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THE SODIUM CONTENT OF THE HEAD OF THE GREAT DAYLIGHT COMET SKJELLERUP 1927 K

ARTHUR ADEL, V. M. SLIPHER AND R. LADENBURG

ABSTRACT

Spectrograms of the great daylight comet, Skjellerup, December, 1927, taken near perihelion passage, reveal the D-lines of sodium in emission with very nearly equal intensity. On the assumption that the D-lines are due to the absorption and re-emission of resonance radiation and that the absorption of this radiation in the interior of the comet is governed by the combined action of the Doppler and radiation damping agencies, the observed intensity ratio $0.9 \leqslant (D_2/D_1) \leqslant 1.1$ yields, for the concentration of normal sodium atoms in the comet's temporary atmosphere, $50 \leqslant N \leqslant 2500/\text{cm}^3$. The concentration of normal sodium atoms is subsequently confined to the region of the lower limit by an analysis of the monochromatic magnitude of the comet.

I. INTRODUCTION

Visual, photographic, spectrographic, and radiometric observations of Skjellerup's brilliant comet (1927 k) were secured on December 16–19 under good conditions of transparency.[1]

On the evening of December 15 an object was reported closely following the sun as it set. The next morning the comet was found to be an easy naked-eye object. The experienced observer saw it readily during that day by extending the hand to shade the eyes from the sun, which was only about 5° southwest of the comet.

[1] V. M. Slipher and E. C. Slipher, *Pop. Ast.*, **36**, 300, 1928; C. O. Lampland, *ibid.*, **36**, 240, 1928.

Venus, to the west of the comet, was available for comparison. Early in the day of December 16, the comet was many times brighter than Venus and was a relatively conspicuous object by reason of its brightness and tail.

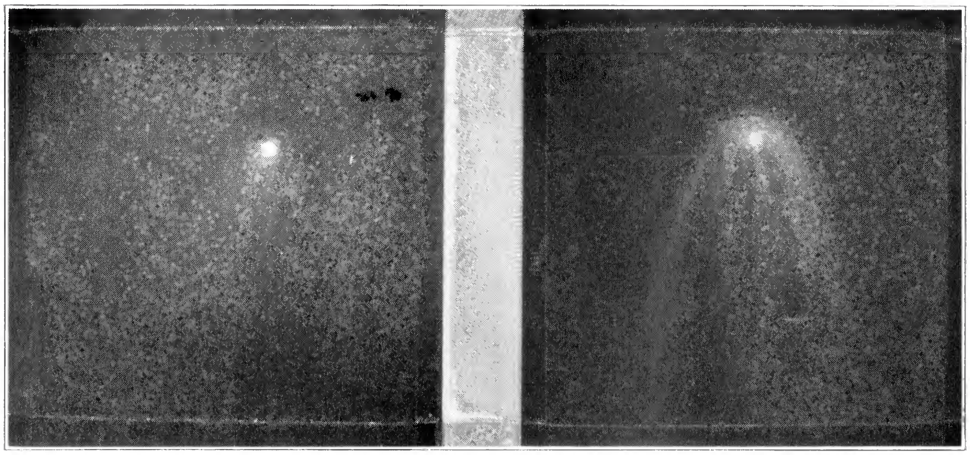
The nucleus was of an orange-yellow color and was small but not quite stellar in form or sharpness. At times it appeared distinctly elongated. The nucleus was enveloped by the arching hoods commonly seen in bright comets. The tail was broad and was about 45′ long. By early afternoon (December 16) the comet had faded, and the tail had also changed in appearance.

After Saturday, December 17, the comet was seen, during the day, only with the aid of the telescope. It was last observed late in the afternoon of December 19. Attempts to direct the telescope on it the next day in full daylight were unsuccessful.

