THE MOTIONS OF THE STARS WITHIN 20 PARSECS OF THE SUN*

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ABSTRACT

The purpose of this investigation is to provide a representative sample of the motions of the stars in the neighborhood of the sun. The three components of the velocities of 444 stars within 20 parsecs from the sun have therefore been computed from their proper motions, radial velocities, and parallaxes.

The constants for the velocity ellipsoids were determined for all the spectral types involved and are referred to the system of galactic co-ordinates. The velocities of the stars were then projected on the "dynamical axes" of the galaxy, the x_1-axis being directed toward galactic longitude 339°, the y_1-axis toward galactic longitude 69°, and the z_1-axis toward the galactic pole. The x_1-axis points approximately toward the center of the galaxy; the y_1-axis is the axis of asymmetry in stellar motions. This asymmetry is very evident among the stars studied.

Attempts were made to establish correlations between the motions of the stars and their masses, the bolometric magnitude M_b being used as a measure of the masses of the stars. For all the velocity components studied a definite correlation was established, although the variation with mass is not very pronounced. The z_1-components, referred to the galactic plane and taken without regard to sign, show a higher speed for the less massive stars than for the more massive. Both the x_1-components and the y_1-components yield definite values for which the mass is a maximum. These values are $x_1 = +5$ and $y_1 = -9$ km/sec, both referred to the sun. Values of $v^2 = (x_1 + 17)^2 + (y_1 + 300)^2 + (z_1 + 7)^2$ were computed and were assumed to represent the velocity of a star relative to the center of the galaxy in a nonrotating system of co-ordinates. These values show a maximum mass for a root-mean-square value of v equal to 295 km/sec. All these effects can be understood on the hypothesis that the more massive a star, the greater is the probability that it moves in an exact circle around the center of the galaxy and that its orbit lies in the galactic plane.

The observed relationship between stellar motions and stellar masses has been interpreted as the effect of internal friction in a prestellar system of gas or dust, in accordance with theories previously advanced.

Many studies of the motions of the stars have been made on the basis both of radial velocities and of proper motions. The stars studied have ordinarily been selected on the basis of apparent brightness, large proper motion, spectral characteristics, or association with particular star systems. Because of the great dispersion in stellar motions and the many established or suspected correlations involved, it is important to find a representative sample of the motions of all the stars in a particular region of the galaxy. Several questions connected with the dynamics of the galaxy and with the causes of the present distribution of stellar velocities would be elucidated if such a sample were available and, in particular, if a definite effect of the masses of the stars involved could be established. The region of the galaxy for which we have the most complete data is, of course, the neighborhood of the sun. The present study aims, therefore, at providing such a sample; it relates to all the stars within a distance of 20 parsecs for which both proper motions and radial velocities are available.

OBSERVATIONAL DATA AND METHODS OF COMPUTATION

The data used in this study were collected by Dr. Ralph E. Wilson some time ago, and I am greatly indebted to him for letting me use his compilation. All those stars for which the weighted mean parallax is equal to or greater than 0".05 were included, except a few for which radial velocities or proper motions had not been determined. The total number of stars available was 444. The proper motions from the *General Catalogue*[1] and for some of the fainter stars from the catalogues of the Cincinnati Observatory[2] were converted into linear tangential velocities. Since the parallaxes are relatively large, the effect

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[1] *Carnegie Inst. Washington, Pub.* 468, 1937. [2] *Pub. Cincinnati Obs.,* No. 18, 1918; No. 20, 1930.

of the errors in the parallaxes could be neglected. Weighted means were formed for the radial velocities taken from all available sources, mainly from Moore's Catalogue.[3] For double stars the velocity of the center of mass was used. The visual absolute magnitudes (M_v) were derived from the apparent magnitudes (m_v), which for the brighter stars were taken from the *H.D. Catalogue*. For double stars the magnitude and the spectral type of the brighter component were used. The visual absolute magnitude was converted into absolute bolometric magnitude (M_b) with the aid of data determined by Pettit and Nicholson.[4] The actual reduction (ΔM) used in this paper is given in Table 1. It is well known that the bolometric magnitude of a star is correlated with its mass. The latest determination of this mass-luminosity relation is that by G. P. Kuiper.[5] The values of the geometric mean mass (m) in Table 2 in terms of the sun's mass have been read off from his diagram.

TABLE 1

REDUCTIONS TO BOLOMETRIC MAGNITUDE

Sp.	ΔM	Sp.	ΔM	Sp.	ΔM	Sp.	ΔM
A0	−0.3	dG5–dG9	0.0	dM0	−1.3	gG3	−0.1
A2	−0.1	dK0	−0.1	dM3	−1.9	gK1	−0.4
A3–A4	0.0	dK3	−0.4	dM5	−2.5	gK5	−1.1
A5–dG4	+0.1	dK6	−0.7	dM6	−2.8	gM2	−1.8

TABLE 2

RELATIONSHIP BETWEEN BOLOMETRIC ABSOLUTE MAGNITUDE AND MASS

$\bar{M}_b$	$\underline{m}$	$\bar{M}_b$	$\underline{m}$	$\bar{M}_b$	$\underline{m}$	$\bar{M}_b$	$\underline{m}$
+10	0.10	+7	0.56	+4	1.20	+1	2.40
9	0.26	6	0.76	3	1.48	0	3.2
+ 8	0.40	+5	0.96	+2	1.86	−1	4.3

Since the relationship between bolometric magnitude and mass is statistical and since there still may be systematic errors in the mass-luminosity curve, the mean bolometric magnitude has been retained as the principal measure of the mean mass of the stars in the various groups.

The formulae for computing the velocity components, relative to the sun, of a star in right ascension α and declination δ are given below.

$$\left.\begin{aligned} \xi &= V \cos\alpha \cos\delta - \frac{k}{\pi}(\mu_1 \sin\alpha + \mu_2 \cos\alpha \sin\delta) , \\ \eta &= V \sin\alpha \cos\delta + \frac{k}{\pi}(\mu_1 \cos\alpha - \mu_2 \sin\alpha \sin\delta) , \\ \zeta &= V \sin\delta + \frac{k}{\pi}\mu_2 \cos\delta . \end{aligned}\right\} \qquad (1)$$

Here V is the radial velocity of the star; $\mu_1 = \mu_\alpha \cos\delta$ and $\mu_2 = \mu_\delta$ are the components of its proper motion in the direction of increasing α and δ, respectively, expressed in

[3] *Pub. Lick Obs.*, Vol. **18**, 1932.

