

THE PHYSICAL STATE OF INTERSTELLAR HYDROGEN

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ABSTRACT

The discovery, by Struve and Elvey, of extended areas in the Milky Way in which the Balmer lines are observed in emission suggests that hydrogen exists, in the ionized state, in large regions of space. The problem of the ionization and excitation of hydrogen is first considered in a general way. An attempt is then made to arrive at a picture of the actual physical state of interstellar hydrogen. It is found that the Balmer-line emission should be limited to certain rather sharply bounded regions in space surrounding O-type stars or clusters of O-type stars. Such regions may have diameters of about 200 parsecs, which is in general agreement with the observations. Certain aspects of the problem of the ionization of other elements and of the problem of the relative abundance of the elements in interstellar space are briefly discussed. The interstellar density of hydrogen is of the order of $N = 3 \text{ cm}^{-3}$. The extent of the emission regions at right angles to the galactic plane is discussed and is found to be small.

I

The recent discovery, by Struve and Elvey,¹ of extended regions in the Milky Way showing hydrogen-line emission has opened up new and highly important possibilities for the study of the properties of interstellar matter. From the observed strength of $H\alpha$ in the emission regions, Struve² has calculated the density of interstellar hydrogen. Also, he has analyzed the problem of the ionization of interstellar calcium and sodium, taking account of the presence of ionized hydrogen.

In the present paper the problem of the ionization and excitation of interstellar hydrogen is first considered in a general way. Based upon the observed intensities of interstellar hydrogen emission, an attempt is then made to arrive at a picture of the actual physical state of interstellar hydrogen. Finally, certain aspects of the problem of the ionization of other elements and of the problem of the relative abundance of the elements in interstellar space are briefly discussed.

II

Eddington³ has considered the problem of the ionization of interstellar hydrogen and has expressed the opinion that, in a normal re-

¹ *Ap. J.*, **88**, 364, 1938; **89**, 119, 1939.

² *Proc. Nat. Acad. Sci.*, **25**, 36, 1939.

³ *M.N.*, **95**, 2, 1934; *Observatory*, **60**, 99, 1937.

gion of interstellar space, hydrogen is entirely un-ionized, the reason being that the ionizing ultraviolet radiation is strongly absorbed by the interstellar hydrogen. The result of the analysis given below tends to confirm Eddington's view with the modification, however, that high-temperature stars, and especially clusters of such stars, are capable of ionizing interstellar hydrogen in regions large enough to be of importance in problems of interstellar space.

In the immediate neighborhood of a star, interstellar hydrogen will be ionized. With increasing distance from the star the proportion of neutral hydrogen atoms increases, and hence the absorption of the ionizing radiation increases. Ultimately, the ionizing radiation is so much reduced that the interstellar hydrogen is un-ionized. Our problem is to derive an expression for the extent of the ionized region as a function of the temperature and absolute magnitude of the star and the density of interstellar hydrogen. A somewhat similar problem has been considered by the author in another connection.⁴

Consider a point in interstellar space at the distance s from a star of temperature T and radius R . Let the number of neutral hydrogen atoms per unit volume be N' ; the number of hydrogen ions, N'' ; and that of free electrons, N_e . The degree of ionization at s is determined⁴ by the equation

$$\frac{N''N_e}{N'} = \frac{(2\pi m_e)^{3/2}}{h^3} \frac{2q''}{q'} (kT)^{3/2} e^{-I/kT} \cdot \sqrt{\frac{T_{el}}{T}} \cdot w e^{-\tau_u}. \quad (1)$$

Here I is the ionization potential; q'' and q' are the statistical weights of the ion and the ground state of the atom, respectively; $\sqrt{T_{el}/T}$ is a correction factor to allow for the difference between the temperature T of the exciting star and the electron temperature T_{el} at s (cf. Rosseland⁵); while $e^{-\tau_u}$ measures the reduction in the ionizing ultraviolet radiation due to absorption, τ_u being the optical depth from the ionizing star to the point s . Finally, w is the dilution factor at s , given by

$$w = \frac{R^2}{4s^2}. \quad (2)$$

⁴ G. P. Kuiper, O. Struve, B. Strömgren, *Ap. J.*, **86**, 570, 1937; sec. III, especially pp. 593 f. and 612.

⁵ *Theoretical Astrophysics*, chap. xxii, Oxford, 1936.

We shall assume that practically all the free electrons are furnished by ionization of hydrogen, so that $N'' = N_e$. The validity of this assumption for actual interstellar space will be discussed in section V. Further, let

$$\left. \begin{aligned} N &= N' + N'' , \\ N'' &= xN , \\ N' &= (1 - x)N , \\ N_e &= xN , \end{aligned} \right\} \quad (3)$$

so that x is the degree of ionization of hydrogen. Introducing numerical values, equation (1) can now be written in the form

$$\frac{x^2}{1 - x} N = C_1 \cdot \frac{1}{s^2} \cdot e^{-\tau_u} , \quad (4)$$

with

$$\left. \begin{aligned} C_1 &= 10^{-0.51 - \theta I} \cdot \frac{2q''}{q'} \sqrt{\frac{T_{e1}}{T}} T^{3/2} \cdot R^2 , \\ \theta &= \frac{5040^\circ}{T} . \end{aligned} \right\} \quad (5)$$

The numerical factor of C_1 corresponds to the following choice of units: 1 parsec = $3.08 \cdot 10^{18}$ cm for s ; the solar radius for R ; and cm^{-3} for N .

Let the absorption coefficient for the ionizing radiation per neutral hydrogen atom be a_u . We simplify the problem by assuming a_u to be independent of the frequency and equal to its value at the absorption edge. The relevant range of frequency is relatively small, so that the accuracy will be sufficient for our present purpose. Then, by definition,

$$d\tau_u = (1 - x)N a_u \cdot 3.08 \cdot 10^{18} ds , \quad (6)$$

the factor $3.08 \cdot 10^{18}$ being derived from the choice of the parsec as the unit of s .

