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## AN INVESTIGATION OF STELLAR MOTIONS

## I. THE McCORMICK PROPER MOTION CATALOGUES

By A. N. VYSSOTSKY and EMMA T. R. WILLIAMS
Summary. These two catalogues contain the proper motions and photovisual magnitudes of 29,000 stars in 782 small sample regions of the sky scattered from the north pole to declination $-30^{\circ}$, and covering about 400 square degrees in all. Spectral classification is given for 14,000 of the stars. The Second Catalogue contains more regions than the First and more stars with known spectra, whereas the First contains more stars fainter than eleventh magnitude. The accuracy of the motions is somewhat better in the Second Catalogue. Corrections are given to reduce the motions in each region to the system of the $\mathrm{FK}_{3}$. Methods are described and errors evaluated in detail.
I.I Introduction. It was not without certain misgivings that we undertook the preparation of a Second Catalogue of proper motions of faint stars ten years ago, for the results of the discussion of the First Catalogue had been disappointing. This time, at Professor J. Schilt's suggestion, we decided to discuss the motions in right ascension independently from those in declination; this afforded a number of valuable checks on the reliability of our findings at various stages. The change from the GC system to that of the FK3, upon Dr. H. R. Morgan's advice, involved months of extra work on the First Catalogue; but without this change the results would have been just as disappointing as those of the first discussion. Finally, the publication of proper motions in the Cape Zones provided much needed observations in the southern sky. These changes are largely responsible for the very great difference between the present results and those of the earlier discussion.
The two McCormick proper motion catalogues ${ }^{1}$ contain the motions, magnitudes and spectra of stars from eighth to twelfth magnitude in 782 different regions of the sky. The regions, each of which covers about half a square degree, are fairly uniformly distributed from the north pole to declination $-25^{\circ}$ (Fig. I.I). In each region there is at least one bright star whose absolute proper motion appears either in $\mathrm{FK}_{3}$ (Dritter Fundamentalkatalog) ${ }^{2}$ or in GC (Albany General Catalogue). ${ }^{3}$

Two characteristics of the McCormick work distinguish it from most other investigations of the proper motions of stars fainter than ninth magnitude. First, we have reduced our motions rigorously to a fundamental system of absolute motions. Second, we have endeavored to obtain a representation of as much of the sky as is within our reach, with a sufficient number of small samples to render sampling fluctuations insignificant in the discussion; thus, there are five times as many McCormick regions between the north pole and declination $-25^{\circ}$ as there are Kapteyn areas in the same part of the sky.

These two characteristics enable us to obtain a different type of information from that which is available from other proper motion investigations of faint stars. In particular, we obtain insight into the nature of the solar motion and galactic rotation which is at present available in no other way. As will be seen from the discussion of the observational errors (Section 1.6), no very high accuracy is claimed for the individual proper motions. In fact, since the reduction to the $\mathrm{FK}_{3}$ system in a given region is frequently supplied by the motion of one star, all the motions in this region will be affected by the same error. We could have achieved somewhat greater individual accuracy had we measured fewer regions but measured them in duplicate; even so, the accuracy in high latitudes cannot be greatly improved with a field the size of ours, because of the scarcity of reference stars and their large "cosmi-
cal errors." However, for our chief purposes, the large number of samples is more important because of the irregularities in galactic structure.

As compared with an earlier analysis ${ }^{4}$ of the motions of the First Catalogue, the present discussion benefits particularly by the fact that the motions are now reduced to the $\mathrm{FK}_{3}$ system, and by the addition of the motions of the Cape Zone Catalogues ${ }^{5}$ also reduced to the FK3 system. It may be noted that the FK3 and the Cape Zone Catalogues were not available at the time of the earlier analysis.

The various appendices of the Second Catalogue contain tabular summaries of groupings, motions, etc., incidental to the discussion presented in this series of papers. These are designed to facilitate the use of the material by other investigators.

