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THE ROTATION OF THE ANDROMEDA NEBULA*

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I. INTRODUCTION

One of the important problems of astronomy is the determination of the internal motions of the extra-galactic nebulae. An analysis of such motions for a single system of this kind should lead to a knowledge of the total mass within it, and of the distribution of this mass. When such studies have been made for many of the spirals, correlation of the various dynamical characteristics with nebular type may serve to establish the evolutionary processes undergone by these systems. The internal motions of the nebulae must be closely linked with the geometry of the spiral arms, and any theory of the formation of the arms must depend for confirmation upon observation of the motions. Finally, the intricate problem of deciphering the dynamics of the galactic system from within can probably be vastly clarified by a comparison with data from some of the external galaxies.

One of the nearest and brightest of these objects is the Andromeda Nebula, also known as Messier 31 and NGC 224. Because such a large body of knowledge concerning it has accumulated through the work of many observers in the past, and because it is favorably situated for spectrographic observation of its internal velocities, it is a natural choice for a detailed study of this kind.

The most comprehensive discussion of the Andromeda Nebula has been given by Hubble in his paper *A Spiral Nebula as a Stellar System, Messier 31*,[1] and in his book *The Realm of the Nebulae*,[2] pp. 134–137. The nebula is assumed to approximate in shape a greatly flattened spheroid, or disk, and to be inclined to the line of sight. The inclination gives it the apparent outline of an ellipse, as seen on the celestial sphere, and from an estimate of the ratio of axes of this ellipse (about 3 or 4 to 1), Hubble has given 15° as the approximate inclination of the equatorial plane of the spiral to the line of sight. He also found that the major axis of the apparent ellipse lies in position angle 36°.7. The unresolved central region measures about 10′ by 30′, and at the center is the bright quasi-stellar nucleus, but this is lost in most reproductions owing to lack of photographic gradation. The region immediately surrounding the nucleus, having a radius of some 4′ or 5′, and assumed from its appearance to be nearly spherical, will be referred to as the "core" of the nebula. Outside of the central region are the two spiral arms, which on good negatives are largely resolved into their brighter stars, star clusters, and nebulous patches. The arms can be traced outward from the core through some two revolutions, to a distance of more than 1½°, and on original negatives, traces of spiral structure can be followed inward to within one minute of the nucleus. For the central portion, however, the structure appears to be rather irregular, and there is no very obvious division or condensation into two distinct arms. In Hubble's classification, the nebula is an intermediate type spiral (Sb).

The distance of M31 that Hubble has obtained from studies of Cepheid variables within it is 210,000 parsecs (680,000 l.y.), in which the appropriate correction for galactic obscuration has been included. This distance yields the convenient scale for absolute distances in the nebula of very nearly 60 parsecs per minute of arc, which has been adopted for use throughout this paper.

Several observers have obtained spectrograms of the bright nuclear region of M31, sufficient to yield radial

* This paper is based largely upon a dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of California.

[1] *Mount Wilson Contr.* No. 376; *Ap. J.* **69**, 103, 1929.

[2] Yale University Press, 1936.

velocities of this part, which can be taken as the radial velocity of the spiral as a whole, apart from its rotation. Some of these are listed below:

Slipher	−300 km/sec
Wolf	−350
Wright	−304
Pease	−316
Pease and Adams (I)	−329
Pease and Adams (II)	−297
Humason	−220

The first six velocities are quoted by Pease[3] in his paper on the rotation and radial velocity of the central part of the Andromeda Nebula; the last, by Humason, is mentioned by Hubble,[4] and is listed by Moore in his *General Catalogue of the Radial Velocities of Stars, Nebulae, and Clusters*.[5] Humason's velocity was obtained with a spectrograph of considerably higher dispersion than those used by the other observers listed, and the fact that his determination is markedly less negative than the others may be significant for this reason. It will be noted that the mean of the above velocities lies in the neighborhood of −300 km/sec.

The rotation of the nuclear region of the Andromeda Nebula was first detected spectrographically by V. M. Slipher[6] at the Lowell Observatory in 1914. He noticed that the spectrum lines were inclined when the slit of the spectrograph was placed on the major axis of the nebula, with the nucleus at the middle. Pease,[3] at the Mount Wilson Observatory, measured the rotation out to a radius of 2½ minutes of arc from the nucleus on spectrograms made in a similar way with the 60-inch reflector in 1917. His results were expressed by the formula

$$y = -0.48x - 316,$$

x being in seconds of arc and y in km/sec., showing that the central portion appears to rotate with constant angular velocity. These measures were limited to one plate with an exposure time of 79 hours. Another spectrogram taken with the slit on the minor axis showed no inclination of the lines.

Within the past two years it has been possible to extend the study of the rotation to the extreme limits of the observed spiral arms. Velocities in the line of sight have been measured along the major axis to a distance of 1°6 from the nucleus. Interesting anomalies in the rotation have been detected just outside the bright central core of the nebula, and the approach to constant angular velocity discovered for the outer spiral arms is hardly to be anticipated from current theories of galactic rotation.

II. Observations of the Central Part of M31

The spectrographic observations of nebular rotation made for this study may be divided into two classes:

1. Observations of the continuous solar-type spectrum of the unresolved central region for displacements of the absorption lines, and

2. Observations of small isolated "emission nebulosities," or gaseous nebulae in the resolved portions of the spiral arms, for displacements of the bright lines in their spectra.

The instrument used for both classes of work was the two-prism nebular spectrograph designed by Messrs. Wright and Mayall.[7] This instrument is normally mounted at the prime focus of the 36-inch aluminized mirror of the Crossley telescope, so that secondary reflections are avoided. The optical parts are of Jena UV glass, and the especially-designed Zeiss camera lens has a focal ratio of 1.33, with a resulting dispersion of 309 angstroms per mm at λ3950. The spectrograph is equipped with a curved slit having an available length of ⅜ inch, which at the focus of the Crossley mirror is equivalent to 6.2 minutes of arc. This curvature was calculated by the designers to give straight spectrum lines at H and K, and measures of test plates show that from λ3500 to λ4700 there is no significant curvature of the lines. Two separate sparks are used for the comparison spectrum, cadmium and common solder for the blue-violet, and palladium for the ultra-violet. A Wratten No. 2 filter cuts out the lines of the cadmium-solder spark to the violet of λ4000. The whole head of the telescope tube can be rotated, so that the slit may be set to any desired position angle.

In the focal plane of the Crossley mirror, the image of the Andromeda Nebula is about one foot long, which is a convenient scale for a detailed study. Observations of the unresolved central portion were made with the slit 6.2 minutes long in position angle 37°, so that it lay on the major axis of the image. The first plates were taken with the nucleus at the center of the slit; succeeding ones with the nucleus at opposite ends of the slit. Following this, spectrograms were made with the slit farther and farther from the nucleus, but still on the major axis. In this manner, observations were made practically without a break out to distances of more than 15′ on the south-preceding and north-following sides of the nucleus. Duplicate plates were obtained over most of this region.

