1938Obs....61..268T

JAMES GREGORY (1638–75).

By H. W. Turnbull.

The International Congress held by the Edinburgh Mathematical Society at St. Andrews July 4–14, 1938, provided a fitting opportunity to honour the memory of James Gregory, F.R.S. (1638–1675) in this the tercentenary year of his birth. Gregory has long been famous as the first inventor of the reflecting telescope: he was also a mathematician of supreme brilliance, who was outstanding even in the golden age of the history of science—the fifty years from the publication of the epoch-making books of Galileo and Descartes in 1638 until that of the *Principia* of Isaac Newton. The celebrations began with a meeting of the Edinburgh Royal Society on 4 July, when papers on the life and mathematical work of Gregory were read or communicated by various historians of science: while on Tuesday 5 July a graduation ceremonial took place in the Upper Library Hall of the University of St. Andrews, in the room where Gregory carried out so much of his work, both mathematical and astronomical. Besides many distinguished guests, who were present as members of the mathematical congress itself, there were others who came for this special ceremony as representatives of other Universities, of the Manse of Drumoak (which was Gregory's birthplace) and of the Grammar School at Aberdeen where Gregory was educated.

No account of Gregory would be complete without reference to the extraordinary and widespread talent of the family to which both he and his wife belonged. They were grandchildren of David Anderson of Aberdeen, a man of considerable scientific and mechanical sagacity who earned the name "Davie-do-a'-thing". He was the Archimedes of Aberdeen, who constructed the spire of St. Nicholas Church, raised its great bells into position, and removed "Knock Maitland', a submerged rock, from the harbour by harnessing it to the tide. His brother Alexander, who went to Paris as Professor of Mathematics, was a friend of Vieta and himself a talented geometer. Janet, the daughter of David Anderson, married the Rev. John Gregory, of Drumoak on Deeside, twelve miles from Aberdeen, and thereafter for two hundred years

© The Observatory • Provided by the NASA Astrophysics Data System

1938Obs....61..268T

their descendants occupied the chairs of mathematics, medicine or philosophy—twenty-four in number—in an unbroken sequence at Scottish Universities and occasionally at Oxford, always with vitality and distinction.

James, the greatest of all in this remarkable family, was the third son of John and Janet Anderson. He was born in November 1638 and died prematurely in October 1675 of a sudden illness accompanied by blindness, which befell him as he was observing the Satellites of Jupiter in company with his students at Edinburgh.

By the age of twenty-four years, James Gregory had written his *Optica Promota*, which he took to London, where it was published in 1663. It contains an elegant and geometrically accurate account of mirrors and lenses, beginning with the rediscovery of the sine-law (of Snellius and Descartes), which Gregory supported first by mathematical argument and next by careful experiments; this was followed by an account of a reflecting telescope. In London where he hoped to find an optician capable of constructing his telescope Gregory met Collins and Hooke, who put him in touch with a celebrated craftsman Reive. An attempt was made, but the resulting mirrors were a failure, and the project was abandoned—until Hooke in February 1674 succeeded, when he presented the first Gregorian telescope to the Royal Society; and the same form was universally employed in the eighteenth century. Meantime Isaac Newton, working independently at Cambridge or in retirement in the country near Grantham, had succeeded in making a reflecting telescope. His second such instrument which was presented to the Royal Society in 1672 brought him fame, and incidentally led, through the mediation of Collins, to an animated and friendly correspondence between these two rival inventors.

After leaving London Gregory spent three years in Italy (1664–1667), chiefly at Padua, where the great Galileo had taught. Gregory was deeply impressed with the doctrine of "indivisibles" of Cavalieri and his pupil de Angelis. This turned Gregory's attention towards geometry and particularly to what is nowadays termed the integral calculus. He wrote and published two short but remarkable books, one in 1667, the *Vera Circuli et Hyperbolae Quadratura*, and the next in 1668, the *Geometriae Pars Universalis*. The one was a bold attempt

© The Observatory • Provided by the NASA Astrophysics Data System

1938Obs....61..268T

to prove the transcendence of the well-known irrational numbers π and e, a task which none could complete until Lindemann found the clue late in the nineteenth century. Gregory approached these numbers by considering the area of a sector of a circle and of an hyperbola. In the second book he gave a systematic *geometrical* account of both differential and integral calculus, which he appears to have developed from the embryo state in which Cavalieri and Fermat had left the subject. The book contains the earliest proof that Fermat's method of tangents (differentiation) and Cavalieri's method of quadratures (integration) are converses of each other. The book profoundly influenced Barrow, whose masterpiece, the *Geometriae Lectiones* was published two years later in 1670.