The observational techniques employed in the Second Catalogue (regions 342 to 782) differ in certain respects from those used in the First Catalogue. As before, one pair of plates has been measured for each region, and the brightness of the $\mathrm{FK}_{3}$ or the GC star has been reduced by means of the rotating sector approximately to magnitude 10.3 in order to minimize errors arising from difference in brightness. In the following sections the changed techniques which characterize the Second Catalogue will be outlined.
I.2 Material: In the Second Catalogue more regions were measured than in the First, but the
total number of faint stars is only in,300 as compared with 17,900 in the First. This reflects a change in policy. We now endeavor to measure all stars brighter than magnitude 11.5 pv , but to observe the fainter stars only in cases where they are needed to insure a sufficient number of reference stars. This change was adopted since it is not possible to be sure of completeness to a limit much fainter than magnitude II.5, neither is it practical to obtain spectra for the fainter stars; moreover, the accidental errors of measurement are considerably larger for them.

The choice of regions is practically limited by the number and distribution of McCormick parallax regions which are more than ten years old. It is seen from Figure i.I that the First Catalogue contains more regions south of $-20^{\circ}$ than the Second. This reflects the establishment of parallax observatories in the southern hemisphere, so that after 1925 the McCormick Observatory chose very few stars in the south for parallax determination. Wherever possible we have observed pairs of regions which partially overlap, since these give us a good means of evaluating our observational errors.

Another important change in policy in the Second Catalogue is the restriction of the measurable area of the plates to those portions which are more than one centimeter from the edge. Thus, the area is cut from 0.56 square degree to o.46, when a rotating sector is used; it is cut from 0.65 to 0.55 square degree in the infre-


Figure 1.I. Showing the distribution of the McCormick proper motion regions in galactic coordinates. The regions of the First Catalogue are represented by dots, those of the Second Catalogue by open circles. The curve indicating the approximate southern limit of the McCormick regions, represents the declination circle at $\mathbf{- 2 5 ^ { \circ }}$. The Cape zone from $-40^{\circ}$ to $-52^{\circ}$ is indicated by cross-hatching.
quent cases when no rotating sector is used. This restriction has greatly diminished the number of large errors as will be indicated in Section i.6.

It should also be mentioned that in the present discussion we have used the Harvard galactic pole at $12^{\mathrm{h}} 40^{\mathrm{m}},+28^{\circ}$, instead of the van Rhijn pole at $12^{\mathrm{h}} 56^{\mathrm{m}},+25^{\circ} \cdot 5$, which had been used in the previous discussion in the First Catalogue.

Other pertinent general information is collected in Table I.I.

| Item | Gal. Lat. | First Catalogue | Second Catalogue |
| :---: | :---: | :---: | :---: |
| Iverage interval between first and second epoch |  |  |  |
| Number of regions | $0^{\circ}-20^{\circ}$ | 134 | 193 |
|  | $2 \mathrm{I}-40$ | 94 | 104 |
|  | $41-90$ | 113 | 144 |
|  | all | 341 | 441 |
| Number of stars fainter |  |  |  |
|  | 21-40 | 3,362 | 2,497 |
|  | 41-90 | 2,406 | 2,166 |
|  | all | 17,927 | 11,320 |
| Number of spectra | $0^{\circ}-20^{\circ}$ | 3,297 | 4,540 |
|  | $2 \mathrm{I}-40$ | 1,048 | 1,614 |
|  | 41-90 | 918 | 1,450 |
|  | all | 5,263* | 7,604 |
| Average pv. magnitude | $0^{\circ}-20^{\circ}$ | 11.4 | 10.8 |
|  | $2 \mathrm{I}-40$ | 11.3 | II. 0 |
|  | $4 \mathrm{I}-90$ | 11.3 | II.I |
|  | all | II. 3 | I 1.0 |
| Number of square degrees in each region | all | 0.56 | 0.46 |
| Number of reduction stars: |  |  |  |
| from FK3 | all | 212 | 197 |
| from GC | all | 326 | 426 |
| Total | all | 538 | 623 |
| Total weight of reduction |  |  |  |
|  | $0^{\circ}-20^{\circ}$ $2 \mathrm{I}-40$ | 74.5 | 123.4 |
|  | $21-40$ $41-90$ | 52.9 62.5 | 59.5 81.0 |
|  | all | 189.9 | 263.9 |

* In addition, more than 1,000 unclassified stars in 85 regions of the First Catalogue have subsequently been assigned spectral classification and have been included in the present discussion (cf. Appendix C of Second Catalogue).
1.3 Measurement and Reduction of Relative Proper Motions. There has been practically no change in technique in so far as measurements are
concerned, from that which is described in the introduction to the First Catalogue.