When this had been accomplished, it was thought desirable to repeat the observations with a smaller scale image of the nebula, for the double purpose of improving the precision of the final results with economy of time, and of detecting systematic errors, if any. Accordingly, the spectrograph was removed from its normal position at the prime focus of the Crossley reflector, and mounted at the Newtonian focus of a six-inch mirror of 32 inches focal length.[8] The spectrograph, along with the mirror and Newtonian flat, both aluminized, was carried in a substantial wooden box bolted to the side of the Crossley

[3] *Proc. National Academy of Sciences*, **4**, 21, 1918.
[4] The Realm of the Nebulae, p. 140 (footnote).
[5] *Publ. Lick Obs.*, Vol. XVIII, 1932.
[6] *Lowell Bull.*, **2**, 65, 1914; *Pop. Astron.*, **25**, 36, 1917.
[7] *Publ. Astr. Soc. Pac.*, **48**, 14, 1936.
[8] I am indebted to Mr. Ferdinand Ellerman for the loan of this mirror.

telescope in such a manner that the slit of the spectrograph assumed the desired position angle of 37°. A viewing arrangement was provided so that the nebula could be centered on the slit, and guiding was accomplished by means of a star picked up in the Crossley reflector. Since the small mirror had the same effective ratio of aperture to focal length as the Crossley, exposure times on a luminous nebular surface remained unchanged, except for a slight loss due to the introduction of the Newtonian flat, and, as the focal length of the small mirror was 32 inches, the length of the slit of the spectrograph corresponded to 40 minutes of arc, instead of only 6.2 minutes on the Crossley telescope. As a result, the spectrum of over half a degree of the major axis of M31, from the nucleus to the faint outer parts, could be photographed with one exposure. A "decker" was provided in front of the slit. This could be moved in or out, so as to cut off the bright central core of the nebula intermittently during long exposures, and thus tend to equalize the density along the spectrum lines. Four spectrograms of M31 were made with the small mirror arrangement, two with the nucleus at the middle of the slit, and two with the nucleus at the south-preceding and north-following ends of the slit, respectively.

In order to avoid possible systematic errors, the same slit width, 0.20 mm, was used for all but five exposures on the unresolved portion of M31. As a further precaution, several control plates were made of the spectrum of the sky. The spectrum of scattered sunlight is very similar to the G-type spectrum of the nebula, and its zero velocity yields convenient standards for settings on the H and K lines and the G-band. These control plates were taken at intervals throughout the observing season, and with both the optical systems described, in order to allow for possible changes in adjustment or differences in the effects of collimation.

Through the kindness of Dr. W. S. Adams, Director of the Mount Wilson Observatory, it was possible to secure two supplementary spectrograms with the 60-inch reflector and nebular spectrograph at Mount Wilson. These exposures, one on the nucleus of M31 and one at nine minutes from the nucleus on the south-preceding major axis, serve as partial checks on the form of the velocity curve.

TABLE 1

SPECTROGRAMS OF UNRESOLVED PARTS OF M31

Plate No.	Date	Region	Exposure time	Quality
	1937			
102	Aug. 8	Major axis 3 NF to 3 SP	1h34m	G
103	Aug. 10	Major axis 3 NF to 3 SP	3 40	E
104	Aug. 14, 15	Major axis 3 NF to 3 SP	7 20	E
110	Sept. 3	Minor axis 0±	0 25	P
111	Sept. 4, 5	Major axis 0 to 6 SP	14 14	E
112	Sept. 7	Major axis 11 SP to 17 SP	7 37	G
114	Sept. 8	Major axis 0 to 6 SP	8 23	E
116	Sept. 10, 11, 12	Major axis 14 NF to 19 NF	14 13	G
118	Oct. 4, 5	Major axis 7 SP to 13 SP	13 00	G
119	Oct. 5, 6	Major axis 6 NF to 12 NF	14 25	E
120	Oct. 23, 24	Minor axis 0 to 3 SF	4 10	F
121	Oct. 25	Minor axis 0 to 4 NP	4 00	G
122m	Oct. 28	Major axis 20 NF to 20 SP	8 10	F
123m	Oct. 29, 30	Major axis 20 NF to 20 SP	15 30	F
124m	Nov. 1, 2	Major axis 0 to 40 SP	19 48	F
125m	Nov. 4, 5, 6	Major axis 0 to 40 NF	19 40	G
126	Nov. 8	Major axis 7 SP to 13 SP	6 43	F
129	Dec. 6	Minor axis 0 to 6 SF	7 30	G
	1938			
147MW	Aug. 25	Nucleus ±	6 37	F
148MW	Aug. 26, 27, 28	Major axis 9 SP	14 48	P
153	Sept. 17, 18, 19	Major axis 7 SP to 13 SP	10 30	P
154	Sept. 19, 20, 21	Major axis 7 NF to 13 NF	21 45	P
156	Sept. 28, 29	Major axis 4 NF to 10 NF	12 00	G

Table 1 lists the spectrograms of the unresolved central portion of the nebula. A letter "m" affixed to the plate number in the first column indicates that the exposure was made with the 6-inch mirror arrangement just described; "MW" means that the two plates so designated were obtained with the 60-inch telescope and nebular spectrograph of the Mount Wilson Observatory. All other plates were taken with the Crossley telescope. In each instance, the slit was set on the major or minor axis of the nebula as indicated in the third column, where "NF" stands for the part of the major axis north-following the nucleus, and "SP" for the part south-preceding. The numbers refer to distances from the nucleus in minutes of arc, and indicate the positions of the ends of the slit on the image of the nebula.

The four exposures with the slit on the minor axis were made with the possibility in mind of detecting radial motions in the nebula in the vicinity of the nucleus. Although some irregular displacements of the lines on these plates were suspected, the data are insufficient to warrant a discussion at present.

The diffuse nebulosity in and surrounding the core of the nebula exhibits a solar type spectrum. With the low dispersion employed in this investigation, the only absorption lines strong enough to be depended upon for velocity measurements are the H and K lines and the G-band at λ4303. On a few plates Hδ was also measurable in absorption, and it was suspected that the relative strength of this line increased with distance from the nucleus.

A peculiarity of the spectrum in the region immediately surrounding the nucleus is the strongly-suspected presence of the bright λ3727 radiation of [O II]. While apparently not present in the spectrum of the nucleus itself, this unresolved pair seems to arise in the neighboring region, within three minutes of arc. Measurements of it yielded velocities in fair agreement with those from the absorption lines. The same bright line was also found to be present in the spectrum of the nucleus of the spiral nebula M81, and to be strong in the ring surrounding the nucleus of NGC 4736. In addition, it was suspected in the spirals NGC 5055 and NGC 7331. The occurrence of λ3727 was not entirely unexpected in any of these nebulae, for Mayall reported its presence in approximately half of the first 15 extra-galactic nebulae investigated by him with this same spectrograph.[7]

Figure 1 on Plate IV is a reproduction of an enlargement of spectrogram No. 103. The slit was on the major axis of the nebula, with the nucleus at the middle. The three strongest absorption features are K, H, and the G-band, and the faint bright line at λ3727 can also be seen. About one minute below the nucleus is the narrow continuous spectrum of a foreground star.

