THE INTERPRETATION OF ϵ AURIGAE

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ABSTRACT

The light-curve of this eclipsing binary is rediscussed with the help of the spectrographic elements determined at the Yerkes Observatory. The ratio of the radii and the mass ratio are determined with the assumption that the visible component falls upon the empirical mass-luminosity-curve. The orbital inclination is found to be about 70°. The spectrum is formed by the smaller, F2 star. The larger component has a temperature of about 1200°–1400° and gives no appreciable light in the region covered by the spectroscopic observations.

The spectroscopic and photometric observations during the eclipse show that the infrared star is semitransparent and that its nonselective opacity for visual and photographic light is concentrated in an outer shell. It is suggested that this effect is produced by photo-electric ionization from the F2 star.

The spectral lines of the F2 star are visible throughout the total phase of the eclipse. In addition, there are lines produced in the ionized shell of the infrared star. The latter are displaced by the rotation of this component, and the displacements and intensities are in accord with the hypothesis. The close proximity, before mid-eclipse, of the rotationally displaced lines of the infrared star to the normal lines of the F2 star results in a relatively small increase of the equivalent breadth. After mid-eclipse, when the two lines are better separated, the increase in the equivalent breadth is more pronounced. The equatorial velocity of rotation of the infrared star is approximately 50 km/sec.

The problem of the formation of an absorbing layer in the atmosphere of the infrared component by the radiation of the other component is investigated. Relations giving the extent of the region of the infrared star ionized by the outside radiation are derived. The optical thickness of the ionized region in visual and photographic light is calculated as a function of the maximum density along the ray traversing the atmosphere. An approximately constant optical thickness is found for a range of maximum density sufficiently wide to account for the observed constant minimum. A considerable excess of the amount of ionizing radiation over that calculated from Planck's law is required in order to give an optical thickness equal to that observed. The opacity of the non-ionized region of the infrared component between the ionized region considered and the observer is calculated. The effect of this opacity is found to be sufficiently small, provided the relative hydrogen content of the atmosphere of the infrared component is below a certain limit. Some aspects of the problem of line absorption in the atmosphere of the cool companion are considered. The results obtained are summarized, and the possible importance of a source of opacity other than the electron scattering and photoelectric transition opacity is briefly discussed.

INTRODUCTION

As is well known, the combination of the photometric and the spectroscopic data on ϵ Aurigae seems to lead to contradictions unparalleled in the study of other eclipsing systems. A beautiful constant minimum is found from the visual observations by Schmidt, reduced by Ludendorff,[1] or from the photoelectric observations[2] published by Huffer and by Miss Güssow. The light-curve suggests

[1] *A.N.*, **192**, 389, 1912.

[2] *Ap. J.*, **76**, 1, 1932; Veröff. Berlin-Babelsberg, **11**, No. 3, 193[illegible].

that the system is an eclipsing binary with a period of about twenty-seven years. This interpretation is in agreement with the spectroscopic orbit published by Ludendorff[3] and with an orbit just derived at the Yerkes Observatory, which includes recent observations. The epoch of the eclipse agrees approximately with the predicted time based upon the spectroscopic orbit. Hence, there can be no doubt about the binary nature of ϵ Aurigae.

But there is a wide discrepancy between the ratio of the surface brightnesses of the components derived from the light-curve and that derived from the spectral types. Since the depth of the constant minimum is about 0.8 mag., it would follow that the components are about equally bright. The ratio of the radii, k, depends, of course, upon the assumed inclination of the orbit. For $i = 90^\circ$, Shapley, from the Schmidt-Ludendorff observations, found[4] $k = 0.35$; and Huffer, from his photoelectric observations, found $k = 0.318$. Since $k^2 = 0.1$, and the total brightnesses of the stars are equal, the ratio of the surface brightnesses is about 0.1. If the inclination is smaller than 90°, this ratio as Shapley has shown, decreases and becomes 0.01 for $i = 76^\circ$. These results lead to four apparent contradictions:

1. The extreme ratio of the surface brightnesses suggests a large difference between the spectral types, but in reality the spectral type during the constant minimum is nearly identical with that observed outside of the eclipse.

2. The conclusion that the two components are equally bright cannot be reconciled with the fact that only one of the F-type spectra is seen at maximum light, even at phases where doubling of the lines must be expected.[5]

3. The F-type spectrum measured at maximum shows small fluctuations in radial velocity superimposed over the regular orbital velocity, and these fluctuations persist during the constant minimum.[5] Also, the light variations found by Huffer and Stebbins are present during the minimum. Although not impossible, it is very improbable that two stars, having quite different radii and probably very different densities, have such similar fluctuations both in brightness and velocity.

[3] *Sitz. Acad. Berlin*, p. 49, 1924.

[4] *A.N.*, **194**, 229, 1913.

[5] Struve and Elvey, *Ap. J.*, **71**, 136, 1930; *Pub. Yerkes Obs.*, **7**, Part II, 1932.

4. The asymmetry of the lines, which should probably be attributed to a rotational effect produced by the atmosphere of the occulting star, extends into the constant part of the minimum,[5] where the total eclipse should, geometrically, remove all light from the eclipsed star.

These four inconsistencies may be removed by supposing, as has been done in the past, that the occulting star is semitransparent. But this model introduces new difficulties. As Struve and Elvey have pointed out,[5] the mass of the secondary is considerable, and a large opacity should be expected. Furthermore, even if the density gradient inside the star were zero, and hence if the star were homogeneous, the minimum could not be constant. Also, it is not clear why the spectrum during the constant minimum should be so similar to that outside of the minimum; many additional lines would be expected to be present.

In Section I, by Mr. Kuiper, it is shown, first, that the inclination of the orbit cannot be close to 90° but must be about 65°–70°; second, that this inclination results in a grazing eclipse, not a nearly central one; third, that only a *shell* of absorbing material surrounding the eclipsing star will be able to produce the observed type of minimum; and fourthly, that the physical conditions in the system are such that in the upper layers of the eclipsing star a shell of highly ionized material will be formed which is identical to the absorbing shell required by the light-curve.

In Section II, by Mr. Struve, the spectral features are discussed in the light of the new hypothesis.

In Section III, by Mr. Strömgren, the astrophysical aspects of the conditions existing in the atmosphere of the eclipsing star are developed.

I

Since the orbit of ϵ Aurigae is eccentric, the spectroscopic elements must be used in the photometric solution. We have adopted the following values, derived in Section II.

P = 9890 days = 27.08 years (Güssow)	K_1 = 15.7 km/sec
e = 0.33	T = 1924.2
ω = 350°	$a_1 \sin i$ = 2,014,000,000 km
V = −2.5 km/sec	$f(m) = 3.34\odot$

These elements are quite similar to those previously derived by Ludendorff. They relate to the cF2 component.

The value of k, the ratio of the radii, is smaller than 0.3, the value obtained for $i = 90°$. As will be seen later, k is probably smaller than 0.1. On account of the small value of k and the fact that fluctuations of small amplitude are superimposed on the general light-curve, it is impossible to obtain both i and k from the observed light-curve. We shall make solutions for different values of the inclination and then limit the range of probable inclinations from the results obtained.

The method used is believed to be considerably simpler than that used in the study of a similar system, ζ Aurigae.[6] This method is an adaptation of the Innes–van den Bos method of computation commonly used for orbits of visual binaries. The procedure is briefly described here.

The rectangular co-ordinates in the tangential plane of the eclipsing star with respect to the eclipsed star are[7]

$$x = AX + FY\,; \qquad y = BX + GY\,.$$

For a spectroscopic binary we may put conventionally $\Omega = 0°$. Taking the semimajor axis of the relative orbit, a, as unit, we have

$$A = +\cos\omega\,, \qquad B = +\sin\omega\cos i = -F\cos i\,,$$

$$F = -\sin\omega\,, \qquad G = +\cos\omega\cos i = +A\cos i\,.$$

If $i = 90°$, $B = G = 0$, and $y = 0$; the motion is then along the x-axis. From the value of ω (belonging to the eclipsing star) an estimate may be made of the range of values of the mean anomaly, M, that will apply to the neighborhood of the minimum. For a few representative values of M we compute the values of X and Y with the aid of the *Union Observatory Tables* of X and Y, and the resulting x-values are found. The problem is now to find two x-values, x_D and x_d, such that the time intervals between $-x_D$ and $+x_D$, and $-x_d$ and $+x_d$, are equal to D and d days, respectively. (D is the duration of

[6] Guthnick, Schneller, and Hachenberg, *Abh. Preuss. Akad. d. Wiss., Phys. Math. Kl.*, 1935.

[7] *Union Obs. Circ.*, No. 68, 1926; *B.A.N.*, **3**, 149, 1926.

the eclipse; and d, of totality.) This is most readily done if D and d are first converted into degrees of mean anomaly. From

$$R_1 + R_2 = x_D \qquad \text{and} \qquad R_1 - R_2 = x_d$$

the radii are found, expressed in terms of a, and hence in kilometers.

If the inclination is different from 90°, the projected distance between the centers of the stars is no longer x but $\Delta = \sqrt{x^2 + y^2}$. In this case Δ_D and Δ_d are found, similarly to x_D and x_d above.

In finding solutions for different values of the inclination it will be seen that the x-co-ordinates remain unchanged and that the y-co-ordinates are proportional to $\cos i$. The whole operation is reduced to a few multiplications and interpolations. Since ω in the spectrographic orbit is somewhat uncertain, the computation has been carried out for two extreme values, $\omega = 340°$ and $\omega = 0°$.

Table 1 gives the solutions for five different values of the orbital inclination, based upon the spectroscopic elements quoted and upon Huffer's constants of the minimum, $D = 754$ days and $d = 360$ days. The first four columns contain, respectively: (1) the assumed inclination; (2) the radius of the eclipsed F2 star, expressed in terms of a; (3) the radius of the eclipsing star in terms of a (the letter "I" is used to designate an infrared star); and (4) the maximum value of the dip of the center of the F star under the occulting limb of the I star as seen from the earth, expressed in a as unit.

Since $a_1 \sin i$ is known from the spectroscopic orbit, for each line of Table 1 the quantity $a_1 = [m_2/(m_1 + m_2)]a$ is known. Hence the radii of the two stars are known except for the factor $m_2/(m_1 + m_2)$. By adopting a value for this factor, the radii are fixed, and also the masses, because of the known mass function. For the F2 star we know the approximate effective temperature, $\log T_e = 3.80$, which, in connection with the radius, fixes the luminosity. We may now obtain an approximate value of the mass ratio by the condition that the F2 star shall lie on the empirical mass-luminosity relation. This relation is derived in a paper to be published soon. It appears that for each value of the inclination the mass ratio is fixed between rather narrow limits. The reason is that a change in the mass ratio affects the masses strongly but the luminosity only slightly. Hence, only a small range of mass ratios is compatible with the mass-luminosity relation, and in particular the luminosity should be well determined. The only exception is the solution for $i = 90°$, which leads to such a high luminosity that the nearly horizontal part of the mass-luminosity relation is reached. In this case the mass ratio is uncertain; but the luminosity should still be fairly accurate, at least for $\omega = 340°$.