As to the reductions, the only important change has been in the choice of reference stars in high latitudes, where it has been found wiser to use more reference stars, even though the four quadrants do not always appear "well balanced." Nearly all available stars were used excepting only those with very large motions. This change was made because it was found that the choosing of reference stars so that the quadrants were "well balanced" actually introduced larger errors than would result from a random choice.
I.4 Reduction of Relative Motions to FK_ System. The decision to refer our motions to the $\mathrm{FK}_{3}$ system has necessitated the derivation of new reductions for the motions of the First Catalogue to supersede those published therein (cf. Appendix A of Second Catalogue).

In a region in which the parallax star is an $\mathrm{FK}_{3}$ star, the reduction is simply:

> FK3 motion for $1900-$ McCormick relative motion.

When a star appears in the GC but not in the $\mathrm{FK}_{3}$, we have:

> GC motion for $1900-$ McCormick relative motion + Kopff corrections.

The Kopff corrections are taken from Tables 4, 5 , and 6 of his paper on the relation between the GC and the FK3 systems. ${ }^{6}$ The 1900 motions must be used since the McCormick plates were oriented and measured with respect to the 1900 equator.

In many regions there are two or more stars whose motions are given in either $\mathrm{FK}_{3}$ or GC. In order to combine the several reductions to a single value to be adopted for the region in question, we have used a weighting formula similar to the one explained at the bottom of page 2 of the First Catalogue but modified in one respect:

$$
W_{1}=\frac{10 \times\left(\mathrm{O}^{\prime \prime} . \mathrm{I}\right)^{2}}{\left(4^{\prime \prime} .8 / T\right)^{2}+\left(\mathrm{I} \cdot 4_{a}\right)^{2}}
$$

Here, the probable error of the difference between the position of the reduction star on the two plates is assumed as $\pm 0.0048 ; T$ is the number of years in the interval between the two plates; and $r_{a}$ is the probable error of the absolute motion as published in the GC. The factor I. 4 has been introduced in accordance with Schlesinger's finding ${ }^{7}$ that the probable errors
listed for the proper motions in GC are systematically too small. In the case of bright stars situated outside the area covered by the rotating sector, the scheme of weighting adopted in the First Catalogue was replaced by the following:

|  | Visual Magnitude |  |  |
| :--- | :---: | :---: | :---: |
|  | $6.0-7.4$ | $7.5-8.9$ | $9.0-$ |
| Stars near center | 0.25 | 0.5 | 1.0 |
| Stars toward edge | 0.25 | 0.25 | 0.5 |

In all there are $409 \mathrm{FK}_{3}$ stars in the two catalogues, and there are 752 additional stars whose motions are given in the GC. The total weights of these reductions to absolute motion are given in Table I.I.
1.5 Reduction of Motions for Secondary Investigations. In low and intermediate latitudes it is possible to strengthen considerably the reduction of any individual region by comparing the mean motions of all stars in the region with that which we should expect from our knowledge of the constants of solar motion, precession, and galactic rotation derived either from our general solution (Paper IV of this series) or from some reliable outside source. For this purpose we have used the constants listed in Table i.II. Nat-

urally, such a supplementary reduction is not suitable for a general solution but only for secondary investigations such as those involving secular parallaxes or peculiar motions. The derivation of the weights, $W_{2}$, at the bottom of the table is explained in the next section.

