ZEEMAN EFFECT IN STELLAR SPECTRA*

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ABSTRACT

Some stars of early spectral type which exhibit sharp metallic lines may be in rapid rotation, with their axes directed nearly toward the observer. On the possibility that strong magnetic fields are correlated with rapid rotation in such objects, the Zeeman effect for a normal triplet line, as observed through a circular analyzer, has been integrated over the visible hemisphere of the star. Reasonable assumptions are made regarding field distribution and limb darkening. It is found that the integrated Zeeman effect for a normal triplet should give a displacement of about 3×10^{-6} A/gauss at λ 4600.

Observations of the star 78 Vir (type A2p), made with a differential circular analyzer in front of the slit of the coudé spectrograph of the 100-inch telescope, show displacements of the metallic absorption lines that are interpreted as a Zeeman effect resulting from a general magnetic field having a strength of 1500 gauss at the pole. The control star ϵ Peg (type K0) shows no effect.

A comparison of the magnitude of the sun's general magnetic field[1] and its equatorial speed of rotation[2] (about 50 gauss and 2 km/sec, respectively) with the average equatorial speed[3] established for stars of type B, A, and early F (about 60 km/sec) suggests the possibility that the rapidly rotating stars may possess relatively strong magnetic fields. If the field strength be supposed to increase directly with the equatorial speed of rotation,[4] fields of the order of 1500 gauss may be predicted.

A small proportion of the early-type stars have very sharp lines, showing little or no rotational broadening; and some of these stars are presumably in rapid rotation, with their axes directed nearly toward the observer. If strong general magnetic fields occur in stars of this sort, the Zeeman effect may be sufficiently large to affect line widths perceptibly; moreover, if reasonable assumptions are made regarding the characteristics of the field, it can be shown that the integrated Zeeman effect for spectrum lines will be preferential in one direction, i.e., the components of the lines transmitted through analyzers for right-handed and left-handed circularly polarized light will be displaced in opposite directions by amounts proportional to the field strength and to the atomic Zeeman effects for the particular lines.

In making a preliminary calculation of the magnitude of the Zeeman effect that might possibly be observed in rapidly rotating stars, only absorption lines will be considered (the inverse Zeeman effect). Simplifying assumptions will be made as follows:

1. The rotational and magnetic axes of the star coincide and are directed toward the observer.
2. The star is spherical.
3. The coefficient of limb darkening[5,6] is 0.45.

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[1] G. E. Hale, *Mt. W. Contr.*, No. 71; *Ap. J.*, **38**, 27, 1913: Hale, Seares, van Maanen, and Ellerman, *Mt. W. Contr.*, No. 148; *Ap. J.*, **47**, 206, 1918. In 1915, Hale expressed the hope that the 100-inch reflector would permit tests for stellar magnetism to be applied (*Ten Years Work of a Mountain Observatory* [Carnegie Institution of Washington, 1915], p. 45).

[2] W. S. Adams, *Mt. W. Contr.*, No. 33; *Ap. J.*, **29**, 110, 1909.

[3] C. T. Elvey, *Ap. J.*, **71**, 221, 1930.

[4] The association of magnetic fields with rotation in astronomical bodies appears to have been suggested first by Schuster (*Report B.A.A.S.* for 1892, p. 634; *Proc. Phys. Soc. London*, **24**, 121, 1912).

[5] A. Pannekoek, *M.N.*, **95**, 733, 1935.

[6] G. E. Kron, *Ap. J.*, **96**, 173, 1942.

4. The equivalent widths of the absorption lines are constant over the projected hemisphere of the star presented to the observer.

5. The distribution of the magnetic field over the surface of the star is similar to that of a uniformly magnetized sphere.

This last assumption was made by Hale and his collaborators in their investigation of the general magnetic field of the sun.[1] Their results depended upon the measurement of very small Zeeman displacements in a number of lines that differed considerably in intensity. The stronger lines, arising higher in the solar atmosphere, gave smaller measured displacements than did the weak lines, and this was provisionally taken to mean that the field decreased rapidly with increasing height. Hale noted, however, that "caution must be exercised in accepting the hypothesis that the general field decreases rapidly in intensity with increasing elevation inasmuch as the measures may perhaps be affected by systematic errors depending upon line intensity." S. Rosseland, later interpreting the Mount Wilson measurements to mean that the sun's general field differed very radically from that of a uniformly magnetized sphere, arrived at a value three times as great as Hale's for the horizontal component in the equatorial regions.[7] Recently, word has been received of the work of G. Thiessen,[8] who used a modification of an interference method described by H. D. Babcock.[9] To quote from the available summary of Thiessen's paper:

> The preliminary measurement of the iron line 6173.348 A gave a magnetic field strength at the poles of about 53 ± 12 gauss under the assumption of a dipole structure of the sun's field. The investigation of the chromium line 5247.576 A gave similar results. Thus, I was not able to confirm the very weak field of about 15 gauss found by Hale and collaborators from measurements of the same iron line 6173.348 A. This seems to be of some importance, because the hypothesis of the rapid decrease of the field strength with height in the solar atmosphere was partly based on these older measurements.

The best evidence at present, then, seems to favor the assumption that the field can reasonably be represented as similar to that of a uniformly magnetized sphere; but, if Rosseland's type of field were used in the following development, a result of the same order of magnitude would be obtained, although with reversed sign.

If we consider the distribution of the field of a uniformly magnetized sphere over its surface (equivalent to the field of a dipole at the center) according to the following formulae:

$$H = \tfrac{1}{2} H_p \left(1 + 3 \sin^2 \phi\right)^{\frac{1}{2}} , \qquad (1a)^{10}$$

$$H_x = \tfrac{3}{2} x z H_p , \qquad (1b)^{11}$$

$$H_y = \tfrac{3}{2} y z H_p , \qquad (1c)^{11}$$

$$H_z = \left\{ z^2 - \tfrac{1}{2} \left(x^2 + y^2\right) \right\} H_p , \qquad (1d)^{11}$$

where ϕ is the stellar latitude and z is the rectangular co-ordinate measured parallel to the axis from the equatorial plane, it will be seen that the field vector at the pole of the star is parallel to the axis and that, in passing from the pole to the equator, it turns through 180°, at the same time decreasing to one-half its value at the pole (Fig. 1). At latitude 35°.6, the field vector is perpendicular to the polar axis. From this we see that the component of the field vector parallel to the axis of the star, when integrated over the visible hemisphere, has a value different from zero, thus presenting the possibility of

[7] *Mt. W. Contr.*, No. 302; *Ap. J.*, **62**, 387, 1925; *Theoretical Astrophysics* (Oxford: Clarendon Press, 1936), p. 219.