TABLE 1

i (1)	R_F (2)	R_I (3)	Dip (4)	$\frac{m_F}{m_I}$ (5)	log R_F (6)	log R_I (7)	log m_F (8)	log m_I (9)	M_{pv}(F) (10)	T_e(I) (11)
	$\omega = 340°$									
90°	0.0572	0.1646	0.1646	3±	2.82	3.28	2.20	1.73	−9.9	2400
75	.0283	.3080	.0492	1.65	2.35	3.39	1.63	1.42	7.5	1450
70	.0221	.3776	.0366	1.40	2.21	3.45	1.51	1.37	6.8	1270
65	.0176	.4474	.0282	1.15	2.08	3.49	1.38	1.32	6.2	1120
60	0.0136	0.5143	0.0223	0.97	1.95	3.53	1.29	1.30	−5.5	1040
	$\omega = 0°$									
90	0.0657	0.1855	0.1855	*	......	......	......	......	......	......
75	.0382	.2958	.0655	2.4±	2.59	3.48	2.01	1.63	−8.7	1730
70	.0303	.3535	.0506	1.7	2.40	3.47	1.70	1.48	7.8	1460
65	.0243	.4127	.0405	1.4	2.27	3.50	1.56	1.41	7.1	1270
60	0.0194	0.4699	0.0322	1.15	2.15	3.53	1.44	1.38	−6.5	1190

* No solution of the mass ratio can be found which will bring the star on the average mass-luminosity relation.

The other columns of Table 1 contain: (5) the value of the mass ratio, found as described above; (6 and 7) the logarithms of the resulting radii (with the sun as unit); (8 and 9) the logarithms of the resulting masses (with the sun as unit); (10) the absolute photovisual magnitude of the F star, which happens to be equal to the absolute bolometric magnitude, since the bolometric correction is zero; (11) the computed effective temperature for the I star, based on the assumption that this star also falls on the mass-luminosity-curve (in this case the luminosity is found from the mass, and the luminosity and the radius define the effective temperature; even if this assumption is not strictly correct, the temperature found should be approximately correct, since errors in the luminosity are reduced by at least a factor of 4 in the temperature.)

The next step is to select from the solutions of Table 1 the most probable solution. The higher inclinations ($i > 70°$) lead to excessive absolute magnitudes for the F2 star. Since the color index of ϵ Aurigae is 0.4 mag., and stars of absolute photographic magnitude brighter than −6.5 are extremely rare in extragalactic systems,[8] we may conclude that very probably the inclination is not greater than 70°.

It is not only possible to give an upper limit for i, but a lower limit may also be found. A solution for $i = 30°$ (not included in Table 1) shows that for this inclination no eclipse takes place at all, even if the I star is as large as $R_I = (1 - e)a$, or $0.67a$, that is, extends up to the periastron distance of the F star, which, of course, is dynamically impossible. For $i = 45°$, a grazing eclipse is present if $R_I = 0.67a$ (for $\omega = 340°$). But since only a considerably smaller radius will lead to a tidally stable system, the inclination must be considerably larger than 45°. Even $i = 60°$ seems to be excluded by the stability argument. For this inclination the masses of the components are roughly equal, and by reason of symmetry for any stellar model the condition that the large star be stable even at periastron requires that $R_I < \frac{1}{2}(1 - e)a = 0.335a$.

Table 1 shows that this condition is not fulfilled. Inclinations higher than 72° or 73° make R_I smaller than this limit, but for these inclinations a different mass ratio is obtained which decreases the limit below $0.33a$.

Considering together the evidence of the two restrictions on the inclination, we believe that a value near 65° or 70° is the most probable. It is true that this value leads to an instability of the outermost layers of the I star at periastron. But the density of these layers is probably small enough to render the loss in mass insignificant, in spite of the fact that these layers have sufficient density to cause the eclipse. A more detailed discussion of this point will be given later.

[8] Hubble, *Ap. J.*, **84**, 164–165, 1936.

From the computations underlying Table 1 it follows that for $i = 70°$ the mean anomaly at the time of mid-eclipse is $69^\circ.0$ when $\omega = 340°$ and $50^\circ.8$ when $\omega = 0°$; for $i = 65°$ it is $67^\circ.2$ when $\omega = 340°$ and $49^\circ.4$ when $\omega = 0°$. There is a continuous change in this mean anomaly with the inclination, and in principle the inclination could be determined from the spectroscopic elements and the time of the eclipse Owing to the limited accuracy of the spectroscopic elements (largely because of the irregular fluctuations in velocity, hereinbefore mentioned), this is not feasible in practice.

If we adopt 70° as the most probable inclination and 350° as the most probable longitude of periastron, the mean anomaly at mid-eclipse is $59^\circ.4$; and from the photometric ephemeris, $P = 1929^a.326 +$

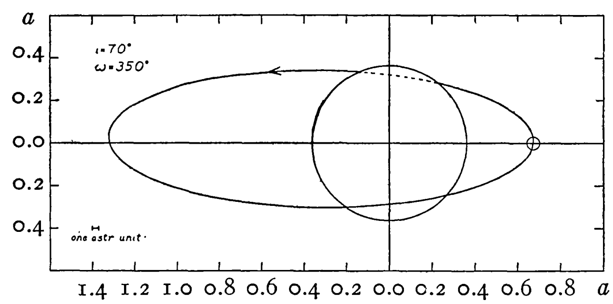

FIG. 1.—The projected orbit of ϵ Aurigae

$27^a.077E$, the epoch of periastron is found to be $T = 1924.86$. The fair agreement of the spectroscopic determination $T = 1924.2$ with the photometric determination shows that the spectroscopic and the photometric elements are consistent.

We shall now examine the consequences of the conclusion that the orbital inclination is close to 65° or 70°. The eclipse produced in this way is a grazing one, as is seen from the fourth column of Table 1, or from Figure 1, which is drawn for $i = 70°$. For this inclination the maximum value of the dip of the center of the F star below the limb of the I star is only 0.097 R_I when $\omega = 340°$ and 0.143 R_I when $\omega = 0°$; for 65° it is 0.063 R_I when $\omega = 340°$ and 0.098 R_I when $\omega = 0°$. This means that only the outermost layers of the I star are responsible for the observed eclipse. It seems possible, therefore, that these layers should be semitransparent, so that the F star is still visible during the minimum But the approximate constancy of the minimum seems to contradict this possibility. It is clear that a homogeneous sphere with a constant mass-absorption coefficient cannot produce a constant minimum. If there were an increase of density toward the center, the condition would be still more unfavorable. *Only a shell of absorbing material surrounding the I star* will be able to produce the type of minimum observed.

We have therefore to examine whether the physical conditions in the system will give rise to such a shell. This appears to be the case. The two stars have nearly equal mass, and probably comparable luminosity. If anything, the F star is slightly more massive and probably more luminous. The dimensions in the system are such that an element in the atmosphere of the I star on the side of the F star is nearly at the same distance from both stellar centers. Hence the amounts of radiation received from the two stars will be comparable. But the quality of the two radiations is very different, since the effective temperatures of the sources are about 6300° and 1300°, respectively. The ionization of the atmosphere of the I star, as far as it is exposed to the F star, will be almost completely governed by the radiation of the F star. It seems likely that a shell of ionized material will be formed in the exposed part of the atmosphere of the I star which, by electron scattering, reduces the intensity of the continuous light of the F star, seen through it. This explains why the visual and the photoelectric observations show the same depth of minimum, about 0.8 mag.: charged particles scatter in a nonselective manner. Furthermore, it seems probable that this ionized layer is in such a condition that it can enhance the intensity of a number of absorption lines of the F2 spectrum. In this way both the character of the photometric minimum, and the spectral changes, are at least qualitatively accounted for.

In Sections II and III a quantitative treatment of these problems is given. The dimensions of the system may be taken from Table 1, and to these may be added the distance of the center of the F star to the terminator of the illuminated part of the I star. From the mean anomaly at mid-eclipse for $i = 70°$ and $\omega = 350°$, viz., 59°, we find for the distance between the centers of the stars at mid-eclipse, $0.93a$. For that same inclination, $R_I = 0.37a$ and $R_F = 0.026a$ (Table 1). Hence, the distance of the center of the F star to the terminator is 0.85a, and the radius of the F star, as seen from the terminator, is 0.0306 radians = $1^{\circ}.8$. In other words, the diameter is seven times that of the sun as seen from the earth, and the surface temperature is slightly higher.

As we shall see, Section II leads to the same hypothesis which has been put forward here, and Section III is also consistent with it. We may, therefore, conclude that the model given follows logically from the observational data: the spectroscopic orbit, the light-curve, and the spectral type; that it removes the apparent contradictions found previously and explains the known phenomena satisfactorily; and that it presents us with a new astronomical phenomenon, an eclipse by a stellar Heaviside layer.

II

The elements of the orbit were derived[9] from the radial velocities plotted in Figure 2a. Open circles represent Potsdam results, while dots represent velocities obtained at the Yerkes Observatory. Each value represents the mean of several plates; an attempt was made to group the observations in such a way that the short-period oscillation would be smoothed out.

The spectroscopic phenomena during the eclipse have already been described.[10] A short time before the first contact nearly all strong lines of the F2 star become slightly unsymmetrical on the red side. This is accompanied by an appreciable increase in the equivalent widths of these lines. The asymmetry—never very large—reaches a maximum roughly at second contact. It diminishes rapidly and becomes zero at mid-eclipse. However, the intensities of the lines at mid-eclipse are somewhat stronger than those outside of eclipse. After mid-eclipse the lines become again unsymmetrical, with a strong core on the violet side. This asymmetry increases rapidly until it is very large at about third contact. Some of our best plates, taken in November and December, 1929, show the lines clearly double; but no attempt has been made to measure the com-

[9] Miss Frances Sherman assisted in this work.

[10] Struve and Elvey, *Ap. J.*, **71**, 136, 1930; Frost, Struve, and Elvey, *Pub. Yerkes Obs.*, **7**, 81, 1932; Adams and Sanford, *Pub. A.S.P.*, **42**, 203, 1930; McLaughlin, *Ap. J.*, **79**, 235, 1934; *ibid.*, **82**, 95, 1935.

ponents separately because the wings always overlap. The measures by Adams and Sanford prove that the duplicity is real. Our measures refer to the deepest point of the blended contour. After the end of the eclipse the strong violet components gradually fade out, and the spectrum resumes its normal aspect. The equivalent breadths reach a maximum near third contact.