[4] *Mt. W. Contr.*, No. 369; *Ap. J.*, **68**, 279, 1928.

[5] *Ap. J.*, **88**, 489, 1938.

seconds of arc per year; π is the parallax; and k is the factor 4.74 km year/sec. The ξ-axis is directed toward the vernal equinox (1900.0), the η-axis is in the equator and in a right ascension equal to 90°, and the ζ-axis is directed toward the north celestial pole.

The equatorial velocity components were converted into galactic components with the aid of the following formulae:

$$\left.\begin{aligned} x &= -\xi \sin A + \eta \cos A \,, \\ y &= -\xi \cos A \sin D - \eta \sin A \sin D + \zeta \cos D \,, \\ z &= +\xi \cos A \cos D + \eta \sin A \cos D + \zeta \sin D \,. \end{aligned}\right\} \tag{2}$$

Here A and D are the right ascension and declination of the north galactic pole. The x- and y-axes are in the galactic plane in galactic longitude 0° and 90°, respectively. The z-axis is directed toward the north pole of the galaxy. Using the standard values $A = 190°$ and $D = +28°$ (1900.0), we find

$$\left.\begin{aligned} x &= +0.1736\xi - 0.9848\eta \,, \\ y &= +0.4624\xi + 0.0815\eta + 0.8829\zeta \,, \\ z &= -0.8695\xi - 0.1533\eta + 0.4695\zeta \,. \end{aligned}\right\} \tag{3}$$

Finally, the velocities were projected on what we may call the "dynamical axes of the galaxy." The x_1-axis in this system of co-ordinates is directed toward the mass center of the galaxy; the y_1-axis is also in the galactic plane and perpendicular to the x_1-axis; and the z_1-axis is directed toward the north pole of the galaxy. The y_1-axis coincides, at least approximately, with what I have called "the axis of asymmetry in stellar motions." The galactic longitude of the center of the galaxy is known to be in the neighborhood of 330°, and the axis of asymmetry has a longitude between 60° and 70°. The longitude of the x_1-axis has been made to coincide with that of the major axis of the velocity ellipsoid when all the stars are combined. The value found for this longitude, L, is 339°, as seen in Table 5. We have, then,

$$\left.\begin{aligned} x_1 &= x \cos L + y \sin L \,, \\ y_1 &= -x \sin L + y \cos L \,, \\ z_1 &= z \,. \end{aligned}\right\} \tag{4}$$

It lies in the nature of things that the orientation of the dynamical axes in the galactic plane cannot be determined with high accuracy. An uncertainty of several degrees is, however, of no great importance in the present study.

RESULTS OF COMPUTATIONS

After the stars were grouped according to spectral type, the velocity ellipsoids were computed from the galactic-velocity components. The stars actually used in this computation numbered 419. Only 16 giants and subgiants were found in the volume with a radius of 20 parsecs; these are listed in Table 3. Their apparent and absolute visual magnitudes are given, as well as their galactic-velocity components in kilometers per second.

In Table 4 are listed the stars which have unusual velocities in any one of the galactic co-ordinates. These stars, together with most of the stars in Table 3, were excluded, although the few giant stars in the list do not exhibit any marked peculiarities in their motion when compared with the dwarf stars in general. Only one isolated white dwarf (Cin. 58) was included in the list. It is of interest to note that this star and another faint F star (Cin. 706) have abnormally large velocities perpendicular to the galactic plane.

Table 5 shows the constants for the velocity ellipsoids. The galactic components of the group motion are indicated by the symbols $\bar{x}$, $\bar{y}$, and $\bar{z}$. The opposite velocity vector is the sun's motion relative to the particular group and is given in equatorial co-ordinates. The velocity dispersions along the principal axes of the velocity ellipsoids are denoted by σ_1, σ_2, and σ_3. The first represents the maximum dispersion, and the last the dispersion along the principal axis which has the highest galactic latitude. The directions of the principal axes are given in galactic co-ordinates.

TABLE 3

GIANTS AND SUBGIANTS

Star	Sp.	m_v	M_v	x	y	z
GC 158	G7	3.9	2.6	−13	− 11	+11
865	G7	2.2	1.0	−19	+ 6	−12
2339	G4	3.7	2.7	−51	+ 16	+11
2538	K1	2.2	0.8	−14	− 21	+ 4
4517	G9	3.8	2.8	−36	− 38	−24
5194	K5	4.4	3.2	0	− 25	−19
5605	K8	1.1	0.1	−50	+ 14	−22
6427	G1	0.2	−0.4	−38	+ 7	− 8
10438	G9	1.2	1.2	−11	+ 12	−24
18666	M6	4.4	3.1	+ 5	− 39	+15
19033	G9	2.3	1.1	−48	− 28	−23
19242	K0	0.2	0.2	−39	−103	− 2
22502	G4	3.6	2.1	+12	− 3	+ 2
25046	G8	3.4	1.9	+ 1	− 82	+15
30289	K0	3.7	2.7	−10	− 35	−13
32875	K1	3.4	2.4	− 1	− 44	0

TABLE 4

STARS WITH UNUSUAL VELOCITIES

Star	Sp.	m_v	M_v	x	y	z
GC:						
1360	G4	5.3	5.9	−116	−107	− 32
6369	K2	8.5	10.6	−137	−248	− 54
15183	M2	7.6	10.5	+ 9	− 71	− 76
16253	G5	6.5	6.6	+157	−297	− 16
22728	G9	6.8	5.7	+ 9	−138	+ 5
Cin. 18:						
58	F3	12.3	14.3	− 32	+ 86	−228
153	M3	10.5	9.8	+103	− 83	−107
217	K5	11.0	9.7	+ 1	−203	− 19
462	M6	12.5	13.4	+ 23	−161	− 51
706	F2	8.8	7.3	− 14	−111	+123
1136	M4	11.2	11.3	− 48	+ 9	− 80

Figure 1 shows the position of the intersection of the velocity ellipsoids with the galactic plane. Figure 2 shows the intersections of the velocity ellipsoids with a plane perpendicular to the galactic plane and through the axis of asymmetry, that is, through the axis y_1 in galactic longitude 69°.