The comet's spectrum was first examined visually on the morning of December 16. However, it showed—on the sky-spectrum background—only a strong continuous spectrum of the nucleus, of the solar type. Examination with both low and high dispersion then showed the same result, disclosing none of the usual cometary bright bands. The next day, it was seen that the dark D-lines of sodium of the sky spectrum were bordered with faint bright D-lines of the comet. On December 18 these had strengthened until they were a very conspicuous pair of bright lines of equal intensity superposed upon the sky spectrum. The following day they were still brighter. They were long and very considerably displaced to the red side of their normal position, proving that the comet was receding from the earth at the rate of about 90 km/sec. The emitting sodium spread to a considerable distance from the nucleus and was most intense on December 19, the last day of observation. The two lines were very sharp and were conspicuous because they were so nearly of equal strength.

Although sought, no trace was found, visually or photographically, of the usual cometary bright bands. The long series of spectrograms included the spectrum region of two of these bands, 5165 A and 5635 A, the former being perhaps the most capable of the series of revealing itself under the conditions. It should be borne in mind, however, that the sky spectrum might have been effective in masking

PLATE VIII

a  b

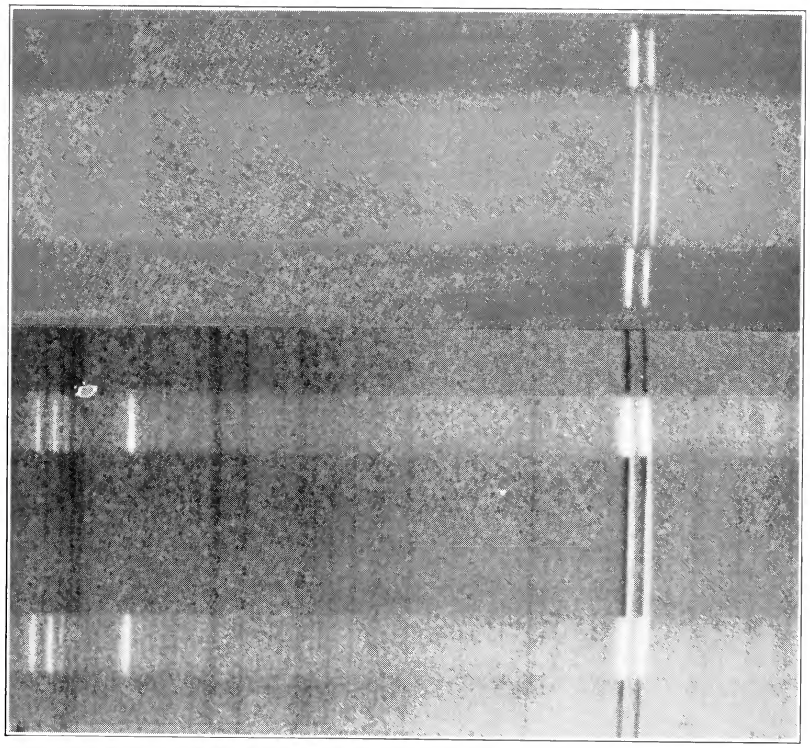
a) Comet (1927 k) Skjellerup

Photographed by E. C. Slipher at midafternoon, December 16, 1927, when the comet was only 5° from the sun.

b) Drawing of Comet Skjellerup

Made by E. C. Slipher on December 17, 1927

PLATE IX



THE SPECTRUM OF COMET SKJELLERUP (1927 K)

The upper spectrum (*a*) with short exposure to record the comet's bright sodium D-lines with the least intensity of sky (solar) spectrum. This displays the strange equally intense D_1 and D_2 in the comet as compared with the usual unequal strength given by the D-lines in the comparison above and below those of the comet.

The lower spectrogram (*b*) with long exposure to record the comet's bright D-lines over the dark lined solar (sky) spectrum background. Superposed are to be seen short bright lines of the comparison spectrum. This spectrogram was made with a long slit for comet (and sky) to allow the bright comet lines to show their extension from the comet's nucleus (which occupied the middle of the slit). Both spectrograms reveal the marked displacement of the comet's lines in accord with its rapid recession from the earth at the time. The comparison spectrum is an iron-vanadium spark into which was dropped a pinch of salt.

weak emissions, inasmuch as all observations were made against a daylight sky, very close to the sun.