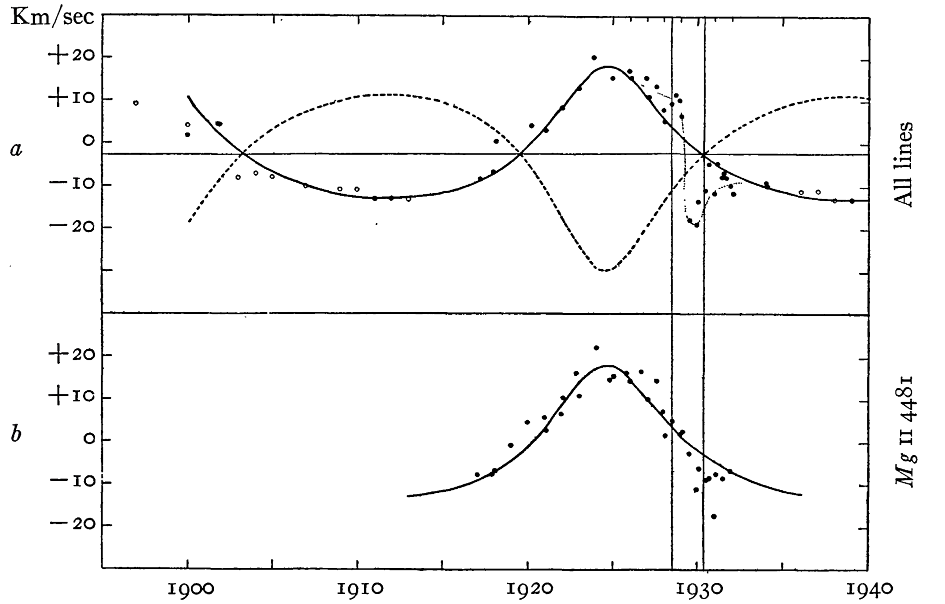

FIG. 2.—Velocity-curve of ϵ Aurigae. The two vertical lines indicate approximately the first and last contacts of the eclipse.

Since we do not know whether there are any lines of the F2 star which are unaffected at eclipse, we have derived the spectrographic elements from that part of the velocity-curve which lies outside the ranges 1927–1932 and 1900–1905. The elements are given in Section I. The velocity-curve of the F2 star, shown in Figure 2, represents these elements. The velocity-curve of the I star is shown for comparison. A mass ratio of $m_F/m_I = 1.3$ was assumed. The eccentricity is well determined, but the longitude of periastron is less certain. It must lie between 340° and 0°. The most probable value is 350°.

Figure 2*b* shows the velocities derived from the line *Mg* II 4481. The systematic departures preceding and following mid-eclipse are less marked, but the large negative residuals in 1930 and 1931 suggest that, although this line is less affected by the asymmetries, it does not exactly follow the velocity-curve. It is, therefore, useless to refer all measures to *Mg* II 4481, as was done in our former work.[10] Instead, the departures must be measured from the ephemeris.

We interpret the spectrographic phenomena in the following way:

1. When the F2 star begins to encroach upon the atmosphere of the I star, new lines, due to the latter, appear in the spectrum.

2. These lines are approximately—but not identically—the same as the normal lines of the F2 star. The strong ionized lines of *Fe* II and *Ti* II are strong in the atmosphere of the I star. *Mg* II is relatively weak. All faint lines are very weak in the I star; *Sr* II is probably relatively weaker in the I star. These phenomena can be explained as a combination of two effects: greater turbulence in the atmosphere of the I star than in that of the F2 star,[11] and a small real difference in spectrum. It is significant also that *Mg* II was absent in the expanding shells of 17 Leporis.[12]

3. Axial rotation shifts the lines of the I star from their normal position on the velocity-curve of this component of the binary. The rotation is therefore direct. The fact that the new lines fall above the velocity-curve of the F2 star in the first half of the eclipse is irrelevant: it simply means that the component of rotation in the line of sight at the proper latitude of the I star is larger than the orbital velocity of the F2 star.

4. The visibility of the rotationally displaced lines of the I star during the phase of the total eclipse proves that the continuous spectrum of the F2 star remains visible even at total phase.

5. The visibility of the normal F2 lines throughout eclipse also proves that the I star is semitransparent.

6. The beginning of the partial phase of the photometric eclipse does not coincide with the date of the first appearance of the additional lines. This suggests the existence in the I star of an outer atmosphere which produces appreciable line absorption but practically no continuous absorption.

7. The total equivalent widths of the blended lines vary continu-

[11] *Ap. J.*, **79**, 409, 1934.

[12] Struve, *ibid.*, **76**, 85, 1932.

ously, reaching a maximum near third contact. This seems at first sight to be in contradiction to the flat minimum of the photometric curve. In reality it is a consequence of the difference in velocity between the F2 lines and the rotationally displaced I lines. If the normal contour of an F2 line is $i_1(\lambda)$ and that of the I line is $i_2(\lambda + \Delta\lambda)$, where $\Delta\lambda$ is the velocity shift between the two lines, then the equivalent width of the blended line is

$$E = \int_{-\infty}^{+\infty} [1 - i_1(\lambda) i_2(\lambda + \Delta\lambda)] d\lambda \, .$$

Only when $\Delta\lambda$ is large, does

$$E = \int_{-\infty}^{+\infty} [1 - i_1(\lambda)] d\lambda + \int_{-\infty}^{+\infty} [1 - i_2(\lambda + \Delta\lambda)] d\lambda = E_1 + E_2 \, .$$

For overlapping lines, $E < (E_1 + E_2)$. The instrumental contour produced by the three-prism Bruce spectrograph of the Yerkes Observatory is easily obtained with the help of the curves published by Ornstein and Minnaert.[13] The focal length of the collimator is 100 cm, and its focal ratio is 1:19. Consequently, the parameter, σ, equals 5.8π. The corresponding intensity-curve for an infinitely narrow line may then be used to derive from the observed contour of a line in ϵ Aurigae the corresponding true contour. The theoretical contour for an infinitely narrow line is found to lie between the curves drawn by Ornstein and Minnaert for $\sigma = (16/3)\pi$ and $\sigma = (20/3)\pi$. This curve is almost to its full extent contained between the limits of $\pm 0.20 A$. Even allowing for additional broadening caused by imperfections in the optical parts of the spectrograph, the entire

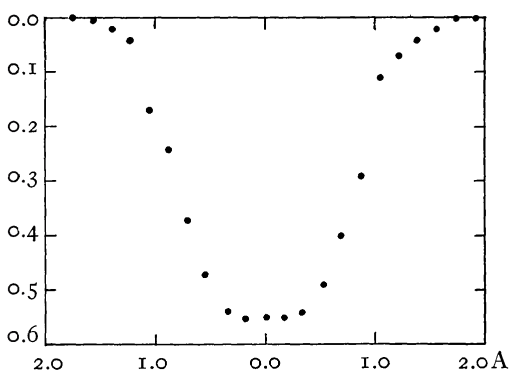

FIG. 3.—Contour of the absorption line *Ti* II 4400 in ϵ Aurigae, outside of eclipse.

[13] *Zs. f. Phys.*, 44, 404, 1927. Unpublished data by Messrs. Rust and Ebbighausen indicate that the labels for σ in Fig. 2 of the paper quoted should be multiplied by 0.5.

instrumental contour should lie within the limits of $\pm 0.25A$. The observed contour of the line *Ti* II 4400 (outside of the eclipse) is shown in Figure 3. The true contour differs so little from the observed contour[14] that it has not been drawn separately. It is clear that the contour has the typical shape of a line produced largely by turbulence,[11] and we shall therefore assume for the F2 star that

$$i_1 = 1 - ae^{-u\lambda^2} ,$$

where we shall understand under λ the difference $(\lambda - \lambda_0)$ measured from the center of the line due to the F star. The contours of the I lines are not accurately known, but it is probable that they are also dominated by turbulence. Accordingly,

$$i_2 = 1 - be^{-v(\lambda+\Delta\lambda)^2} .$$

The total equivalent breadth is

$$E = a\int_{-\infty}^{+\infty} e^{-u\lambda^2} d\lambda + b\int_{-\infty}^{+\infty} e^{-v(\lambda+\Delta\lambda)^2} d\lambda - ab \cdot e^{-v\Delta\lambda^2}\int_{-\infty}^{+\infty} e^{-(u+v)\lambda^2 - 2v\Delta\lambda \cdot \lambda} d\lambda = E_1 + E_2 - E_3 .$$

The last term is

$$E_3 = \sqrt{\frac{\pi}{u+v}}\, ab \cdot e^{-[(uv)/(u+v)]\Delta\lambda^2} .$$

Let us use a specific line, the central intensity of which is 0.4 in the F2 star and 0.2 in the I star. Let the total equivalent breadth of the F2 line be about 0.7A. Then $a = 0.6$, $u = 1.2$, and

$$E_1 = 0.6\sqrt{\frac{\pi}{2.1}} = 0.73\text{A} .$$

The contour of the I line is not known. From the measured contours it seems probable that, although the central absorption is

[14] This is also apparent from a comparison of the width of the absorption line λ 4400 in Plate III, *Pub. Yerkes Obs.*, 7, Part II, 1932, with the widths of faint comparison lines.

greater than that of the F2 line, the equivalent breadth is not larger. If we assume that $E_2 = E_1$, we have $b = 0.8$, $v = 3.8$, and

$$E_2 = 0.8\sqrt{\frac{\pi}{3.8}} = 0.73A .$$

We now compute E_3 for different values of $\Delta\lambda$:

$\Delta\lambda$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9A
E_3	0.25	0.25	0.24	0.22	0.20	0.18	0.16	0.13	0.11	0.08A
$\Delta\lambda$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8A	
E_3	0.07	0.05	0.04	0.03	0.02	0.01	0.01	0.00	0.00A	

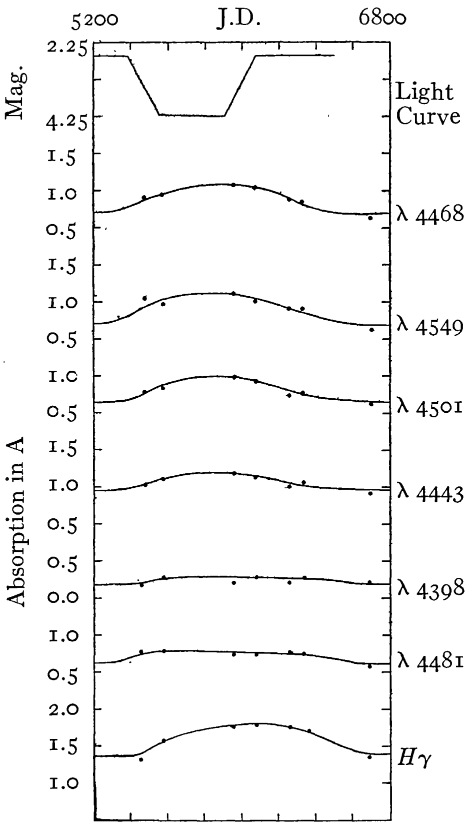

FIG. 4.—Equivalent breadths of typical lines in ε Aurigae, during the eclipse.