The distribution of the individual velocity vectors referred to the sun as origin and projected on the galactic plane is shown in Figure 3. The distributions of the dynamical

TABLE 5

VELOCITY ELLIPSOIDS

Quantity	Spectrum and Number of Stars					
	A(26)	F(83)	G(104)	K(108)	M(98)	All (419)
$\bar{x}$ (km/sec)	− 10.0	− 14.8	− 30.9	− 24.6	− 21.5	− 22.6
$\bar{y}$ (km/sec)	− 2.0	− 3.6	− 11.1	− 15.1	− 16.2	− 11.3
$\bar{z}$ (km/sec)	− 7.8	− 6.2	− 7.9	− 5.1	− 7.7	− 6.8
V_0 (km/sec)	12.9	16.5	33.8	29.3	28.0	26.2
α_0 (degrees)	249.1	265.7	276.7	286.1	282.0	278.2
δ_0 (degrees)	+ 24.9	+ 21.8	+ 23.7	+ 32.4	+ 39.7	+ 30.2
σ_1 (km/sec)	17.2	25.1	39.9	45.5	48.4	40.4
L_1 (degrees)	350.4	334.2	346.7	338.5	330.8	338.9
B_1 (degrees)	+ 8.0	− 6.3	− 5.1	− 4.3	+ 0.9	− 2.7
σ_2 (km/sec)	5.6	16.2	24.4	24.8	28.0	24.4
L_2 (degrees)	77.8	69.1	74.5	68.5	60.6	58.8
B_2 (degrees)	− 17.3	− 37.4	+ 21.4	− 2.2	− 10.0	+ 1.3
σ_3 (km/sec)	8.2	18.9	18.9	17.5	18.9	18.3
L_3 (degrees)	104.9	56.2	269.5	5.7	65.4	312.6
B_3 (degrees)	+ 71.4	+ 51.8	+ 67.9	+ 85.2	+ 80.0	+ 87.0

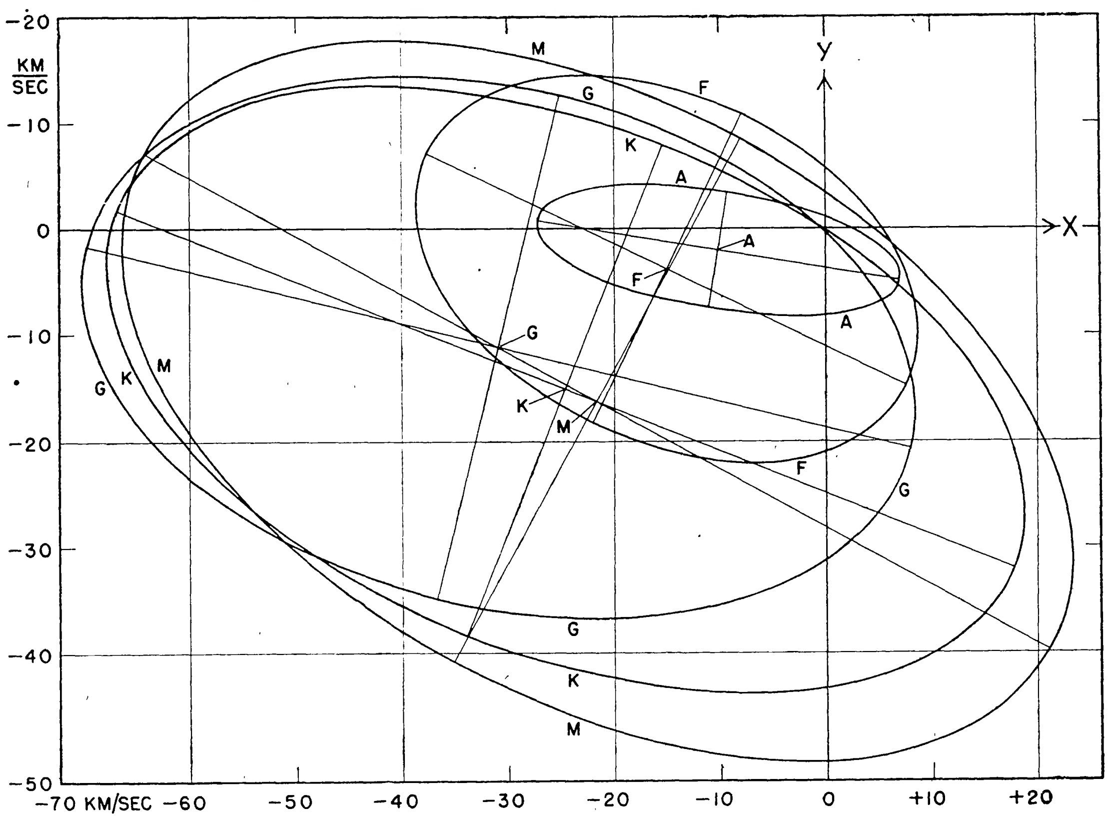

FIG. 1.—Projections of the velocity ellipsoids for different spectral types on the galactic plane. The sun is at the origin, and the x-axis is in galactic longitude 0°.

velocity components, when all the types are combined, are given in Table 6 and are shown graphically in Figure 4. The distribution in the normal error-curves based on the constants in Table 5 and projected on the dynamical axes is given in tabular and graphic forms.

The results of this analysis agree closely with those derived from the motions of the stars in general. There is a steady increase in velocity dispersion as we proceed along the main sequence to stars of later spectral types, and at the same time there is an increase in the numerical value of the group motion. The last phenomenon is a result of the now well-established asymmetry in stellar motions. This asymmetry is clearly shown in Figure 3