[8] A summary of Thiessen's work is given by G. P. Kuiper, *Pop. Astr.*, **54**, 263, 1946, based on a report on file at the Yerkes Observatory.

[9] *Pub. A.S.P.*, **53**, 237, 1941.

[10] Foster and Porter, *Electricity and Magnetism* (London, 1903).

[11] Rosseland, *op. cit.*, p. 219. A common factor, $A(r)$, has been omitted, and the equations have been normalized.

observing the Zeeman effect in spectrum lines arising in the star's atmosphere. Limb darkening tends to suppress the contributions to the integrated Zeeman pattern arising in low latitudes, where the field is opposite and weaker than at the pole, and hence increases the net effect.

The Zeeman effect for a line exhibiting the normal triplet pattern, (0.00) 1.00, results in the appearance of two components equally displaced in opposite directions from the normal position of the line by an amount

$$\Delta\nu = 4.67 \times 10^{-5}\ \text{cm}^{-1}\ \text{gauss}^{-1}\ . \tag{2}$$

These two components are characterized by circular polarization in opposite directions when the line of sight is parallel to the field. If the field vector is inclined to the line of sight, the undisplaced plane-polarized component of the normal triplet appears, while the displaced components are elliptically polarized, although their displacements remain

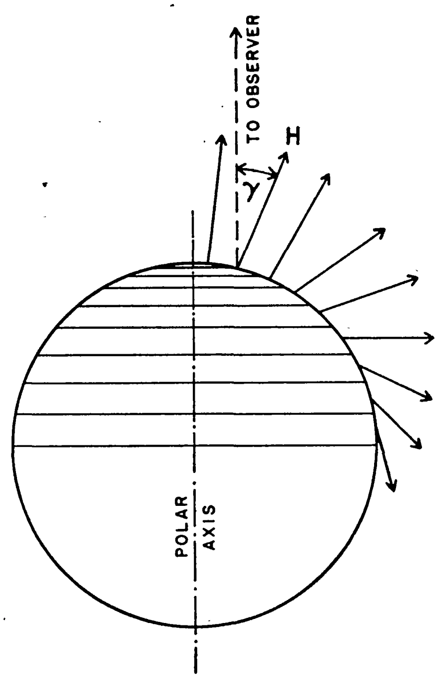

FIG. 1.—Variation of the direction and strength of the magnetic field with latitude on the surface of a star, on the assumption that the field is similar to that of a uniformly magnetized sphere. The field vector H makes an angle γ with the line of sight.

proportional to the field strength. The discovery of magnetic fields in sunspots by Hale[12] in 1908 stimulated interest in the variation of the intensities of the components of the pattern in the inverse Zeeman effect as the angle between the field vector and the line of sight is altered. This was the subject of investigations by P. Zeeman and B. Winawer,[13] W. Voigt,[14] and H. A. Lorentz,[15] among others. The problem becomes quite complex when the absorption is strong and when the magnetic field is not sufficiently intense to resolve the components completely. In the present investigation we shall apply only the more elementary theory that pertains to the relative intensities of Zeeman components in emission or in very weak absorption lines. This should be sufficient to permit an approximate calculation of the magnitude of the integrated effect to be expected in stars.

F. H. Seares has discussed the appearance of Zeeman patterns observed at discrete points at various latitudes on the surface of the sun, resulting from the sun's general

[12] *Mt. W. Contr.*, No. 30; *Ap. J.*, **28**, 315, 1908.

[13] *Ap. J.*, **32**, 329, 1910.

[14] *Magneto- und Elektrooptik* (1908).

[15] *Proc. R. Acad. Sci. Amsterdam*, **12**, 321, 1909.

magnetic field.[16] He expressed the relative intensities of the three components of a normal Zeeman triplet, as observed through an analyzer for circularly polarized light, as follows:

$$I_V = \tfrac{1}{4}\,(1 \pm \cos\gamma)^2\,; \qquad I_M = \tfrac{1}{2}\sin^2\gamma\,; \qquad I_R = \tfrac{1}{4}\,(1 \pm \cos\gamma)^2\,, \tag{3}$$

where γ is the angle between the magnetic field vector and the line of sight. These formulae are represented graphically in Figure 2, which has been taken from the *Handbuch der Astrophysik,* **4,** 189.

In the present case, where the resulting Zeeman pattern must be integrated over the whole visible hemisphere of the star and where the components are blended together by overlapping and low resolution, a small shift in the center of gravity of the blended pattern in changing from a right-handed to a left-handed circular analyzer may lend itself to detection and measurement if the stellar field is sufficiently strong. To obtain, first, a qualitative picture of the distorted components of the Zeeman pattern, we may consider the radiation emitted in the direction of the observer from zones equally wide in

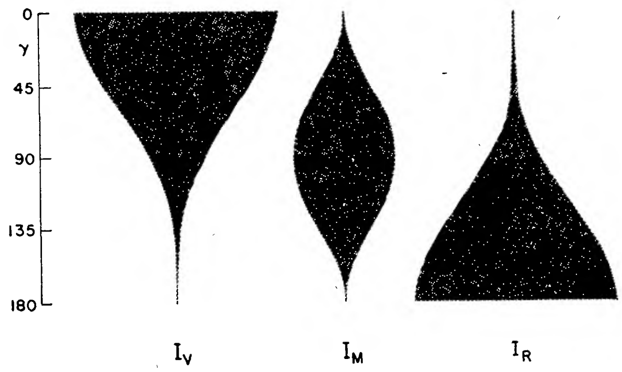

FIG. 2.—Relative intensities of the components of a Zeeman triplet, seen through a circular analyzer, as a function of the angle γ between the field vector and the line of sight (see eq. [3]). Intensity is represented by the width of each component.

stellar latitude. Bearing in mind the projected areas of the latitude zones, the limb darkening, and the variation in strength and direction of the field vector with latitude, we see that the Zeeman pattern for a normal triplet line will be distorted from that of Figure 2 to something resembling the pattern of Figure 3. The displacement, δ, of the center of gravity of the blended Zeeman pattern will evidently be only a fraction of the maximum displacement of the satellites from the normal position of the line.