III. Measurement and Reduction of Plates of the Unresolved Central Portion of M31

Owing to the necessarily wide slit and to the low dispersion, as well as to the microscopic nature of the shifts under investigation, the task of measuring the plates was a difficult one. This is illustrated by the large velocity factor of 22.6 km/sec. per micron for the K line, which relates velocities to measured displacements on the photographic plate. Unavoidable graininess of the emulsion was perhaps the most important factor limiting the accuracy of measurement.

The eyepiece of the measuring engine was provided with a fixed vertical wire and with fixed and movable horizontal wires. The plates were so adjusted on the machine that the fixed horizontal wire traversed the middle of the spectrum, while the movable wire could be shifted to aid in estimating the position of the particular point on the nebular line which was being measured. With a little practice, it was found possible to estimate the position along the line (i.e., distance from the nucleus) to a third of a minute, the length of a line being 6.2 minutes on spectrograms taken with the Crossley. Plates taken with the six-inch mirror had a line length of 40 minutes, and estimates of position were made to the nearest two minutes. Generally, from five to eight points or short sections were selected for measurement along the length of each line. These were chosen so as to be as well spaced as possible, yet so as to avoid any defects, spectra of foreground stars, or portions of the line that were suspected of being adversely affected by grain structure. On each point so chosen, four or more settings were made, and the mean reading was recorded on a greatly enlarged plot of the spectrum. Settings of low weight were so marked at the time of measurement. Plates were measured direct and reversed, and an effort was made to measure in each direction without an interruption, in order to avoid possible systematic effects arising from temperature changes or other causes. It is estimated that the total number of individual line settings made in measuring these plates was in excess of 12,000.

The comparison spectrum, as usual, appears on each side of the nebular spectrum. In ordinary measuring, it is permissible to take the average reading of the screw for settings on the two parts of the comparison line. Here, however, where small inclinations of the nebular lines are found, further precautions must be taken, because the vertical wire may not be in perfect adjustment. The upper and lower comparison spectra were therefore measured and reduced separately, and appropriate corrections were later made by linear interpolation to compensate for lack of parallelism between the vertical wire and comparison lines.

The majority of the plates were measured twice, and the more important ones three times. Successive measures of given plates agreed well, the probable error of a single measured point being of the order of 20 km/sec. for a spectrogram measured three times. These are only errors of measurement, however, and it is believed that errors due to non-uniform grain of the emulsion are more serious.

Reduction of the measurements was carried out with the aid of a standard dispersion table, computed from wave lengths of all the comparison and nebular lines. The standard settings of the nebular lines corresponding to zero velocity were checked by measurements of sky spectra. Terms for the reduction of the radial velocities to the sun were computed with the aid of Herrick's tables,[9] and applied before the velocities were plotted in graphical form.

Plots of the velocities from all the Crossley plates were made separately for H, K, and the G-band, to see whether the main features of the velocity curve were followed by all three spectral lines. A fair degree of con-

[9] *Lick Obs. Bull.* **17**, 85, 1935.

sistency was revealed, with the G-band showing the greater deviations and irregularities. This was expected, for the dispersion at the position of the G-band is considerably lower than at H and K; also the G-band is somewhat fainter, and, because it is a blend, possible changes in effective spectral class in proceeding outward from the nucleus of the spiral would probably have some effect on the measured velocities.

Medium dense, weak, and very weak exposures of sky spectra, when measured for velocities, gave values differing from zero, as indicated in Table 2. Those for the H and K lines vary progressively in opposite directions with increasing density of exposure, indicating that on weaker plates, these two lines appear farther apart to the measurer. The fact that the "sky velocities" measured for the G-band all have definite positive values is accounted for by the arbitrary standard wave-length that was used for this blend of absorption lines. The H line on these small scale spectra is of course really a blend of H and Hϵ, but in the absence of data as to differences in spectral class for different parts of the nebula, the approximation is made that the spectrum overall is like that of the sun. Accordingly, appropriate corrections, depending upon the strength of the exposures, were made to the nebular velocities by the use of Table 2. Next, for all the Crossley plates, the curves first plotted were smoothed, and means were taken, giving H and K measures unit weight, and the G-band half weight. The resulting normal velocities are plotted as dots in Plate III.

TABLE 2

Mean Measured "Velocities" from Sky Spectra

	K	H	G
Medium spectrum.......	+34	− 23	+100
Weak spectrum.........	−11	+ 12	+ 43
Very weak spectrum....	−27	+100	+ 62

Measured velocities from the plates made with the six-inch mirror were then plotted plate by plate for the three spectral lines, and means were taken. Sky spectra made with the spectrograph in conjunction with this mirror showed a mean "velocity" of about −20 km/sec. The mean points, corrected for this effect, are plotted in Plate III as open circles.

TABLE 3

Emission Nebulosities Observed in M31

Plate No.	m(*pg*)	Exposure time	Number of bright lines measured	x	y	4y	R	V_o	V_c
158	14.5	19^h22^m	7	−70′.0	− 0′.3	− 1′.2	70′.0	− 45±12	271
160	16.7	8 52	3	+64.5	− 5.7	−22.8	68.5	−568±25	290
161	17	20 45	6	+96.0	+ 2.4	+ 9.6	96.5	−664±26	375
163	15.2	17 50	9	− 0.7	−11.2	−44.8	44.8	−266±17	...
164	15.2	7 00	5	+28.5	− 3.7	−14.8	32.1	−478±22	203

IV. Outlying Emission Nebulosities

Some excellent direct photographs of the Andromeda Nebula taken by Mayall with the Crossley revealed several small, faint, irregular objects in the spiral arms, which he suspected from their diffuse appearance might be gaseous emission nebulosities associated with stars in M31. The appearance of these objects on the direct photographs was sufficient to distinguish them from globular clusters or ordinary stars belonging to the spiral. Spectra obtained by the writer exhibited in every case bright lines superposed on a faint continuous spectrum, characteristic of gaseous nebulae. Although faint, these objects are naturally well suited to yield radial velocities on account of their bright-line spectra. The emission nebulosities appear to be concentrated along the lines of the spiral arms, and from their distribution and close association with resolved stars in the spiral, as well as from their appearance, it is concluded that they must be members of the spiral system of the Andromeda Nebula. The symmetrical distribution of their velocities as plotted in Plate III confirms this conclusion.

Spectra and velocities were obtained with the Crossley reflector and nebular spectrograph for the five emission nebulosities listed in Table 3. Although the number is limited, the favorable distribution of the observed objects is such as to make practicable a preliminary survey of the rotation of the outer spiral arms. Spectra of all five of these objects were quite similar, and closely resemble the spectrum of the Orion Nebula as photographed with the same instrument. The strongest bright line in every instance is λ3727, which is fortunate, as it lies at the end of the spectrum where the dispersion is greater. The next brightest line is Hβ, followed by other members of the Balmer series in decreasing intensity. The most strongly exposed spectrum showed six hydrogen lines. The chief nebular lines, N_1 and N_2, of [O III], were also present, though faint. In Figure 2, Plate IV, is shown an enlarged spectrum of one of these emission nebulosities. The strong continuous spectrum just above that of the bright-line object is that of a foreground star, while the faint, wide, continuous spectrum extending over the full length of the slit is due principally to the diffuse nebulosity of the spiral. Figure 3 is an enlargement from one of Mayall's photographs showing the emission nebulosity 29′ from the nucleus near the south-preceding major axis. The brighter stars are all foreground objects.