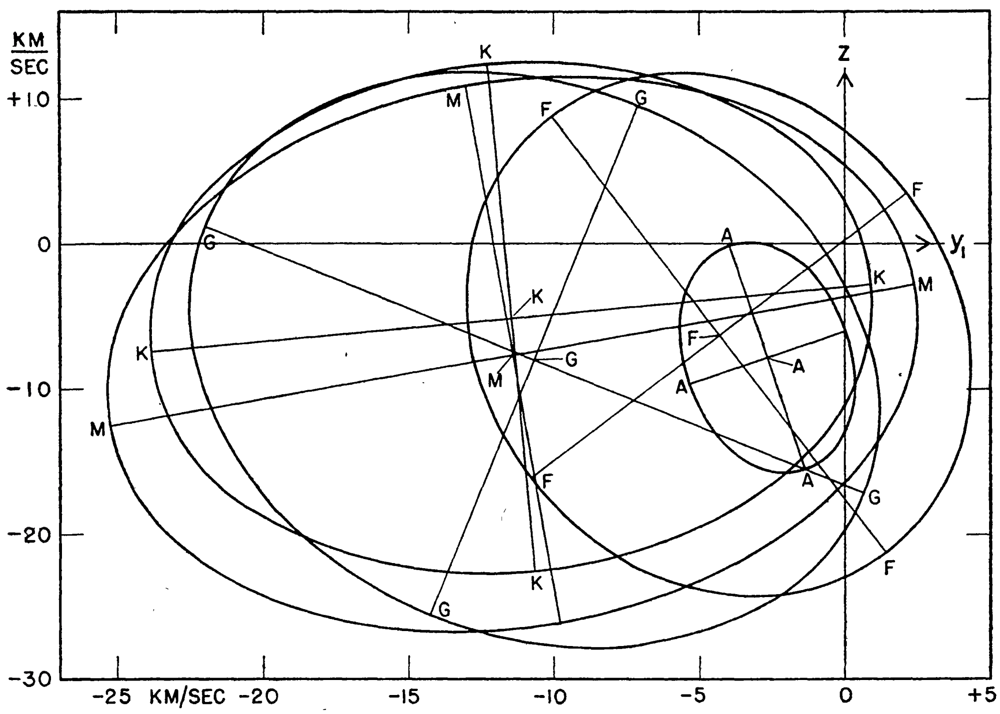

FIG. 2.—The velocity ellipsoids for different spectral types projected on a plane in galactic longitude 69° and perpendicular to the galactic plane. The sun is at the origin.

as a wide scattering of high-velocity stars in the third quadrant and the complete absence of stars with a velocity, relative to the sun, greater than 100 km/sec in the first quadrant. The asymmetry is brought to light in the most striking way in the distribution of velocities projected on the axis of asymmetry (the y_1-axis) when all the stars are combined. As shown in Table 6 and in Figure 4, the distribution of the y_1-components is exceedingly asymmetric and can by no means be represented by a normal error-curve.

STELLAR MOTIONS AND MASSES

The asymmetry in the motions of the stars in our region of the galaxy has previously been established for practically all types of stars. References to its significance in the dynamics of the galaxy and in the theory of the formation of the galaxy will be given later. In order to give a dynamical interpretation of stellar motions it is essential to ascertain whether and to what extent they can be correlated with the masses of the stars in-

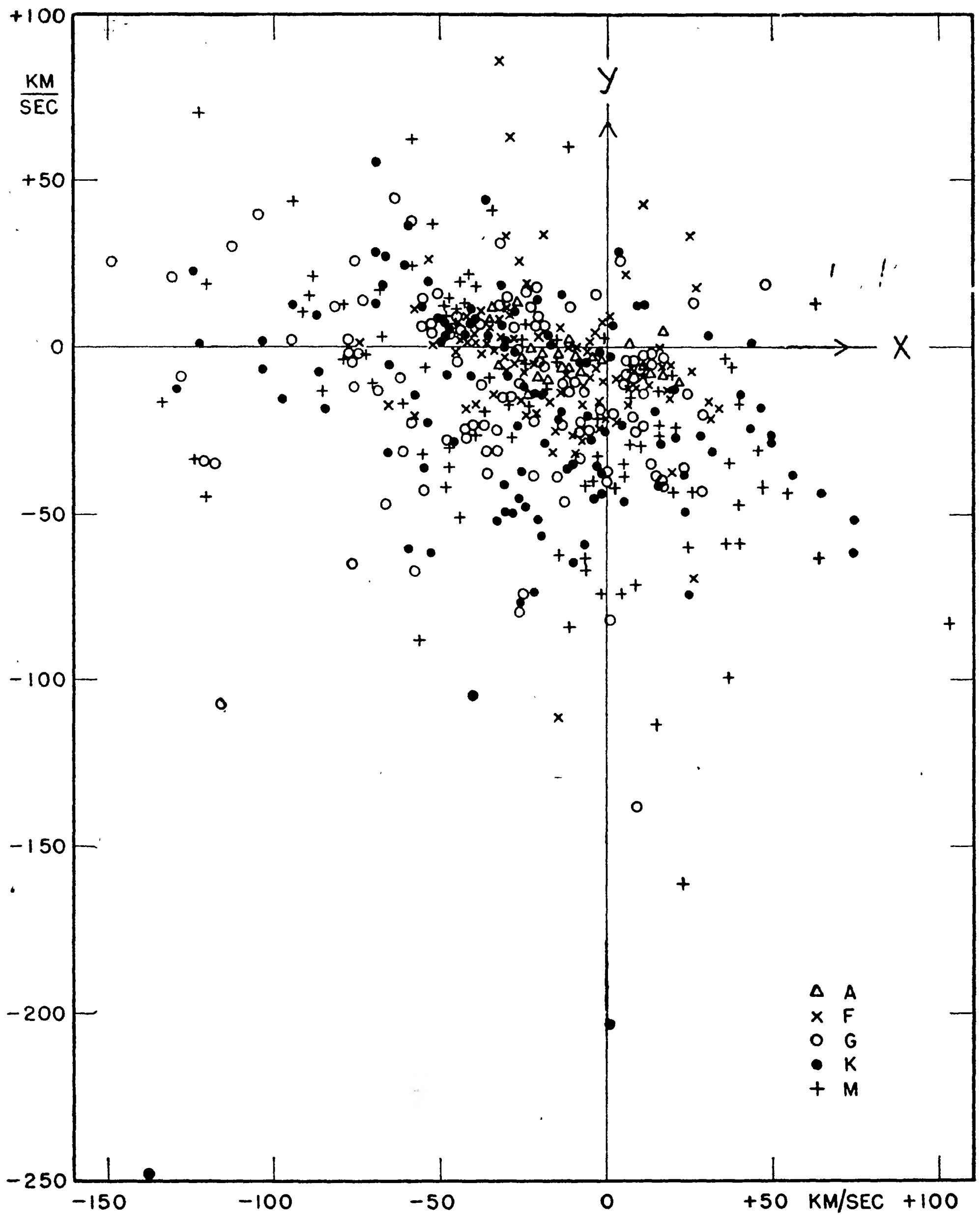

FIG. 3.—The velocities of the stars projected on the galactic plane. The sun is at the origin, and a line from the origin to a particular point indicates the velocity vector of a star projected on the galactic plane and referred to the sun.

