorption appears in the spectra of the nuclei of NGC 1535 and IC 3568, but other lines have not been found because of the low dispersion and the intense superposed nebular lines. Until satisfactory higher-dispersion plates of these objects have been obtained, we shall not be able to assign them spectral classes.

III. PHOTOMETRIC PROCEDURE

Mention may be made here of two devices employed for the photometric calibration of the plates—a spot sensitometer and a wedge slit. The second method involves replacing the slit of the Cassegrain spectrograph by a wedge slit and illuminating it by artificial light reflected from a white diffusing screen. Calibration-curves derived by the two procedures are in good agreement. I traced the plates with the Beals-type microphotometer of the Yerkes Observatory.

The measurement of the intensity of a stellar absorption line upon which a strong nebular emission is superposed presents a formidable problem. Invariably, $H\beta$ was unmeasurable, $H\gamma$ and $H\delta$ were partly masked, and sometimes only rough intensity estimates were possible. In NGC 6891, $H\gamma$ and $H\delta$ could be measured with seeming confidence. In other objects in which these absorption lines could be recognized, I had the feeling that the measured intensities tended to be too low; but there appears little possibility of securing improved values without going to higher dispersion. The object IC 418 has such strong hydrogen lines that none of the nuclear absorption lines are observable, and only the strongest *He* II lines can be seen. In a number of nuclei the higher members of the Balmer series are readily measurable.

The Eberhard effect complicates the measurement of absorption lines underlying strong nebular emission; but, if it is present, the strong forbidden nebular lines will also show spurious underlying absorptions, and upon such a plate only those absorption lines that are not associated with nebular emissions will be measurable. Fortunately, the Eberhard effect was encountered on only one plate.

Table 2 lists the intensities of the absorption lines measured in the central stars of NGC 2392, IC 418, NGC 6543, NGC 6891, IC 4593, NGC 6826, and NGC 6210. The emission lines measured in the spectra of these stars will be discussed later. In a star such as the NGC 2392 nucleus, which shows good absorption lines, the accidental error in the measured line intensities is of the order of 15 per cent for the stronger lines. The weaker lines and lines like $H\delta$ or $H\gamma$, which are confused with nebular emissions, may be seriously in error.

IV. THE HYDROGEN LINES

Aside from the observational difficulty arising from the superposition of the nebular emission, the interpretation of the Balmer line intensities is complicated by their confusion with the Pickering *He* II lines. From a comparison of alternate members of the Pickering series with the Balmer lines, which represent *H* I+*He* II absorption, we may estimate the amount by which the Balmer lines are to be corrected for the contribution

[4] *Lick Obs. Bull.*, **18**, 57, 1937.

[5] *Russ. A.J.*, **24**, 88, 1947.

of *He* II. This crude procedure is seemingly justified for the higher members of the Balmer series—certainly, within the limits of the present observational accuracy. Its application to $H\gamma$ and $H\delta$ can give us only a lower limit to the intensities that these lines would have in a pure hydrogen atmosphere at the same temperature and electron pressure.

If we can suppose that the hydrogen lines are formed in a thin layer, then the equivalent width, W_λ, of the line corresponding to the (2–n) transition will be related to the number of atoms in the second level above the photosphere by

$$W_\lambda = \frac{\pi \epsilon^2}{m c^2} \lambda^2 N_{02} H f_{2n} , \tag{1}$$

TABLE 2

ABSORPTION LINES IN CENTRAL STARS OF PLANETARY NEBULAE
(Intensities in Equivalent Angstroms)

Wave Length	Identity	NGC 2392	IC 418	NGC 6543	NGC 6891	NGC 4593	NGC 6826	NGC 6210
4541.58	*He* II	0.85	0.55	0.47	0.67	0.76	0.80	0.85
4471.6	*He* I	0.54	....	....	....	....	....	....
4340	*H* I, *He* II	1.20	....	....	1.8	1.53	1.55	1.47
4200	*He* II	0.67	1.36	....	0.73	0.45	0.51	0.28
4100	*N* III, *He* II, *H* I	1.82	....	....	1.94	1.10	1.28	....
4025.6	*He* II	0.70	....	....	....	....	....	....
3969	*H* I, *He* II	0.72	....	....	....	1.23	1.05	1.36
3965	*He* I	0.19	....	....	....	....	....	....
3933	*Ca* II (ints)	0.41	....	0.36	0.33	0.28	0.30	0.38
3923.5	*He* II	0.47	0.58	....	0.28	0.43	0.30	0.19
3889.0	*H* I, *He* II	0.66	....	1.30	1.46	1.30	1.15	1.10
3858.0	*He* II	0.31	0.65	....	1.1:	0.20	0.30	0.24
3835.2	*H* I, *He* II	0.65	0.81	0.56	0.95	1.00	0.92	1.00
3813.5	*He* II	....	0.4	....	....	....	....	....
3797.3	*H* I, *He* II	0.74	1.4	0.70	1.06	1.20	0.67	0.67
3781.3	*He* II	....	0.36	....	....	....	....	....
3770.6	*H* I, *He* II	0.45	....	....	0.50	0.78	0.55	....
3750.1	*H* I	0.36	....	....	0.41	0.52	0.40	....
3734.4	*H* I, *He* II	0.27	....	....	0.58	0.40	0.35	....
3721.9	*H* I	....	....	....	0.28	....	....	....
3712.0	*H* I	....	....	....	0.24	....	....	....
3705.1	*He* I	0.23	....	....	....	....	....	....
3634.3	*He* I	0.11	....	....	....	....	....	....
3554.5	*He* I	0.46	....	....	....	....	....	....
3483.0	*N* IV	0.45	....	....	....	....	....	....
3478.7	*N* IV	0.26	....	....	....	....	....	....