Equations (4) and (6) define the solution of the problem of deriv-

ing the degree of ionization as a function of the distance from the star. Solving (6) for $1 - x$ and substituting in (4), we get

$$e^{-\tau_u} d\tau_u = \frac{N^2}{C_1} x^2 s^2 \cdot 3.08 \cdot 10^{18} a_u ds. \quad (7)$$

We introduce the following new variables:

$$y = e^{-\tau_u} \quad (1 \geq y > 0), \quad (8)$$

$$dz = \frac{N^2}{C_1} \cdot 3.08 \cdot 10^{18} a_u \cdot s^2 ds, \quad (9)$$

with $z = 0$ for $s = 0$, so that

$$s = \left(\frac{3C_1}{N^2 \cdot 3.08 \cdot 10^{18} a_u} \right)^{1/3} \cdot z^{1/3}. \quad (10)$$

Then (4) and (7) can be written as

$$\frac{dy}{dz} = -x^2, \quad (11)$$

$$\frac{1-x}{x^2} = a \frac{1}{y} z^{2/3}, \quad (12)$$

with

$$a = \left(\frac{9}{NC_1 \cdot (3.08 \cdot 10^{18} a_u)^2} \right)^{1/3}. \quad (13)$$

Since $\tau_u = 0$ for $s = 0$, we have, according to (8) and (10),

$$y = 1 \text{ for } z = 0. \quad (14)$$

In the cases of actual interest, a is a small quantity. When $a \ll 1$, it follows from (12) that, as long as y is not small compared to 1, $1 - x \ll 1$, so that x is equal to 1, very nearly. In that case it follows from (11) and (14) that

$$y = 1 - z \text{ for } 1 - x \ll 1. \quad (15)$$

Consequently, when α is small, ionization is strong until z is nearly equal to 1, so that (cf. [15]) y becomes a small quantity. With increasing z the degree of ionization now decreases very abruptly, so that, for z slightly greater than 1, hydrogen is practically un-ionized.

The value of s corresponding to $z = 1$, which we shall call s_0 , is, according to (10),

$$s_0 = \left(\frac{3C_1}{N^2 \cdot 3.08 \cdot 10^{18} a_u} \right)^{1/3}. \quad (16)$$

The result of the analysis can be stated as follows: for small α the ionization is nearly complete up to the distance from the ionizing

TABLE 1

x^2	s/s_0		
	$\alpha = 0.001$	$\alpha = 0.01$	$\alpha = 0.1$
1.0.....	0.000	0.000	0.00
0.8.....	1.000	0.988	0.82
0.6.....	1.002	1.009	0.97
0.4.....	1.003	1.020	1.05
0.2.....	1.004	1.028	1.12

star s_0 , given by (16), while there is almost no ionization for distances greater than s_0 .

The abrupt change of the ionization can be interpreted in the following way. Once the proportion of neutral atoms begins to increase, the absorption of the ionizing radiation increases, leading to an accelerated increase of neutral atoms. (For a somewhat more detailed discussion of this phenomenon see the investigation by the author quoted above.⁴)

The exact dependence of the degree of ionization x upon the distance s from the ionizing star has been derived for three different values of α by numerical integration according to (11), (12), and (14). The results are shown in Table 1. For $\alpha = 0.01$ the results are given in greater detail in Table 2. Finally, it may be noticed that for small α the following relation, obtained by integrating (11),

(12), and (14), putting $z = 1$, holds very nearly in the transition region between almost complete ionization and negligible ionization:

$$z = \text{Const} - \alpha \left\{ \frac{1}{1-x} + 2 \ln \frac{x}{1-x} \right\} (\alpha \ll 1, |1-z| \ll 1). \quad (17)$$

Numerical values of $z + \text{constant}$, normalized to 0 for $x^2 = 0.5$ and calculated, according to (17), as a function of x^2 , are shown in Table 3. The table shows, for instance, that with $\alpha = 0.001$ the

TABLE 2

 $\alpha = 0.01$

z	s/s_0	$y = e^{-\tau u}$	τu	$1-x$	x^2
0.0.....	0.00	1.00	0.00	0.000	1.00
0.2.....	0.58	0.80	0.22	.004	0.99
0.4.....	0.74	0.60	0.50	.009	0.98
0.6.....	0.84	0.41	0.89	.017	0.97
0.8.....	0.93	0.22	1.52	.037	0.93
0.9.....	0.97	0.13	2.06	.064	0.88
1.0.....	1.00	0.046	3.07	.15	0.72
1.1.....	1.03	0.0018	6.3	0.67	0.11

TABLE 3

x^2	$z + \text{Constant}$ Normalized to 0 for $x^2 = 0.5$	x^2	$z + \text{Constant}$ Normalized to 0 for $x^2 = 0.5$
0.1.....	+5.3 α	0.6.....	-1.7 α
.2.....	3.8	.7.....	4.2
.3.....	2.6	.8.....	8.6
.4.....	+1.3	0.9.....	-20.1
0.5.....	0.0		

change from $x^2 = 0.5$ to $x^2 = 0.1$ takes place for a change in z equal to 0.0053, corresponding to a change in s of about one-sixth of 1 per cent.

For small α the relative extent of the transition region is proportional to α . From (13) and (16) it follows that αs_0 is independent of C_1 , i.e., of the properties of the ionizing star. Therefore, as long as α is small, the absolute width of the transition region does not vary with the properties of the ionizing star.