In Table 3 the list of observed emission patches gives the estimated photographic magnitudes of these objects, based on Hubble's magnitude scale for globular clusters in M31.[10] The number of measured emission lines in each object is also given, and the positions of the objects themselves are listed in the rectangular coördinate system used by Hubble in giving the positions of the globular clusters identified by him. The major axis of the apparent ellipse of the spiral is taken as the x axis, with the positive end south-preceding the nucleus. The south-following end of the y axis, which passes through the nucleus, was chosen by Hubble as the positive end. Units are minutes of arc.

V. The Rotation Curve

The measured velocities of the emission nebulosities were reduced to the sun and then, with the exception of the one on the minor axis, were plotted in Plate III, along with the velocities of the unresolved central portion of the nebula. The principal results of the investigation are represented in this figure, where the horizontal scale is given both in minutes of arc and in parsecs, the latter dimensions being based on a distance of 210,000 parsecs for the spiral. Ordinates are in km/sec. No correction has been made in Plate III for the inclination of the plane of the spiral to the line of sight, nor for the fact that the emission nebulosities do not lie precisely on the major axis. Velocities of the diffuse solar-type nebulosity are plotted as small dots for observations with the 36-inch Crossley reflector, and as small open circles for observations with the 6-inch mirror. The velocities of the four emission nebulosities close to the major axis are indicated by ringed dots, and to emphasize the symmetry of the velocities on the two sides of the spiral, these have been "reflected" from one side to the other. Each velocity point has been transferred to an equal distance on the opposite side of the center, and its velocity with respect to that of the nucleus has been reversed. The positions of these "reflected" points are indicated by the dotted circles.

The reproduction of the Andromeda Nebula above the graph has been pieced together from three direct photographs made with the Crossley reflector by Mayall, and very kindly placed at my disposal. On the reproduction, the five gaseous nebulosities of which velocities have been obtained are marked by circles. The velocity of the one nebulosity near the minor axis is not indicated, but it is listed with the others in Table 3. From the observations plotted in the figure, it is judged that the best value for the radial velocity of the spiral as a whole is −300 km/sec. This is in good agreement with the mean of the earlier observations quoted on page 42.

With regard to the general accuracy of the velocity curve, it is believed that the velocities of the bright-line emission nebulosities, all of which have probable errors of the order of 25 km/sec. or less, are considerably more reliable than those of the central, solar-type nebulosity. The difficulty of measuring the absorption lines in the spectrum of the central part of the nebula leads to estimated probable errors of about 50 km/sec. for all except the relatively bright region within two or three minutes of the nucleus, where the accuracy is greater. Every effort was made in exposing and measuring the spectrograms to avoid systematic errors, and if one may judge from the general symmetry of the distribution of the points in Plate III, it seems that these efforts were in some measure successful, at least so far as dependence of measured velocity upon luminosity is concerned. The discrepancy between the Crossley velocities and those of the 6-inch mirror at a distance of 10′ from the nucleus of the south-preceding major axis is evidently of a systematic nature, and is perhaps due to the difference in scale of the two images, but, in general, the consistency of the measures is better than might have been expected. It is unfortunate that the measures are of lowest weight in the interesting parts of the curve where the rotational velocities appear to drop to a value indistinguishable from zero, but it is believed, nevertheless, that the general trend of the curve has been established even in these regions.

Before making quantitative use of the rotation data, it is necessary to correct for the inclination of the plane of the spiral to the line of sight, and to allow for the fact that the outlying emission nebulosities are not located precisely on the major axis of the nebula.

In correcting for the inclination, Hubble's value of 15° for the angle between the plane of the spiral and the line of sight is adopted. The x and y coördinates, as before, apply to the projected image of the nebula, and in allowing for the foreshortening, the values of x will remain unchanged. The y coördinates, however, must be multiplied by 4. Then, for an observed point, the radius vector in the plane of the spiral is given by

$$R = (x^2+16y^2)^{1/2} \qquad (1)$$

and, if we let θ designate the angle in the equatorial plane between the x axis and the radius vector,

$$\tan\theta = 4y/x\,. \qquad (2)$$

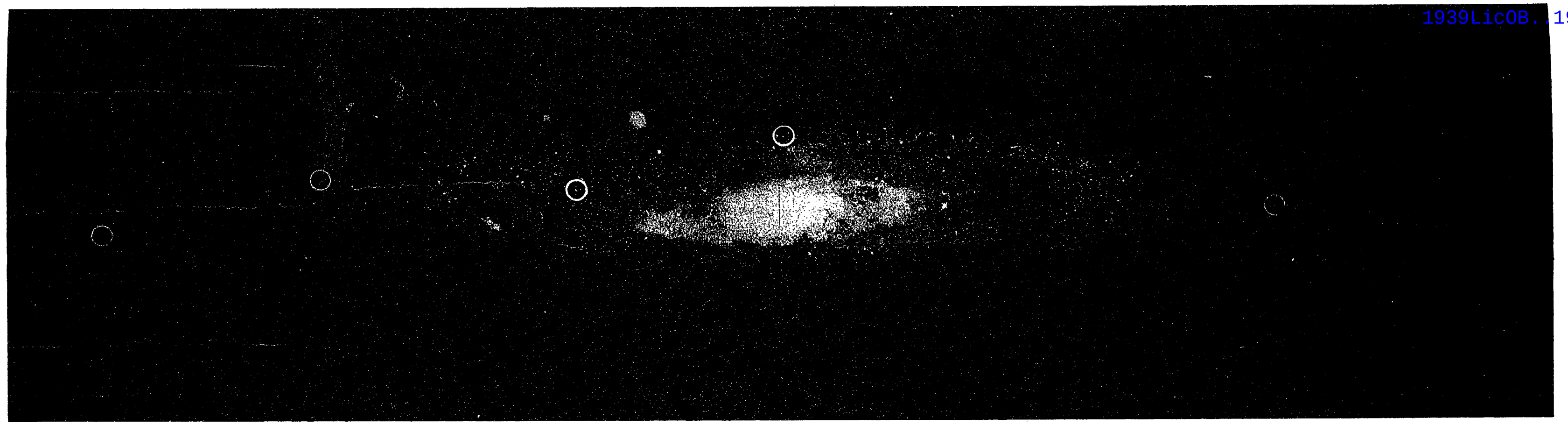
Assuming that the observed points are all moving in circular orbits around the nucleus with velocities V_c, and taking the velocity of the nucleus as −300 km/sec., we have