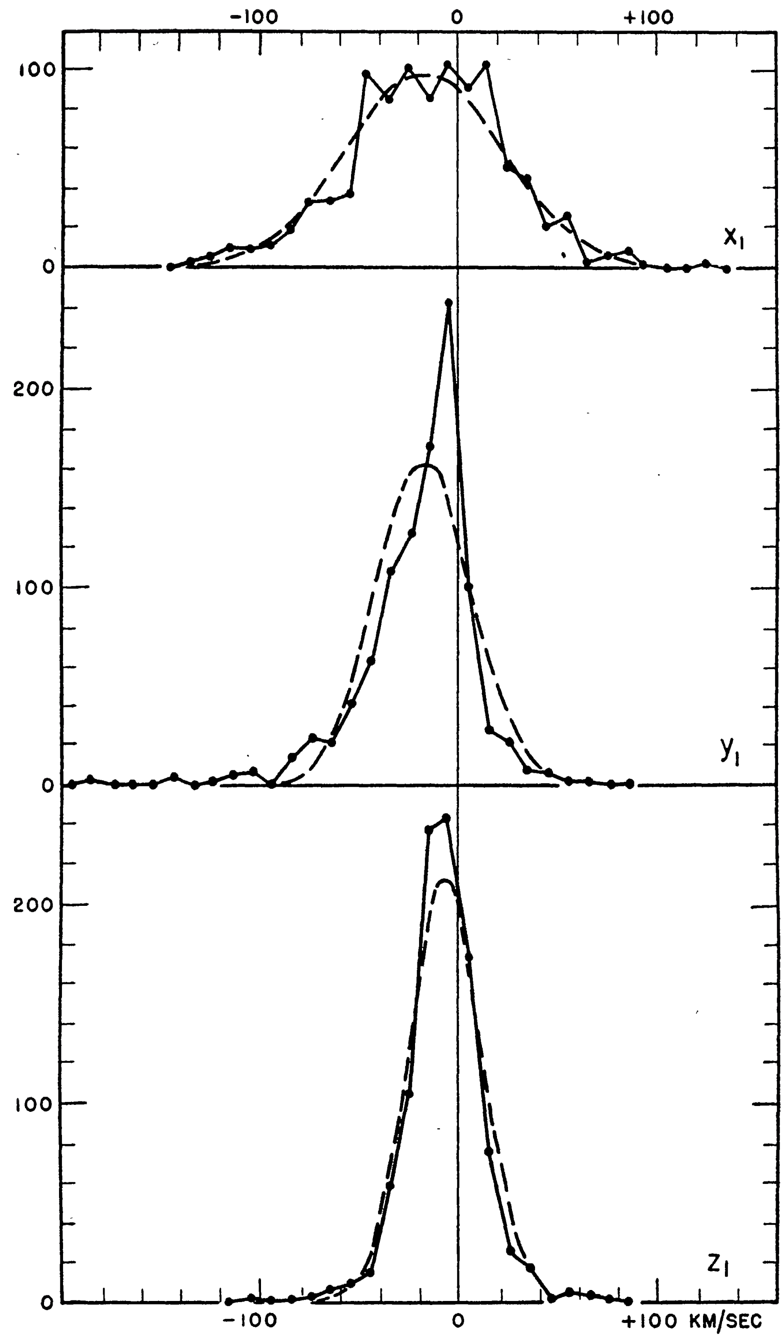

FIG. 4.—Distribution-curves for the velocities projected on the dynamical axes. The x_1-axis is in galactic longitude 339° and points approximately toward the center of the galaxy; the y_1-axis is in galactic longitude 69° and coincides with the axis of asymmetry in stellar motions. The z_1-axis points toward the north pole of the galaxy. The dashed curves are normal error-curves.

TABLE 6

Distribution of Velocities

Velocity (Km/Sec)	A						F					
	$F(x_1)$		$F(y_1)$		$F(z_1)$		$F(x_1)$		$F(y_1)$		$F(z_1)$	
	O	C	O	C	O	C	O	C	O	C	O	C
Between − ∞ and −100	....	...	....	...	....	...	....	...	12	...	11	...
−100 − 90	....	...	....	...	....	...	....	1	0	...	0	...
− 90 − 80	....	...	....	...	....	...	....	2	0	...	0	...
− 80 − 70	....	...	....	...	....	...	6	8	0	...	0	...
− 70 − 60	....	1	....	...	....	...	12	18	0	1	12	1
− 60 − 50	....	6	....	...	....	...	46	38	12	7	0	6
− 50 − 40	....	24	....	...	....	...	116	69	6	27	12	23
− 40 − 30	107	71	....	...	36	4	128	107	52	73	35	63
− 30 − 20	143	148	36	12	18	66	93	140	145	147	70	129
− 20 − 10	196	219	196	228	239	325	81	159	227	213	279	195
− 10 0	303	229	696	556	500	432	180	151	308	222	233	218
0 + 10	36	169	54	195	71	157	128	124	157	169	221	181
+ 10 + 20	179	89	18	9	36	16	105	87	12	92	75	111
+ 20 + 30	36	33	....	...	....	...	47	52	23	36	6	51
+ 30 + 40	....	9	....	...	....	...	46	26	6	11	23	17
+ 40 + 50	....	2	....	...	....	...	6	12	29	2	0	4
+ 50 + 60	....	...	....	...	....	...	6	4	0	...	0	1
+ 60 + 70	....	...	....	...	....	...	....	2	11	...	0	...
+ 70 + ∞	....	...	....	...	....	...	....	...	....	...	23	...

Velocity (Km/Sec)	G						K					
	$F(x_1)$		$F(y_1)$		$F(z_1)$		$F(x_1)$		$F(y_1)$		$F(z_1)$	
	O	C	O	C	O	C	O	C	O	C	O	C
Between − ∞ and −100	45	29	36	1	....	...	27	35	26	1	....	...
−100 − 90	9	21	0	1	....	...	26	20	0	2	....	...
− 90 − 80	22	32	27	5	....	...	26	30	9	8	....	...
− 80 − 70	49	45	40	15	....	1	39	40	35	18	....	...
− 70 − 60	36	61	9	32	18	3	35	51	9	38	....	1
− 60 − 50	63	76	22	63	0	13	35	62	110	70	9	5
− 50 − 40	58	88	89	102	27	37	114	73	79	107	9	19
− 40 − 30	71	98	139	140	71	80	61	82	114	142	66	56
− 30 − 20	156	100	134	162	138	138	66	87	140	159	74	120
− 20 − 10	80	98	161	160	210	186	88	87	140	152	259	191
− 10 0	76	88	192	131	214	196	105	85	189	124	246	221
0 + 10	98	76	89	91	152	161	114	77	70	86	210	189
+ 10 + 20	116	61	31	54	125	104	61	68	53	51	44	119
+ 20 + 30	49	45	22	26	9	52	48	57	26	25	48	55
+ 30 + 40	36	32	9	12	18	21	35	44	....	11	17	18
+ 40 + 50	9	21	....	3	9	6	44	34	....	4	0	5
+ 50 + 60	9	13	....	2	9	2	31	25	....	2	9	1
+ 60 + 70	0	8	....	...	....	...	0	16	....	...	9	...
+ 70 + ∞	18	8	....	...	....	...	45	27	....	...	....	...