Combining equations (5) and (16), we find

$$\left. \begin{aligned} \log s_0 = -6.17 + \frac{1}{3} \log \left(\frac{2q''}{q'} \sqrt{\frac{T_{el}}{T}} \right) - \frac{1}{3} \log a_u - \frac{1}{3} \theta I \\ + \frac{1}{2} \log T + \frac{2}{3} \log R - \frac{2}{3} \log N. \end{aligned} \right\} \quad (18)$$

This relation holds for any element. Introducing the proper numerical values for hydrogen, viz., $q''/q' = \frac{1}{2}$, $I = 13.53$ volts, and $a_u = 6.3 \cdot 10^{-18} \text{ cm}^{-2}$, we get

$$\left. \begin{aligned} \log s_0 = -0.44 + \frac{1}{3} \log \left(\sqrt{\frac{T_{el}}{T}} \right) - 4.51\theta + \frac{1}{2} \log T \\ + \frac{2}{3} \log R - \frac{2}{3} \log N. \end{aligned} \right\} \quad (19)$$

Table 4 gives $\log s_0$ for hydrogen for $R = 1$ and $N = 1$ as a function of T . The temperatures have been so chosen as to correspond to the spectral types from O5 to B5, according to the temperature

TABLE 4

Sp.	$\log T$	θ	$\log s_0$ for $R=1$ and $N=1$	s_0	$\log a$ for $R=1$ and $N=1$
O5.....	4.90	0.063	1.73	54 parsecs $\times R^{2/3} N^{-2/3}$	7.46-10
O6.....	4.80	.079	1.60	40	7.59
O7.....	4.70	.100	1.46	29	7.73
O8.....	4.60	.126	1.29	20	7.90
O9.....	4.50	.158	1.10	13	8.09
B0.....	4.40	.200	0.86	7.2	8.33
B1.....	4.36	.219	0.75	5.6	8.44
B2.....	4.31	.245	0.62	4.2	8.57
B3.....	4.27	.269	0.49	3.1	8.70
B4.....	4.23	.295	0.35	2.2	8.84
B5.....	4.19	0.324	0.20	1.6	8.99

scale recently derived by Kuiper.⁶ In making the calculation the factor T_{el}/T has been put equal to 1. With $T_{el}/T = \frac{1}{4}$ (cf. Rosse-land⁵), all values of $\log s_0$ would have to be decreased by 0.10, which would correspond to a decrease of the s_0 -values of 21 per cent. Table 4 also gives a according to (13) for $R = 1$ and $N = 1$; a is proportional to $R^{-2/3}$ and to $N^{-1/3}$.

For any given temperature of the ionizing star, s_0 increases with

⁶ *Ap. J.*, **88**, 429, 1938.

the stellar radius R as $R^{2/3}$. If a region of interstellar space is ionized by a cluster of n similar stars, close together, then s_0 has to be calculated with an equivalent R equal to $n^{1/2}R$. This follows immediately from equation (2) for the dilution factor w . Consequently, s_0 for such a cluster is equal to $n^{1/3}$ times s_0 , calculated for the individual star in the cluster. This may also be expressed by saying that the volume of interstellar space ionized by a cluster of stars close together is equal to the sum of the individual volumes that would be ionized by the stars if placed so far apart in interstellar space that the volumes did not overlap.

Table 5 gives s_0 for main-sequence stars of spectral types O5–B5. The assumed visual magnitudes⁷ from which R was computed, using

TABLE 5

Sp.	T	M_{vis}	s_0
O5.....	79,000°	-4 ^m 2	140 parsecs $\times N^{-2/3}$
O6.....	63,000	-4.1	110
O7.....	50,000	-4.0	87
O8.....	40,000	-3.9	66
O9.....	32,000	-3.6	46
B0.....	25,000	-3.1	26
B1.....	23,000	-2.5	17
B2.....	20,000	-1.8	11
B3.....	18,600	-1.2	7.2
B4.....	17,000	-1.0	5.2
B5.....	15,500	-0.8	3.7
A0.....	10,700	+0.9	0.5

Kuiper's bolometric corrections,⁶ are also given. The values of R range from about 4 to 7 solar radii. An increase in brightness of 1 mag. would lead to an increase of s_0 by a factor of 1.36.

The increase in the ionized volume as one passes from low-temperature stars to high-temperature stars is so pronounced that it may be concluded that the total volume ionized by all high-temperature stars (hotter than about B2) is much larger than that ionized by all low-temperature stars, in spite of the much greater number of the latter. For instance, one O7 star ionizes a volume equal to that ionized by about two thousand B3 stars or by about five million A0 stars.

⁷ R. Trumpler, *Lick Obs. Bull.*, No. 420, 1930.

In deriving the values given above, certain simplifying assumptions have been made. The effect of a possible decrease of the factor $\sqrt{T_{el}/T}$ has already been discussed. A resulting decrease of the s_0 -values on this account by more than, say, 20–30 per cent is not very probable. It has also been pointed out that a certain mean value of a_u , taken over the relevant frequency range of the ionizing radiation, should have been used instead of a_u at the absorption edge. It is estimated that this effect will lead to an increase of s_0 of probably less than, say, 20–30 per cent. Finally, the ionizing radiation emitted by the interstellar hydrogen has been neglected in calculating the ionization. An upper limit to the change of s_0 resulting from this effect is obtained in the following manner. Upon absorption of a quantum of ionizing radiation by an interstellar hydrogen atom there may follow reradiation of a quantum of the same frequency through a capture process to the ground state. In the case of hydrogen under the general conditions in question it is known that about one-third of the captures of free electrons by ions are to the ground state, the rest being to excited states. Therefore, about two-thirds of the absorption processes from the ground state are effective in removing entirely the quantum of ionizing radiation. The remaining one-third of the absorption processes also act to reduce the resulting ionization, through the scattering involved. The upper limit referred to is obtained by neglecting the effect of the scattering, thus introducing an effective absorption coefficient equal to $\frac{2}{3} a_u$ in (16). The maximum influence is therefore an increase of about 15 per cent in s_0 . Altogether, it is estimated that the s_0 -values calculated are sufficiently accurate for a first survey of the problem. It would not, however, present very great difficulties to develop a theory that would take the effects mentioned more accurately into account.