When $\Delta\lambda = 0$, the total equivalent breadth is $E = E_1 + E_2 - E_3 = 1.21$A. When the separation of the two lines corresponds to about 0.5A (or 33 km/sec) the total equivalent breadth increases to about 1.28A. This agrees fairly satisfactorily with the intensity curve[15] of *Ti* II 4468 in Figure 4. We observe from Figure 2*a* that $\Delta\lambda$ is of the order of 0.1A before mid-eclipse but reaches about 0.5A after mid-eclipse. The computed increase in the equivalent breadth of 0.07A between the beginning and the end of the eclipse is a little less than the actual amount observed for lines which outside of eclipse have equivalent breadths of about 0.7A. But by making the contours of both

[15] The equivalent breadths of all lines shown on our three-prism spectrograms have been measured during the eclipse and after its end. The lines in Figure 4 have been chosen at random to illustrate the behavior of strong and of weak lines.

lines narrower and deeper, we could have easily obtained a close approximation to the observed curves.

An entirely similar result is obtained if, by following Elvey, we use triangular contours for the two lines. By adjusting the available constants, we can fit, as accurately as we wish, the curves in Figure 4.

8. Figure 5 shows, for four inclinations, the projection of the orbit of the F2 star upon the disk of the I star. The component of axial

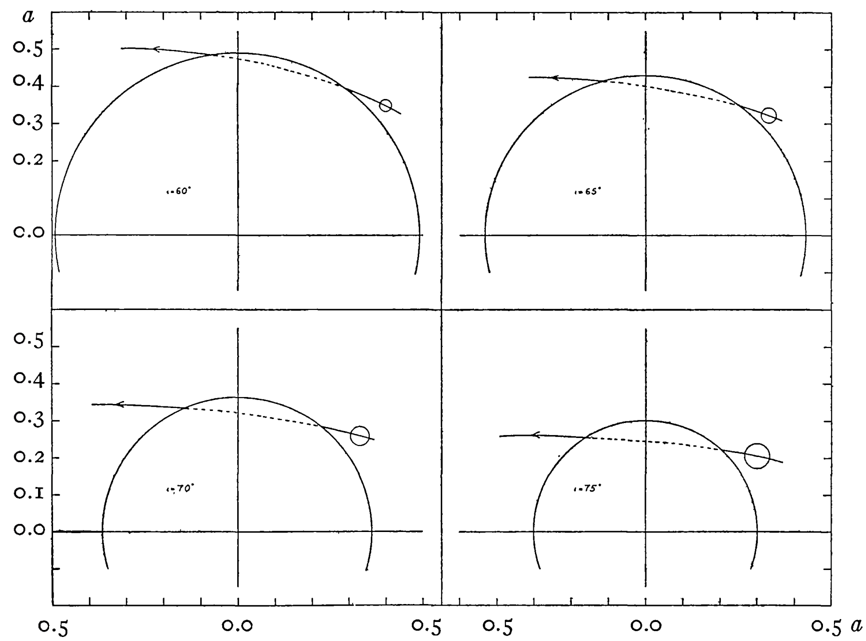

FIG. 5.—Illustration of the asymmetry in the rotation effect for $\omega = 350°$

rotation at the points of intersection is proportional to the x-coordinate of each point, provided the rotation is parallel to the plane of the orbit. The rotational effects with respect to the center of the I star are unsymmetrical. If $\omega = 350°$, we find[16] the ratio of the components of rotational velocity as shown in the following table. Since m_F/m_I depends upon i, we determine separately for

[16] The following figures refer to the points at which the center of the F2 star crosses the limb of the I star. These points are quite close to the times of maximum departures from the velocity-curve. Similar results would have been obtained if, instead of determining the rotational components for the times of disappearance and appearance of the center of the F2 star, we had determined them for second and third contact, respectively.

each value of the inclination the corresponding departure of the measured I line from the corresponding velocity-curve. If we assume

i	Ratio of Components of Rotational Velocity	i	Ratio of Components of Rotational Velocity
60°	4.1:1	70°	1.6:1
65	2.3:1	75	1.3:1

that the Yerkes measures refer to the I lines, corresponding to the deepest point on the contour, we find from the dotted curve in Figure 2*a* that the maximum residuals with respect to the center of the I star are in the ratio of 1.4:1, which would support a value of i close to 70°.

Adams and Sanford's measures gave:

	I Lines	F2 Lines
Nov. 15, 1929	−36 km/sec	+4 km/sec
Feb. 5, 1930	−35	+6

At the same time, the Yerkes measures for the blended lines gave −19 km/sec. It is probable, however, that Adams and Sanford's measures should be corrected for the effect of overlapping wings. We shall adopt −30 km/sec for the velocity of the I lines.

The Mount Wilson plates do not separate the components before mid-eclipse. We are therefore forced to adopt a somewhat arbitrary correction for the Yerkes measures. After mid-eclipse (December, 1929), we have:

Computed velocity of F2 star *minus* Yerkes measure of blended line	+20 km/sec
Computed velocity of F2 star *minus* Mount Wilson measure of I line	+31

The ratio is 1.55. Before mid-eclipse we have:

Computed velocity of F2 star *minus* Yerkes measure of blended lines	−6 km/sec

The factor of correction before mid-eclipse should be $1.55 < f < 2$, since with decreasing displacement between two unequal lines the blended wave length will lie more and more nearly halfway between the two centers. Adopting $f = 1.8$, we find, for the true difference, 11 km/sec. Referring these velocities now to the center of the I star, we compute for the times of the disappearance and reappearance of the centers of the F2 star the velocity of the center of the I star, making due allowance for the fact that m_F/m_I depends upon i and also upon ω. The measured components, corrected as has been explained, *minus* the velocity of the center of the I star are shown in Table 2. Almost the same effect would have been obtained if we had computed the departures, not for the times of appearance and disappearance of the center of the F2 star, but for second and third contact. The asymmetry is in the right direction, and its amount seems to favor $i = 70°$ or $75°$. But this result depends upon our assumption that the axis of rotation of the I star is perpendicular to the plane of the orbit. The velocities are not sufficiently precise to determine i accurately, but $i = 70°$ is probably a good compromise.

TABLE 2

		$\omega = 340°$	$\omega = 0°$
$i = 60°$	Disappearance	+31 km/sec	+34 km/sec
	Reappearance	−26	−27
$i = 65°$	Disappearance	+31	+35
	Reappearance	−26	−26
$i = 70°$	Disappearance	+32	+36
	Reappearance	−26	−26
$i = 75°$	Disappearance	+33	...
	Reappearance	−26	...

9. The spectral lines of the F2 star outside of eclipse are relatively deep. The central intensity of $H\beta$ is of the order of 0.3. At the node the velocity shift between the two components should be about 50 km/sec. Accordingly, if the conventional photometric solution were correct, the lines of the F2 star should be appreciably filled in by the continuous spectrum of the other component. The fact that there is no such filling-in proves that the continuous spectrum of the companion does not contribute an appreciable fraction of the light.

10. A new search for the lines of the second component has been made on good three-prism spectrograms at and near the time of passage through the ascending node of the F2 star. No trace of such lines could be found.

It is clear that the hypothesis of a semitransparent infrared star explains all spectrographic observations. The equatorial rotational velocity of this star is about 50 km/sec if $i = 70^\circ$ and if the axis is perpendicular to the plane of the orbit. It is surprising that, although the atmosphere of the I star is ionized by the F2 star, the spectral type of the rotational I lines is somewhat similar to that of the F2 star. But we have an analogous phenomenon in the expanding shells of 17 Leporis, P Cygni stars, and novae. In all of these objects the absorption lines adjust themselves rather closely to that spectral type which we would expect from the energy-curves of the stars.

III

As has been shown in the preceding sections, the analysis of the photometric and spectroscopic observations of ϵ Aurigae leads to the following conclusions:

1. Throughout the entire eclipse the light observed is due to the F component.

2. At light minimum about one-half of the light of the F component is absorbed during its passage through the outer part of the I component. In the visual and photographic regions the absorption is approximately independent of the wave length.

3. Within a certain range of dip of the F component under the effective boundary of the I component, corresponding to the phase of constant minimum, the absorption is, to a close approximation, independent of the dip.

It has been pointed out in Section I that, on the surface of the I star, owing to its very low temperature, the flux of ultraviolet radiation from the F star is very much greater than the corresponding flux from the F star. It was further pointed out that this circumstance might be expected to lead to the formation of a semiopaque layer in the outer part of the I star, of such a nature as to produce absorption of the visual and photographic light of the F star, as described in the foregoing summary.

A complete theory of the atmospheres of the components of a stellar system like ϵ Aurigae would necessarily be very complex. An attempt is made in this section to explain the problem of the absorption of the light of the F star in the atmosphere of the I star, through a discussion of certain simplified models. Though we cannot yet claim to know with certainty the details of the mechanism of absorption, we believe that the main features of the situation have been correctly understood.

We shall consider primarily the following mechanism of absorption of the light of the F star in the I star. The ultraviolet radiation of the F star produces ionization in a region of the I star that would, if the F star were not present or if it were at a much greater distance, be only slightly ionized and practically transparent to visual and photographic light. As the ultraviolet radiation penetrates farther into the I star, it is gradually absorbed. Thus, the region of the I star, ionized by the ultraviolet radiation of the F star, is limited as a consequence of the absorption of the ultraviolet radiation. The free electrons in the ionized region of the I star weaken the passing light of the F star. The weakening, due to electron scattering, is the same for all wave lengths.

In order to see whether this is a true picture in the case of ϵ Aurigae, we must investigate the following questions:

1. Does the picture lead to a sufficiently wide range in dip, within which the absorption is amply independent of the dip?

2. Is the absorption in visual and photographic light of the correct magnitude?

3. Would the region of the I star, to which the absorption has to be attributed, be transparent to visual and photographic light if the F star were absent (i.e., is the region of the I star between the ionized region and the observer transparent to visual and photographic light)?