a formula used extensively by Unsöld;[6] the same principle was applied to emission lines of the solar chromosphere by Menzel.[7] Here f_{2n} is the oscillator strength for the Balmer line in question, and the other symbols have their usual meaning. If we measure the wave length, λ, in centimeters and W_λ in angstrom units, we may write equation (1) as

$$\log N_{02}H = 4.05 - \lambda^2 f_{2n} + \log W_\lambda = C_n + \log W_\lambda . \tag{2}$$

We plot $\log N_{02}H$ against n and find that, for the earlier members of the series, the lines give successively greater values of $\log N_{02}H$, since these lines are not formed in a thin layer and W is not proportional to the number of atoms. Then the curve flattens, the lines begin to coalesce, we tend to draw the continuum too low and measure W too small, and the curve begins to fall. An extrapolation of the curve, in the manner suggested by

[6] *Zs. f. Ap.*, **21**, 38, 1941.

[7] *Pub. Lick Obs.*, Vol. **17**, 1931: "A Study of the Solar Chromosphere."

Unsöld, gives an estimate of log $N_{02}H$, the number of hydrogen atoms above the photosphere and in the second level. Table 3 gives, for each central star, the equivalent width corrected for blending with *He* II, log $N_{02}H$, computed from each spectral line, and the finally extrapolated value of this quantity for the star.

For comparison we may mention results obtained by Unsöld[8] for several hot stars, including 10 Lac, discussed also by the writer.[9] For the supergiants, λ Ori (07.5) and 9 Cam (09), log $N_{02}H$ was 15.8 and 15.7, respectively, while for the main-sequence star, 10 Lac, log $N_{02}H = 15.8$. For the central stars of the planetary nebulae, the mean value of log $N_{02}H = 15.16$, which means that there seems to be about a fourth as much material above the photosphere of the central star of a planetary as above main-sequence or supergiant stars of the same spectral types. The different stars appear to give fairly concordant results; it seems possible that our average for this group of five stars is correct to within a factor of 2.

TABLE 3

CORRECTED INTENSITIES OF THE BALMER LINES

n	λ	LOG C_a	NGC 6891		NGC 6826		NGC 2392		IC 4593		NGC 6210	
			W	log $N_{02}H$	*W*	log $N_{02}H$	*W*	log $N_{02}H$	*W*	log $N_{02}H$	*W*	log $N_{02}H$
5	4340	14.13	1.10	14.17	0.90	14.09	0.45	13.78	0.75	14.01	0.91	14.09
6	4101	14.48	1.24	14.57	.63	14.28	.30	13.96	0.52	14.20	.....	....
7	3970	14.75	.....	......	.65	14.56	.43	14.39	0.72	14.61	1.02	14.76
8	3889	14.96	0.88	14.90	.85	14.89	.40	14.56	1.00	14.96	0.82	14.88
9	3835	15.14	0.57	14.90	.62	14.93	.39	14.73	0.80	15.04	0.75	15.02
10	3797	15.31	0.63	15.11	.47	14.98	.44	14.96	0.90	15.27	0.50	15.01
11	3770	15.45	0.30	14.93	.38	15.03	.27	14.88	0.58	15.22	.....	....
12	3750	15.57	0.25	14.96	.28	15.02	.22	14.90	0.39	15.16	.....	....
13	3734	15.60	0.35	15.24	0.24	15.07	0.16	14.90	0.30	15.07	.....	....
14	3722	15.79	0.17	15.02	.....	......	.....	......	.....	......	.....	....
15	3712	15.87	0.14	15.03	.....	......	.....	......	.....	......	.....	....
Final log $N_{02}H$	.....	......	.....	15.16	.....	15.12	.....	14.96	.....	15.35	.....	15.10

A second item of interest is the electron density in the atmosphere—this is the quantity which helps distinguish supergiants, main-sequence stars, and subdwarfs. For a normal early-type star two methods are available for estimating the electron density. One method is based on the fact that the equivalent widths of the earlier members of the Balmer series, e.g., $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$ depend not only on the number of atoms in the second level but also on the electron pressure which determines the broadening coefficient of the line.[6] The greater the electron pressure, the greater the equivalent width. Unsöld has employed this method to estimate the mean electron densities in the atmospheres of early-type stars. From the theoretical standpoint we can see that it gives only a rough log N_ϵ, since the assumption is made that the line is formed in an isothermal atmosphere of constant density. A more serious objection to its application in the present problem is the circumstance of the blending of $H\gamma$ and $H\delta$ with the Pickering lines and the influence of overlying nebular lines upon the measured intensities. The measured equivalent widths are almost certainly too small; hence the derived values of the electron density will be a lower limit. The most accurate values of W_λ are those determined for NGC 6891, while NGC 2392 appears to give poor W_λ's.