We have tacitly assumed that ionization by electron collisions is unimportant compared with photo-ionization. The justification for this assumption lies in the fact that, in interstellar space, the radiation from the ionizing stars is the primary energy source. Without making detailed calculations, we assume that any tendency for collisional ionization to become of importance is checked by a corre-

sponding decrease of the electron temperature. The mean free path of the electrons is so small as not to interfere appreciably with such an adjustment of the electron temperature.

Applications of the theory developed in this section are given in sections IV and V.

III

We shall consider in this section the problem of the excitation of the higher energy-levels of hydrogen in interstellar space. We shall limit ourselves mainly to the consideration of the level $n = 3$, which determines the emission of $H\alpha$ radiation. The discussion is naturally divided into two parts, dealing with ionized and nonionized regions, respectively.

In the almost completely ionized region surrounding a high-temperature star or a cluster of such stars, the mechanisms of excitation of one of the higher levels—say $n = 3$ —can be classified as follows, leaving out of consideration mechanisms that are entirely negligible: (*a*) capture of a free electron by an ion to the state considered; (*b*) capture of a free electron by an ion to a higher state, followed by one or more transitions leading through the state considered; (*c*) Lyman-line absorption from the ground state to the state considered, and Lyman-line absorption from the ground state to higher states, followed by one or more transitions leading through the state considered; (*d*) Balmer-line absorption from the state $n = 2$, either directly to the state considered or to a higher state followed by cascading, as before; and (*e*) excitation by electron collision from the ground state to the state considered or to higher bound states, followed by cascading.

In investigating these mechanisms we can largely make use of the results of investigations of the analogous problem in the case of planetary nebulae.

Consider first mechanism (*a*). The effective cross-section $\pi\sigma^2(n)$ for capture of a free electron of kinetic energy E_e to the state with quantum number n of hydrogen is given by a well-known formula (cf., e.g., Cillié⁸) which follows immediately from the Sugiura-

⁸ *M.N.*, **92**, 820, 1932; **96**, 771, 1936.

Gaunt formula for the continuous absorption cross-section and the Milne relation between continuous absorption cross-section and capture cross-section for reverse processes:

$$\sigma^2(n) = 1.43 \cdot 10^{-33} \frac{g \nu_g}{n \nu} \frac{1}{E_e}, \quad (20)$$

where g is the Gaunt correction factor, which is approximately equal to 1 ($g = 0.91$ for $n = 3$), while ν_g and ν are the frequency of the absorption edge and of the radiation emitted at the capture, respectively (so that $h\nu = h\nu_g + \chi_n$, when χ_n is the binding energy of the state considered).

With the aid of the well-known gas-kinetic formula, giving the number of captures corresponding to a given capture cross-section (cf., e.g., Fowler⁹), and using the mean lifetimes for the various states of hydrogen, as tabulated, for example, by Bethe,¹⁰ the number of atoms per unit volume N_n in any state n can now be calculated easily (cf. Cillie⁸). For $n = 3$ one finds

$$\frac{N_3}{N''N_e} = 6.3 \cdot 10^{-20} \cdot T^{-1/2} \cdot J \text{ (excitation by mechanism [a] only)}. \quad (21)$$

Here J denotes the proper mean value of the factor ν_g/ν in (20). For very small temperatures, J is approximately equal to 1; for $T = 5000^\circ$, $J = 0.81$; for $T = 20,000^\circ$, $J = 0.57$; and for $T = 50,000^\circ$, $J = 0.40$.

It may be noted that, in the particularly simple case of excitation by mechanism (a) there is no difficulty at all in treating the substates of the state with $n = 3$ separately. When this is done, the numerical factor in (21) is increased by a factor of about 1.5, the number N_3 being so defined that the actual number of transitions to the state $n = 2$ is given by the product of N_3 and the standard transition probability τ_{32} , obtained by averaging according to the statistical weights.

The case of excitation by both mechanisms (a) and (b) has been

⁹ *Statistical Mechanics*, Cambridge, England, 1936.

¹⁰ *Handb. d. Phys.*, 24, 1, chap. iii, 1933.

treated by Menzel and by Menzel and Baker.¹¹ These authors give the ratio $N_n/N''N_e$ in terms of the corresponding ratio for thermodynamical equilibrium. In thermodynamical equilibrium we have

$$\frac{N_3}{N''N_e} = 10^{-14.44+1.50\theta} \cdot T^{-3/2} \text{ (thermodynamical equilibrium).} \quad (22)$$

In the case of excitation by mechanisms (a) and (b)

$$\left. \begin{aligned} \frac{N_3}{N''N_e} &= b_3^{(A)} \cdot 10^{-14.44+1.50\theta} \cdot T^{-3/2} \\ &\text{(excitation by mechanisms [a] and [b]),} \end{aligned} \right\} \quad (23)$$

where $b_3^{(A)}$ is tabulated by Baker and Menzel¹¹ (their hypothesis A_2) as a function of the electron temperature T_{el} . The values of N_3 given by (23) are naturally larger than those given by (21). For $T_{el} = 5000^\circ$ the increase is given by a factor about equal to 2; with increasing temperature the factor tends toward 1.

The case of excitation by mechanisms (a), (b), and (c) has been considered by Cillie⁸ and by Menzel and Baker.¹¹ Following Zanstra,¹² they treated the case in which the exciting Lyman-line radiation, as well as the ionizing radiation, which determines the relative number of atoms in the ground state (from which the excitation considered takes place), is given by a common dilution factor times the Planck intensities for a common temperature. In this case the transitions from any excited state to the ground state are exactly balanced by the reverse Lyman-line excitations, and the treatment is analogous to the treatment of the case (a) + (b), except that all transition probabilities to the ground state are put equal to zero.