4. Is electron scattering, influenced by the diluted radiation of the F star, the main cause of visual and photographic opacity in the region of the I star considered?

We shall now deal with these questions. The result is that the picture considered may be true if certain assumptions, which, of necessity, are as yet somewhat arbitrary, are taken for granted. Other possible mechanisms are also discussed in less detail.

Before developing the theory of the mechanism of absorption described, we shall discuss a few features of the situation in the case of ϵ Aurigae, making use of the data of the system derived in Section I. Only rough numerical values are needed for our purposes. In our calculations we shall use the following numerical values, which correspond to a value of the inclination of the orbit of 70°:

$R_I = 2.10^{14}$ cm	$T_I = 1400°$
Maximum dip of center of F star $= d = 2.4 \times 10^{13}$ cm	$\log m_I = 1.42$
$R_F = 1.4\ 10^{13}$ cm.	$\log R_I = 3.46$

From the values of m_I and R_I we can calculate the density gradient of the outer, practically isothermal, part of the atmosphere of the I star, assuming that it is given by the well-known relation valid for the case of hydrostatic equilibrium:

$$N = N_0 e^{\alpha_s h} , \tag{1}$$

$$\alpha_s = \frac{m_H \mu g}{kT} . \tag{2}$$

Here N denotes the number of particles per unit volume; h, the depth below a certain arbitrary boundary sphere where the value of N is N_0; μ, the mean molecular weight of the gas; g the surface gravity, i.e., Gm/R^2; k/m_H, the gas constant ($k/m_H = 8.26 \times 10^7$); and T, the temperature.

With $\mu = 1$, corresponding to a mixture in which hydrogen predominates, the following value of α_s is found from the data given above:

$$\log \alpha_s = -12.2 . \tag{3}$$

The assumption of a somewhat greater value of μ would not affect the final conclusions. This value of α_s leads to a density increase along a distance equal to the radius of the F star by a factor of about 10^4, and to a density increase by a factor of about 10^7 along a distance equal to the maximum dip. With the hydrostatic value of α, therefore, the cut-off of the I star is so sharp that the partial phase of the light-curve is due practically to the extent of the F star, while the change of density within the region which obstructs the light of the F star during the constant minimum is so high as to correspond to a factor of 10^7.

It is well known that the gradient in tenuous atmospheres due to turbulent motions or other deviations from the static case may be considerably smaller than the hydrostatic gradient. The sun's chromosphere is one example. Supergiant atmospheres have been considered recently from this point of view by Menzel[17] and by Pannekoek.[18] We describe the density distribution approximately with the aid of an exponential relation, as in the hydrostatic case:

$$N = N_0 e^{\alpha h} . \qquad (4)$$

The value of α may be considerably smaller than α_s. Menzel and Pannekoek consider values of α about one hundred times smaller than α_s. Now, in the case of ζ Aurigae, which is considered by Menzel, the assumption of an α-value that is so small apparently contradicts the photometric data, leading, when combined with the known radius of the B star of the system, to a duration of the partial phase which is too long (cf. Kuiper[19]). Still, a diminution factor, say of the order of magnitude 10, is entirely possible.

In the case of ϵ Aurigae the known duration of the partial phase similarly sets a lower limit to the density gradient, which can be assumed. This again leads to a minimum value of the density variation in the region through which the F star shines during the constant minimum. This density variation can hardly be assumed less than, say, 20.

We have thus made one of our problems (cf. 1, p. 588) more definite. The absorption has to be constant for a range in dips that corresponds to a range of density of the gas passed by at least a factor of 20.

From the foregoing considerations it follows that the numerical value of the constant α in (4), which governs the density distribution, probably lies somewhere between 10^{-12} and 10^{-13}.

It will appear from the following analysis that the accuracy with

[17] *Harvard Circ.*, No. 417.

[18] *B.A.N.*, **8,** 175, 1937.

[19] *Ap. J.*, unpublished.

which α is thus known is sufficient for the purposes of numerical discussion of the theory, the constant α being, in the final equations in question, only raised to the power of $\frac{1}{4}$.

We shall now develop the theory of ionization in the atmosphere of the I star by the ultraviolet radiation of the F star and of the electron scattering opacity thereby produced.

The order of magnitude of the electron pressure in the region with which we shall be concerned can be estimated at once. The electron-scattering coefficient is $0.4\ m_H$, or 7×10^{-25}, per electron. The optical depth corresponding to the electron scattering along the path of the radiation from the F star through the I star must be of the order of magnitude 1 in order to produce the observed absorption. On the other hand, the path along which the electron density is appreciable will be considerably smaller than the radius of the I star. It will be of the order of magnitude of the maximum dip. With a geometrical path of 10^{13} cm we find that a mean absorption coefficient of the order of 10^{-13} is required, and hence a mean number of electrons of the order of magnitude 10^{11} per cm^{-3}. The corresponding electron pressure is of the order of 10^{-1} dynes, or 10^{-7} atmospheres.

The number of free electrons and ions is therefore so high that the ionization equilibrium in a given element of volume is reached practically instantaneously when the radiation field changes. This simplifies the discussion of the ionization of the atmosphere of the I star by the radiation of the F star. The order of magnitude of the degree of ionization can also be estimated at once. As the ionizing radiation of the F star penetrates into the I star, it is gradually absorbed. At an optical depth in the wave lengths of the ionizing radiation (which we shall refer to as "ultraviolet" for the sake of brevity) of the order of magnitude 10, the ionizing radiation will be cut down so much that the ionization is inappreciable. We thus see that the ultraviolet absorption coefficient must not be more than about ten times larger than the absorption coefficient corresponding to electron scattering; otherwise the ionized region will not be sufficiently extended to give the required electron-scattering optical depth of the order of magnitude 1.

The ratio of the electron-scattering absorption coefficient to the ultraviolet absorption coefficient is equal to the product of the ratio of the scattering coefficient per electron, 0.4 m_H, to the ultraviolet absorption coefficient a_u per absorbing atom and the ratio of the number of electrons to the number of absorbing atoms. Now, the numerical value of a_u is of the order of magnitude $10^6 m_H$ (we shall deal with the question of the value of a_u in more detail later). We conclude that the number of absorbing atoms must be less than about 10^{-5} times the number of free electrons in order that the ratio of ultraviolet and electron scattering absorption be smaller than the required limiting value. The degree of ionization in the region which obstructs light by electron scattering must therefore be quite high.

It is clear that the derived value of the electron pressure, together with the degree of ionization just found, gives the order of magnitude of the amount of ionizing ultraviolet radiation from the F star which is required in order to produce the observed opacity in the visual and photographic regions.

We now proceed to the analytical discussion of the problem. We shall consider first the case that the ionizing F star, which is also the star for which we want to find the light loss in the visual and photographic region, is practically a point source as seen from the other star (the I star). Consider a ray from the point source through the atmosphere of the I star. The degree of ionization at any point along the ray is given with sufficient accuracy for our purpose by the following equation:

$$\frac{N''N_e}{N'} = 10^{15.4-\theta I}T^{3/2}we^{-\tau_u}\,. \tag{5}$$

The notation used is explained below:

N'' = Number of ions per unit volume
N' = Number of neutral atoms per unit volume
N_e = Number of free electrons per unit volume
T = Temperature
θ = $5040^\circ/T$
I = Ionization potential
w = The dilution factor of the ionizing star as seen from the I star
τ_u = The ultraviolet optical depth along the ray from the ionizing star to the point considered

Equation (5) is the ionization equation of the thermodynamical equilibrium modified to take account of dilution in the usual way and, further, to take account of the reduction of the ionizing radiation due to its passage through part of the I star.

For our purpose it will be sufficiently accurate to lump the metals together into one group with a certain mean ionization potential. Hydrogen is practically nonionized in the region in question. The role played by the ionization potential and the abundance of an element in the problem will become more clear at a later point in the discussion.

Let N be the sum of the number of ions N'' and the number of atoms N'; and let x, as usual, denote the degree of ionization:

$$\left.\begin{aligned} N'' &= xN \\ N' &= (1 - x)N \\ N_e &= xN \end{aligned}\right\} . \tag{6}$$

Introducing, further, the constant C,

$$C = 10^{15.4 - \theta T} T^{3/2} w , \tag{7}$$

we can write equation (5) in the form

$$\frac{x^2}{1 - x} N = Ce^{-\tau_u} . \tag{8}$$

The optical depth τ_u in the ionizing ultraviolet region can be expressed in terms of the absorption coefficient in the ultraviolet a_u per atom and the number of neutral atoms $N' = (1 - x)N$ per unit volume, which absorb the ionizing radiation:

$$d\tau_u = (1 - x)Na_u ds . \tag{9}$$

Here ds is the line element along the ray considered. Equations (8) and (9) now enable us to find the degree of ionization x as a function of position s on the ray considered. The analytical process of finding $x(s)$ is simplified by the following circumstance. As was shown by the qualitative discussion of the problem, the ionization in the region of the I star, where the ultraviolet radiation of the F star has not yet suffered serious loss by absorption, must be quite high. In fact, $1 - x$ must be as low as about 10^{-5}. According to equation (8), this means that e^{τ_u} is of the order of magnitude 10^4 before x begins to differ appreciably from 1. When e^{τ_u} has reached this magnitude, the absorption coefficient in the ultraviolet is so high [cf. (8 and 9)] that a relatively very small advance Δs along the ray cuts the ultraviolet radiation down so much that the degree of ionization x is practically zero. Consequently, the transition region in which x falls from a value practically equal to 1 to a value practically equal to zero is relatively very narrow. In calculating the optical depth due to electron scattering, we therefore obtain sufficient accuracy by dividing the path along the ray considered in two parts: one, $s \leqq s_0$, for which $x = 1$; and the other, $s > s_0$, for which $x = 0$.