A weighted combination of this supplementary reduction of each region with the original reduction (preceding section) has been computed after
applying to the original reduction the adopted corrections, $\Delta k$ and $\Delta n$, as listed in Table 1.II and also removing the effects of galactic rotation consistent with the values adopted for $P$ and $Q$. The resulting reduction, of weight $W_{1}+W_{2}$, is published in the Second Catalogue for all the regions in both catalogues; this is not a reduction to the $\mathrm{FK}_{3}$ system nor to any absolute system in the usual sense, since the effects of galactic rotation on the resultant motions have been removed. On the other hand, it is necessary to remove these effects before certain secondary investigations are possible. Such motions may be designated as "conditioned motions."
I. 6 Accidental Errors in the Motions. Our total accidental error, $\epsilon$, is made up of three different types of error, such that $\epsilon=\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}+\epsilon_{3}{ }^{2}\right)^{\frac{1}{2}}$. Here, $\epsilon_{1}$ is the McCormick error of observation, i.e. film, bisection and screw errors; $\epsilon_{2}$ is the error introduced by the plate constants; and $\epsilon_{3}$ is the error of the reduction to absolute motion, which is a combination of the error in the GC or $\mathrm{FK}_{3}$ and of the McCormick observational error for the reduction star. The component, $\epsilon_{2}$, depends on the number of reference stars and on the dispersion in their motions; it is small in low latitudes and large in high latitudes; a considerable increase in the interval between the early and recent plates leaves it very little affected, whereas $\epsilon_{1}$ is considerably diminished thereby and $\epsilon_{3}$ is also somewhat diminished.

Various methods of determining our errors are available. We have used the following:
for $\epsilon_{3}$, comparison of overlapping regions;
for $\epsilon_{3}$, comparison of two independent reductions in the same region;
for $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$, comparison of overlapping regions;
for $\epsilon_{1}$, comparison from two pairs of plates of the same region;
for $\epsilon$, computation of the "motions" in galactic latitude of certain A stars.

Determination of $\epsilon_{3}$. In the two catalogues there are altogether 39 cases in which two regions partially overlap. Their distribution in galactic latitude is indicated in Table r.III. It is seen
$\left.\begin{array}{ccc}\text { TABLE I.III. DISTRIBUTION OF PARTIALLY } \\ \text { OVERLAPPING REGIONS }\end{array}\right]$.
that, although ten per cent of our regions are represented in these cases, the total number of stars involved is relatively small. From a comparison of the motions in common, we find for the probable error of a reduction to absolute motion whose weight ( $W_{1}$ ) is equal to unity:
in R.A., $\epsilon_{3}=0^{\prime \prime} .39 \pm 0^{\prime \prime} .04^{6}$ ) for centennial
in Decl., $\left.\epsilon_{3}=0.20 \pm 0.03^{3}\right\}$ motions.
We have made rather better determinations of $\epsilon_{3}$ from the interagreement of two independent reductions to absolute motion in the same region, where, in addition to the "parallax star," there is another GC star. In the 54 cases chosen for investigation, the additional star is of apparent magnitude 9.0 or fainter and is situated at least 35 mm from the edge of the plate in the direction of right ascension and 25 mm from the edge in the direction of declination. From the 54 regions, we find:

$$
\begin{aligned}
& \text { in R.A., } \epsilon_{3}=0^{\prime \prime} .26 \pm 0 \text { ". } 02, \\
& \text { in Decl., } \epsilon_{3}=0.23 \pm 0.02 .
\end{aligned}
$$

Combining the results from the two determinations, we find:

$$
\begin{aligned}
& \text { in R.A., } \epsilon_{3}=0 \text { ". } 30 \pm 0 " 03, \\
& \text { in Decl., } \epsilon_{3}=0.22 \pm 0.02 .
\end{aligned}
$$

Weight of supplementary reduction. We are now in a position to compare the values of $\epsilon_{3}$ just derived, with the probable error, $r$, of a single equation in our general solution as found in Paper IV of this series. The quantity, $r$, includes "cosmical errors" and errors in the $\mathrm{FK}_{3}$ system as well as our $\epsilon_{3}$; since the value of $r$ as derived from the equations in right ascension is the same as its value derived from the equations in declination, we have assumed that the value of $\epsilon_{3}$ in both right ascension and declination is equal to $\mathrm{o}^{\prime \prime}$ 26. In general, three regions have been combined in the formation of each observation equation of the general solution, the average weight, $W_{1}$, of the reduction in each region being 0.6 ; consequently, in Table i.IV, we have compared

|  | $0^{\circ}-10^{\circ}$ | $\underset{1 \mathrm{I}^{\circ}-20^{\circ}}{\text { Galactic }} \underset{2 \mathrm{I}^{\circ}-40^{\circ}}{\text { Latitude }}$ |  | $41^{\circ}-90^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: |
| $r \sqrt{3}$ | 0". 49 | o'. 59 | o". 68 | 0. 99 |
| $\epsilon_{3} / \sqrt{0.6}$ | 0.34 | 0.34 | 0.34 | 0.34 |
| $\left(3 r^{2}-\epsilon_{3}{ }^{2} / 0.6\right)^{\frac{1}{2}}$ | 0.35 | 0.49 | 0.59 | 0.93 |
| $\epsilon_{3} /\left(W_{1}+W_{2}\right)^{\text {g }}$ | 0.25 | 0.27 | 0.29 | 0.34 |