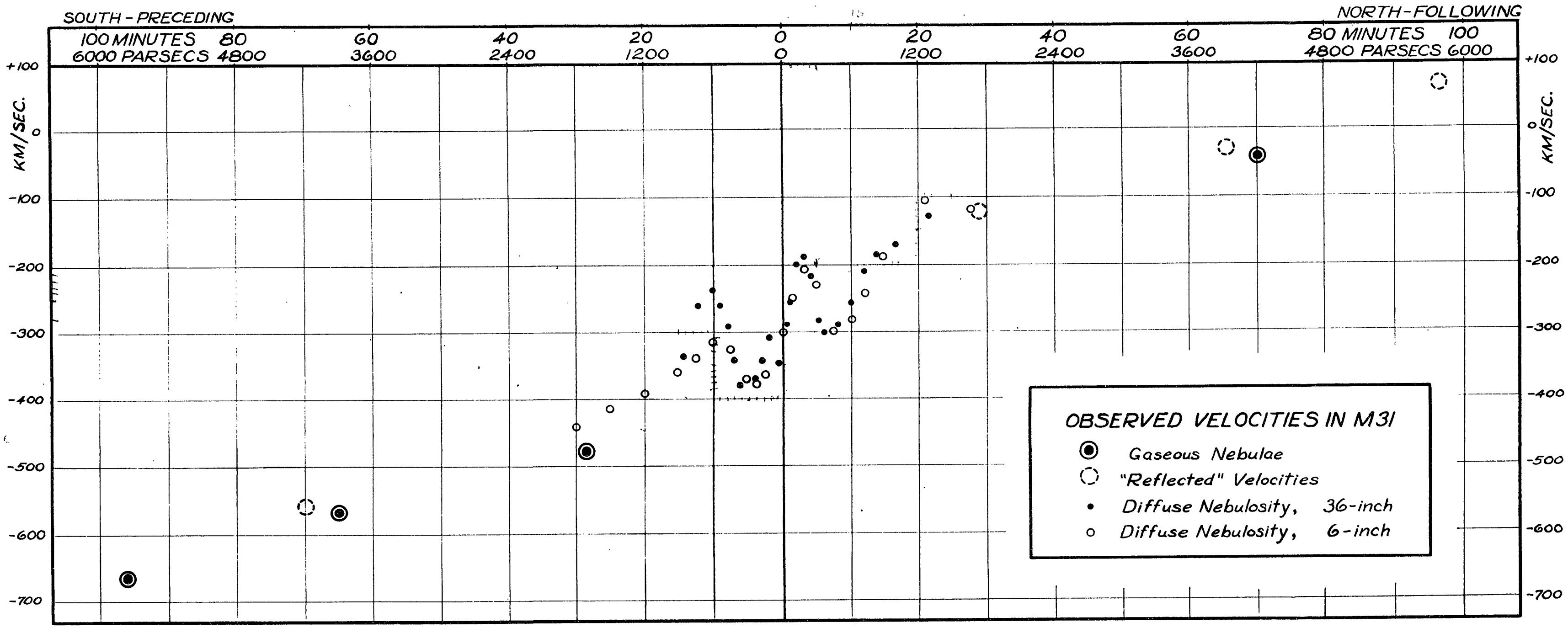
$$V_c = (V_o + 300)\sec\theta\sec 15° \qquad (3)$$

where V_o is the observed velocity. The values of the circular velocity, V_c, are given in Table 3 for the four emission nebulosities which lie close to the major axis.

In considering the velocities measured for the unresolved central portion of the nebula, the question of

[10] *Mount Wilson Contr.* No. 452; *Ap. J.*, **76**, 44, 1932.





The Andromeda Nebula, with observed velocities plotted below. Smoothed mean velocities of the unresolved central portion are represented by small dots for observations with the Crossley reflector, and by small open circles for observations with the 6-inch mirror. In the photograph, the bright-line emission nebulosities are encircled, and their velocities, with the exception of the one on the minor axis, are plotted as ringed dots. *(Photograph by Mayall)*

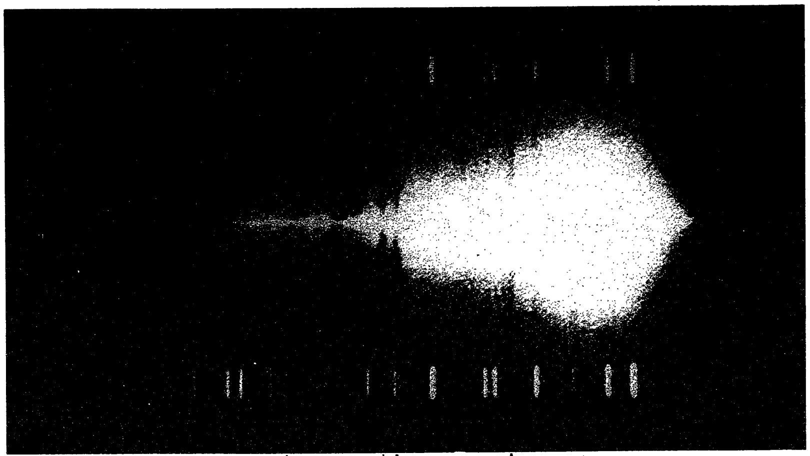


Fig. 1. Seventeen-fold enlargement of Plate No. 103. Nucleus at center o slit. Exposure 3^h 40^m.

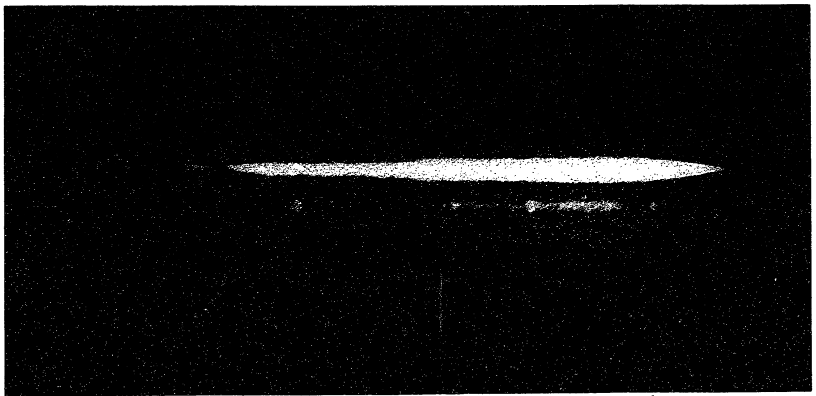


Fig. 2. Enlargement of Plate No. 163 of bright-line spectrum of emission nebulosity at $x = -0.7$, $y = -11.2$. The strong continuous spectrum is that of a foreground star.