II. THE D-LINES IN EMISSION

The amount of vaporized material per unit volume of a comet's atmosphere may be considered to be minute. Accordingly, in discussing the processes governing the distribution of energy in spectral lines emitted, it is safe to assert that collision damping, pressure effects, and Stark effect may be neglected. The breadth of the D-lines, or the width at half-intensity, is therefore controlled by the combined action of radiation damping and the Doppler effect due to random motions. It is of interest to determine which of these mechanisms is dominant.

The Doppler half-intensity breadth is given by the familiar expression[2]

$$\omega_d = \frac{2\nu}{c}\left\{\frac{2RTln2}{M}\right\}^{\frac{1}{2}},$$

from which it follows that

$$\omega_d = 2.08 \times 10^9 \text{ sec}^{-1} .$$

The radiation damping half-intensity breadth is the sum of the half-intensity breadths of the resonance and ground levels. The half-intensity breadth of the ground level is substantially zero, apropos of the long mean life of the atom in this state. The upper-level half-intensity breadth is given by

$$\omega_r = \sum_m C_n 4\pi e^2 (\nu_n - \nu_m)^2 \frac{1}{3\mu c^3},$$

where

$$C_n = 3\sum_m f_{nm}\frac{g_m}{g_n}.$$

[2] The value of T will be taken equal to $(277° + 329°)/2\sqrt{r}$, where $277°/\sqrt{r}$ is the temperature obtained when the nuclear masses are assumed to be approximately spherical black bodies absorbing solar radiation in an amount corresponding to their cross-sections, and reradiating this energy in an amount corresponding to their entire surfaces; and where $329°/\sqrt{r}$ is the value obtained when reradiation occurs only from the hemisphere exposed to the sun. r (taken $\backsim 0.17$) is the heliocentric distance in astronomical units. The meteoric masses are assumed to be in near equilibrium with the solar radiation, and the gas, having been effused by these masses, is assumed to have partaken of their temperatures. $T = 750°$ K.

The sums are extended over all levels for which $E_n > E_m$, E being the energy. For a resonance level this becomes

$$\omega_r = \frac{3f_{no}g_0}{g_n} \times \frac{4\pi e^2 \nu_n^2}{3\mu c^3},$$

where f_{no} is the oscillator strength of the transition, and g_0 and g_n are the weights of the normal and resonance states, respectively. The transitions in question are

$$\begin{aligned} &D_1, \qquad {}^2P_{1/2} \rightarrow {}^2S_{1/2} \\ &D_2, \qquad {}^2P_{3/2} \rightarrow {}^2S_{1/2}, \end{aligned}$$

for which we have

$$g_{S_{1/2}} = 2, \qquad g_{P_{1/2}} = 2, \qquad g_{P_{3/2}} = 4;$$

and[3]

$$f_{P_{3/2}} = 2f_{P_{1/2}} = 0.71 .$$

It follows that ${}_{D_1}\omega_r = {}_{D_2}\omega_r = 10^7 \text{ sec}^{-1}$. (The value of ω_r has actually been observed for the D-lines by R. Minkowski.[4] He has found $2\pi\omega_r = 0.63 \times 10^8$, from which it follows that $\omega_r = 10^7$.)

The large ratio $\omega_d/\omega_r = 208$ indicates that the distribution of energy in the cometary D-lines is predominantly of Doppler origin.

III. THE COMETARY D-LINES

We seek an explanation for the observed equality of intensity of the D-lines emitted by the sodium in the temporary atmosphere of Comet Skjellerup. The intensity in question is clearly the total or integrated emission, since it is manifest from Section II and from the fact that the slit width was 0.78 A that the spectrograph was incapable of resolving the true contours of the emissions.

Schwarzschild and Kron[5] have satisfactorily accounted for the luminosity of comet tails by the mechanism of absorption and re-emission of sunlight. Zanstra[6] has been similarly successful in the

[3] R. Ladenburg and E. Thiele, *Zs. f. Phys.*, **72**, 697, 1931.

[4] *Ibid.*, **86**, 839, 1926.

[5] *Ap. J.*, **34**, 342, 1911.