The number of resolvable lines of the Balmer series will give us an additional estimate of the electron density. If n_m is the principal quantum number of the last still-resolved

[8] *Zs. f. Ap.*, **23**, 100, 1944.

[9] *Ap. J.*, **104**, 347, 1946.

Balmer line, the concentration of free electrons (or ions) per cubic centimeter is given by the Inglis-Teller formula:[10]

$$\log N = 23.26 - 7.5 \log n_m .$$

Unfortunately, the confluence of the stellar lines often underlies that of much stronger nebular emission, and it is not possible to establish n_m with certainty. In IC 4593 and NGC 6826 the last Balmer line appears to be recognizable, while in NGC 2392, where the nebular spectrum is much weaker, it seems quite definite that $n_m = 14$, λ 3722, is the last line of the Balmer series.

Table 4 summarizes the estimates of log N_ϵ based on $H\gamma$ and $H\delta$ and n_m, together with the values adopted for further computations. The agreement between the electron densities computed by the two methods is good for NGC 6826, fair for IC 4593, and poor for NGC 2392. Since the determination of n_m seems most accurate for NGC 2392, we may conclude that the log N_ϵ estimated from n_m is the more reliable. Inspection of the plates and tracings suggests that $H\gamma$ and $H\delta$ are measured systematically too weak; if W_λ for $H\gamma$ is in error by a factor of 2, log N_ϵ would be 14.0—in fair agreement for the values of the others. In this star, $H\delta$ appears to be badly blended with λ 4089 *Si* IV, as

TABLE 4

ELECTRON DENSITIES DERIVED FROM BALMER LINES

OBJECT	LOG N_ϵ		n_m	LOG N_ϵ FROM n_m	LOG N_ϵ (ADOPTED)
	$H\gamma$	$H\delta$			
NGC 6891........	14.04	14.26	....	....	14.15
NGC 6826........	13.87	13.56	15	13.76	13.75
NGC 2392........	13.28	12.92	14	14.66	14.5
IC 4593........	13.67	13.45	15:	13.8	13.65
NGC 6210........	13.90	....	....	....	13.9

well as *N* III λ 4097 and λ 4103. The adopted values of log N_ϵ would appear to represent lower limits in most stars. In the supergiants λ Ori and 9 Cam, Unsöld finds log N_ϵ = 14.17 and 13.75, respectively. In 10 Lac the electron density is higher; Unsöld found log N_ϵ = 14.37, while the writer obtained 14.10. The electron densities appear to be comparable with those in other stars of the same spectral class.

As an alternative mode of treating the higher members of the Balmer series, I have computed theoretical equivalent widths on the basis of Pannekoek's[11] assumption that the Stark pattern produced by the interatomic fields gives a uniform spreading-out for a given field and that the ratio of line to continuous absorption is constant with depth. As parameters I have employed the number of atoms per gram in the second energy level n_{02}, divided by the continuous absorption coefficient, k, and the electron pressure. It turns out that, for the higher members of the series, the equivalent width is insensitive to the electron pressure, but we may obtain log n_{02}/k. The accompanying tabulation

Object	log n_{02}/k	log $N_{02}H$	Optical Depth	Object	log n_{02}/k	log $N_{02}H$	Optical Depth
NGC 2392........	15.48	14.96	0.30	NGC 6826........	15.62	15.12	0.31
NGC 6891........	15.68	15.16	0.30	NGC 6210.......	15.72	15.10	0.24

gives the results of such calculations. A comparison of log n_{02}/k and log $N_{02}H$ will give the optical depth effective for the formation of the hydrogen lines. Notice that the ef-

[10] *Ap. J.*, **90**, 439, 1939.

[11] *M.N.*, **98**, 701, 1938.

fective value of the optical depth is near C.3, in harmony with the result found by Strömgren for an atom in the sun that becomes easily ionized with depth, e.g., sodium.[12]

V. TEMPERATURES AND SPECTRAL CLASSES OF THE PLANETARY NUCLEI

The temperatures of the central stars of the planetary nebulae may be estimated either from their spectral classes (when known) or from a comparison of the luminosity and spectrum of the surrounding nebula with that of the central star.