Gregory spent three months of 1668 in London—a fruitful time when he first heard of the infinite series for $\log(1+x)$ newly discovered by Nicolas Mercator. Thereafter he turned from the geometrical to the analytical aspect of the calculus. He published a few of his immediate results in the autumn of 1668 entitling his booklet the *Exercitationes*, which contained a remarkable method of reversing series, a rigid proof of Mercator's theorem on the expansion of $\log(1+x)$, formulæ for integrating both $\sec x$ and $\tan x$, and the rule of approximate integration usually attributed to Simpson. By integrating $\sec x$ Gregory cleared up the mystery of the rhumb spiral, explaining why Wright's *Nautical Tables* of 1599 actually presented a set of logarithms, of which the author was unaware.

In 1668 Gregory was appointed first professor of mathematics in St. Andrews, where he resided until 1674. Partly owing to a conflict with Huygens over the *Vera Quadratura* and partly through hearing of Newton's success in the "analytics", Gregory never again published his mathematical work, beyond adding one short note to a vehement little book on the *New Art of Weighing Vanity*, written by a colleague, William Sanders, a regent at St. Leonard's College, St. Andrews. This book was directed against George Sinclair, a retired professor of natural philosophy at Glasgow, who had quarrelled with Sanders. The mathematical ammunition of the book was supplied by Gregory, while the appended note contained

© The Observatory • Provided by the NASA Astrophysics Data System

1938Obs....61..268T

the expansion of an elliptic integral in a doubly infinite series, solving the finite motion of a circular pendulum. In 1674 Gregory was appointed to the chair of mathematics in Edinburgh, which he held only for a year before he died. He married, in 1669, Mary, daughter of George Jamieson, the painter, and widow of Peter Burnet of Aberdeen, and they had two daughters and a son, James, afterwards professor of physic in King's College, Aberdeen (*d.* 1731).

At St. Andrews Gregory planned an observatory, the first of its kind to be erected in Britain, almost on the day when its prototype in Paris, begun some eight years earlier in 1664, was completed. Greenwich observatory was endowed and founded four years later, in 1676. Occasionally Gregory visited Edinburgh or London or Aberdeen; on one famous occasion he persuaded his townsfolk to hold a church-door collection throughout Aberdeen to supply instruments for the observatory at St. Andrews! The actual building stood for many years, situated about a hundred yards east of south from the Library, until in the early nineteenth century it was demolished when a road was widened. It is, however, doubtful whether Gregory ever worked in the observatory itself, since he left St. Andrews for Edinburgh in 1674. He certainly made observations from the long upper hall of the library, through its high windows facing the open country southward. His pendulum clock, made by Joseph Knibb of London is still in working order, with its large dial curiously divided into seconds and its small dial into hours. The quiet ticking breaks the silence and reminds us that, but a few years before the clock was made, Huygens had discovered the secret of the pendulum. Across the floor runs the meridian line, slightly askew, but truly north and south. It is directed towards an iron trident affixed to a stone and set on a hill about a mile away as an aid to Gregory's transit observations. A large bracket on the wall at the window still remains, upon which Gregory mounted his quadrant or his telescope. The bracket still bears traces of the screw adjustment whereby the axis was brought to a true level. A twelve-foot long sectional parchment tube still remains as evidence of his Galiliean telescope, but the lenses are lost. Gregory was in close touch with Cassini at the observatory in

© The Observatory • Provided by the NASA Astrophysics Data System

1938Obs....61..268T

Paris, and with Flamsteed of Derby, another young enthusiast. They compared notes, as, for example, of eclipses. A mighty snowstorm throughout Scotland robbed Gregory of cherished results during the Solar Eclipse, foretold by Flamsteed, of 19 April 1670, which, however, was seen in Paris. A joyful letter four years later tells us how Gregory and his friends in Paris made simultaneous observations of a lunar eclipse, which enabled Gregory to work out the longitude of St. Andrews, a difficult feat in days before the invention of the chronometer. In several places in his books and letters Gregory considers astronomical problems—sometimes physical and sometimes more purely mathematical. For instance, he observes the importance of making observations on the transit of the planets Venus or Mercury across the Sun for the purpose of measuring parallax: he deals with the problem of the distance to the nearest fixed stars by comparing the apparent brightness of Sirius with that of the planet, Jupiter, and arguing therefrom that the distance to Sirius must be many thousand times the diameter of the solar system. He also sought evidence of movement among the fixed stars due to the orbital movement of the Earth. This was excellent in theory, but impossible to carry out experimentally until modern refinements of lenses had been achieved.