[6] *M.N.*, **89**, 178, 1928.

case of the comet heads. It seems appropriate, therefore, to assume that the sodium atoms in the atmosphere of Comet Skjellerup were excited by solar radiation. The problem then resolves itself into determining the set of conditions which prevailed in the cloud to make it possible for the gas of sodium atoms to absorb in such manner as to emit equally in the regions D_1 and D_2. It is sufficient, as in Section II, to consider radiation damping and the Doppler effect as the agencies governing the absorption of sunlight. It has been assumed, of course, that in so rare a medium the resonance absorption and emission are equal for each volume element.

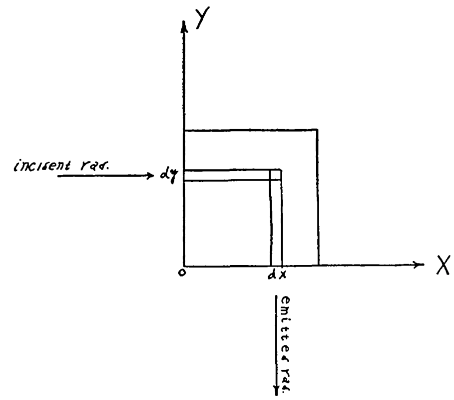


FIG. 1.—Model of comet's head

To take into account the differential absorption suffered by the re-emitted resonance radiations as they pass through the body of the cloud to the boundary, it suffices to represent the atmosphere of the comet as a two-dimensional rectangular layer illuminated in the X-direction by the D-region of the solar spectrum. We shall determine the intensity of the resonance radiation emitted by the sodium vapor in the comet in the Y-direction. The absorption coefficient of the sodium vapor is a function of the frequency ν and will be designated by $\alpha(\nu)$. With negligible error, we assume that the intensity of the sunlight incident upon the vapor is constant over the small spectral region D_1 and over the small spectral region D_2 of the vapor. Denote this by ${}_{D_1}I_0$ and by ${}_{D_2}I_0$. Neglecting the effect of scattering from surrounding space elements, the amount of radiation absorbed by the element $dxdy$ is proportional to

$$I_0 e^{-\alpha(\nu)x}\alpha(\nu)dxdy \, .$$

A quantity proportional to $e^{-\alpha(\nu)y}$ of this amount will escape through the boundary $y = 0$ and will proceed, let us say, toward the earth. In consequence, the total radiation of frequency ν emitted in the direction of the negative y-axis is proportional to the integral of the product of these factors taken over the area occupied by the sodium cloud. That is to say,

$$I(\nu) = I_0 \iint \alpha(\nu) e^{-\alpha(\nu)(x+y)} dx dy \, .$$

If l is taken to be the length of the layer in both the X- and Y-directions, it follows that

$$I(\nu) = \frac{I_0 \{1 - e^{-\alpha(\nu) l}\}^2}{\alpha(\nu)} \, .$$

The total energy radiated as a D-line is consequently represented by

$$P = \int_0^{\infty} I(\nu) d\nu = I_0 \int_0^{\infty} \frac{\{1 - e^{-\alpha(\nu) l}\}^2}{\alpha(\nu)} d\nu \, .$$

The absorption coefficient determined by the combined action of radiation damping and the Doppler effect may be stated explicitly[7]:

$$\alpha(\nu) = \frac{NK\delta M c^2}{2RT\nu_0^2 \pi^{1/2}} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{(\omega - y)^2 + D^2} dy \, ,$$

where

N = the number of normal atoms per cm³,

$$K = \frac{f_{no} e^2}{\mu c} \, ,$$

$$\delta = \frac{f_{no}}{4\pi\tau} = \tfrac{1}{2}\,\omega_r \, , \quad \tau = \text{mean life,}$$

$$\omega = (\nu_0 - \nu) \frac{c}{\nu_0} \left(\frac{M}{2RT}\right)^{1/2} = \frac{(\nu_0 - \nu)}{\omega_d} \, 2\, (ln\ 2)^{1/2} \, ,$$

$$y = u \left(\frac{M}{2RT}\right)^{1/2} , \quad u = \text{the velocity of an atom in the line of sight,}$$

$$D = \frac{\delta c}{\nu_0} \left(\frac{M}{2RT}\right)^{1/2} = \frac{\omega_r}{\omega_d} (ln\ 2)^{1/2} \, .$$

[7] Cf. S. Rosseland, *Theoretical Astrophysics*, p. 100, 1936.