Velocity (Km/Sec)	M						All					
	$F(x_1)$		$F(y_1)$		$F(z_1)$		$F(x_1)$		$F(y_1)$		$F(z_1)$	
	O	C	O	C	O	C	O	C	O	C	O	C
Between − ∞ and −100	49	38	29	3	10	...	29	20	25	...	4	...
−100 − 90	10	20	0	6	0	...	11	15	0	2	0	...
− 90 − 80	29	28	19	13	5	...	19	25	14	4	1	...
− 80 − 70	38	38	19	26	14	1	33	35	24	11	3	...
− 70 − 60	58	48	72	47	0	2	34	49	21	27	7	2
− 60 − 50	14	58	19	73	29	11	37	64	41	54	9	7
− 50 − 40	134	68	77	104	14	33	98	78	62	91	15	26
− 40 − 30	82	76	144	126	63	77	85	89	108	130	59	68
− 30 − 20	77	80	111	140	159	138	101	97	126	156	106	133
− 20 − 10	58	83	159	135	187	191	85	99	170	161	239	195
− 10 0	10	81	183	115	221	202	103	94	243	140	245	213
0 + 10	43	77	106	87	154	166	91	85	99	103	176	176
+ 10 + 20	115	68	14	58	72	103	104	72	28	64	77	108
+ 20 + 30	62	60	19	35	43	51	51	57	21	34	26	49
+ 30 + 40	77	49	19	18	19	18	45	43	8	15	18	18
+ 40 + 50	29	38	0	9	0	6	21	30	6	6	2	4
+ 50 + 60	67	29	10	3	0	1	27	20	2	1	5	1
+ 60 + 70	14	22	....	1	10	...	3	13	2	1	4	...
+ 70 + ∞	34	39	....	1	....	...	23	15	....	...	4	...

volved. The absolute bolometric magnitudes (M_b) were therefore derived, as mentioned before, and the stars were grouped according to certain characteristics of their motion, the mean bolometric magnitude being derived for each group.

First, the distribution of the bolometric magnitudes for each spectral type was determined. The results are given in Table 7 as percentages. The first grouping was made to ascertain whether or not there was any tendency toward equipartition of kinetic

TABLE 7

DISTRIBUTION OF BOLOMETRIC MAGNITUDES

M_b	A	F	G	K	M	All
Between − 1.0 and 0.0	....	....	1	....	....	0.2
0.0 + 1.0	14	....	0	2	1	1.7
+ 1.0 + 2.0	21	1	3	0	0	2.5
+ 2.0 + 3.0	50	14	4	2	0	7.3
+ 3.0 + 4.0	4	49	7	2	0	12.0
+ 4.0 + 5.0	7	29	39	3	0	16.6
+ 5.0 + 6.0	4	5	36	30	1	17.9
+ 6.0 + 7.0	....	0	8	39	24	17.8
+ 7.0 + 8.0	....	1	2	17	35	13.3
+ 8.0 + 9.0	....	0	....	3	24	6.3
+ 9.0 +10.0	....	0	....	1	9	2.4
+10.0 +11.0	....	0	....	1	3	1.0
+11.0 +12.0	....	0	....	....	3	0.8
+12.0 +13.0	....	0	....	....	....	0.0
+13.0 +14.0	....	0	....	....	....	0.0
+14.0 +15.0	....	1	....	....	....	0.2
Reduced numbers	100	100	100	100	100	100.0
Actual numbers	28	86	112	114	104	444

TABLE 8

CORRELATION BETWEEN BOLOMETRIC MAGNITUDE AND z_2

M_b	No.	$\bar{M}_b$	$\bar{z}_2$	z_2	No.	$\bar{z}_2$	$\bar{M}_b$
$\leqslant$3.9	101	2.74	10.7	$\leqslant$7	147	3.4	5.20
4.0 to 5.9	155	4.97	15.0	8 to 13	116	9.5	5.46
6.0 to 7.5	119	6.73	14.1	14 to 20	78	16.0	5.70
$\geqslant$7.6	69	8.71	21.4	$\geqslant$21	103	36.1	5.89
All	444	5.52	14.8	All	444	14.8	5.52

energy. The kinetic energy can most readily be studied in the velocity component perpendicular to the galaxy because in this component we have a well-defined origin to which the velocities can be referred. As reference frame I used a plane relative to which the sun has a velocity of 7 km/sec toward the north galactic pole, in accordance with the value of $\bar{z}$ in Table 5. Values of $z_2 = z + 7$ were then computed, and the stars were grouped according to M_b and also according to the values of z_2, with positive and negative values combined. The values thus found are given in Table 8, where all the spectral types are combined.

The data in Table 8 are shown in Figure 5, where the two regression lines are drawn. The coefficient of correlation between $\bar{M}_b$ and z_2 is equal to $+0.24$. The correlation between mass and motion perpendicular to the galactic plane is therefore rather small, although there can be no doubt that a positive correlation exists; hence there is only a very slight tendency toward equipartition of energy in this velocity component.

The stars were also grouped according to the size of the velocity components along the x_1-axis, which is supposed to be directed toward the center of mass of the galaxy as a whole. Among the A stars we have long had evidence of group motions along this axis,

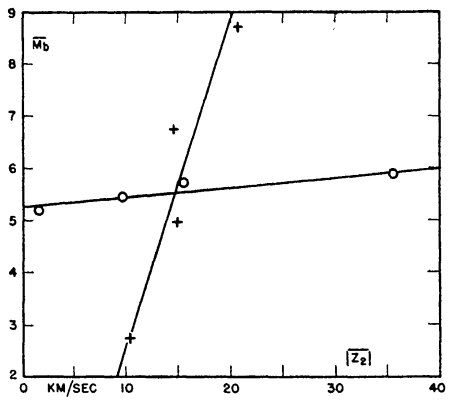

FIG. 5.—Correlation between bolometric magnitudes and the absolute values of $z_2 = z_1 + 7$. The two regression lines are shown.