In the particular case of excitation by mechanisms (a), (b), and (c) just considered,

$$\left. \begin{aligned} \frac{N_3}{N''N_e} &= b_3^{(B)} \cdot 10^{-14.44+1.50\theta} \cdot T^{-3/2} \\ &\text{(particular case of excitation by mechanisms [a],[b], and [c]),} \end{aligned} \right\} \quad (24)$$

¹¹ D. H. Menzel, *Ap. J.*, **85**, 330, 1937; D. H. Menzel and J. G. Baker, *Ap. J.*, **86**, 70, 1937; J. G. Baker and D. H. Menzel, *Ap. J.*, **88**, 52, 1938.

¹² *Pub. Dom. Ap. Obs., Victoria*, **4**, 200, 1931; *Zs. f. Ap.*, **2**, 1, 1931.

where $b_3^{(B)}$ is tabulated by Baker and Menzel (their hypothesis B) as a function of the electron temperature.

With regard to the importance of excitation by mechanism (d), we shall only consider two limiting cases. If the excitation of the state $n = 2$ is normal for interstellar space, the role of mechanism d is entirely negligible. If, however, the excitation of the state $n = 3$ is supernormal to such an extent that the ionized region becomes opaque in the frequencies of the Balmer lines, then the relative population of the state $n = 3$, excited by mechanisms (a), (b), (c), and (d), approaches the value given by expression (22), valid for thermodynamical equilibrium, with the temperature about equal to that of the ionizing star. (For the possibility of superexcitation of the state $n = 2$, owing to the metastability of the $2s$ -substate, see Chandrasekhar and Stoy, as quoted by Cillié.⁸)

With regard, finally, to the importance of excitation by mechanism (e), it is easily seen that in an almost completely ionized region, and for temperatures of the order of those to be expected in actual applications to problems of interstellar space, the mechanism is unimportant. The average capture cross-section for the capture of a free electron by an ion, to the state $n = 3$, will, for the temperatures in question, be of the order of magnitude 10^{-21} . The cross-section for excitation by an electron of a neutral atom in the ground state to the state $n = 3$ is of the order of magnitude 10^{-17} , if the energy of the electron is high enough to excite the state $n = 3$ at all. The fraction of electrons that have sufficient energy for excitation is of the order of magnitude 10^{-3} . When, as in the case under consideration, the ionization is so high that the number of ions is, say, 10^2 – 10^4 times the number of neutral atoms, then mechanism (e) will be unimportant compared with mechanisms (a) and (b).

It is interesting to note, as can easily be shown, that even in such cases of less complete ionization, when mechanism (e) is of importance, the ratio $N_3/N''N_e$ will approach the value given by equation (23), the reason being that, for high-level states like $n = 3$, the cross-section for electron ionization from the ground state is considerably larger than the cross-section for electron excitation of that particular state from the ground state (cf. Bethe¹⁰). In interstellar space, moreover, we assume that electron ionization is unimportant com-

pared with photo-ionization as a consequence of an adjustment of electron temperature. This effect also limits the importance of mechanism (e) in the case considered.

Table 6 illustrates equations (21), (22), (23), and (24), valid for the various cases considered. However, the table does not give the ratio $N_3/N''N_e$ directly. It gives, with a view to the application in the following section, the value of N_e , calculated with $N'' = N_e$ and $N_3 = 3 \cdot 10^{-21}$. For other values of N_3 , N_e may be obtained by

TABLE 6*

θ	T	(1) Thermodynamical Equilibrium; Special Case of Excitation (a)+(b)+(c)+(d)	(2) Special Case of Excitation (a)+(b)+(c) (Eq. [24])	(3) Excitation (a)+(b) (Eq. [23])	(4) Excitation (a) (Eq. [21])	(4') Excitation (a); Sub- states Con- sidered Separately
1.0.....	5,040°	0.10 cm ⁻³	1.0 cm ⁻³	1.4 cm ⁻³	2.0 cm ⁻³	1.7 cm ⁻³
0.8.....	6,300	0.16	1.1	1.6	2.2	1.8
0.6.....	8,400	0.28	1.2	1.8	2.4	2.0
0.4.....	12,600	0.55	1.4	2.2	2.8	2.3
0.2.....	25,200	1.3	2.0	3.0	3.8	3.1
0.1.....	50,400	2.5	2.8	4.3	5.2	4.2

* The table gives N_e for $N'' = N_e$ and $N_3 = 3 \cdot 10^{-21}$ cm⁻³.

multiplication with the square root of the factor by which N_3 differs from $3 \cdot 10^{-21}$.

It is interesting to note that for the whole range of excitation mechanisms considered, and also for the relevant range of temperatures, the values of N_e come out approximately the same. (As was noted above, in the special case of excitation [a] + [b] + [c] + [d] the temperature approaches the temperature of the ionizing star, so that the relatively small numbers in the upper left corner of Table 6 are irrelevant. For mechanism [a] + [b], say, the temperature in question is the electron temperature, and so may be considerably lower than that of the star; but here the change in the temperature affects N_e very little.) The relative insensitivity of the ratio $N_3/N''N_e$ here encountered is not surprising. We may mention the analogous example of the approach of the central intensities of stellar

absorption lines to the value for thermodynamical equilibrium, for temperatures high compared with the binding energy.

Further, it is of interest to note that for mechanisms (a), (b), (c), and their combinations the ratio $N_3/N''N_e$ is independent of the distance from the ionizing star. This is not true for mechanism (e), which, however, as we have seen, is unimportant in the present connection.