I have employed the criteria suggested by R. M. Petrie[13] to estimate the spectral classes from the absorption lines. Except for NGC 2392, it is possible to apply only the *H*/*He* II ratio. Since the Balmer lines tend to be confused with the nebular emissions and are sometimes, therefore, measured too weak, the derived spectral classes may tend to be systematically too early. For the objects studied, the spectral classes seem to fall between O5 and O7, and the corresponding temperatures range from 33,000° to 35,000° K.

Second, one may utilize the magnitude difference between the central star and the nebula in conjunction with the theory of nebular luminosity,[4] in the manner suggested by Zanstra. Thus one may put

$$m_s - m_n = \delta(T) ,$$

TABLE 5

TEMPERATURES AND SPECTRAL CLASSES OF PLANETARY NUCLEI

Nebula	Spect.	Temperature			
		Spect. Cl.	Berman	Stoy	Adopted
NGC 6891......	O7	32,900	30,000	40,000	32,000
NGC 6826......	O6	34,600	30,000	48,000	32,000
NGC 2392......	O5.5	35,000	35,000	70,000	35,000
IC 4593......	O6.5	34,000	25,000	40,000	25,000–34,000
NGC 6210......	O7	32,900	30,000	57,000	32,000

where $\delta(T)$ can be calculated as a function of the temperature. Part of the nebular emission comes from the recombination of protons and free electrons, liberated by photoionization from radiation emitted beyond the Lyman limit in the central star, and part comes from forbidden lines excited by inelastic impacts by the free electrons. The higher the color temperature of the star beyond the Lyman limit, the greater will be the magnitude difference between the nebula and the star. Berman employed this method to estimate the temperatures of the planetary nuclei.

Third, one may employ a method due to Stoy,[14] wherein one compares the sum of the intensities of the Balmer lines with the sum of the intensities of the forbidden lines. In accordance with the arguments of Zanstra and Menzel, the former is determined by the number of quanta radiated by the central star beyond the limit of the Balmer series, while the latter is determined by the amount of energy radiated beyond the Balmer limit. Hence a comparison of the two should give the color temperature in the far ultraviolet, on the assumption that the stars radiate there like black bodies.

Table 5 gives the temperatures estimated by the three methods. The temperatures in the third column correspond to the spectral types and Petrie's temperature scale, those in the fourth column are taken from Berman's work, while those in the fifth column are derived from nebular line intensities measured upon plates taken at the

[12] *Festschrift für Elis Strömgren* (Copenhagen: E. Munksgaard, 1940), p. 218.

[13] *Pub. Dom. Ap. Obs., Victoria*, **7**, 321, 1947.

[14] *M.N.*, **93**, 588, 1933.

Lick Observatory; the intensity of $H\alpha$ was extrapolated from the observed Balmer decrement. In the application of the Stoy method, I have assumed that the electron temperature of the gaseous nebula is near 10,000° K. The temperatures derived from the spectral classes tend to be larger than those obtained by Berman, while the Stoy method seems to give temperatures that are much too high. The discordances suggest that in the far ultraviolet the planetary nuclei do not radiate like black bodies but may have extended chromospheres with strong bright lines, whose principal probable members have been suggested by Swings.[3] The nebula NGC 2392 shows a high excitation spectrum with [*Ne* v] lines. Other planetary nebulae of comparable excitation tend to have nuclei showing only a continuous spectrum (e.g., NGC 3242), while in some instances the spectrum of the central star is not observed at all (e.g., NGC 7027). Hence there appears to be a good chance that this star will show strong deviations from a black body in the ultraviolet, and similar departures probably exist for the other stars as well. The appearance of emission lines in the stellar spectra hint that bright-line radiation may be important in the far ultraviolet.

The adopted temperatures are based primarily on the Petrie temperature scale, with the Berman temperatures used as a check. For IC 4593, I have carried out calculations for two values of the temperature, to illustrate the uncertainty of this quantity upon estimates of bolometric luminosity, radius, and mass.

VI. ESTIMATION OF EFFECTIVE SURFACE GRAVITIES

Our interest in the intensities of the Balmer lines in the central stars of the planetary nebulae arises partly from the possibility that from the mass of material above the photosphere and from the electron pressure, which we have attempted to estimate from the temperature and electron density, we may be able to estimate the effective surface gravities of these stars. If the stellar radii are known, the effective surface gravities may then give us a lower limit to the stellar masses.