In order to derive δ quantitatively, on the basis of the foregoing assumptions, for a star having a polar field strength H_p, use is made of the following additional relationships:

$$\text{Projected area of a latitude zone} = 2\pi r^2 \sin\phi\cos\phi\, d\phi\,, \tag{4}$$

$$\text{Projected surface brightness } I(\phi) = 0.55 + 0.45\sin\phi\,, \tag{5}$$

$$\cos\gamma = \frac{H_z}{H}\,. \tag{6}$$

Now (see Fig. 3) let the maximum displacement of the satellites of the normal triplet Zeeman pattern (at the pole) be aH_p. Then, for a zone of stellar latitude, $d\phi$, we set up, for the three parts of the distorted pattern, an equation of moments about the new center of gravity:

$$I_{V_\phi}(aH - \delta_\phi)\,d\phi - I_{M_\phi}\delta_\phi\, d\phi - I_{R_\phi}(aH + \delta_\phi)\,d\phi = 0\,, \tag{7}$$

[16] *Mt. W. Contr.,* No. 72; *Ap. J.,* **38,** 99, 1913.

whence

$$\delta_\phi = \frac{aH\,(I_{V_\phi} - I_{R_\phi})\,d\phi}{(I_{V_\phi} + I_{M_\phi} + I_{R_\phi})\,d\phi} = aH\cos\gamma\,. \tag{8}$$

We now desire to find the mean value of δ_ϕ over the hemisphere, taking account of the projection factor and projected surface brightness. Using equations (1*a*), (4), (5), and (8), we have

$$\delta = \frac{aH_p}{2}\,\frac{\int_0^{\pi/2}\sin\phi\cos\phi\,(1+3\sin^2\phi)^{1/2}\,(0.55+0.45\sin\phi)\cos\gamma\,d\phi}{\int_0^{\pi/2}\sin\phi\cos\phi\,(0.55+0.45\sin\phi)\,d\phi}\,. \tag{9}$$

Upon integrating the numerator numerically and the denominator directly, we obtain

$$\delta = \pm\,0.311\,aH_p\,.^{*} \tag{10}$$

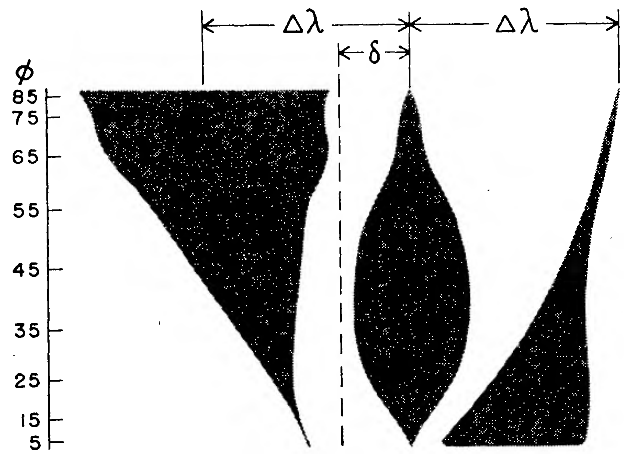

FIG. 3.—Intensities of the three analyzed Zeeman components, weighted according to projected areas of latitude zones, limb darkening, and variation in strength of magnetic field. The displacement of the center of gravity of the group from the normal position of the line is indicated by δ.

We should, therefore, expect the integrated effect to be about 31 per cent as great as that which would result if light from the pole of the star could be isolated.

From equations (2) and (10) we obtain

$$\Delta\lambda = 1.45\times10^{-5}\lambda^2 H \tag{11}$$

for the integrated case. If we assume that the wave length employed is λ 4600, the shift for a normal triplet integrated over the stellar hemisphere would be 3.1×10^{-6} A/gauss.

Thus far we have been concerned with the normal inverse Zeeman effect, the pattern of which is customarily designated (0.00) 1.00, where figures in parentheses represent the displacements of the "parallel" components from the normal position of the line and the following figures represent the displacements (positive and negative) of the "perpendicular" components, which, when viewed in the direction of the field, are circularly polarized. In the anomalous Zeeman effect, which is actually much more common than the normal effect, the several components are represented by their relative displacements, the most intense being printed in boldface type. For example, the notation for the Zeeman effect corresponding to the $^3D_1 - {^3D_2^\circ}$ transition is (**0.00**, 0.67) 0.50, 1.17, **1.83**. A discussion of the notation and a list of all common patterns are given by Kiess and

* The sign depends upon the use of a right- or a left-handed circular analyzer.

Meggers in "Tables of the Theoretical Zeeman Effect."[17] Occasional lines exhibit large Zeeman effects, two to three times as great as the normal; a few are unaffected.

For an emission line arising in a magnetic field, the direct Zeeman effect will be observed; and, as seen through a given circular analyzer, the displacement will be opposite to that found in the inverse effect.[18] In a stellar line exhibiting both emission and absorption components, this could lead to complications.

It is of interest to inquire whether the longitudinal Zeeman effect would be capable of giving the relationship between the polarity of a stellar magnetic field and the direction of rotation of the star. It appears that this might be possible if a measurable Zeeman effect can be found in one of the components of a visual binary where the direction of orbital motion is known, on the assumption that the sense of rotation of the star on its axis is the same as that of the revolution in its orbit.

We can now compute approximately the smallest stellar magnetic field that could possibly be detected with a suitable analyzer used in connection with the coudé spectrograph of the 100-inch telescope.[19] The long-focus (114-inch) camera of the coudé spectrograph has a dispersion of 2.9 A/mm in the second order. If two adjacent spectra of the star are photographed through right-handed and left-handed circular analyzers, the measurements can be made differentially, and any displacements will be effectively doubled. Finally, it is perhaps reasonable to assume that lines can be chosen having Zeeman effects 1.5 times the normal and that differential displacements of 0.001–0.002 mm on the plate can be detected if a number of lines are available for measurement. The minimum polar field observable is then found to be of the order of 500 gauss. Since the Zeeman effect increases with the square of the wave length, a considerable gain can be expected in going to the red or perhaps the infrared region of the spectrum, if other factors remain constant. With the grating currently in use in the coudé spectrograph, this is not feasible; but the restriction is only temporary.

In the optical system of the 100-inch telescope, all reflections are essentially at normal incidence, with the exception of the reflection from the coudé flat. Here the angle of incidence is 45° for an object on the celestial equator. This reflection alters somewhat the ellipticity of any reflected elliptically polarized light; the amount of the change will depend to some extent upon the condition of the reflecting surface, as well as upon the angle of incidence. For the coudé flat mirror the effect is small for objects not too far north of the celestial equator. The result will be to decrease somewhat any observed Zeeman effects; the displacement equation (11) should properly be written

$$\Delta\lambda = 1.45 \times 10^{-5} \epsilon \lambda^2 H \,, \tag{12}$$

where ϵ is a polarization factor, generally a little less than unity.