We shall now consider briefly the actual mechanism of excitation in the ionized regions around the high-temperature stars. The problem is not quite simple, but it is suggested that, in the major part of the ionized region, Lyman-line radiation is present almost only in the form of $L\alpha$, the radiation in the other Lyman lines having been converted by repeated fluorescence processes into $L\alpha$, Balmer lines, Paschen lines, etc.; also, that the ionized region is transparent in the frequencies of the Balmer lines (cf. above), so that the excitation of the state $n = 3$ and of higher states is by mechanism (a) + (b) (Table 6, case 3).

In the paper quoted at the beginning of this article, Struve² calculated the ratio $N_3/N''N_e$ in an ionized region of interstellar space by a procedure leading to equation (22) (Table 6, case 1), namely, the use of the Boltzmann and the Saha formulae with a common dilution factor which drops out in forming $N_3/N''N_e$. It follows from the previous discussion that the value of the ratio obtained by Struve will not differ very much from those obtained on any of the other hypotheses discussed. The improved method for calculating $N_3/N''N_e$ briefly outlined by Struve is essentially equivalent to that used here in case 3, i.e., the case which probably corresponds most closely to actual conditions.

We finally have to discuss the excitation of hydrogen in the practically un-ionized regions considered in section II. Nearly all the radiation beyond the Lyman limit, and also that in the Lyman lines $L\beta$, $L\nu$, etc., has here been converted into $L\alpha$ and low-frequency lines. Free electrons, supplied by elements of lower ionization potential than hydrogen, will all have relatively low kinetic energy. Therefore, in these parts of interstellar space there is almost no excitation of the state $n = 3$ or of higher states. The state $n = 2$ is probably superexcited by a factor of 10^3 , very roughly, owing to the

conversion of continuous radiation into $L\alpha$, while the metastability of the substate $2s$ is of no consequence here.

Eddington³ has suggested that hydrogen in un-ionized regions of interstellar space is present in molecular form. It is not intended to attempt a complete solution of this problem here, but a few points may be noted. If the equilibrium between hydrogen atoms and molecules is determined principally by one process and its reverse process, the ratio of atoms to molecules will be given by an equation analogous to (1). (In interstellar space the lowest vibrational and rotational level only will be excited. The reduced mass $\frac{1}{2}m_H$ occurs instead of m_e , however.) If the frequency of the radiation absorbed in the dissociation process considered were less than that of the Lyman limit, the factor corresponding to $e^{-\tau u}$ would be of the order of magnitude 1. In that case the ratio of molecules to atoms would be, roughly, $10^{-9}N$; i.e., the number of molecules would be negligibly small. If, however, the frequency of the dissociation process is greater than that of the Lyman limit—and this is certainly the case if the dominant dissociation and recombination processes are the same as in the laboratory—then the number of dissociations is practically zero. On the other hand, the reverse recombination process, which involves the collision of one normal and one excited hydrogen atom, is extremely slow in interstellar space. Using the approximate value for the effective cross-section of this process given by Terenin and Prileshajewa¹³ and by Rompe,¹⁴ namely, 10^{-5} times the gas-kinetic cross-section, it is found that every hydrogen atom collides to form a molecule once in (very roughly) 10^{21} years. This means that molecule formation during any accepted time-scale is negligible. This, of course, does not definitely settle the question. It should be added that the mixture of material in the galactic system will tend ultimately to bring any molecules that may exist into ionized regions, where they will quickly dissociate. A sufficiently strong mixture will thus lead to practically complete dissociation of molecules into atoms.

IV

According to the analysis of the preceding sections, one would expect—if the density of interstellar hydrogen is within a certain range

¹³ *Phys. Zs. Soviet Union*, 3, 337, 1932.

¹⁴ *Phys. Zs.*, 37, 807, 1936.

—to find Balmer-line emission limited to certain rather sharply bounded regions in space surrounding high-temperature stars or clusters of high-temperature stars, the emission being of uniform strength, without any concentration toward particular stars, within these regions. With a view to the relative ionizing efficiency of O and B stars, as discussed in section II, one would identify the high-temperature stars in question principally with O stars.

Comparing this picture with the observations by Struve and Elvey,¹ referred to at the beginning of this article, of extended regions of $H\alpha$ emission, it is tempting to interpret the observed areas of uniform $H\alpha$ emission as the projections of bounded regions of interstellar space ionized principally by clusters, or condensations, of O stars. We shall, for the present, accept this interpretation as a working hypothesis.

Our working hypothesis may be regarded as a specialized version of the more general hypothesis put forward by Struve² in the investigation previously referred to.

Several clusters or clusterings of O stars are known.¹⁵ In accordance with our working hypothesis we connect the observed extended $H\alpha$ emission region in Cygnus with the cluster of O stars having its center at $l_g = 42^\circ$, $b_g = +1^\circ$. About ten of the known O stars presumably belong to this cluster. Likewise, we connect the extended $H\alpha$ emission in Cepheus with the condensation of O stars around galactic longitude 70° .

We can now immediately make use of the analysis of section III to derive the density of interstellar hydrogen in the regions in question. Struve^{1,2}, from the observed strength of the $H\alpha$ emission in Cygnus and Cepheus, derived $N_3 = 3 \cdot 10^{-21}$ cm⁻³, assuming the effective length of the emitting column to be 1000 parsecs. Using this number we estimate, with the aid of Table 6, that the density N of hydrogen is equal to $2-3$ cm⁻³. We have here assumed that practically all the free electrons result from hydrogen ($N'' = N_e$, and also $N'' = N$, the ionization being nearly complete). This is probable a priori and is further supported by reasons stated below. With an estimated depth of the emitting column of 300 parsecs,

¹⁵ C. H. Payne, "The Stars of High Luminosity," *Harvard Obs. Monograph*, No. 3, chap. vi, Cambridge, Mass., 1930; J. S. Plaskett, *Pub. Dom. Ap. Obs., Victoria*, 2, 339, 1924; O. Struve, *A.N.*, 231, 17, 1927.

instead of 1000 parsecs, one would find N equal to about 4 cm^{-3} . The resulting N is proportional to the square root of the assumed value of N_3 .