$r \sqrt{ } 3$ with $\epsilon_{3} / \sqrt{ } 0.6$. The vectorial difference between these two quantities, which is given in the third line, represents the probable error resulting
from the combined effect of a possible error in the $\mathrm{FK}_{3}$ and the "cosmical error" in the mean motions of all the stars in an average region. It is seen that in low latitudes these errors are relatively small and that consequently in these zones we may use the mean motions predicted by the general solution to strengthen the reduction of the motions by the method described in the preceding section. The values of the weight, $W_{2}$, of this supplementary reduction have been obtained from a comparison of the second and third line of the table; they will be found in Table i.II. In the fourth line of Table I.IV is given the probable error of reduction for an average region reduced by a weighted combination of the two methods.
Determination of $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$. This problem is somewhat complicated by the fact that the relative motions of the Second Catalogue are better than those in the First, not only because of the longer average interval in the Second Catalogue, but in particular because of the more severe restrictions imposed with respect to "edge stars" in the Second Catalogue, as mentioned in Section I.2. The restriction has greatly diminished the number of large errors. Thus, in the First Catalogue, out of 843 A stars fainter than magnitude 9.4 there were 56 whose centennial "motions," as observed, were greater than 2 ". 4 whereas the corresponding figures in the Second Catalogue are 1,074 and 30 respectively. Again, there were 459 F stars fainter than magnitude 9.4 in the First Catalogue, and of these 74 had centennial "motions" greater than 2 ". 4 , whereas in the Second Catalogue the figures are 692 and 55 respectively. In other words, probably more than half of the large "motions" for A and F stars in the First Catalogue are in reality large errors. That this interpretation is the correct one was checked by a review of a fair number of individual cases. It was found that a large proportion of them occur in the cases of stars within one centimeter of the edge of the plate. Hence in our determination of $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$ we have first rejected all stars in the First Catalogue which would have been rejected under our subsequent restrictions. Thus, we have determined values appropriate to the Second Catalogue.

In this way, from the 39 pairs of overlapping regions we find the values of $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$ listed in Table i.V. Here, we have combined the values from the motions in right ascension with those from the motions in declination, inasmuch as there was no systematic difference between them.

TABLE I.V. OBSERVED VALUES OF $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$

| $\begin{gathered} \text { Magnitude } \\ \text { Group } \end{gathered}$ | $0^{\circ}-20^{\circ}$ | Galactic Latitude $21^{\circ}-40^{\circ}$ | $41^{\circ}-90^{\circ}$ |
| :---: | :---: | :---: | :---: |
| 7.5-9.4 | o'. 49 (30) | o'. 37 (10) | O". 80 (24) |
| 9.5-10.4 | 0.41 (50) | 0.46 (8) | 0.73 (26) |
| 10.5-11.4 | 0.42 (154) | 0.66 (26) | 0.74 (20) |
| II . 5-12.4 | 0.43 (143) | 0.69 (22) | 0.51 (28) |
| $12.5{ }^{-}$ | 0.46 (8) |  | 1.74 (6) |

The number of items which entered into each mean is indicated in parenthesis.