These considerations can easily be put into an analytical form. Writing equation (8) in the form

$$1 - x = \frac{1}{C} N x^2 e^{\tau_u} \tag{10}$$

and substituting this expression for $1 - x$ in (9), we find

$$d\tau_u = \frac{a_u}{C} N^2 x^2 e^{\tau_u} ds \, . \tag{11}$$

Upon separation of the variables τ_u and s and upon integration this becomes

$$\int_0^{\tau_u} \frac{1}{x^2} e^{-\tau_u} d\tau_u = \frac{a_u}{C} \int_0^s N^2 ds \, . \tag{12}$$

The integral on the left-hand side of (12) increases with τ_u. We know that x does not differ appreciably from 1 until e^{τ_u} has reached a value of the order of magnitude 10^4. Hence we have

$$\int_0^{\tau_u} \frac{1}{x^2} e^{-\tau_u} d\tau_u = 1 - e^{-\tau_u} \, , \qquad \text{for} \qquad 1 \geqq e^{-\tau_u} > 10^{-4} \, . \tag{13}$$

For values of $e^{\tau_u} > 10^4$, x begins to decrease, and hence $1/x^2$ begins to exceed 1. While x decreases to $1/10$, the integrand stays less than 10^{-2}, and hence the increase of the integral is inappreciable compared with the value of the integral itself, which is very close to 1. When $x < (1/10)$ it is sufficiently accurate to put $1 - x = 1$ in (10), which gives

$$\frac{1}{x^2} = \frac{1}{C} N e^{\tau_u} , \qquad \text{for} \qquad x < \tfrac{1}{10} . \tag{14}$$

Inserting this in the integral on the left-hand side of (12), we find for this integral

$$\int_{\tau'_u}^{\tau_u} \frac{1}{x^2} e^{-\tau_u} d\tau_u = \int_{\tau'_u}^{\tau_u} \frac{N}{C} d\tau_u , \qquad \text{for} \qquad x < \tfrac{1}{10} . \tag{15}$$

Now N/C is of the order of magnitude, say, $10^{-5} - 10^{-3}$ [cf. (10)], and, consequently, the integral increases extremely slowly with τ_u. In fact, even an increase of, say, 20 in τ_u, which, according to (14), will reduce x from a quantity less than $1/10$ to a quantity less than $(1/10)e^{-10}$, or less than 10^{-5}, will not increase the value of the integral appreciably.

We therefore see that, as the integral on the right-hand side of (12) increases with s, x remains practically equal to 1, until the value of that integral becomes very nearly equal to 1. Let us define s_0 so that

$$1 = \frac{a_u}{C} \int_0^{s_0} N^2 ds . \tag{16}$$

Then x remains practically equal to 1 until s is practically equal to s_0. A very small increase of s, leading to a very small increase of the integral on the right-hand side and, hence, of the integral on the left-hand side in (12), corresponds to a large increase in τ_u and a very rapid reduction of x from 1 to a value practically equal to 0.

Thus, the result of the analysis is that

$$\left. \begin{aligned} x &= 1 \text{ for } s \leqq s_0 - \epsilon_1 \\ \text{and} \qquad x &= 0 \text{ for } s \geqq s_0 + \epsilon_2 \end{aligned} \right\} , \tag{17}$$

where s_0 is defined by (16) and ϵ_1 and ϵ_2 are so small as to be entirely negligible in our problem

We can now calculate the optical depth τ corresponding to the electron scattering. We have

$$d\tau = 0.4 m_H x N ds , \tag{18}$$

or

$$\tau(s) = 0.4 m_H \int_0^s x N ds . \tag{19}$$

From (17) and (19) we now find

$$\tau(s) = 0.4 m_H \int_0^{s_0} N ds , \qquad \text{for} \qquad s \geqq s_0 , \tag{20}$$

which, in particular, gives the optical depth τ along the total path through the atmosphere of the I star:

$$\tau = 0.4 m_H \int_0^{s_0} N ds . \tag{21}$$

Equations (16) and (21) contain the solution of our problem. For any given density distribution $N(s)$, equation (16) determines s_0 and equation (21) then determines τ.

In order to get a general insight into the problem we shall now, with the aid of equations (16) and (21), study three models of different density distributions. The last of these models probably corresponds rather closely to the atmosphere of the I component of ϵ Aurigae.

First, let us consider a region of constant density N, of arbitrary shape, subject only to the conditions that the region should be practically opaque in the ultraviolet for all the traversing rays considered. For this model, (16) becomes

$$1 = \frac{a_u}{C} N^2 s_0 , \tag{22}$$

and (21)

$$\tau = 0.4 m_H N s_0 . \tag{23}$$

From (22) we find

$$s_0 = \frac{C}{a_u} \frac{1}{N^2} . \tag{24}$$

Inserting in (23), we get

$$\tau = 0.4 m_H \frac{C}{a_u} \frac{1}{N} . \tag{25}$$

The opacity due to the electron scattering is then the same for all directions. It is inversely proportional to the density. Thus, the decrease of the number of electrons per unit volume with decreasing density is more than compensated by the increase in the extent of the ionized region with decreasing density. The opacity τ is further proportional to the constant C, which, according to (7), is a measure of the amount of ionizing radiation and is inversely proportional to the atomic absorption constant a_u in the ultraviolet.

It should be noted that the variation of τ with N is in the opposite direction to the usual one and is also less pronounced than the ordinary exponential variation.

For a given region, when N tends toward zero, the region will ultimately become transparent in the ultraviolet. Then the opacity τ is no longer given by (25) but decreases with the density, ultimately becoming proportional to it.

Next we consider a model of plane-parallel stratification of the matter, with a density increasing exponentially in the direction of the normal to the stratification. Along a ray making the angle θ with the normal, the relation between s and N is then

$$N = N_0 e^{a \cos\theta \cdot s} , \tag{26}$$

the geometrical path being counted from some point on the ray where the density N_0 is negligibly small, compared with the other densities that may enter into the problem. For this model equation (16) becomes

$$1 = \frac{a_u}{C} \int_0^{s_0} N_0^2 e^{2a \cos\theta \cdot s} ds , \tag{27}$$

or

$$1 = \frac{a_u}{C} \frac{1}{2a \cos\theta} N_0^2 (e^{2a \cos\theta \cdot s_0} - 1) . \tag{28}$$

Neglecting N_0^2 in comparison with $N^2(s_0)$, this becomes

$$1 = \frac{a_u}{C} \frac{1}{2\alpha \cos \theta} N^2(s_0) . \tag{29}$$

Equation (21) takes the form

$$\tau = 0.4 m_H \int_0^{s_0} N_0 e^{\alpha \cos \theta \cdot s} ds , \tag{30}$$

or, again neglecting N_0 in comparison with N,

$$\tau = 0.4 m_H \frac{1}{\alpha \cos \theta} N(s_0) . \tag{31}$$

Eliminating $N(s_0)$ in (31), with the aid of (29), we get

$$\tau = 0.4 m_H \sqrt{\frac{C}{a_u}} \frac{\sqrt{2}}{\sqrt{\alpha \cos \theta}} . \tag{32}$$

The opacity τ is now proportional to the square root of the constant C, measuring the amount of impinging ionizing radiation. The opacity increases with the angle between the ray and the normal, but only as $\sqrt{\sec \theta}$.

Finally, we consider a model which probably corresponds rather closely to the actual conditions in the atmosphere of the I component of ϵ Aurigae. We assume that the density distribution is symmetrical around a center and falls off exponentially with the distance r from the center:

$$N = N_0 e^{-\alpha r} . \tag{33}$$

The region corresponding to the atmosphere is the only region of the model that interests us. The numerical value of the constant α is limited to the range $10^{-12} - 10^{-13}$ cm^{-1} (cf. p. 591).

Consider, now, a ray which traverses the atmosphere in such a way that the minimum distance from the center of the star to the ray is y. For the rays that interest us, y will not be very different from the radius of the I star. A change in the dip of the ray is equivalent to a change in y of the same amount. Let s be counted along the ray from the point of smallest distance from the center, positive in the direction toward the observer. Then it follows from (33) that the dependence of N upon s is given by the relation

$$N = N_0 e^{-a\sqrt{y^2+s^2}} . \tag{34}$$

Introducing

$$N_c = N_0 e^{-ay} , \tag{35}$$

the maximum density on the ray, and

$$t = \frac{s}{y} , \tag{36}$$

we can write (34) in the form

$$N = N_c e^{-ay(\sqrt{1+t^2}-1)} . \tag{37}$$

We now have to introduce this relation between N and s into the general equations (16) and (21). We thus find:

$$1 = \frac{a_u}{C} N_c^2 \int_{-\infty}^{s_0} e^{-2ay(\sqrt{1+t^2}-1)} ds , \tag{38}$$

or

$$1 = \frac{a_u}{C} N_c^2 y \int_{-\infty}^{t_0} e^{-2ay(\sqrt{1+t^2}-1)} dt , \tag{39}$$

and

$$\tau = 0.4 m_H N_c y \int_{-\infty}^{t_0} e^{-ay(\sqrt{1+t^2}-1)} dt . \tag{40}$$

Now, the smallest possible value of ay is about 20, corresponding to $a = 10^{-13}$. This means that the integrands of the integrals in (39) and (40) fall off rapidly with t. In fact, for $t = 1$, they are at most of the order of magnitude 10^{-7} and 3×10^{-4}. This means that the effective extent of the atmosphere along the rays considered is small compared with the radius of the I star. Analytically, it means that we can obtain useful asymptotic expansions of the integrals in

(39) and (40) by expanding $\sqrt{1+t^2}$ in the exponents in powers of t. In fact, the first term of the asymptotic expansion gives sufficient accuracy. (A similar asymptotic expansion has been used by Menzel[17] to evaluate an extinction integral occurring in the discussion of ζ Aurigae.) In this way we find, from (39) and (40),

$$1 = \frac{a_u}{C} N_c^2 y \int_{-\infty}^{t_0} e^{-\alpha y t^2} dt \tag{41}$$

and

$$\tau = 0.4 m_H N_c y \int_{-\infty}^{t_0} e^{-\frac{1}{2}\alpha y t^2} dt \,. \tag{42}$$

Using $erf(x)$ to denote the error function:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx \,, \tag{43}$$

we finally get from (41)

$$N_c = \sqrt{\frac{C}{a_u}} \frac{1}{\sqrt{y}} (\alpha y)^{1/4} \frac{\sqrt{2}}{\pi^{1/4}} \frac{1}{\sqrt{1 - erf(-\sqrt{\alpha y}\, t_0)}} \,, \tag{44}$$

and from (42)

$$\tau = 0.4 m_H N_c y \frac{\sqrt{2}}{\sqrt{\alpha y}} \frac{\sqrt{\pi}}{2} [1 - erf(-\sqrt{\tfrac{1}{2}} \sqrt{\alpha y}\, t_0)] \,. \tag{45}$$

Equation (44) relates the maximum density N_c along the ray with $t_0 = s_0/y$, which measures the relative extent of the ionized region, while equation (45) allows τ to be found from N_c and t_0. Remembering that N_c is a function of the dip of the ray under the surface of the I star, we see that (44) and (45) determine the opacity τ as a function of the dip. Eliminating N_c from (45) with the aid of (44), we find

$$\tau = 0.4 m_H \sqrt{\frac{C}{a_u}} \sqrt{y} \frac{1}{(\alpha y)^{1/4}} \pi^{1/4} \frac{1 - erf(-\sqrt{\tfrac{1}{2}} \sqrt{\alpha y}\, t_0)}{\sqrt{1 - erf(-\sqrt{\alpha y}\, t_0)}} \,. \tag{46}$$

Let us now consider τ as a function of the dip. When the dip increases, N_c increases according to (35). The minimum distance y from the center of the I star for the ray considered is decreased; however, the relative variation of y is small, the maximum dip being small compared to the radius of the I star (cf. p. 590). For our purposes it is sufficiently accurate to consider y in equations (44), (45), and (46) as a constant equal to R_{I}. Equation (44) then gives t_0 as a function of N_c, and equation (46) gives τ as a function of t_0.