Assuming, for the present, that $N = 3$, we find, with the aid of Table 5, that the diameter of the region ionized by a cluster of n stars of the average spectral type O7 is about $80 n^{1/3}$ parsecs. With $n = 10$ (cf. above) we find a diameter of about 200 parsecs. This agrees about as well as could be expected with the estimated diameter of the Cygnus emission regions of, say, 200–300 parsecs.

The total intensity of a hydrogen-line emission area is proportional to the number n of ionizing O stars. The total area is proportional to $n^{2/3}$, the depth, and hence the surface intensity, to $n^{1/3}$.

With regard to the extension of the areas of $H\alpha$ emission in the direction at right angles to the galactic plane, consideration of the concentration of hydrogen toward the galactic plane is of importance. Calculations based on the assumption of a static interstellar hydrogen cloud may perhaps serve as a rough guide. Using the empirical field of force in the direction at right angles to the galactic plane as derived by Oort,¹⁶ it is found that the density of hydrogen decreases with the height above the galactic plane approximately according to a Gaussian error law, with a dispersion of about $1.3 \text{ parsecs} \times T^{1/2}$, where T is the temperature of the hydrogen gas. With $T = 10,000^\circ$, a dispersion of 130 parsecs results. Though the true distribution will probably differ from this, owing to the effects of turbulence, the order of magnitude should be correct.

It is, of course, very important to find how large a fraction of the relevant galactic stratum of hydrogen is ionized. From the estimated density of O and B stars, in connection with Table 5, assuming $N = 3 \text{ cm}^{-3}$, we are led to the very rough estimate that about one-tenth of the stratum is ionized. The numerical value of the fraction is inversely proportional to the square of the density of interstellar hydrogen.

As the observational material on extended areas of hydrogen-line emission accumulates, it will be possible to test the working hypothesis formulated in this section. The necessity of a critical test is obvious, particularly in view of the fact that a revision of the temperature scale might lead to a somewhat different picture of the relative

¹⁶ *Bull. Astr. Inst. Netherlands*, 6, 249, 1932.

importance of O and B stars and that a decrease in the accepted value of N might lead to a less pronounced separation between ionized and nonionized regions.

Further observations of hydrogen-line emission areas would be especially valuable, as they would lead to an improved knowledge of the relative extent of that part of the galactic stratum in which hydrogen is ionized.

At the present moment it can be said that a number of determinations, or estimates, of the density of interstellar hydrogen lead to values in approximate mutual agreement. First, Dunham's¹⁷ discovery of interstellar lines of neutral calcium and the discussion of this phenomenon by Dunham and Struve suggest as the most plausible choice of the value of the hydrogen density the maximum value compatible with the results of Oort's dynamical investigations, i.e., $N = 2-4 \text{ cm}^{-3}$ (cf. Struve²). The analysis based upon the strength of interstellar $H\alpha$ emission leads to a value of N around 3 cm^{-3} . The observed extent of $H\alpha$ emission areas confirms the approximate correctness of this value. Finally, it may be mentioned that Eddington's estimate, based upon the average separation of nebular condensations, when modified to correspond to a mean molecular weight of 1, leads to $N = 5 \text{ cm}^{-3}$.

V

We shall consider briefly the state of ionization of elements other than hydrogen. In those regions of interstellar space where hydrogen is ionized, the electron density N_e is equal to N . All ionizing radiation of wave lengths shorter than that of the Lyman limit comes from stars within the ionized region, i.e., particularly from the same stars which ionize the hydrogen. This radiation is not very much reduced by hydrogen absorption except near the boundary of the ionized region (cf. sec. II). In the case of sufficiently abundant elements of high ionization potential the ionization may be checked in a way analogous to that in which the ionization of hydrogen is checked. With obvious modification, the analysis in section II holds for such cases. Thus, it is seen that, if helium were as abundant as, or more abundant than, hydrogen, ionized helium would be restricted to a volume considerably smaller than that of ionized hydrogen. If, on the other hand, helium is considerably less abundant than hydro-

¹⁷ *Pub. A.S.P.*, **49**, 26, 1937; *Nature*, **139**, 246, 1937.

gen, the ionization of $He\ I$ to $He\ II$ will usually be almost complete up to the boundary of the region of hydrogen ionization; but in that case, of course, helium contributes considerably less to the number of free electrons than hydrogen. This argument tends to confirm the validity of our assumption that the free electrons are contributed mainly by hydrogen in those regions where hydrogen is ionized. It also shows that helium D_3 -line emission is handicapped in comparison with hydrogen Balmer-line emission.

Struve and Elvey^r observed the emission line $\lambda\ 3727$ of $[O\ II]$ in many of the areas where $H\alpha$ was seen, while N_1 and N_2 of $[O\ III]$ were rarely observed. As the mechanism of excitation of the lines is similar, this probably indicates that oxygen is present mainly as $O\ II$. Further observations on the occurrence of N_1 and N_2 may lead to valuable results, particularly so if it should turn out that the abundance of oxygen is high enough to restrict the ionization of $O\ II$ to $O\ III$ (ionization potential 34.9 volts) by a mechanism similar to the hydrogen mechanism.

Ultimately, the observed relative intensity of interstellar hydrogen and oxygen emission lines will lead to a reliable value of the relative abundance of interstellar hydrogen and oxygen. A very rough estimate is easily made. The effective cross-section for the excitation of the upper state of $\lambda\ 3727$ by electron collision is estimated to be somewhat smaller than 10^{-17} cm^2 . Assuming that somewhat less than one-tenth of the electrons have sufficient energy to excite $\lambda\ 3727$, and putting the cross-section for capture of a free electron to the state $n = 3$ equal to about 10^{-21} cm^2 , we derive from the observed fact that $\lambda\ 3727$ is somewhat fainter than $H\alpha$ that there are, roughly, 10^2 – 10^3 H ions to every $O\ II$ ion. Since hydrogen is largely ionized, and oxygen is mostly present as $O\ II$, the resulting interstellar abundance is 10^{-2} – 10^{-3} oxygen atoms to every hydrogen atom.