ANALYZERS

Two slightly different analyzers for circularly polarized light have been used in making observations with the coudé spectrograph. In each analyzer the optical parts are held in a metal mounting directly in front of the slit. The analyzer generally used consists of a suitably oriented quarter-wave plate of mica, of a thickness chosen as correct for λ 4600, followed by a plane-parallel crystal of calcite. The calcite divides the incident beam into two beams that emerge parallel and are separated by a distance proportional to the thickness of the crystal. The calcite is so oriented that both transmitted images of the star fall on the slit, while its thickness is so chosen that the separation of

[17] *Bureau Standards J. Research*, **1**, 641, 1928.

[18] This point is discussed in Jenkins and White, *Physical Optics* (1st ed.; New York: McGraw-Hill, 1937), p. 425.

[19] W. S. Adams, *Mt. W. Contr.*, No. 638; *Ap. J.*, **93**, 11, 1941.

the images yields two closely adjacent spectra on the plate. If any right-handed circularly polarized light is received by the analyzer, it will be directed into only one of the photographed spectra, while left-handed circularly polarized light will be directed into the other. Unpolarized light will be equally divided between the two. This type of analyzer was described and used by Zeeman.[20] It is found that the grating of the coudé spectrograph does not exhibit preferential effects for plane-polarized light parallel or normal to the slit; hence the two spectra are of very nearly equal intensity. The deviation of light transmitted by the analyzer is not over 1 minute of arc. The *Fe* comparison spectrum, extending relatively far on either side of the stellar spectra, is photographed in the ordinary way.

A second type of analyzer used for a few plates differs from the first in that the calcite is replaced by two pieces of Polaroid with their axes perpendicular to each other and mounted with the dividing line perpendicular to the slit. The mica quarter-wave plate is used as before. This analyzer produces essentially the same effect as the calcite analyzer if the star is trailed along the slit, yielding two contiguous spectra having the desired properties, but it is much more wasteful of light.

A Fresnel rhomb might replace the mica quarter-wave plate to advantage, but greater care would have to be exercised to insure proper optical alignment.

OBSERVATIONS

The only previous accounts known to the writer of attempts to observe the Zeeman effect, or circularly polarized light, in stellar spectra are those of W. H. Wright[21] and of P. W. Merrill.[22] They used plane and circular analyzers in connection with the Mills spectrograph attached to the 36-inch refractor of the Lick Observatory. Attention was directed mainly to components of bright hydrogen lines; no positive results were obtained.

Beginning in April, 1946, a number of selected stars have been observed by the writer, using the analyzers described, with the 32-inch (10 A/mm) and 114-inch (2.9 A/mm) cameras of the coudé spectrograph of the 100-inch telescope. In general, the stars selected have been members of types B, A, and F that exhibit sharp absorption lines (apart from *H* and *He* lines, which may be broadened by Stark effect). The observing list included several stars listed by W. W. Morgan[23] as belonging to a peculiar branch of the spectral sequence in the interval B8–F0. It is desired, of course, to select stars that are probably in rapid rotation about axes directed nearly toward the observer, even though there may be no direct basis for such a selection. The proportion of early-type stars which have sharp lines is small and, according to Miss Christine Westgate,[24] is in agreement with the assumption of random distribution of the direction of their axes. A small inclination of the axis, resulting in a slight rotational broadening of the lines, would not be expected to interfere seriously with the search for the Zeeman effect. A few early-type stars exhibiting emission lines, in addition to sharp absorption lines, have been placed on the observing program, as O. Struve[25] has shown that emission may indicate the existence of an extended ring or shell resulting possibly from rapid rotation.

The present discussion is confined primarily to the star 78 Virginis ($\alpha = 13^{h}31^{m}35^{s}$; $\delta = +3°55'$ [1950]; mag. 4.93; spectral type A2p), of which observations were made as shown in Table 1. The lines are quite sharp, even with a dispersion of 2.9 A/mm. The region λλ 3800–4800 is covered on the well-exposed plates.

[20] *Researches in Magneto-optics* (New York: Macmillan Co., 1913), p. 47.

[21] *Lick Obs. Bull.*, **6**, 60, 1910.

[22] *Lick Obs. Bull.*, **7**, 162, 1913.

[23] *Ap. J.*, **77**, 330, 1933.

[24] *Ap. J.*, **77**, 141, 1933.

[25] *Ap. J.*, **72**, 1, 1930; **73**, 94, 1931.

From the first, relative displacements between the right-handed and left-handed spectra were suspected for some of the lines in this star. On some of the plates taken with rather poor seeing, the image of the star on the slit was so large that the two spectra appear partially blended, with no dividing line between them. Under these conditions the affected lines appear very slightly inclined. The plates were measured by carefully aligning the vertical wire of the measuring machine with the arc lines of the comparison spectrum; settings were then made on the stellar lines, first on the upper part and then on the lower, and the scale difference was noted for each line. After the wave lengths were computed and the lines identified, the term designations were found by reference to Miss C. E. Moore's *Revised Multiplet Table*.[26] Then the mean Zeeman effects for the perpendicular components of each pattern were determined with the aid of "Tables of Theoretical Zeeman Effects," by Kiess and Meggers.[17] For the anomalous effect Russell's rule was used, that the center of gravity of an asymmetrical blended pattern may be taken to lie one-quarter of the way from the strong component to the extreme weak component. Finally, the results for each plate were represented by plotting the measured differential displacement for each line, in angstroms, against the theoretical Zeeman

TABLE 1

OBSERVATIONS OF 78 VIRGINIS

1946	Plate No.	Camera	Analyzer		Remarks
			Type	Position	
April 24........	Ce 4254	114 inch	Mica-calcite	Erect	Weak
May 7........	4257	32	Mica-Polaroid	Erect	
May 8........	4262	32	Mica-calcite	Erect	
May 8........	4263	114	Mica-Polaroid	Erect	Weak
June 15........	4293	114	Mica-calcite	Inverted	
June 16........	4299	114	Mica-calcite	Erect	

effect for the line. The measured displacements were found to be predominantly in one direction, even on the plates of lower dispersion. For Ce 4262, made with the mica-calcite analyzer, 86 lines were measured; for Ce 4257, made with the mica-Polaroid analyzer, 59 lines were measured. After rejecting a few lines from each plate for uncertain identification or blends, it was found that for 28 lines common to the two plates the measured displacements agreed within 0.002 mm. The mean values for these lines are plotted in the diagram of Figure 4.