On the mathematical aspects of astronomy Gregory had much to say. He solved many a geometrical problem arising from the still comparatively new theory of elliptical orbits due to Kepler. There was the well-known Kepler's Problem: in its simplest form it can be stated thus:—To find the angular position of a planet at a given time. Since equal areas are swept out by the radius from the Sun in equal times, the problem reduces to one of elliptical sectors, or still more simply to drawing a straight line through a given point in the diameter of a semicircle so as to divide the area in a given ratio. Christopher Wren solved this geometrically in terms of a cycloid and, about twelve years later, Gregory gave its analytical equivalent in the form of an infinite series which he sent to Collins in April 1672; but it was far from obvious how the series was obtained, for Gregory gave no clue to his method. This has, however, been rediscovered, thanks to his own rough notes which have been preserved on the backs of the

© The Observatory • Provided by the NASA Astrophysics Data System

1938Obs....61..268T

letters from Collins to Gregory, which first he and subsequently his family carefully preserved. These notes and letters came into the possession of the University Library, St. Andrews, about ten years ago, and careful scrutiny has unravelled their contents. The method employed by Gregory was that of expansion by successive differentiation, and thus a famous theorem which has hitherto been attributed to Brook Taylor in 1715 was actually known to Gregory forty-five years earlier. The notes also contain several other instances of the use of this theorem, besides a number of complicated examples of integration, and some beautiful proofs of problems in the Theory of numbers which Fermat and his school of French arithmeticians had set as challenges to the mathematical world.

One of these problems came to light very dramatically upon the back of a letter which reached Gregory in the summer of 1675, to which he sent a solution, as we learn from his friend James Frazer, in Paris, within two or three weeks of receiving it. It was the Six Square Problem—to find three whole numbers whose sums or differences, taken in pairs, should always form perfect squares. The problem was sent out as a general challenge, and Gregory alone solved it (one or two mathematicians a century later, including Euler, were the next to solve it). The result so pleased his friends in Paris that they sent him in return a problem on Cubes, of the same nature. Actually this has proved to be insoluble—but the proof has only been found in quite recent times. The attempt, which Gregory made within a few days of his death, to solve the problem, lay, like a Choral Prelude of J. S. Bach, unfinished on his desk. Gregory was indeed taken in his prime: and with his passing Scotland lost a man of genius who was reckoned among his peers to be second only to Newton.

The reputation of Gregory has suffered from eclipse by that of Newton, who stands unrivalled in the history of science. Yet many of Newton's greatest discoveries were also made by Gregory: notably the general formula for interpolation and the binomial theorem for a fractional exponent. There is a startling likeness to the famous Lemmas at the beginning of the *Principia* in a manuscript

© The Observatory • Provided by the NASA Astrophysics Data System

1938Obs....61..268T

of about a dozen propositions in the handwriting of James Gregory at the University Library of Edinburgh, written certainly ten years before the *Principia* appeared. As Collins once remarked, Newton showed a great "wariness to impart": consequently it was almost impossible for his contemporaries to find out just how far Newton had advanced in his speculations. Gregory learnt enough from Collins to realise that Newton had already discovered the differential and integral calculus some years before 1670, the year in which Barrow's book on the subject came out, when Gregory also made many of his chief discoveries in the same allied subjects. Gregory admitted the help which he derived from both these his rivals and friends, not realising that in respect of Taylor's series he had advanced further in the subject than either. Gregory considered that Newton, his junior by four years, had first hit upon the method of the calculus a year or two before he himself had done so, and therefore it was right to withhold publication until Newton had made known his own discoveries. But the reticent Newton never broke the silence until after the death of Gregory in 1675. Consequently the masterpiece of Gregory has lain hid throughout the centuries. Indeed, it is likely that, had Gregory been spared to live another few years, the controversy between Newton and Leibniz over priority in the discovery of the calculus, which dragged on so wearily at the close of the century, would never have taken place. With Newton, Barrow and Leibniz, Gregory takes an honoured place as a founder of the calculus.

CORRESPONDENCE.

To the Editors of 'The Observatory'.

Localized Solar Activity and Terrestrial Effects.

Gentlemen,—

The remark by Mr. H. W. Newton in the *Solar Notes* of the *Observatory* of 1938 May, that there had been a sequence of magnetic disturbances corresponding at the time of their respective commencements to solar longitudes between 80° and 180° from the beginning of 1937,

© The Observatory • Provided by the NASA Astrophysics Data System