At the base of the layers producing the stellar absorption lines, the gas pressure, P_g, must balance the weight of the overlying layers,

$$P_g = m_H g_{\text{eff}} \Sigma \mu NH , \tag{3}$$

where NH is the total number of atoms above the photosphere, μ is the molecular weight, and g_{eff} is the effective surface gravity. The gas pressure at the base must be twice the mean gas pressure $\bar{P}_g$ and $\bar{P}_g = 2P_\epsilon$, since the material, predominantly hydrogen, is singly ionized. From equation (3) we obtain, by multiplication by P_e,

$$4P_\epsilon^2 = m_H g_{\text{eff}} \Sigma NHP_\epsilon \mu . \tag{4}$$

For the hydrogen atoms we derived NHP_ϵ from the number of second-level atoms above the photosphere by means of the combined Boltzmann and Saha equation, viz.,

$$\log \frac{NHP_\epsilon}{N_{02}H} = -\frac{5040}{T}(I - \chi_2) + \frac{5}{2}\log T + \log \frac{2\varpi_{1,0}}{\varpi_{0,2}} - 0.48 , \tag{5}$$

where I is the ionization potential, χ_2 is the excitation potential of the second level, T is the adopted temperature, $\varpi_{1,0}$ is the statistical weight of the ionized atom, and $\varpi_{0,2}$ is the statistical weight of the second level. Except for helium, the other elements contribute very little by mass or volume to the stellar atmosphere.

The numerical value of the term on the right-hand side of equation (4) will depend on the assumption made concerning the composition of the atmosphere. I have carried out calculations of the surface gravity on the assumption that the number of helium atoms is one-tenth the number of hydrogen atoms. Then μ is 1.29 and $m_H \Sigma NH\mu = 1.42\, NHm_H$. If we suppose that there are five times as many hydrogen as helium atoms, μ is 1.5, and the coefficient is 1.80 instead of 1.42. Our computed effective surface gravities then must be diminished by 20 per cent.

If we can suppose that the atmosphere is in static equilibrium, the acceleration experienced by a mass at the surface of a star is the resultant of the usual surface gravity, g, due to the mass of the star and the radiation pressure which we may denote as g'. That is

$$g_{\text{eff}} = g - g' \tag{6}$$

where

$$g = g_0 \frac{\mathfrak{M}}{R^2} . \tag{7}$$

Here $\mathfrak{M}$ and R are, respectively, the mass and radius of the star in terms of the sun, and g_0 is the acceleration of gravity at the solar surface. If we can suppose that the contribution of line absorption to the radiation pressure can be neglected, then

$$g' = \frac{\bar{k}F}{c} = \bar{k}\sigma \frac{T_e^4}{c} , \tag{8}$$

TABLE 6

ESTIMATED SURFACE GRAVITIES OF PLANETARY NUCLEI

Object	T_e	log NHP_ϵ	log P_ϵ	log g_{eff}	log $\bar{k}$	log g'	log (g_e+g')
NGC 6891......	32,000	24.83	2.80	4.99	+0.30	3.66	5.01
NGC 6826......	32,000	24.79	2.40	4.23	+ .06	3.42	4.29
NGC 2392......	35,000	24.77	3.20	5.75	+ .50	3.98	5.76
IC 4593......	{34,000	25.09	2.32	3.77	− .12	3.28	3.89
	{25,000	24.59	2.19	4.01	+ .30	3.08	4.06
NGC 6210......	32,000	24.77	2.55	4.55	+0.10	3.46	4.58

where $\bar{k}$ is the straight mean value of the absorption coefficient,

$$\bar{k} = \frac{1}{F} \int_0^\infty k_\nu F_\nu d\nu , \tag{9}$$

and T_e is the effective temperature of the star. Table 6 gives $N(H)HP_e$, i.e., the number of hydrogen atoms above the photosphere; log P_ϵ; the resultant g_{eff}; $\bar{k}$ as computed from the effective temperature and the electron pressure of each star; log g'; and, finally, log g = log $(g_{\text{eff}} + g')$.

VII. THE APPARENT MASSES OF THE PLANETARY NUCLEI

We may call the mass estimated from equation (7) the "apparent" mass of the star. To obtain an estimate of this quantity, we must know the temperature and the absolute magnitude, whence we may derive the radius by means of the well-known expression:

$$M_p = -0.83 - 5 \log R + \frac{36,600}{T} + 2.5 \log (1 - 10^{-14,600/T}) . \tag{10}$$

This formula is based on the assumption that the star radiates like a black body throughout most of its spectrum. Hence there is some question as to precisely what temperature ought to be employed. A consistent procedure would appear to be to employ the temperature from Table 5. We shall further suppose that deviations from a black body are small, except perhaps in the far ultraviolet.

The major difficulty in the application of equation (10) lies not in the uncertainty in the temperature but in the uncertainty in M_p, which depends on relatively rough values of the photographic magnitude, crude distance estimates, and cruder corrections for space absorption. I have taken the photographic magnitude of the central star, m_p, from the work of Berman and Parenago. These values may sometimes be in error by as much as half a magnitude, but, unfortunately, better values are not available.

The distance in parsecs is derived from the work of Berman and Parenago[15] and from the intensity of the interstellar K line interpreted with the aid of Sanford's[16] distance-equivalent width correlation. Berman's values are derived from the radial velocities of the planetary nebulae, together with the theory of galactic rotation, while Parenago's estimates are obtained from statistical parallaxes. Both distance estimates are highly uncertain. Parenago finds the known planetary nebulae to be confined to regions close to the sun, while Berman found them distributed over the whole galaxy. Minkowski's recent investigations suggest that the planetary nebulae are found at great distances—at least in the direction of the galactic center. The distances given by Berman may be of the correct order of magnitude, even though the individual values may be appreciably in error. With new proper-motion data, a re-analysis of the whole problem could be undertaken and the distances of these objects placed upon a more secure basis.