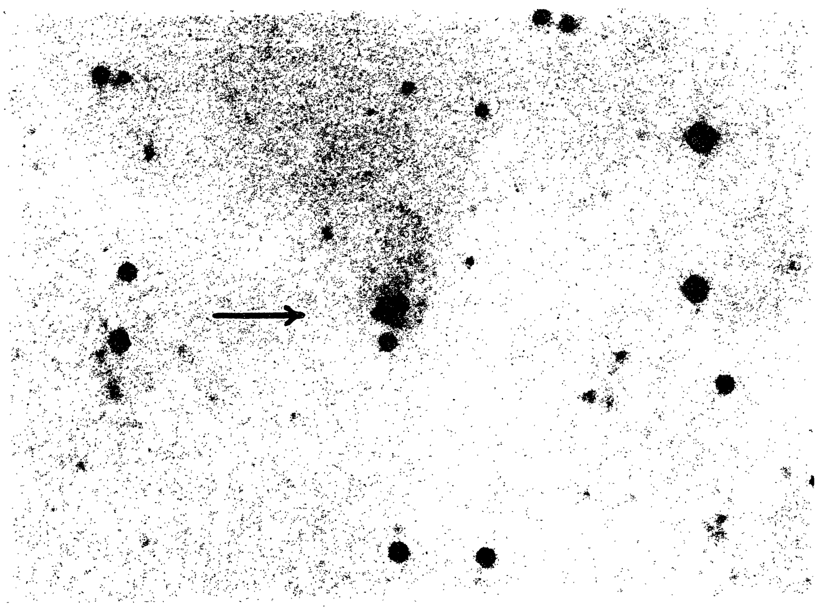


Fig. 3. Diffuse emission nebulosity in M31 at $x = +28.5$, $y = -3.7$. Mag. 15.2 (pg).

optical depth arises.[11] Do the velocities apply to material that is essentially in the equatorial plane? Hubble has considered the transparency of the central region of M31,[12] and he finds that among the novae and Cepheids there is no definite relation between luminosity of these stars and distance from the nucleus. He points out that the nucleus is sharply visible, although buried to a depth of many hundreds of parsecs in nebular material, and concludes that there is no considerable absorption in the unresolved luminous parts of the nebula, although obscuration by well defined dark markings is conspicuous. The intensity gradient in the nuclear region of the nebula is quite steep, as is evident from an inspection of almost any original direct photograph of this part, and as shown more precisely by the photometric measures of Redman and Shirley,[13] among others. These considerations seem to justify the assumption that to a first approximation, the observations on the major axis really indicate the velocities of rotation in the equatorial plane, when corrected for the inclination of this plane to the line of sight. These observed velocities then need only be added to 300 km/sec. (the velocity of the nucleus) and multiplied by 1.036, the secant of 15°. Of course this last correction is practically negligible.

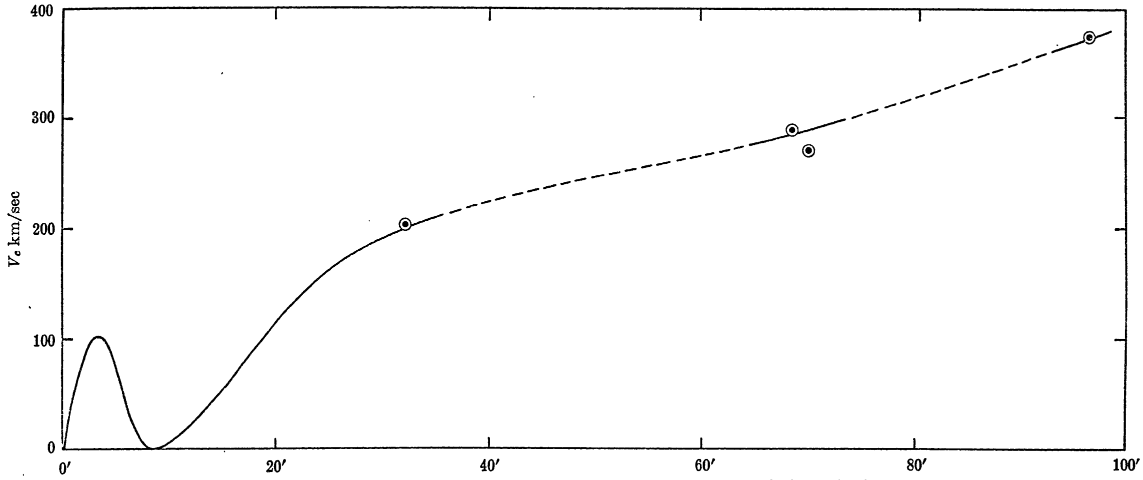


Fig. 4. Mean velocities of rotation in the plane of the spiral.

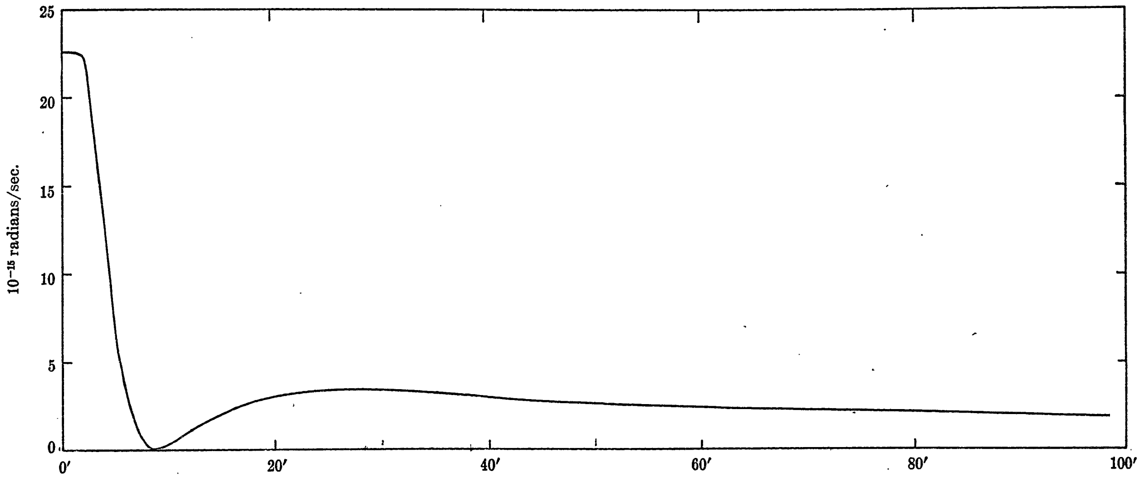


Fig. 5. Angular velocity, ω, in units of 10^{-15} radians per second, plotted against radius.

In Figure 4 the circular orbital velocities have been plotted against radius. Mean values have been used for the unresolved nebulosity on each side of the nucleus, but the circular velocities of the bright-line emission nebulosities have been plotted individually. The parts of the curve between these outer points are shown dotted, as they indicate only the probable form of the curve in these regions between the observed points.

Since the angular velocities of the parts of the system are also of interest, these have been plotted in Figure 5. Ordinates are in units of 10^{-15} radians/sec., and are shown as a function of the radius in minutes of arc. For purposes of conversion, it may be noted that this unit is equivalent to 0.00651 seconds of arc per year.

Perhaps the most striking features of the two curves

[11] Since this Bulletin went to press, an important paper dealing with this question and with the distribution of mass, light and absorption within galaxies has been published by E. Holmberg, *Mon. Not. R. A. S.* **99**, 650, June, 1939.

[12] *Mount Wilson Contr.* No. 376, p. 46, 1929.