The two best plates of high dispersion (2.9 A/mm), Ce 4293 and Ce 4299, were then measured in the same way. Table 2 lists the measured stellar wave lengths corrected for velocity, the measured differential displacements for the lines on each plate in microns, and the mean value of the displacements corrected to λ 4600 on the assumption that the displacements are due to a cause varying as the square of the wave length. A minor correction is also made for the decrease in efficiency of the quarter-wave plate toward the ends of the measured spectrum. This is necessary because the phase retardation for wave lengths other than λ_0, for which the plate is designed, is approximately

$$\delta = \frac{\pi}{2} \frac{\lambda_0}{\lambda},$$

and the undesired component of circularly polarized light will be transmitted with an intensity nearly proportional to $\sin^2 (\delta - [\pi/2])$. The fifth column of Table 2 gives the

[26] *Contr. Princeton U. Obs.*, No. 20, 1945.

shift in angstroms corresponding to the measured displacements. The laboratory wave lengths and identification are then listed, followed in the last column by the theoretical Zeeman effect for the perpendicular component.

Figure 5 is the characteristic plot of displacement against Zeeman effect, where a few lines have been omitted because of blending. The encircled points are those for which agreement was closest between results from the two plates. It is perhaps not surprising that there should be considerable dispersion in the measured quantities, since these quantities are so small and are affected by the grain of the plate. Furthermore, the dispersion might yet be somewhat reduced by a more exhaustive study of blending effects. I am grateful to Dr. O. C. Wilson for independently measuring a few of the lines showing the largest displacements. Figure 6 is a microphotometer tracing of the region near λ 4177 on a plate (Ce 4299) of 78 Vir.

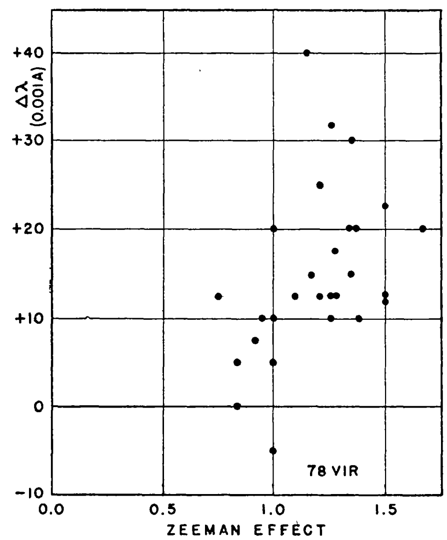

FIG. 4.—Differential displacements plotted against Zeeman effect for 78 Vir. Each point represents the mean value for a line measured on two plates having a dispersion of 10 A/mm.

As indicated in Table 1, Ce 4293 was made with the analyzer "inverted," while Ce 4299 was made with the analyzer "erect." This means that the analyzer was rotated through 180° between the two exposures so that the right- and left-handed properties of the analyzer were interchanged with respect to the upper and lower spectra on each plate. The sign of the differential displacements was found to be different on the two plates, as would be expected for the Zeeman effect. Furthermore, the measured displacements in angstroms were found to be essentially the same for the mica-calcite and the mica-Polaroid analyzers and for the 32- and 114-inch cameras. As a check, an exposure was made in which, instead of photographing the spectrum of a star, light from the iron arc was reflected diffusely into the slit through the analyzer; no outstanding displacements were found.

To provide a further check on the reality of the results for 78 Vir, a plate (Ce 4328) of the K0 star, ϵ Pegasi, was taken with the mica-calcite analyzer and the 114-inch camera. Since no appreciable rotation has been detected for stars of types as late as G and K, it is to be expected that ϵ Pegasi would possess no strong general magnetic field[27] and that the Zeeman effect should not be observed in its spectrum. This plate was

[27] ϵ Peg is listed by Aitken (A15268) as possessing two fainter companions at distances of 82″ and 141″, respectively. The observed changes in position angle are, at most, a few tenths of a degree in thirty-one years. This is entirely compatible with our assumption of a low rate of rotation.