TABLE 9

CORRELATION BETWEEN VELOCITIES PROJECTED ON THE x_1-AXIS AND BOLOMETRIC MAGNITUDES

x_1	No.	$\bar{x}_1$	$\bar{M}_b$	x_1	No.	$\bar{x}_1$	$\bar{M}_b$
<-46............	87	-84.4	6.14	$>+13$............	103	$+37.9$	6.18
-46 to -17......	126	-31.4	5.25				
-16 to $+13$......	128	-1.2	4.83	All...........	444	-15.0	5.52

and the main groups can be identified with the generalized Taurus group and the generalized Ursa Major group. These groups are now left as the only direct evidence of Kapteyn's two star streams, the rest of these streams being engulfed in the large velocity dispersion in the x_1-axis. In the present small sample of A stars the existence of the two moving groups of stars is indicated by a minimum in the frequency of the x_1-components at $+5$ km/sec. It also explains the abnormally high dispersion in the x_1-components of the velocities of the A stars.

Table 9 gives the mean bolometric absolute magnitudes for the stars when grouped according to velocities projected on the x_1-axis.

The data indicate a definite maximum mass for x_1 at about $+5$ km/sec (Fig. 6). Such a maximum mass is to be expected on the theory, discussed later, according to which a maximum should occur for a value of the x_1-component of the velocity of a star moving in an exactly circular orbit around the center of the galaxy.

The result of a grouping of the stars according to the value of the velocity components along the y_1-axis is shown in Table 10. The data show a maximum mass for y_1 at about -9 km/sec (Fig. 7), which, according to theory, represents the difference between an exact circular motion around the center of the galaxy and that of the sun along the y_1-axis.

The sun's velocity relative to the center of the galaxy is about 300 km/sec in the approximate direction of the y_1-axis here used. The best value available at present is prob-

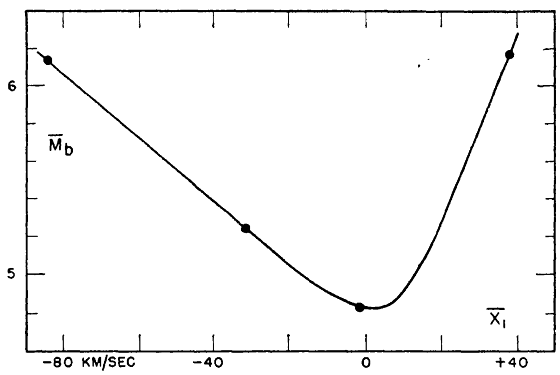

FIG. 6.—Mean bolometric magnitudes for stars grouped according to the values of x_1

TABLE 10

CORRELATION BETWEEN VELOCITIES PROJECTED ON THE y_1-AXIS AND BOLOMETRIC MAGNITUDES

y_1	No.	$\bar{y}_1$	$\bar{M}_b$	y_1	No.	$\bar{y}_1$	$\bar{M}_b$
<-40	81	-69.3	6.23	>0	78	$+12.3$	5.75
-40 to -20	100	-29.8	5.68				
-19 to 0	185	-10.0	5.02	All	444	-21.3	5.52

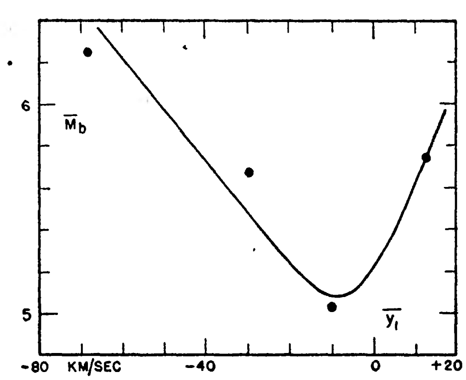

FIG. 7.—Mean bolometric magnitudes for stars grouped according to the values of y_1

ably that determined by A. H. Joy[6] from the radial velocities of the Cepheid variables. He found a value of 296 km/sec in a longitude assumed to be 55°, which is sufficiently close to the value of 69° used in this study.

I have added exactly 300 km/sec to the y_1-components of the stellar velocities when referred to the sun, in order to refer them to a nonrotating reference frame. By adding appropriate corrections, the x_1- and the z_1-components have been corrected for the deviation of the sun's orbit from an exact circle in the galactic plane. For each star I have computed a "galactic velocity," v, defined by the equation

$$v^2 = (x_1 + 17)^2 + (y_1 + 300)^2 + (z_1 + 7)^2 .$$

The stars were then grouped according to the values of v and the mean bolometric magnitudes determined. The results are given in Table 11. The results show a definite minimum of $\bar{M}_b = 5.03$ for a root-mean-square velocity of 295 km/sec, differing by 5 km/sec from the orbital velocity of the sun (Fig. 8). It should be noted that this cannot be regarded as a method of determining the circular velocity of the galaxy, which must be determined by other means. It gives, however, some data from which to study the distribution of the kinetic energy of the stars in our neighborhood.

TABLE 11

CORRELATION BETWEEN GALACTIC VELOCITIES AND BOLOMETRIC MAGNITUDES

$v^2/100$	No.	$\bar{v}^2/100$	$\sqrt{\bar{v}^2}$	$\bar{M}_b$	$v^2/100$	No.	$\bar{v}^2/100$	$\sqrt{\bar{v}^2}$	$\bar{M}_b$
<700	84	586.4	242.2	6.22	>900	98	985.4	313.9	5.77
700–800	102	753.0	274.4	5.44					
800–900	160	854.8	292.4	5.04	All	444	809.5	284.5	5.52

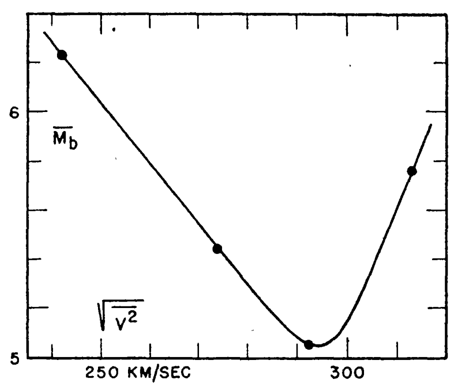

FIG. 8.—Mean bolometric magnitudes for stars grouped according to $v^2 = (x_1 + 17)^2 + (y_1 + 300)^2 + (z_1 + 7)^2$.