Distances estimated from the intensities of the interstellar lines and a mean calibration-curve can have, at best, but a rough statistical significance. It would be better to establish the interstellar line-intensity–distance relation for the vicinity of each nebula by observations of the K line in distant B stars.

Second, one must consider the effects of space absorption. I have taken Stebbins and Huffer's measures of the colors of B stars[17] near the objects in question and have tried to infer from them the color excess per kiloparsec and, finally, the total absorption from the A/E ratio of 8.1 suggested by Greenstein and Henyey.[18] The interstellar material is so extremely patchy that stars but a few degrees apart show a considerable difference in reddening. Nevertheless, even though the uncertainties are large, it seems preferable to estimate the space absorption per kiloparsec from the stars near each object rather than to adopt some kind of a mean absorption coefficient.

Because of our ignorance of the distances of the planetaries, I have carried out the calculations of the radii and masses of their nuclei with both the Parenago and Berman distances. Table 7 lists the apparent photographic magnitude, m_p; the distance in parsecs as given by Parenago, by Berman, and as estimated from the intensity of the interstellar K line; the absorption in magnitudes per kiloparsec as deduced from the Stebbins-Huffer measures; then M_p, $\log R$ (from eq. [10]), and $\log \mathfrak{M}$ from equation (7) for the Parenago and Berman distances. For IC 4593, two values are given because there are two estimates of the surface temperature. For comparison, Table 8 gives the bolometric absolute magnitudes[19] of these stars and corresponding masses deduced from the mass-luminosity law.

From a comparison of Tables 7 and 8 we note that NGC 6891 and NGC 2392 have the highest apparent masses—giving the values closest in agreement with the predictions of the mass-luminosity law. For these objects it would appear that the determination of the $\log P_\epsilon$ and $\log g$ were the most reliable. For the other objects the earlier Balmer lines were probably measured systematically too weak; hence the corresponding g-values and apparent masses seem certain to be lower limits. The influence of a small error in W_λ upon $\log g_{\mathrm{eff}}$ is very large. Thus, if we take $\log P_\epsilon$ from $H\delta$ instead of from λ 4340 in NGC 6891, i.e., 2.91 instead of 2.80, $\log \mathfrak{M}$ would come out 0.95. An error of a factor of 2 in $\log W_\lambda$ in $H\gamma$ and $H\delta$ in NGC 6826 would result in $\log P_\epsilon$ being increased by 0.65

[15] *Russ. A.J.*, **23**, 78, 1946

[16] *Ap. J.*, **86**, 136, 1937.

[17] *Pub. Washburn Obs.*, **15**, 217, 1934.

[18] *Ap. J.*, **93**, 327, 1941.

[19] The bolometric corrections are obtained from Kuiper's table, *Ap. J.*, **88**, 453, 1938.

and log $\mathfrak{M}$ by 1.30; and the mass of the star would then agree with the predictions of the mass-luminosity law. Because the derived masses are so very sensitive to the computed electron pressure, there appears to be no hope of getting improved estimates until much higher-dispersion material is available.

If we can suppose that the atmospheres of these stars are similar to those of main-sequence objects of the same spectral class, the derived masses may not be without meaning. Unsöld applied this method to main-sequence early B stars and showed that the masses derived from the surface gravities were in good agreement with the predictions of the mass-luminosity law.

TABLE 7

ESTIMATED RADII AND MASSES

Object	m_p	Distance			A/Kpc	M_p		log R		log $\mathfrak{M}$	
		P	B	Int.		P	B	P	B	P	B
NGC 6891	11.6	1400	2750	800	0.00	+0.85	−0.60	−0.20	+0.09	+0.17	+0.75
NGC 6826	10.7	660	1050	600	.24	+1.44	+0.35	− .32	− .10	− .79	−0.35
NGC 2392	10.5	580	860	1000	.80	+1.24	+0.11	− .31	− .08	+ .70	+1.08
IC 4593	10.2	1200	2000	800	.00	−0.20	−1.30	{+ .10	+ .32	− .18	+0.26
								{− .01	+ .21	− .57	−0.13
NGC 6210	11.7	1050	1750	1000	0.16	+1.43	+0.22	−0.31	−0.08	−0.48	−0.02

TABLE 8

BOLOMETRIC MAGNITUDES AND CORRESPONDING MASSES

Object	Bolometric Magnitude		Predicted log Mass	
	P	B	P	B
NGC 6891	−2.0	−3.4	+0.76	+0.91
NGC 6826	−1.4	−3.2	+ .70	+ .89
NGC 2392	−1.9	−3.0	+ .75	+ .87
IC 4593	{−2.6	−3.7	+ .82	+ .94
	{−3.2	−4.4	+ .89	+ .90
NGC 6210	−1.4	−2.6	+0.70	+0.80

To within the limits of the observational uncertainties, we may say that the spectra of the central stars of the planetary nebulae appear consistent with the suggestion that these objects are submain-sequence dwarfs with masses appropriate to their luminosities in accordance with the mass-luminosity law.