TABLE 2

MEASUREMENTS IN THE SPECTRUM OF 78 VIRGINIS

λ	ΔS Ce 4299	−ΔS Ce 4293	Mean |ΔS| (λ 4600)	Δλ	Identification	Zeeman Effect
3878.66......	0	....	0	0.000	8.663 *Fe* I	0.88
3899.72......	9	....	12	.035	9.709 *Fe* I	1.50
3900.55......	0	....	0	.000	0.546 *Ti* II	1.11
3902.95......	4	....	6	.017	2.948 *Fe* I	1.20
3913.50......	4	....	6	.017	3.464 *Ti* II; 3.635 *Fe* I	0.89; 1.50
3920.26......	8	....	11	.032	0.260 *Fe* I	1.50
3922.93......	2	....	3	.009	2.914 *Fe* I	1.50
3926.00......	− 1	....	− 1	− .003	6.001 *Fe* I	1.38
3933.67......	1	....	1	.003	3.664 *Ca* II	1.33
3935.93......	− 2	....	− 3	− .009	5.942 *Fe* II	1.05
3938.34......	0	....	0	.000	8.400 *Mg* I	1.00
3939.01......	1	....	1	.003	8.969 *Fe* II	0.90
3945.21......	8	....	11	.032	5.119 *Fe* I; 5.21 *Fe* II (*P*)	1.78; 1.80
3948.80......	2	....	3	.009	8.779 *Fe* I; 8.901 *Ca* I	1.01; 2.00
3951.16......	1	....	1	.003	1.154 *Nd* II	1.04
3983.95......	2	....	3	.009	3.960 *Fe* I	1.00
3997.42......	3	8	7	.020	7.43 *Y* II; 7.394 *Fe* I	1.04; 1.00
4002.54......	7	....	9	.026	2.549 *Fe* II	1.07
4003.30......	....	10	13	.038	3.33 *Cr* II	0.95
4005.26......	4	....	5	.014	5.246 *Fe* I	1.50
4012.47......	0	0	0	.000	2.467 *Fe* II	0.40
4020.05......	....	3	4	.012	0.05 *Fe* I (*P*)	1.50
4020.91......	....	8	11	.032	0.898 *Co* I	1.33
4033.00......	7	....	9	.026	3.073 *Mn* I	1.94
4037.99......	0	5	3	.009	8.03 *Cr* II	1.04
4045.82......	7	....	9	.026	5.815 *Fe* I	1.25
4051.94......	1	....	1	.003	1.923 *Fe* I; 1.97 *Cr* II	0.67; 1.14
4063.60......	3	3	4	.012	3.597 *Fe* I	1.08
4070.82......	3	....	4	.012	0.766 *Fe* I; 0.90 *Cr* II	1.50; 1.05
4071.74......	4	0	3	.009	1.740 *Fe* I	0.67
4072.56......	1	2	2	.006	2.56 *Cr* II	0.83
4086.14......	2	....	3	.009	6.14 *Cr* II	1.33
4110.98......	0	....	0	.000	1.01 *Cr* II	1.10
4122.64......	0	1	1	.003	2.638 *Fe* II	1.31
4128.12......	1	....	1	.003	8.14 *Mn* II	0.87
4128.74......	8:	5	7	.020	8.735 *Fe* II	1.80
4132.05......	1:	5	5	.014	2.060 *Fe* I	1.50
4136.96......	4	....	5	.014	7.002 *Fe* I	1.00
4143.43......	0	....	0	.000	3.418 *Fe* I	1.00
4145.78......	2	3	3	.009	5.77 *Cr* II	1.20
4150.99......	....	1	1	.003	0.97 *Zr* II	1.29
4153.89......	− 2	8	4	.012	3.906 *Fe* I	1.50
4163.65......	4	0	2	.006	3.655 *V* II; 3.658 *Cb* I; 3.644 *Ti* II	0.84; 1.47; 1.07
4173.48......	1	2	2	.006	3.450 *Fe* II	1.48
4177.66......	15	1:	13	.038	7.70 *Fe* II; 7.597 *Fe* I; 7.59 *Co* I	1.66; 1.30; 2.28
4178.88......	0	5	3	.009	8.855 *Fe* II	0.80
4179.41......	− 3	5	1	.003	9.43 *Cr* II; 9.419 *V* I	1.21; 1.41
4195.42......	1	7	5	.014	5.41 *Cr* II	1.40
4202.02......	3	− 1	1	.003	2.031 *Fe* I	1.15
4210.34......	11	4	9	.026	0.352 *Fe* I; 0.352 *Sm* II	3.00; ......
4217.08......	....	1	1	.003	7.07 *Cr* II	0.84
4219.36......	0:	4	4	.012	9.364 *Fe* I; 9.383 *W* I	1.08; 1.50
4224.13......	11	10	13	.038	4.09 *Cr* II (*P*); 4.176 *Fe* I	1.93; 1.80
4224.85......	....	10	12	.035	4.85 *Cr* II	0.80
4227.39......	1	....	1	0.003	7.42 *Fe* I (*P*)	0.75

TABLE 2—*Continued*

λ	ΔS Ce 4299	−ΔS Ce 4293	Mean |ΔS| (λ 4600)	Δλ	Identification	Zeeman Effect
4229.83......	....	16	19	0.055	9.81 *Cr* II	1.90
4233.17......	8	− 2	4	.012	3.167 *Fe* II	1.22
4242.34......	1	1	1	.003	2.38 *Cr* II	1.17
4250.78......	0	8	5	.014	0.790 *Fe* I	0.91
4252.62......	....	3	4	.012	2.62 *Cr* II	1.20
4261.90......	− 4	5	1	.003	1.92 *Cr* II	1.07
4269.27......	2	3	3	.009	9.28 *Cr* II	0.80
4271.12......	− 1	10	5	.014	1.159 *Fe* I	1.50
4274.80......	....	4	5	.014	4.803 *Cr* I	1.96
4275.54......	....	4	5	.014	5.57 *Cr* II	0.91
4278.14......	....	1	1	.003	8.128 *Fe* II	0.21
4289.71......	8	2	6	.017	9.721 *Cr* I	1.67
4294.10......	3	8	6	.017	4.101 *Ti* II; 4.128 *Fe* I	1.20; 1.20
4303.18......	5	7	7	.020	3.166 *Fe* II	1.47
4307.91......	5	2	4	.012	7.906 *Fe* I	1.00
4312.91......	....	6	7	.020	2.861 *Ti* II	1.37
4383.56......	3	....	3	.009	3.547 *Fe* I	1.28
4385.40......	2	....	2	.006	5.381 *Fe* II	1.33
4395.05......	− 5	5	0	.000	5.031 *Ti* II	1.07
4404.78......	4	8	7	.020	4.752 *Fe* I; 4.81 *Zr* II	1.25; 1.26
4443.82......	0	5	3	.009	3.802 *Ti* II	0.94
4447.74......	3	....	3	.009	7.722 *Fe* I	2.00
4451.57......	3	3	3	.009	1.566 *Nd* II; 1.586 *Mn* I	0.97; 1.43
4466.59......	2	7	5	.013	6.57 *Fe* I	1.50
4468.53......	1	2	2	.006	8.493 *Ti* II	1.05
4476.08......	6	1	4	.010	6.082 *Fe* I	1.51
4481.26......	5	4	5	.013	1.261 *Ti* I; 1.327 *Mg* II; 1.273 *Tm* II	1.67; 0.89; ....
4489.18......	0	5	3	.009	9.185 *Fe* II	1.50
4501.31......	7	....	7	.020	1.270 *Ti* II	0.93
4508.30......	4	1	3	.009	8.26 *Fe* II (*P*)	1.25
4522.65......	3	3	3	.009	2.634 *Fe* II	0.91
4528.62......	− 3	....	− 3	− .009	8.619 *Fe* I	1.25
4541.54......	3	1:	3	.009	1.523 *Fe* II	0.80
4555.00......	3	0	2	.006	5.02 *Cr* II	1.33
4558.70......	5	− 6:	0	.000	8.659 *Cr* II	1.17
4563.78......	− 4	....	− 4	− .011	3.761 *Ti* II	0.83
4565.76......	3	....	3	.009	5.78 *Cr* II; 5.73 *Mn* I	0.60; 2.40
4576.34......	2	4	3	.009	6.331 *Fe* II	1.20
4580.07......	5	....	5	.014	0.055 *Fe* II; 0.056 *Cr* I	1.86; 1.75
4588.21......	....	9	9	.026	8.217 *Cr* II	1.07
4589.94......	7	2	5	.014	9.961 *Ti* II; 9.89 *Cr* II	1.07; 2.06
4592.07......	− 2	2	0	.000	2.09 *Cr* II	1.20
4616.64......	− 1	4:	2	.006	6.64 *Cr* II	0.80
4618.65......	− 2	....	− 2	− .006	8.568 *Fe* I; 8.765 *Fe* I	.... ; 1.50
4620.52......	2	8	5	.014	0.513 *Fe* II	1.33
4629.31......	2	5	4	.010	9.336 *Fe* II	1.33
4634.05......	0	....	0	.000	4.11 *Cr* II	0.50
4652.14......	6	....	6	.017	2.158 *Cr* I	1.17
4684.75......	− 2	....	− 2	− .006	4.77 *Cr* II	1.10
4780.06......	2	....	2	.006	9.986 *Ti* II	1.33
4805.20......	1	....	1	0.003	5.18 *Cr* II	1.31