In the correlation between mass and the velocity components, x_1 and y_1, and the galactic velocity, v, it is not practical to group the stars according to the bolometric magnitude, since the regression lines given have a minimum value of $\bar{M}_b$ for a definite value of the velocity variable. In a grouping according to $\bar{M}_b$, values both greater and smaller than this value of the velocity variable would be mixed together, and practically the whole effect would be obliterated.

[6] *Mt. W. Contr.*, No. 607; *Ap. J.*, 89, 356, 1939.

THE INTERPRETATION OF STELLAR MOTIONS

When the asymmetry in stellar motions was first discovered, the writer[7] showed that it could be explained formally as the combined effect of two simultaneously acting velocity restrictions, the combined effect being represented as a product of two symmetrical velocity distributions with greatly differing centers. The sun had a very small velocity, about 10 km/sec, relative to the center of one of these distributions, and a very high one, at least 250 km/sec, relative to the other center. Since the sun's velocity relative to the globular clusters is about 300 km/sec, it was natural to assume that the second restriction represented a velocity distribution centered in the galactic system as a whole. This restriction was clearly manifested by an apparently complete absence of stars of velocities greater than that of escape from the galaxy, that is, by an absence of interlopers from other galaxies. The first restriction was originally interpreted by J. H. Oort[8] as a similar effect in the local star system, for which the escape velocity was about 65 km/sec. B. Lindblad[9] found that a unitary interpretation was possible. He interpreted the asymmetry as due to a predominance of circular motions and to the fact that the frequency of stellar orbits decreases with an increase in their eccentricity. This type of distribution can be expected from kinematic considerations alone, since for circles the variation in the size of the major axis and the variation in the direction of the line of apsides disappear. A decrease in the eccentricity of the orbit of the stars with a decrease in their masses would indicate that there are now, or have been in the past, forces tending to round out the orbits of the stars about the center of the galaxy with all the stars moving in the *same direction*.

At the present time that part of the galaxy which we can survey consists mostly of well-separated stars. There are interstellar clouds, to be sure, but their effect on stellar motions is relatively small, except possibly for large stars of small density. In general, the stars are very small relative to their mutual separation, and therefore the effect of stellar encounters, and even more of actual stellar collisions, is not sufficient to produce group motions or an equipartition of energy. We know that groups of stars exist having almost exactly identical motions, and this can be explained only by assuming that the component stars have been formed from a larger system of nearly uniform mass motion.

The writer[10] has attempted to show, by a method that was probably too simplified, that the observed motions of the stars could to some extent be explained on the assumption that in the prestellar stage the motions in the matter from which the stars were formed had, to a certain extent, become systematic and had a relatively small velocity gradient. The end-result of the effect of the internal friction—which itself decreases with time but never completely disappears—was found to be a motion with uniform speed, that is, $DT/Dt = 0$, where T is the kinetic energy per unit mass and t is the time, and the differentiation is taken along the path of a moving element. The corresponding rate of change, DG/Dt, of the gravitational energy was then found to be equal to $-\varphi$, where φ is the dissipation function, which, as used in this connection, is the rate, per unit mass, at which mechanical energy is converted into heat. Since φ is always positive and approaches asymptotically a finite limit greater than zero,[11] the gravitational energy of a fluid element diminishes steadily, and the whole system therefore contracts, generally and locally. The stars gradually formed as condensations from this system would therefore, after the friction in the interstellar matter had practically disappeared, predomi-

[7] *Mt. W. Contr.*, Nos. 275 and 293; *Ap. J.*, **59**, 228, 1924, and **61**, 363, 1925.

[8] *B.A.N.*, **3**, 275, 1927; **4**, 269, 1928.

[9] *Ark. f. Mat., Astr. och Fysik*, **19A**, 27, 35, 1925, and subsequent publications. A summary is given in "Die Milchstrasse," *Handb. d. Ap.*, **5**, Part 2, 1033–1076, 1933.

[10] *Mt. W. Contr.*, Nos. 492 and 503; *Ap. J.*, **79**, 460, and **80**, 327, 1934.

[11] D. J. Korteweg, *Phil. Mag.*, **16**, 112, 1883; Rayleigh, *Phil. Mag.*, **36**, 354, 1893.

nantly move in orbits of nearly uniform speed and of approximately constant gravitational energy, and all in the same direction. In a system where there is one predominant center of attraction and where the system has an angular momentum, supposed to be produced by tidal effects sufficient to produce a definite plane of maximum density, the stars formed *after* the systematic motions had been fully established could be expected to move in circles in the plane of maximum stellar population. Since such stars were formed from a medium with relatively small internal motions and with relatively high density, they could attain great mass. Circular motions are indicated by a vanishing velocity dispersion in a particular part of the galaxy, although group motions would produce a similar effect. We should therefore expect to find a correlation between the masses of the stars and their velocity dispersion and also a correlation of the masses with the motions along the x_1- and the y_1-axes of the same type as actually found here for the stars studied. A direct correlation between the mass of a star and the time of its formation cannot be made, since we have no direct information concerning this time. The existence of such an additional correlation would, to some extent, at least, account for the lack of any detailed correlation between the mass of a star and its present motion.

The theory of the motions in a compressible, viscous fluid as outlined in *Mt. W. Contr.*, Nos. 275 and 293[7] has been criticized by T. Gustafson and H. Nordström[12] on the ground that it cannot have general validity. To the writer it seems that the investigators mentioned have not realized the specific nature of the equations. These equations were never intended to represent general equations of motion and are valid only when the internal friction is sufficiently large and has acted for a sufficiently long time to establish a stable form of motion *independent of the initial conditions*. It can be shown that the assumptions I have made are equivalent to making that term in Lagrange's equations which varies explicitly with time disappear at all points. The authors apply the theorem to the free fall of a stream of water and show that the theorem does not hold in this case; but this example represents a motion without effective friction and dependent upon the initial level of the water; obviously, the theorem cannot hold then. If, however, the jet of water is falling in a viscous fluid, like air, a constant end-velocity is reached, independent of the height from which the water drops are falling, and the loss in potential energy is balanced, not by a gain in gross kinetic energy, but by a gain in internal heat motion. This represents the final state of affairs that corresponds to the assumptions actually made.

A number of problems connected with the development of a mass of gas or dust into a system of stars should be studied by investigators more familiar with such problems than is the writer of this article. The results given here about the motions of the stars in our part of the galaxy may be useful in formulating and testing theories of the development of our own galaxy and star systems in general.

[12] *Zs. f. Ap.*, **10**, 228, 1935.