It is more difficult to assess the significance of the apparent masses. We have neglected the possible role of radiation pressure in the resonance lines as a factor in the mechanical support of the atmosphere. Furthermore, the atmosphere may not be in equilibrium at all but in some kind of a kinematical steady state, with ascending and descending currents of gas. Hence the derived masses can be strictly regarded only as lower limits.

Dr. Olin Wilson has generously communicated to me the results of his study of the nucleus of NGC 2392 in advance of publication. He finds the different absorption lines to give different Doppler displacements, in the sense that the smaller the excitation of the line, the greater the outward velocity as though the material was accelerated as it moved outward in the atmosphere and the excitation diminished. Conclusions based on the assumption of a static atmosphere, therefore, are to be regarded with reserve.

The earlier discussions by Menzel,[20] Gerasimovič,[21] and Milne[22] suggested that the central stars of the planetaries had bolometric magnitudes near +1 and therefore must be white dwarfs if they followed the mass-luminosity law. Present indications are that they are much brighter than +1. The objects discussed in this paper appear to have surface gravities five to ten times those of main-sequence stars, and radii comparable with that of the sun rather than with a white dwarf—a result in agreement with that of Beals.[23] The densities may therefore be expected to be about ten times that of the sun; and, if the NGC 2392 nucleus were built on the same model as the sun, the central density would be about 1000—still probably below the density where degeneracy sets in. Since radiation pressure must be important in their interiors, these stars are not likely to be built on the same model as the sun, and the central density is less than in the homologous model.

The line-intensity data give no support to the suggestion that these stars are massive objects, i.e., about one hundred times as massive as the sun. From a study of the relative shifts of absorption and emission lines, Cillié[24] suggested that the nucleus of NGC 6826 was a massive star; but there is nothing in the line profiles to suggest a white-dwarf character.

McLaughlin[25] has called attention to the similarity in dimensions, luminosity, and surface temperature between the ex-novae and the planetary nuclei. Both are dense blue stars with radii less than that of the sun. The absorption-line objects discussed in the present paper, however, appear to be cooler, larger, and presumably less dense than the old novae. As a class the planetary nuclei appear to cover an appreciable range in size, surface temperature, and perhaps luminosity and density.

The planetary nuclei appear to represent the high-temperature end of the Type II class of stars. Their photographic magnitudes appear to be comparable with those of cluster-type variables, suggesting that the high-temperature end of the Type II sequence runs horizontally on the Russell diagram.

VIII. HELIUM LINES IN THE PLANETARY NUCLEI

With the present data it is not possible to make a comparison of the abundances of the light-elements in a planetary nucleus and a main-sequence Type I object of the same spectral class. About all that is possible is a comparison of the Pickering *He* II lines with the Balmer lines to obtain an estimate of the relative numbers of doubly ionized helium and ionized hydrogen atoms above the photosphere. If we know what fraction of helium is doubly ionized, we can compare the relative proportions of hydrogen and helium.

For each line of the Pickering series one may compute the number of *He* II atoms in the $n = 4$ level above the photosphere, $N_{1,\,4}H$, and extrapolate for large n as we did for hydrogen:[26]

$$\log N_{1,\,4}H = 4.05 - \log \lambda^2 f_{4n} + \log W_\lambda = \log C_a + \log W_\lambda \,, \tag{11}$$

[20] *Pub. A.S.P.*, **38**, 295, 1926.

[21] *Observatory*, **54**, 108, 1931; see also *Harvard Bull.*, No. 864, p. 13, 1929.

[22] *Observatory*, **54**, 140, 1931.

[23] *J.R.A.S. Canada*, **169**, 1940.

[24] *M.N.*, **94**, 48, 1933.

[25] *Pop. Astr.*, **49**, 292, 1941.

[26] The subscript "1, 4" means we deal with the fourth level of singly ionized atom.

where the value of f may be computed from the hydrogenic formula,[27]

$$f_{n'n}=\frac{2^6}{3\sqrt{3}\pi}\frac{1}{\varpi_n}\frac{1}{\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)^3}\left|\frac{1}{n^3}\frac{1}{n'^3}\right|g_{nn'}\,.$$

Table 9 gives the wave length, n, f-value, and log C_a for the relevant Pickering lines, while Table 10 gives the details of the calculation of log $N_{1,\,4}H$ from the Pickering lines; log $N(He^{++})P_\epsilon H$; log $N(He)/N(He^{++})$, where $N(He)$ denotes the total number of helium atoms; and, finally, the ratio of hydrogen to helium for the assumed temperature and