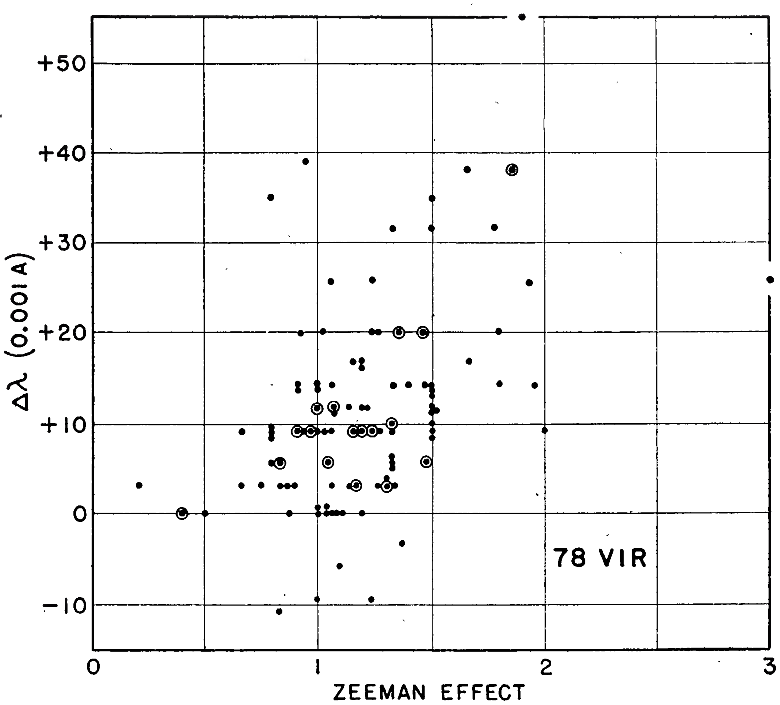

FIG. 5.—Differential displacements plotted against theoretical Zeeman effects for individual lines in high-dispersion spectra (2.9 A/mm) of 78 Vir. Data are from Table 2. Encircled dots represent mean values where the displacements from the two plates agree within 0.003 mm.

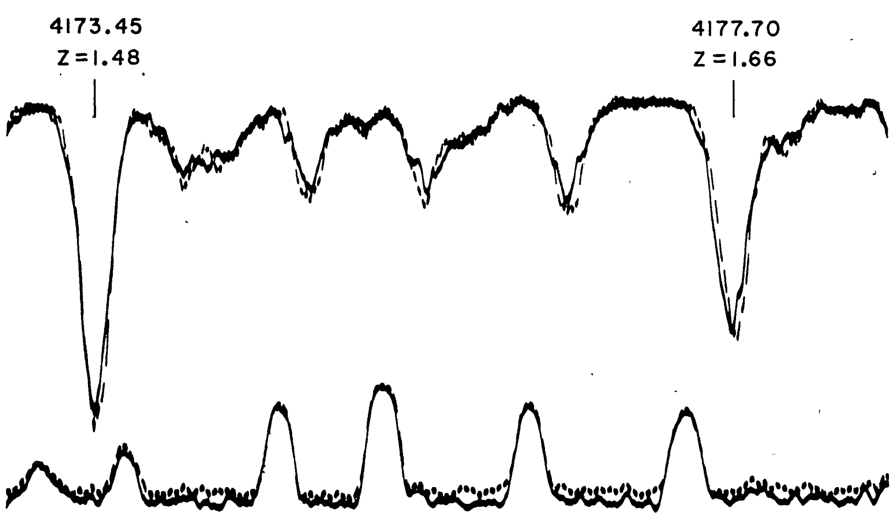

FIG. 6.—Microphotometer tracing of a small section of the spectrum of 78 Vir. The plate was scanned four times, always in the same direction and with no change in the adjustments other than the lateral position of the plate. The two sides of the stellar spectrum (analyzed for right- and left-handed polarization) were scanned separately; for one side the recording beam was interrupted by a light-chopper to give a dotted trace. Note the displacements for the two strong absorption lines (*top*). The Zeeman effect for λ 4173 (*Fe* II) is 1.48; λ 4177 may be a blend of three lines having Zeeman effects of 1.66, 1.30, and 2.28, respectively (see Table 2). The two parts of the comparison spectrum, on opposite sides of the stellar spectrum, were likewise scanned separately; the superposition of the traces (*bottom*) shows that the plate was properly oriented with respect to the slit of the microphotometer.

measured in the same way as the others, settings being made on the upper and lower parts of each line. For 62 lines selected at random, the numbers of positive and negative displacements were about equal, and the average displacement (with regard to sign) was 0.0003 A compared to 0.0110 A for 78 Vir. A plot of measured displacement against Zeeman effect for the lines in ε Peg is given in Figure 7. It seems obvious that the measurements in the spectrum of this star merely reflect the usual photographic and personal errors. We conclude that there is no measurable Zeeman effect in ε Peg and that no spurious effect is introduced by the apparatus.