We have smoothed these observed values, taking into consideration the independent determinations of $\epsilon_{1}$ published on page 45 of the First Catalogue. Table I.VI gives the values which we

| TABLE I.VI. SMOOTHED VALUES OF | $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{3}{2}}$ |  |  |
| :---: | :---: | :---: | :---: |
| Magnitude <br> Group | $0^{\circ}-20^{\circ}$ | Galactic Latitude <br> $2 \mathrm{I}^{\circ}-40^{\circ}$ | $4 \mathrm{I}^{\circ}-90^{\circ}$ |
| $7.5^{-9} 9.4$ | $0^{\prime \prime} .5:$ | $0.6:$ | $00^{\prime \prime} .8:$ |
| $9.5^{-10.4}$ | 0.4 I | 0.5 I | 0.73 |
| $10.5^{-11} .4$ | 0.43 | 0.52 | 0.74 |
| $11.5^{-12.4}$ | 0.47 | 0.56 | 0.76 |
| $12.5^{-}$ | $0.7:$ | $0.8:$ | $0.9:$ |

believe best represent the quantity $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$ for stars of the Second Catalogue.

Total errors. Combining the values of $\epsilon_{3} /\left(W_{1}+W_{2}\right)^{2}$ from Table I.IV with the values of $\left(\epsilon_{1}{ }^{2}+\epsilon_{2}{ }^{2}\right)^{\frac{1}{2}}$ from Table I.VI we obtain the total errors for an average region of the Second Catalogue as listed in Table i.VII.
$\left.\begin{array}{cccc}\text { TABLE I.VII. TOTAL ERRORS IN CENTENNIAL MOTIONS, } \\ \text { SECOND CATALOGUE } \\ \text { Galactic Latitude }\end{array}\right]$.

As previously mentioned, the errors of the Second Catalogue are somewhat smaller than those of the First because of the longer average time interval and because of the exclusion of edge stars. To obtain an idea of the relative size of the errors of the two catalogues as well as an independent check on the values given in Table r.VII we have investigated the peculiar motions in galactic latitude of tenth magnitude A stars between latitudes $+20^{\circ}$ and $-20^{\circ}$. Details of this investigation will be found in Section io.2. For 22I stars of the First Catalogue the average centennial motion (regardless of size) was found to be $0^{\prime \prime} .78$ and for 230 stars of the Second Catalogue it was 0 " 65 . These "motions" are largely errors of observation since from various considerations we know that the average of the true motions is about $0^{\prime \prime} .3 \mathrm{I}$. It follows that the prob-
able error of observation of the First Catalogue is $0.16 \mathrm{I} \pm$ ". 02 for the tenth magnitude stars in low latitudes, and the corresponding figure for the Second Catalogue is $0 . .49 \pm$ ". 02 . This last value confirms the value already derived in Table i.VII.

Combining the results of the comparison just given with the total errors of the Second Catalogue (Table I.VII) we obtain the values listed in Table i.VIII for the errors of the First Cata-

| $\underset{\text { Group }}{\text { Magnitude }}$ | $0^{\circ}-20^{\circ}$ | Galactic Latitude $21^{\circ}-40^{\circ}$ | $41^{\circ}-90^{\circ}$ |
| :---: | :---: | :---: | :---: |
| 7.5-9.4 | 0".7: | - ${ }^{\text {". }} 8$ : | 1. o : |
| $9.5-10.4$ | 0.6 | 0.7 | 0.9 |
| 10.5-II. 4 | 0.6 | 0.7 | 0.9 |
| II . 5-12.4 | 0.612 | 0. $7^{\frac{1}{2}}$ | 0.9 |
| $12.5{ }^{-}$ | 0.8: | 0.9: | 1.0: |

logue. These differ relatively little from the values derived on page 46 of that catalogue.
I.7 Systematic Errors in the Motions. Possible systematic errors in the motions are of much greater importance in this investigation than the accidental errors. They might conceivably arise from a difference in observing conditions between the first epoch and the second epoch, such as a slight change in the collimation. To test for this, we may compare the motions of the Second Catalogue with those of the First, since, as it happens, the second epoch of the First Catalogue is about the same as the first epoch of the Second Catalogue. Accordingly, in the general solution and in the discussion of secular parallaxes, the residuals have been examined to see if those of the First Catalogue differ systematically from those of the Second. There is some evidence of a moderate systematic difference in southern declinations, but apart from this the agreement is very good.

A more fundamental test is applied in Paper IV, where a general solution was carried through from the motions in right ascension entirely independent of a second solution from the motions in declination. Since these two solutions yielded results in substantial agreement with one another, we conclude that the systematic errors affecting our motions are not very serious. Again, the secular parallaxes obtained from the motions in right ascension have been compared zone by zone with those obtained from the motions in declination; details of this comparison are given in Section 8.3. The agreement is perhaps not completely satisfactory but it is within the limits to be expected from the probable errors.