TABLE 3

$\sqrt{ay}\,t_0$	$[1-erf(-\sqrt{ay}\,t_0)]^{-\frac{1}{2}} \sim N_c$	$\dfrac{1-erf(-\sqrt{\frac{1}{2}}\sqrt{ay}\,t_0)}{\sqrt{1-erf(-\sqrt{ay}\,t_0)}} \sim \tau$
−3.0	210.	0.57
−2.5	50.	0.60
−2.0	14.5	0.65
−1.5	5.6	0.73
−1.0	2.5	0.81
−0.5	1.4	0.90
0.0	1.00	1.00
+0.5	0.81	1.12
+1.0	0.74	1.24
+1.5	0.71	1.33
+2.0	0.71	1.38
∞	0.71	1.41

Table 3 gives the quantities $[1 - erf(-\sqrt{ay}\,t_0)]^{-(1/2)}$ and $[1 - erf(-\frac{1}{2}\sqrt{ay}\,t_0)][1 - erf(-\sqrt{ay}\,t_0)]^{-(1/2)}$ as a function of $\sqrt{ay}\,t_0$. According to (44), N_c is proportional to the first of these quantities, while, according to (46), τ is proportional to the latter.

It appears from Table 3 that the opacity decreases with increasing dip, contrary to the case of normal extinction. Furthermore, it is seen that the range of variation of τ is only by a factor of 2.4, corresponding to a range in N_c of 300. In the case of normal extinction, τ would have been proportional to N_c, leading to a tremendous variation in the extinction. We thus see that the decrease of the thickness of the ionized region with increasing density approximately compensates—in fact, slightly overcompensates—the increase in the absorption per unit volume with the density.

Before completing the discussion of equations (44) and (46) we shall consider briefly the influence of the finite disk of the F star, as seen from the ionized region of the I star considered, upon the resulting opacity in the visual and photographic regions.

Let us consider for a moment the case that the ionizing radiation came from the half of the disk of the F star highest above the horizon of the considered region of the I star, and that the obstruction of the photographic and visual light from the lower half was being studied. The ionized region would then extend farther into the I star, and the opacity τ would be increased. This effect would be the more marked the greater the dip. In this way we see that for the lower half of the disk, owing to the effect studied, there will be an increase of opacity and a tendency for the march of τ with N_c to be reversed. For the upper half of the disk the effect will lead to the opposite result. The net result for the whole disk will probably be in the same direction as for the lower half of the disk. It is easy to show that for an angular radius of the F star as seen from the region of the I star considered, equal to $1^{\circ}.8$ (cf. p. 579), the result cannot differ very much from the case of a point source. The sign of the change of τ with N_c could hardly be reversed, and the opacities given in Table 3 could hardly be increased by more than a factor of 2. It would not present great difficulties to treat the case of an extended disk rigorously.

Returning now to the discussion of equations (44) and (46), we next find the value of C/a_u required to give the observed opacity in visual and photographic light. The observed value of τ is 0.7. With $y = R_{\text{I}} = 2.10^{14}$ and $\alpha y = 50$, and with the last factor in (46) equal to 2 to allow for the effect of the finite disk, we find that the required value of $\sqrt{C/a_u}$ is about 6.10^{16}, and thus C/a_u is about 4.10^{33}.

The atomic absorption coefficient a_u for the ionizing radiation can be estimated as follows: Taking sodium as a typical atom and using the Coulomb-field formula to derive a_u (cf., for instance, Unsöld[20]), a value of the order of 10^{-16} is found. In the case of sodium it is well known, however, that the Coulomb-field formula gives a value that is considerably too great, when applied to the continuum originating from the *ground state.* In fact, the application of the sum rule, as well as direct experimental and theoretical determinations of the

[20] *Zs. f. Ap.*, **8**, 225, 1934.

absorption coefficient in question, gives a value about one hundred times smaller than the Coulomb-field value (cf. Trumpy[21]). The situation is not so clear for other atoms involved. We shall assume that the correction to the Coulomb-field value is of the same order of magnitude in these cases also. More accurate values naturally would be of great value. Hence, we shall adopt a_u as of the order of magnitude 10^{-18}.

With $a_u = 10^{-18}$, we find $C = 4.10^{15}$. We have now to compare this value with the value found from (7). Choosing *Ca* with $I = 6.1$ as a typical element, we find from (7), with $T = 6300°$ and $w = 2.5 \times 10^{-4}$, a value of C equal to 10^{13}. We thus find that the value of C required to give the observed opacity is between one hundred and one thousand times larger than the value computed from (7). We conclude that the amount of ultraviolet ionizing radiation of the F star at the surface of the I star is between one hundred and one thousand times greater than the amount calculated from Planck's formula with $T = 6300°$. The same fact may also be expressed by saying that the opacity corresponding to the Planck radiation is about twenty times smaller than the opacity observed.

The value of C found above is naturally rather uncertain, but it seems difficult to escape the conclusion that, if the picture of the mechanism of light obstruction considered here is correct, the ultraviolet radiation of the F star at the I star must be very considerably higher than the Planck value.

A priori, it is not impossible that an excess of ionizing radiation such as has just been considered is really present. The presence of hydrogen emission lines in the spectrum of ϵ Aurigae at the time of the eclipse (cf. Adams and Sanford[10]) is an indication in that direction. Also, in the case of the sun, as is well known, many anomalies can be explained by the assumption of an excess of ultraviolet radiation, though here this assumption is not the only one possible.

We conclude, therefore, that the result obtained from the discussion of the value of C derived from the observed opacity does not rule out the picture that we are discussing here in the greatest detail. On the other hand, we cannot be certain that this picture is

[21] *Zs. f. Phys.*, **71**, 720, 1931.

the correct one; and, therefore, all other possibilities have to be considered carefully.

With the value of C/a_u derived from (46), we now find N_c as a function of t_0 with the aid of (44). The numerical value of the multiplier of the quantity $[1 - erf(-\sqrt{ay}\, t_0)]^{-(1/2)}$ given in Table 3 is found to be 1.10^{10}. To $t_0 \rightarrow \infty$, then, corresponds the density $N_c = 7.10^9$. This means that for a ray with the maximum density $N_c = 7.10^9$ the ionized region just extends along the whole of the ray through the atmosphere of the I star. For a smaller value of N_c, that is, for a smaller dip, the I star is no longer opaque to the ionizing radiation. The ionization is complete ($x = 1$) along any ray with $N_c < 7.10^9$. The opacity τ, due to electron scattering, is now [cf. (42)]:

$$\tau = 0.4 m_H N_c y \int_{-\infty}^{+\infty} e^{-\frac{1}{2} a y t^2} dt \,, \qquad \text{for} \qquad N_c < 7.10^9 \qquad (47)$$

or

$$\tau = 0.4 m_H N_c y \frac{\sqrt{2\pi}}{\sqrt{ay}} \,, \qquad \text{for} \qquad N_c < 7.10^9 \,. \qquad (48)$$

Hence, τ decreases proportionally to N_c for $N_c < 7.10^9$. The approximately constant minimum thus begins when the dip has been reached for which $N_c = 7.10^9$. As N_c increases with increasing dip, τ, according to Table 3, now decreases slowly. The range of increase of N_c during the constant minimum is, as we have seen (cf. p. 591), at least about 20. The corresponding change of τ, according to Table 3, is by a factor of 2. Even an increase of N_c by a factor of 300 above the limit $N_c = 7.10^9$ changes τ by a factor of 2.4 only.

Now a change of τ by a factor of 2, although very much smaller than the change corresponding to ordinary extinction, contradicts the observed constant minimum. As we have seen, the effect of the finite size of the disk tends to diminish the variation of τ with N_c. The effect of the presence of several elements with different ionization potentials goes in the same direction. Further, as we shall see, with increasing N_c other sources of opacity become of importance, hence tending to compensate the small remaining variation. It may well be possible that a sufficiently constant minimum is produced in this manner, so that the picture considered cannot be said to be contradicted by the observational evidence of a very nearly constant minimum.

We now have to investigate whether the region of the I star here considered is practically transparent when the influence of the F star is absent, that is, on the nonionized part of the rays considered. The temperature of that region is about 1400°, the number of free electrons per unit volume being about $10^{10} - 10^{12}$. An inspection of the tables by Unsöld[20] and Pannekoek[22] for the opacity due to photoelectric absorption from excited states shows that it is so low for $T = 1400°$ and electron pressure P_e from 10^{-3} to 10^{-1} dynes that it can be neglected. We shall see, however, that there is one source of opacity which becomes appreciable as the density increases from the limit $N = 7 \cdot 10^9$ to values that are, say, from ten times to one hundred times higher, namely, Rayleigh scattering due to neutral hydrogen atoms.

The atomic absorption coefficient corresponding to Rayleigh scattering for wave lengths that are large compared to the dispersion wave length is known to be

$$a = \frac{128\pi^5}{3} \alpha'^2 \cdot \frac{1}{\lambda^4}, \tag{49}$$

where α' is the polarizability of the atom and state considered. For the ground state of hydrogen we have $\alpha' = 6.63 \times 10^{-25}$ (cf. van Vleck).[23] Thus, we find

$$a_H = 1 \times 10^{-27}\left(\frac{5000\text{ A}}{\lambda}\right)^4. \tag{50}$$

Now, owing to the high ionization potential of hydrogen, the region of ionization of hydrogen in the I star is negligible. We then find [cf. (48)] that the opacity τ_H, owing to Rayleigh scattering along a ray, on which the maximum number of hydrogen atoms per unit volume is $N_{c,H}$, is given by

$$\tau_H = 1 \times 10^{-27}\left(\frac{5000\text{ A}}{\lambda}\right)^4 N_{c,H}\, y \frac{\sqrt{2\pi}}{\sqrt{ay}}. \tag{51}$$

[22] *Astr. Inst. Amsterdam*, No. 4, 1935.

[23] J. H. van Vleck, *The Theory of Electric and Magnetic Susceptibilities*, Oxford, 1932.