We consider next the ionization of elements with ionization potential less than that of hydrogen, in those regions where hydrogen is ionized. In this case the ionizing radiation comes from the entire galactic system, so that the customary calculations of the dilution factor and the effective temperature hold, while the electron density N_e is equal to N . It should be noted that the O stars which ionize the hydrogen do not contribute especially to the ionization of such elements as calcium and sodium.

Outside the regions in which hydrogen is ionized, there will be almost no ionization for ionization potentials larger than that of hydrogen. Of the most abundant elements, helium, nitrogen, and oxygen will be un-ionized, while carbon and the metals will be ionized. Of the abundant metals, only calcium will be doubly ionized. It may be noticed that titanium will be present mostly as $Ti\ II$. This is of interest in connection with Dunham's¹⁷ observations of interstellar $Ti\ II$ lines.

It is quite likely that in the regions of un-ionized hydrogen most of the free electrons are furnished by carbon. With an interstellar carbon abundance of the same order as the abundance derived for oxygen, we find that N_e is about 10^2 - 10^3 times smaller than in the regions where hydrogen is ionized. This figure is, however, as yet extremely uncertain. It is probably best to postpone the detailed discussion of the ionization of the metals until two basic constants are known with greater certainty, namely, the fraction of the galactic stratum in which hydrogen is ionized, and the ratio of electron density in regions of ionized and un-ionized hydrogen, respectively. With the present values for these constants it is found that interstellar $Ca\ I$ lines are almost entirely absorbed in the regions of ionized hydrogen, while $Ca\ II$, $Na\ I$, and $K\ I$ lines are also mostly absorbed here.

Restriction of the absorption of an interstellar line to a fraction of the galactic stratum naturally affects the abundances calculated from the observed intensities of the line. Increasing the abundance of calcium and sodium found by Struve² (forming an average abundance of the two elements) on this account, and using $N_e = 3$ in the absorption region, we arrive at a density of $3 \cdot 10^{-5}\text{ cm}^{-3}$. Adopting, for interstellar space, the relative cosmic abundances of oxygen, calcium, and sodium, as determined by Goldschmidt,¹⁸ the corresponding density of oxygen is found to be $5 \cdot 10^{-2}\text{ cm}^{-3}$. The resulting interstellar abundance is $2 \cdot 10^{-2}$ oxygen atoms to every hydrogen atom, which is of the same order as the value determined above from the strength of the line $\lambda\ 3727$. It is hardly necessary to emphasize the fact that some of the factors entering into this calculation will certainly need considerable revision.¹⁹

¹⁸ *Norske Videnskaps-Akademi, Oslo, Mat.-Naturv. Kl., No. 4, 1937.*

¹⁹ In a more refined calculation of the ionization of an element in interstellar space the effect of recombination processes to excited states, followed practically always by

Struve²⁰ has recently emphasized the relatively much greater sensitivity of the interstellar absorption-line method as compared with the interstellar emission-line method. Even so, it seems already quite clear that the emission-line method is capable of yielding, ultimately, extremely valuable results with regard to the determination of the interstellar density of the most abundant elements, all of which show no interstellar absorption lines in the accessible wavelength region.

The author is much indebted to Dr. O. Struve for placing at his disposal the results of investigations on the subject of this paper before their publication.

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cascading to the ground state, must be taken into account. The effect will always tend to decrease the degree of ionization. For hydrogen the effect is not very marked, leading to an extra factor on the right-hand side of equation (1) estimated to be about equal to $\frac{1}{3}$. This would correspond to a decrease of the s_0 -values calculated in the present paper by about 30 per cent. For elements like sodium and potassium, however, the effect is very marked. It is well known (cf. e.g., n. 4) that for these elements the absorption from the ground state is much smaller than for hydrogen, while, according to an unpublished investigation by Mr. M. Rudkjöbing, the excited states are approximately hydrogen-like with regard to continuous absorption. It follows that for these elements the number of recombinations to excited states is far greater than the number of recombinations to the ground state. In interstellar space the number of dissociation processes is, of course, proportional to the continuous absorption coefficient for the ground state. We conclude that the degree of ionization of sodium and potassium in interstellar space is quite considerably reduced by the effect in question. The abundances of sodium and potassium calculated from the strength of interstellar absorption lines are similarly reduced. According to an unpublished result by Dr. L. C. Green, kindly communicated to the writer, $Ca\ II$ is intermediate between sodium and hydrogen with regard to the continuous absorption from the ground state. Hence, a reduction of the calculated calcium abundance is indicated, but it is smaller than for sodium. Pending more accurate calculations, it is suggested that the remainder (cf. Struve²) of the well-known discrepancy with regard to the relative strength of interstellar $Na\ I$ and $Ca\ II$ lines will disappear when the effect just considered is taken into account. For $Ca\ I$ the continuous absorption coefficient from the ground state has not yet been calculated. It is very probable, however, that it is considerably smaller than for hydrogen, the excited states being more hydrogen-like with respect to continuous absorption. If this is the case, it follows that the interstellar ratio of $Ca\ I$ to $Ca\ II$ is considerably larger than that calculated from the customary equation. This would tend to reconcile the value of the electron pressure derived from the observed strength of $Ca\ I$ and $Ca\ II$ (cf. Dunham¹⁷ and Struve²) with that found in the present investigation.

²⁰ *Zs. f. Ap.*, 17, 316, 1939.