For completeness, it may be well to indicate that there is definite evidence in certain regions of systematic errors depending on magnitude which probably arise from guiding errors on some of the plates; but since these regions are exceptional and since the errors are not always in the same sense among these exceptional regions, they do not systematically affect our results.
I. 8 Magnitudes. The observational technique for determining the photovisual magnitudes which appear in the Second Catalogue differs greatly from that used in connection with the First Catalogue; our experience has shown that the method of direct pole comparison is not satisfactory when a long-focus instrument is used. For the determination of the magnitudes in the Second Catalogue, a coarse grating especially designed for the purpose was placed in front of the 26 -inch objective. Inasmuch as the HPv magnitude of the $\pi$ star has usually been fairly accurately determined, ${ }^{8}$ it was used to fix the zero point; and the first and second order diffraction images of the $\pi$ star and of the other bright field stars have been used to establish the scale. An account of the methods used and an investigation of the system of the magnitudes thus derived are given in Paper II.
I.9 Spectra. In respect to spectra, too, the observational technique differs greatly from that used in the First Catalogue, for the ro-inch prismatic camera which was acquired by the observatory in 1935, has enabled us to classify our stars from our own objective prism plates. The classification system has been described elsewhere, ${ }^{9}$ together with its relation to other systems, and an evaluation of the accidental errors of classification.
As indicated in Table I.I, two-thirds of all the stars of the Second Catalogue have been assigned spectral classification. This figure does not include the doubtful classifications which have been indicated with a colon in the catalogue and which have been omitted in the discussion of the motions according to spectral type. In general, the limiting magnitude of the McCormick spectroscopic plates is 12.0 pg , although many stars brighter than the limit are unclassifiable in lower latitudes because of overlapping images.
The spectral statistics from the Second Catalogue, which represent a fairly homogeneous sampling of two-thirds of the sky, have been analyzed according to galactic latitude and published separately. ${ }^{10}$ A partial analysis of the spectral statistics of both catalogues according to galactic longitude is given in Paper VII.

More than a thousand additional spectral classifications in 85 regions of the First Catalogue are listed in Appendix $C$ to the Second Catalogue. Furthermore, in the present discussion no B stars from the First Catalogue were included unless they had been reclassified from our own spectral plates; Appendix D lists all the B stars of the two catalogues which were used in the discussion. Similarly, in the case of stars of classes Mo and later the only stars included in the present discussion are those which have been classified from our own plates; these are listed in Appendix E. In other words, certain B and M stars of the First Catalogue have been excluded if no McCormick spectral plate was available to verify the classification.

Professor S. A. Mitchell, formerly director of this observatory, was responsible for the support of the work on the Second Catalogue from its inception in the fall of 1936 until his retirement in 1945. Professor P. van de Kamp planned and supervised the early stages of the Second Catalogue for nine months until he left for Sproul Observatory in June 1937. Upon his departure, the present authors first undertook the work, with numerous modifications. Director H. L. Alden, by assigning additional clerical help to the task, has expedited the completion of the discussion.

Many individuals have taken part in the work on the Second Catalogue. In the measures and reductions, the most important single contributor was Mrs. Virginia M. Lee, who measured and reduced more than half of all the motions. Mrs. Bertha L. W. Armistead and Miss Betty Allen Magruder measured and reduced the rest. The extensive computing required for this discussion was performed by Miss Pauline Yancey, Miss Nancy M. Kidder and in particular by Miss Dorothy R. Watson who has also typed the Second Catalogue for photo-offset reproduction. It is difficult to express suitably our great appreciation of the high quality of conscientious work performed by all of these young women.

For the measurement and reduction of the magnitudes we are greatly indebted to Edward R. Dyer, Jr. and R. Gardner Reed, who were Vanderbilt Fellows at the Observatory. The work was done practically independently.

Our investigation in Paper VI would scarcely have been possible without the very helpful attitude of Drs. R. E. Wilson and H. Raymond in sending us their extensive tabulations of the mean motions of the GC. It seems peculiarly