F. Reiche[8] has shown that, when $D \leqslant 0.01$, the absorption coefficient may be expressed in the form

$$\alpha(\nu) = k_0 \left\{ e^{-\omega^2} - \frac{a}{\pi^{1/2}} (1 - 2\omega F(\omega)) \right\} ,$$

where

$$k_0 = \frac{2\pi e^2}{\mu c} N \cdot f \left(\frac{ln\ 2}{\pi}\right)^{1/2} \frac{1}{\omega_d}$$

$$F(\omega) = e^{-\omega^2} \int_0^\omega e^{y^2} dy$$

$$a = 2D = \frac{2\omega_r}{\omega_d} (ln\ 2)^{1/2} .$$

The factor preceding the integral in Rosseland's expression for $\alpha(\nu)$ is identical with the product

$$\frac{k_0 a}{2\pi} .$$

The simplification presented above is applicable to the present problem in virtue of the fact that $D = 0.004$ for Comet Skjellerup.

Before proceeding with the evaluation of the integral P, it will be instructive to find the solution in the first approximation by neglecting the effect of the differential absorption exercised by the sodium cloud upon the escaping radiations. In this approximation, the integral P becomes l times the integral Q:

$$Q = I_0 \int_0^\infty \{1 - e^{-\alpha(\nu) l}\} d\nu .$$

This integral has been evaluated not only for the limiting cases of the Doppler effect and radiation damping[9] but also for the general

[8] Cf. A. C. G. Mitchell and M. W. Zemansky, *Resonance Radiation and Excited Atoms* (New York: Macmillan Co., 1934), pp. 101 and 321.

[9] R. Ladenburg and S. Levy, *Zs. f. Phys.*, **65**, 189, 1930. R. Ladenburg and F. Reiche, *Ann. d. Phys.*, **42**, 181, 1913; *C.R.*, **158**, 1788, 1914.

case of the combined effects.[10] In the general case of the combined effects, the manner in which Q depends upon the optical density of the gas, that is, upon the product Nfl, is depicted by the curves of growth (Fig. 2) taken from the paper by E. F. M. van der Held. The