TABLE 9

OSCILLATOR STRENGTHS FOR THE *He* II LINES

n	Wave Length	f_{4-n}	log C_a	n	Wave Length	f_{4-n}	log C_a
9......	4541	0.01872	14.46	17.....	3858	0.00172	15.64
11......	4199	.008185	14.84	19.....	3813	.001195	15.81
13......	4026	.00436	15.20	21.....	3781	0.000865	15.96
15......	3924	0.00266	15.44				

TABLE 10

THE PICKERING SERIES

n	λ	NGC 2392		IC 418		NGC 6891		IC 4593		NGC 6826		NGC 6210	
		log $W\lambda$	log $N_{1,\,4}H$	log $W\lambda$	log $N_{1,\,4}H$	log $W\lambda$	log $N_{1,\,4}H$	log $W\lambda$	log $N_{1,\,4}H$	log $W\lambda$	log $N_{1,\,4}H$	log $W\lambda$	log $N_{1,\,4}H$
9................	4541	−0.07	14.39	−0.26	14.20	−0.17	14.29	−0.12	14.34	−0.10	14.36	−0.03	14.4
11................	4200	− .17	14.67	+ .1	15.0	− .13	14.71	− .35	14.49	− .29	14.55	− .52	14.3
13................	4026	− .16	15.05	......	......	......	......	......	......	......	......	......	......
15................	3924	− .34	15.10	......	......	−0.55	14.89	− .36	15.08	− .52	14.92	......	......
17................	3858	−0.51	15.13	− .19	15.45	......	......	−0.70	14.94	−0.52	15.12	−0.65	15.0
19................	3813	......	......	− .42	15.39	......	......	......	......	......	......	......	......
21................	3781	......	......	−0.44	15.52	......	......	......	......	......	......	......	......
Final log $N_{1,\,4}H$......	....	......	15.20	......	15.56	......	14.90:	......	15.10	......	15.20	......	15.0:
log $N(He^{++})HP_\epsilon$.....	....	......	24.41	......	......	......	24.0	......	24.24	......	24.27	......	24.0
log $N(He)/N(He^{++})$..	....	......	0.35	......	......	......	0.6	......	0.60	......	0.35	......	0.4
H/He.............	....	......	1.0	......	......	......	2	......	1.8	......	1.5	......	2

electron pressure. The results suggest that helium is comparable in abundance with hydrogen. The determination is rough, however, not only because of the uncertainties in the line intensities and in the electron pressure but also because the excitation of *He* II increases rapidly with the depth. That is, the effective depth at which the helium lines are formed is much greater than the effective optical depth where the Balmer lines are formed. Hence we cannot use the same temperature and electron pressure for calculating the equilibrium of hydrogen and ionized helium. The proper treatment of this problem would require a detailed integration through the atmosphere, taking into account the change of ionization and excitation with depth. With the aid of a model atmosphere computed for 10 Lac, I have estimated the effect of this stratification upon the estimated H/He ratio, and it would appear that the values listed in Table 10 are too small by a factor of about 2. The present data do not justify the more precise calculations necessary to establish this relation better.

[27] See Menzel and Pekeris, *M.N.*, **96**, 77, 1935.

IX. THE CENTRAL STAR OF NGC 246

Among the more interesting of the planetary nuclei with absorption lines is the central star of NGC 246. The nebular emission is so weak that we can observe the practically unobstructed spectrum of the nucleus. Berman gives the distance of this object as 1400 parsecs, while Parenago assigns 950 parsecs. With an adopted distance of 1400 parsecs, the absolute magnitude is +0.5; and, with a temperature of 35,000° K, from the Zanstra theory, the radius turns out to be 0.7 that of the sun; and the bolometric magnitude is about −3.0. The measured line intensities are as shown in the accompanying tabulation. $H\beta$ may be affected with emission. In the stellar spectrum appear the

λ	Ion	W_λ	λ	Ion	W_λ
4861	*H* I, *He* II	0.85	4340	*H* I, *He* II	0.40
4686	*He* II	0.83	4200	*He* II, *N* III	.62
4650	*C* III	1.6	4026	*He* II	.23
4541	*He* II	0.84	3931	*C* IV	0.70
4441	*C* IV	1.35			

λ 3811, λ 3838, emission lines of *O* VI with intensities of 0.63 and 0.57 equivalent angstroms of the underlying continuum. The spectrum seems to be of higher excitation than any other absorption object studied. The assigned temperature of 35,000° K may be too low.

I wish to express my gratitude to Dr. W. A. Hiltner and Dr. D. M. Popper, who obtained spectrograms of the nuclei of NGC 2392, IC 418, NGC 3242, and IC 3568 for me, and to Dr. F. K. Edmondson, who helped me obtain most of the other spectrograms. Drs. R. M. Petrie, D. H. Menzel, O. C. Wilson, and C. S. Beals read the manuscript and offered helpful suggestions.