A most significant point is the appearance of a definite correlation between the magnitude of the measured displacements and the Zeeman effect in 78 Vir (Figs. 4 and 5), although the Zeeman effects for the lines were unknown to the measurer at the time the measures were made. However, an unlooked-for effect is that a line representing the data, if passed through the origin, would not be straight; the displacements appear to in-

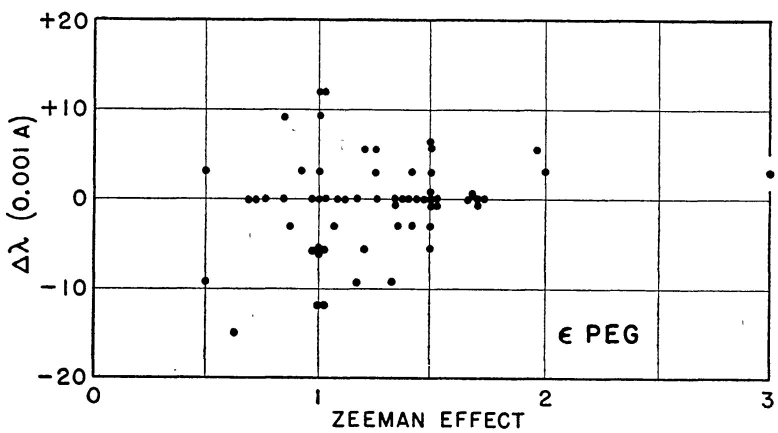

FIG. 7.—Measured displacements plotted against Zeeman effect for lines in the spectrum of the control star ε Peg (type K0), for which no detectable general magnetic field would be expected. The vertical dispersion of the points represents ordinary errors of measurement. Compare with Fig. 5.

crease more rapidly than would be expected for the larger Zeeman patterns. A plausible explanation for this apparent lack of linearity is found in Seares's discussion of the measurement of small Zeeman displacements in solar spectra.[16] Seares points out that, for very small displacements in a comparatively diffuse line where the components are unresolved, settings will probably be made on the center of gravity of the pattern, but that if the pattern is partially resolved, as in a sharper line having a large displacement, there will be a tendency to set on the most intense component of the pattern rather than on the center of gravity. This will apply to the pattern of Figure 3 for the triplet and also to the multiple components of the pattern in the anomalous effect, for which Russell's blending rule was used in computing the mean Zeeman effects.

Since the data appear to satisfy criteria for the Zeeman effect and since no other reason for the occurrence of the displacements in the spectrum of 78 Vir has been found, one feels justified in interpreting the data according to the foregoing theory. The 114-inch camera plates give a differential (double) shift of about 0.009 A for a Zeeman effect of 1.00. This shift is found to be equivalent to a polar-field strength of about 1500 gauss for 78 Vir. It is, of course, pure coincidence that this value is identical with the prediction made in the first paragraph of this paper.

The distribution of points in the characteristic diagram for 78 Vir (Fig. 5), deserves further comment, particularly as the dispersion in measured displacements appears to be somewhat greater than the corresponding dispersion for ϵ Peg. The available data are probably insufficient to permit definite conclusions; but, if more precise results can be obtained in the future, either through the discovery of stars with stronger fields or through improved apparatus, and if careful attention is given to blending effects, the distribution of points in the characteristic diagram may yield a clue to the variation of physical conditions in the stellar atmosphere as a function of latitude or as a function of height. For a given line, let the measured shift be denoted by Δ and the atomic Zeeman effect by Z. The ratio Δ/Z for various points in the diagram may then be of interest. It was shown earlier in this paper that for spectrum lines having uniform intensity as a function of latitude the ratio Δ/Z will be constant in a given star. If a weak absorption line arises dominantly in the polar region of a star (which we assume to be rotating rapid-

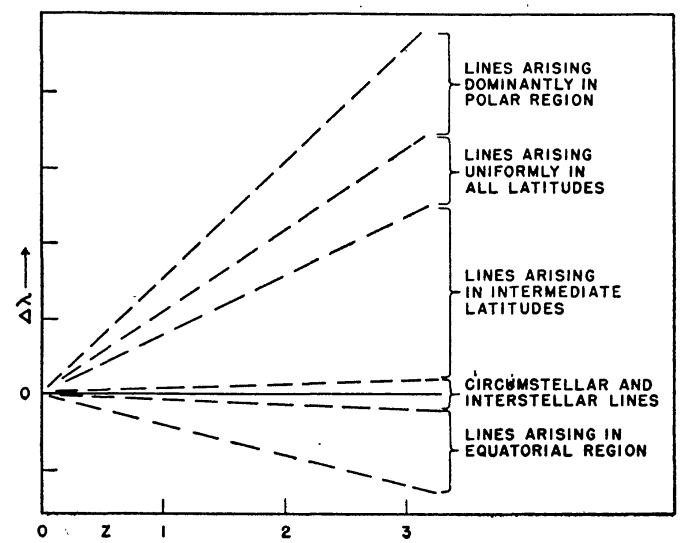

FIG. 8.—Schematic diagram showing for a weak line the relationship between the ratio Δ/Z and the latitude or height at which it arises.

ly about an axis directed nearly toward the observer), its ratio of Δ/Z may be expected to be about twice as great as the normal. On the other hand, a line arising dominantly in the equatorial region, where the surface gravity may be notably lower than at the pole, will have a ratio Δ/Z smaller than the normal and possibly even reversed in sign, since at latitude 35° the component of the field parallel to the line of sight reverses, becoming negative for lower latitudes. Particularly in extended stellar atmospheres, the variation of Δ/Z with height may outweigh any variation with latitude. Circumstellar or shell lines will naturally have a ratio Δ/Z approximately equal to zero, since the field strength (according to our model) decreases as the cube of the distance from the center of the star. The ratio for interstellar lines should, of course, be exactly zero. The diagram of Figure 8 shows these relationships schematically. Some of the physical effects of rapid stellar rotation have been discussed by P. Swings and S. Chandrasekhar,[28] J. A. Hynek,[29] and R. B. Baldwin.[30] In addition to these, a considerable body of literature exists on the

[28] *M.N.*, **96**, 883, 1936.

[29] *Ap. J.*, **83**, 476, 1936.

[30] *Pop. Astr.*, **52**, 134, 1944.

theory of rotationally distorted stars as developed from the study of close binary systems.[31]

The demonstration that some of the absorption lines in the spectrum of 78 Vir exhibit circular polarization interpretable in terms of Zeeman effect opens interesting possibilities for further investigations. The profiles and visibilities of stellar lines may be altered by magnetic fields, and, particularly if varying fields occur, it is possible that the key is at hand to some of the peculiar phenomena of stellar spectra. Further discussion can well be deferred until the results of additional observations become available.

It is a pleasure to express my indebtedness to my father, H. D. Babcock, for much helpful advice.

Note added in proof:

An additional spectrogram of 78 Vir, obtained with the analyzer and 32-inch camera on 1947 Jan. 7, confirms the existence and approximate magnitude of the displacements.

[31] For discussion and bibliography see Z. Kopal, *An Introduction to the Study of Eclipsing Variables* (Harvard University Press, 1946), p. 123.