For $\lambda = 5000$ A, with $y = 2.10^{14}$ and $\alpha y = 50$ as before, we find from (51) that

$$\tau_H = 2.5 \times 10^{-13} N_{c,H} . \qquad (52)$$

Therefore, when $N_{c,H}$ is, say, $1 \times 10^{12} - 2 \times 10^{12}$, Rayleigh scattering becomes quite appreciable. Now, as we have seen, the constant minimum sets in for $N_c = 7.10^9$, the dip increasing until a value of at least $1 \times 10^{11} - 2 \times 10^{11}$ has been reached. We now see that, unless the ratio of the number of hydrogen atoms N_H per unit volume to the number of atoms in the group of ionizable elements N per unit volume is less than about 10, the Rayleigh scattering would become appreciable to such an extent as to destroy the constant minimum. It may be noted that an appreciable obstruction of a smaller part of the disk of the F star by Rayleigh scattering would only tend to flatten the minimum (cf. p. 605).

With the upper limit of the relative hydrogen abundance just mentioned, it is further found that the influence of Rayleigh scattering in the wave-length region of the ionizing radiation is not appreciable in the density region in question [$N > 7 \times 10^9$].

The numerical value 10, which we have found for the upper limit of N_H/N is, of course, subject to some uncertainty. The order of magnitude, however, should be right. The relative abundance of hydrogen thus turns out to be considerably smaller than is now usually assumed for the atmospheres of normal stars of the main sequence. It is not impossible, however, that the true value of the relative hydrogen abundance in the atmosphere of a supergiant like the I component of ϵ Aurigae is as low as, or lower than, the required limit.

We are now in a position to discuss the question mentioned on page 594, which elements contribute appreciably to the opacity in the visual and photographic regions, according to the mechanism under discussion. From equations (7) and (46) we see that elements of high ionization potential are ionized in such a narrow region only that the resulting τ becomes very small. On the other hand, we now see that elements of small relative abundance reach the required N_c for the element [cf. (44) and (46)] only at such depths that Rayleigh scattering of the neutral hydrogen would prevent the occurrence of any constant minimum. Probably, therefore, the opacity is largely due to such elements as *Na*, *Al*, *Ca*, and possibly *Mg* and *Fe*. Doubly ionized ions are probably present in a very narrow region only, and thus the singly ionized ions do not contribute an appreciable amount of free electrons.

We have seen that the density of the region responsible for the light obstruction, to which the picture under discussion leads, is low enough to insure that the constant minimum is not distorted by other sources of opacity, provided the relative hydrogen abundance does not exceed a certain limit. We shall next consider the density arrived at from another point of view. From Table 3 it is apparent that in the denser part of the obstructing region most of the traversing rays go through nonionized matter. It is, therefore, to be expected that the corresponding absorption lines due to resonance lines of neutral atoms would be very strong. In fact, combining the value of the density arrived at with the length of the path through nonionized matter [cf. (48)], we find that the strength of a line such as the resonance line of *Ca* would be extremely strong—much stronger than observed. In order to avoid contradiction with the observations it is, therefore, necessary to assume that the abundances and the amount of ionizing radiation are such as to shift the obstruction effect to elements with higher ionization potentials that have their resonance lines in the unobservable ultraviolet region. This would require an even higher excess of the ionizing radiation over Planck radiation, though the ratio of the ionizing radiation to the total radiation of the F star would not be very much changed.

It will be seen that we have now dealt with the first three of the points mentioned on page 589. We shall finally consider the last point, whether other sources of opacity are of importance in the region of the I star that is influenced by the radiation of the F star. In order to investigate this we shall have to find the state of excitation of the matter in the ionized region. Comparing this with a region in thermodynamical equilibrium at the same density and at a temperature equal to that of the F star, we see that the number and the velocity distribution of the ions and the free electrons are practically the same in the two cases. The number of neutral atoms in the ground state is extremely low in the ionized region but still higher than in thermodynamical equilibrium by a factor equal to the reciprocal of the dilution factor. At the surface of the I star the number of processes of radiation excitation of excited states is about the same as in thermodynamical equilibrium, because the reduction of the intensity of the exciting radiation is compensated by the greater number of atoms in the ground state. Scattering of the exciting radiation will very soon reduce it, however. The contribution of electron captures to the population in the excited state, on the contrary, is practically the same as in thermodynamical equilibrium, throughout the ionized region. The population of the excited states will, therefore, correspond very nearly with that in the highest layers of the F star itself. The opacity due to photoelectric absorption from the excited states will be equal to the value found for thermodynamical equilibrium at the same pressure and at a temperature equal to that of the F star, reduced by a factor approximately equal to the residual intensity of the lines corresponding to transitions between the ground state and the excited states in question.

The absorption coefficient $\kappa_\nu\rho$ per unit volume, owing to photoelectric transitions, is given by the well-known expression (cf., for instance, Unsöld[20])

$$\kappa_\nu\rho = \frac{16\pi^2}{3\sqrt{3}}\frac{e^6}{hc(2\pi m_e)^{3/2}}Z_{eff}^2\frac{e^{h\nu/kT}-1}{\nu^3}\frac{1}{(kT)^{1/2}}N''N_e\,, \tag{53}$$

or

$$\kappa_\nu\rho = c_1 N''N_e\,, \tag{54}$$

that is, cf. (6),

$$\kappa_\nu\rho = c_1 x^2 N^2\,. \tag{55}$$

With $Z_{eff} = 1$, $\lambda = 5000$ A, and $T = 6300°$, the constant c_1 is 1×10^{-36}. We thus see that in thermodynamical equilibrium the photoelectric absorption coefficient per free electron in a practically completely ionized gas ($x = 1$) is smaller than the free-electron scattering coefficient $0.4\ m_H$ when the density N is smaller than $10^{11} - 10^{12}$.

Now, in the ionized region considered, the opacity, as we have seen, is reduced by a factor of the order of the residual line intensities. Therefore, even allowing for a considerable increase in the photoelectric opacity due to a value of Z_{eff} larger than 1, the photoelectric opacity in the ionized region of the I star with $N < 1\text{-}2 \times 10^{11}$ would not be comparable with electron scattering.

It may be noted that the contribution of photoelectric opacity to the obstruction of photographic and visual light is independent of the dip, for a certain range of dips. In fact, the optical depth τ_p corresponding to this process, is [cf. (21)]

$$\tau_p = c_1 \int_{-\infty}^{s_0} N^2 ds \, . \tag{56}$$

Now, according to (16), for all dips for which the I star is opaque to the ultraviolet [$N > (7.10^9)$], this is equal to

$$\tau_p = c_1 \frac{C}{a_u} \, , \tag{57}$$

which is, in fact, constant.

It is interesting to note that in the ionized region of the I star, as we have seen, the distribution over the excited states corresponds closely to that in the highest layers of the F star. We shall not, however, enter here into the rather complicated problem of the influence of the I star upon the intensities of the corresponding lines.

Summarizing the discussion of the mechanism of electron scattering in a region of the I star ionized by the radiation of the F star, we can say that this picture may be the correct one but that we have met with certain difficulties: the required amount of ultraviolet radiation; the required, rather low, upper limit to the hydrogen abundance; and the necessity of avoiding too great a strength of resonance absorption lines in the observable wave-length region, or of the extreme edges of such lines beyond that region (Rayleigh scattering).

In investigating other possibilities of mechanisms to explain the observed obstruction effect, the following considerations may be of importance. With a mechanism that would give a higher mass-absorption coefficient than electron scattering, the obstructing region would be shifted toward smaller densities. The required ultraviolet radiation would be smaller, the influence of hydrogen Rayleigh scattering would also be smaller, and the strength of the resonance lines of neutral atoms would be smaller—possibly very much smaller. In fact, all three difficulties would be lessened.

In investigating the general problem of stellar atmospheres, the author of this section has been led to consider the effect of two-quantum absorption processes on opacity in the continuous spectrum. Without entering into a detailed discussion of the possibility of the importance of this effect in the present connection, we shall show that the mechanism of two-quantum absorption processes leads to constant obstruction for a certain range of dip. Let the absorption coefficient in a frequency of the visual and photographic regions be proportional to the intensity of ultraviolet radiation, not necessarily of the same wave-length region as that considered before:

$$d\tau = c_t N e^{-\tau_{u'}} ds \, , \qquad (58)$$

with

$$d\tau_{u'} = c_{u'} N ds \, . \qquad (59)$$

It follows from (58) and (59) that

$$d\tau = \frac{c_t}{c_{u'}} e^{-\tau_{u'}} d\tau_{u'} \, , \qquad (60)$$

or

$$\tau(s) = \frac{c_t}{c_{u'}} (1 - e^{-\tau_{u'}}) \, , \qquad (61)$$

which means that

$$\tau = \frac{c_t}{c_{u'}} \qquad (62)$$

when the region considered is opaque to the ultraviolet radiation in question.

This example, together with the discussion of the opacity due to electron scattering and also of the opacity due to photoelectric absorption, shows that the independence (or practical independence) of opacity of dip in a certain range of dips is a very general feature when the cause of the opacity is outside radiation. Therefore, though we cannot claim that we yet know the mechanism of the light obstruction in the case of ϵ Aurigae with certainty, it does seem certain that an important characteristic of that mechanism has been established.

In concluding, it may be mentioned that the general discussion given on page 593, of the extent of the region of ionization as a function of the amount of ionizing radiation and the absorption coefficient of that radiation, has V, with an obvious modification, an application also to another problem, namely (cf. Eddington[24]), that of the ionization of hydrogen in interstellar space.

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[24] *Observatory*, **60**, 99, 1937.

Note added in proof.—The computations given in Table 1, page 575, are based on the elements of Huffer's light-curve. If Miss Güssow's elements ($D = 714$ days, $d = 330$ days) are used, which are based on a somewhat larger number of observations, the entries in Table 1 are slightly reduced. For ω 350° and $i = 70°$ we find

$$R_F = 0.025a\,, \quad R_I = 0.357a\,, \quad \frac{m_F}{m_I} = 1.47\,, \quad \log R_F = 2.28\,, \quad \log R_I = 3.43\,,$$
$$\log m_F = 1.56\,, \quad \log m_I = 1.39\,, \quad M_F = -7.2\,, \quad T_e(I) = 1350°\,.$$

These figures make the stability argument given on page 576 slightly more favorable.

Since $m_v = 3.27$ P.D. (Güssow), or 3.10 IPv, the distance modulus of ϵ Aurigae is 10.3 mag. Allowing for a visual space absorption of 0.4 mag. per 1000 parsecs, the parallax is found to be 0″.0010. The total amplitude of the orbital motion of the F component projected on the sky is very nearly $2a_1$, or 0″.030. A verification of this value would at the same time verify the high luminosity of the F component, found in this paper; a lower luminosity would lead to a larger displacement, by a factor of 1.6 for each magnitude.