[13] *Mon. Not. R. A. S.*, **97**, 416, 1937.

are the approach to a constant angular velocity of a considerable magnitude for the outer spiral arms, and the dip to a value indistinguishable from zero in the neighborhood of eight to ten minutes from the nucleus. It may be significant that this minimum occurs in a region of somewhat decreased intensity between the core of the nebula and the first encircling turn of the spiral arms.[14] The idea suggests itself that it may be due to non-uniform distribution of mass along the radius vector of the spiral, and also in azimuth. Furthermore, it is possible that the circular component of velocity just outside the core varies markedly with azimuth in the plane of the nebula. A detailed dynamical analysis, taking into account the spiral distribution of part of the mass of the nebula, might throw light on these points.

The slope of the portion of the curve immediately adjacent to the nucleus is in good agreement with the rotation found for this part by Pease in 1918, and indicates that the core has a period of rotation of about 1.1×10^7 years. The period of the outer arms, on the other hand, is about 9.2×10^7 years, and the obvious interpretation of the nearly constant angular velocity from a radius of 20 minutes of arc outward is that a very great proportion of the mass of the nebula must lie in the outer regions. The problem of mass distribution will be taken up in a later section.

The one emission nebulosity observed on the minor axis is doubtless close to the spiral plane, and therefore about 2700 parsecs from the nucleus, after correction for foreshortening. The spectrum of the object is strong, and the velocity, based on measurements of a plate showing nine bright lines, is -266 ± 17 km/sec. The fact that there is a difference between this and the value of -300 ± 10 km/sec. for the nucleus may be significant, but the true magnitude of the difference must be regarded as very uncertain.

In connection with the rotation of the outer spiral arms, it may be noted that Lindblad, from his theory of the development of the spiral structure,[15] has already arrived at the conclusion that "it seems possible that the spiral formation in the case of heavy spiral structure moves with approximately uniform angular speed of rotation in approximately circular orbits." Lindblad's theory requires that the rotation of the spirals be in such a direction that the concave side of each arm precedes the convex side, and this direction is the opposite of that which both Slipher[16] and Curtis[17] thought most probable from their observational data. The latter observers based their opinions on the spectrographic data for several nebulae, combined with the interpretation that the sides of the spiral exhibiting the more prominent obscuring or occulting effects were the nearer. In this connection, the nebula NGC 7331 was observed spectrographically by the writer. The inclination of the spectrum lines shows that the north end of the major axis is approaching, relatively to the nucleus. This spiral is one in which the arms are prominent, and in which the dark lanes are easily seen, owing to its favorable inclination. If, following Slipher and Curtis, one concludes that the strong dark lanes are more probably on the near side of the spiral, then the rotation is in the direct sense that the convex sides of the arms precede the concave sides.

With regard to future observations of the internal motions of the spiral nebulae, it appears that there should be many more opportunities for profitable observation both of M31 and of similar objects. It seems safe to predict that there are several additional emission nebulosities scattered along the spiral arms of M31 whose velocities can be obtained with existing instruments. These velocities should be of great value in supplementing those already obtained for the study of internal motions. In addition, it would be highly desirable to have an independent check on the velocities of the unresolved nebulosity of the central region of the spiral.

VI. Mass Distribution and Total Mass of M31

In order to make a preliminary calculation of the distribution of mass within the nebula, the latter will be represented by a model consisting of a relatively small sphere surrounded by three concentric, coaxial, similar, greatly flattened spheroids. The densities within the sphere and the three homogeneous shells surrounding it will be evaluated on the basis of Newtonian attraction to produce the central accelerations required to maintain the observed velocities. Since the outer spheroids are similar, the two outer shells are each spheroidal homeoids, and possess the convenient property that the resultant attractions on interior particles are zero. It is assumed that the parts of the nebula revolve in circular orbits, while the spiral distribution of mass and the dip to zero in the rotation curve at 8′ from the nucleus are neglected.

The radius of the central sphere, r_1, is taken as 4′, and this is also taken to be the semi-minor axis of the spheroid next outside it. The semi-major axes of the spheroids are given the values 32′.1, 69′.25, and 96′.5, respectively, corresponding to the radius vectors of the points of measured velocity. The eccentricities of the elliptical cross-sections of all the spheroids are the same, being equal to 0.922.

The central force on a unit mass pursuing a circular orbit with velocity V_c is

$$X = -\frac{V_c^2}{r} \tag{5}$$

For particles at each of the radii given above, this force is to be equated to the gravitational attraction of the mass within the spheroids. Let $S_j(r_i, \sigma_j)$ be the force ex-

[14] This correlation has also been brought out by Lindblad: *Arkiv för Matematik, Astronomi och Fysik*, **27** A, No. 2, 1939.

[15] *Zeits. f. Astrophys.*, **15**, 124, 1938.

[16] *Pop. Astron.* **29**, 272, 1921.

[17] *Publ. Lick Obs.* **13**, 52, 1918.

erted on a unit mass at distance r_i from the center by the j-th spheroid of semi-major axis r_j and density σ_j. Then

$$\left.\begin{aligned} X_1 &= S_1(r_1, \sigma_1) \\ X_2 &= S_1(r_2, \sigma_1 - \sigma_2) + S_2^s(r_2, \sigma_2) \\ X_3 &= S_1(r_3, \sigma_1 - \sigma_2 - \sigma_3) + S_2^d(r_3, \sigma_2 - \sigma_3) + S_3^s(r_3, \sigma_3) \\ X_4 &= S_1(r_4, \sigma_1 - \sigma_2 - \sigma_3 - \sigma_4) + S_2^d(r_4, \sigma_2 - \sigma_3 - \sigma_4) \\ &\quad + S_3^d(r_4, \sigma_3 - \sigma_4) + S_4^s(r_4, \sigma_4). \end{aligned}\right\} \quad (6)$$

The superscript s denotes that the attraction is for a particle on the equator of the spheroid; d, for a distant particle in the equatorial plane.

The expression for X_1 is only approximate, since the attraction of the next outer shell is not zero. However, the error should not be serious, for σ_1 is considerably greater than σ_2. The other attractions in this model are essentially exact.

In every case the attraction of the sphere, S_1, is simply

$$S_1 = -\frac{4}{3}\frac{\pi G r_1^3}{r_i} \quad (7)$$

The attraction of a homogeneous spheroid for a particle on its equator is

$$S^s = -2\pi G\sigma r \frac{\sqrt{1-e^2}}{e^3}\left[\sin^{-1} e - e\sqrt{1-e^2}\right] \quad (8)$$

and the attraction of a homogeneous spheroid for a distant particle in its equatorial plane is

$$S^d = -\frac{4\pi G a^2 c\sigma}{3r^2}\left[1 + \frac{3}{10}\frac{c^2 e^2}{r^2} + \dots\right] \quad (9)$$

where a, b, and c are the semi-axes of the spheroid.

The velocity data are taken from Table 3 and from Figure 4, and they are listed in Table 4. The conversion factor for distances in the nebula is $1' = 1.85\times10^{20}$ cm. G, the constant of gravitation, is 6.66×10^{-8} cm^3 gm^{-1} sec^{-2}.

Upon substituting the forces and the appropriate expressions for the attractions in equations (6), one can solve for the densities of the four concentric shells in turn, beginning with σ_1. The resulting values are listed in Table 4, as are the volumes of the corresponding shells and the masses of each. It may be pointed out that since the eccentricity of the cross-sections of the spheroids is

TABLE 4

Shell	$a=b=r$	c	V_c	X	Volume	Density	Mass
1	4′	4′	100×10^5 cm/sec	13.5×10^{-8} dynes	0.170×10^{64} cm^3	6.54×10^{-22} gm/cm^3	1.11×10^{42} gm
2	32.1	4	203	7.21	10.78	1.79	19.3
3	69.2	8.63	280	6.06	99.05	0.612	60.6
4	96.5	12.3	375	7.90	194	0.62	120.3
							201×10^{42} gm

more or less arbitrary, the thickness of the model and the absolute values of the computed densities are only rough approximations, but the relative values of the densities and the computed masses are not thus restricted. The total mass of M31 is thus found to be 201×10^{42} gm., or $1.02\times10^{11}\odot$. No allowance has been made for the mass of the very faint extensions of the nebula beyond a diameter of $3^\circ.2$.

Zwicky has emphasized the uncertainties in the calculation of masses of nebulae from spectroscopic rotations.[18] These objections apply particularly to the unresolved central portions of the nebulae, where it may be that what is observed is only the resultant of the uncoördinated motions of many stars. Constant angular velocity does not necessarily imply uniform distribution of mass. In the model used above, the masses calculated for the inner sphere and the first surrounding shell may be considered only as minimum values. For the outer regions of the nebula, however, these objections to the calculation of mass from the observed rotations of isolated objects should not be serious, unless the effects of "internal gravitational viscosity" of the spiral are much greater than has heretofore been supposed in systems of this kind.

[18] *Ap. J.* **86**, 217, 1937.

VII. The Mass-Luminosity Ratio in M31

Estimates of the density both of mass and of luminosity, and of the ratio of these quantities for different parts of the nebula, were made by Hubble[1] in 1928. With the data now available, these estimates can be revised and extended somewhat, although they remain rough approximations.

Redman and Shirley[12] have made a photometric study of the Andromeda Nebula, including the distribution of luminosity along the major axis, and the total luminosity. Their result for the latter quantity is $3^{\rm m}.6 \pm 0^{\rm m}.3$ (photographic). This is substantially brighter than the earlier figures, and Redman and Shirley suggest that in round numbers the total luminosity is $4^{\rm m}$, although they are inclined to believe that the nebula is brighter than this. The absolute visual magnitude which follows is -18.7, on the assumption that the color index is $+0^{\rm m}.5$.

The total volume of the nebula, on the basis of the model used in the preceding section, is 1.04×10^{11} cubic parsecs, and the calculated mass is 1.02×10^{11} ☉. It follows that the mean luminosity density, in absolute visual magnitudes, is 8.85 per cubic parsec, and that the average mass per cubic parsec is 0.98 ☉. The total luminosity of M31 is found to be 2.1×10^{9} times the luminosity of the sun, and the ratio of mass to luminosity, in solar units, is about 50. This last coefficient is much greater than that for the same relation in the vicinity of the sun. The difference can be attributed mainly to the very great mass calculated in the preceding section for the outer parts of the spiral on the basis of the unexpectedly large circular velocities of these parts.

An approximate notion of the variation of the mass-luminosity relation from the nucleus to the outer parts of the nebula can perhaps be gained from the available data. The model of flattened spheroidal shells surrounding a small central sphere used in the preceding section will be considered again.

Allowing for the inclination of the plane of the spiral, let the line of sight be directed to the major axis of the nebula at points 0'.5, 15', 50', and 80' from the nucleus. Imagine a column of cross-section one square parsec extending through the nebula along the line of sight at these points. Using the computed densities of the various shells, one can make a rough graphical summation to obtain the masses in solar units of the four columns of matter, as shown in the second row of Table 5. The third row gives the volume of each column in cubic parsecs, and in the fourth row are listed the mean densities in solar masses per cubic parsec. The next row gives the figures of Redman and Shirley for log I at these points on the major axis of the nebula. Their intensity unit is such that log $I=2$ corresponds to a photographic surface brightness of $16^{m}.2$/(second of arc)^2. From values of log I and the volume of each column in the line of sight are computed the luminosities in terms of suns per cubic parsec in the next row, and finally the ratio of mass to luminosity in solar units is given for the different radii in the nebula. The relation between mass and luminosity for the nucleus itself was estimated by Hubble as

$$\text{Mass} = 0.001\ L,$$

and for this there seems to be no possibility of improvement at present.

The coefficients for the mass-luminosity relation given in the table evidently indicate little more than orders of magnitude. The mass densities are especially uncertain in the central core, where they are probably too small, so that the corresponding mass-luminosity coefficients near the nucleus may be considered minimum values. Nevertheless, the great range in the calculated ratio of mass to luminosity in proceeding outward from the nucleus suggests that absorption plays a very important rôle in the outer portions of the spiral, or, perhaps, that new dynamical considerations are required, which will permit of a smaller relative mass in the outer parts.

TABLE 5

MASS-LUMINOSITY RELATIONS IN M31

x (distance from nucleus)	0'	0'.5	15'	50'	80'
Mass of column (☉)		11000	7900	4100	2200
Volume (cu. psc.)		5400	5300	4500	2400
Mass density (☉/cu. psc.)		2.1	1.5	0.9	0.9
Log I (Redman and Shirley)	(2.00)	1.29	0.10	9.44	9.00
I	(100)	19.5	1.26	0.276	0.100
Luminosity density (☉/cu. psc.)		1.25	0.0827	0.021	0.0144
M/L	0.001 (Hubble)	1.6	18	43	62

VIII. Comparison of the Galactic System and M31

Current theories of galactic rotation indicate that the stellar system is in rotation about a center distant some eight or ten kiloparsecs from the sun, and that the period of rotation of the sun about this center is of the order of 10^8 years. The radius of the Galaxy is estimated as 15,000 parsecs, and from the observed rotation effects, its mass has been calculated by various writers to be about 2×10^{11} ☉. Observation and theory seem to demonstrate that, in a wide region around the sun, circular velocities of the stars decrease with distance from the center.

The Andromeda Nebula and the Galaxy have many well-known features in common, but one outstanding discrepancy between the two systems has hitherto been in their diameters. The spiral arms of M31 can hardly be traced to a radius of more than 1°.6 or 6 kiloparsecs. Beyond this radius, no stars, comparable to the brighter stars in the vicinity of the sun, have been reported, although Hubble has discovered some outlying globular clusters[10] which lead him to suggest that the radius of the nebula, as outlined by these objects, may possibly be as great as 3½°. This large figure is supported by the photo-electric measurements of Stebbins and Whitford,[19] and by the micro-photometric measures of Shapley,[20] both of which indicate the existence of a widespread, faint, diffuse luminosity.

A new discrepancy is now directly apparent when the rotations of the two systems are compared, for the

[19] *Proc. National Academy of Sciences*, **20**, No. 2, 93, 1934.

[20] *Harvard Bull.* 895, 19, 1934.

nearly constant angular velocity of the outer parts of M31 is the opposite of the "planetary" type of rotation believed to obtain in the outer parts of the Galaxy.

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