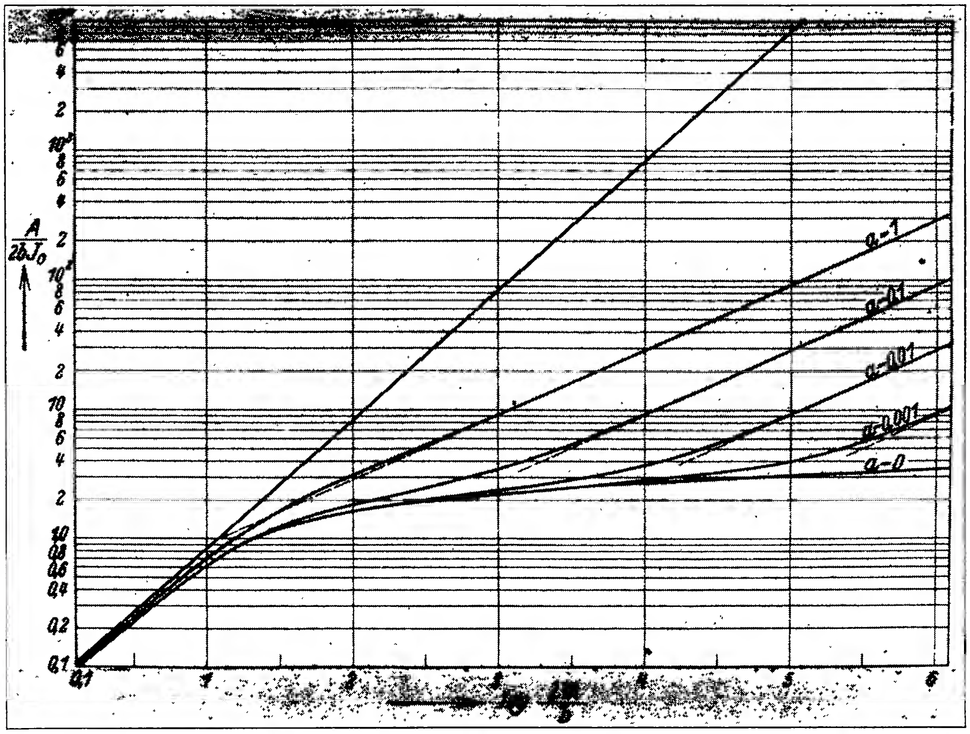


FIG. 2.—Curve of growth in the first approximation

arabic $\mathfrak{N}$ is equal to the product $N \cdot f$. The curve is a plot of $\pi\, Q/b\, I_0$ against log Nfl/b, where

$$b = \frac{2\pi\nu_0}{c}\left(\frac{2RT}{M}\right)^{1/2} = \frac{\pi}{(ln\ 2)^{1/2}}\,\omega_d = 3.77\omega_d\,.$$

The parameter of the family of curves is

$$a = 2\,\frac{\omega_r}{\omega_d}\,(ln\ 2)^{1/2} \equiv \tfrac{1}{2}\,D\ ;$$

[10] W. Schütz, *Zs. f. Phys.*, **64,** 682, 1930; *Zs. f. Ap.*, **1,** 300, 1932. E. F. M. van der Held, *Zs. f. Phys.*, **70,** 514, 1931; see also Rosseland, *op. cit.*, p. 166.

that is, the ratio of radiation damping and Doppler half-intensity breadths multiplied by 1.66. In the case of Comet Skjellerup,[11]

$$\omega_d = 2.08 \times 10^9,\ b = 7.85 \times 10^9,\ l = 5 \times 10^9\ \text{cm},\ f_{D_2} = 2f_{D_1} = 0.71\ .$$

Consequently, for D_2,

$$\frac{Nf_2 l}{b} = N \times 0.454 = \frac{N}{2.2}\ ,$$

and for D_1,

$$\frac{Nf_1 l}{b} = \frac{N}{4.4}\ .$$

The curves of growth express the fact, which was first pointed out by Ladenburg and Reiche, that the total absorption is proportional to the optical density for small values of the latter regardless of whether the absorption coefficient is determined by radiation damping, Doppler effect, or both; whereas, in the event that radiation damping is an appreciable agent for the absorption ($a > 0.01$), the total absorption is proportional to the square root of the optical density for large values of the latter.

In the case of Comet Skjellerup

$$a = \frac{2 \times 10^7\ (ln\ 2)^{1/2}}{2.08 \times 10^9} = 0.008\ ,$$

and we are therefore concerned with the intermediate portion of the curves of growth. This section, in which Nfl/b ranges between 10^2 and 2×10^4, supplies the clue for the approximately equal total emission of D_2 and D_1. It is evident that in this region the emission increases very slowly with increasing optical density. Specifically, the ratio of the total emission of the two lines, for which the ratio of the optical density is 2:1, differs from unity by only 10–15 per cent.

A more exact determination of the optical density demands recognition of the fact that the intensity of sunlight $I_{0,2}$ in the cometary spectral region of D_2 will, in general, differ from $I_{0,1}$ in the region D_1. This is due to the different contours of the D-lines in the solar

[11] In this calculation the comet's atmosphere has been assumed to be spherical and its diameter has been taken to be l, as determined from the length of the D-lines.

TABLE 1

$\log Nlf/b$	Nlf/b	$A/2bI_0=Q\pi/bI_0$		
		$a=0$	$a=0.001$	$a=0.01$
0.2	1.585	0.155	...	...
0.3	2.0	.177	...	...
0.4	2.51	.218	...	...
0.5	3.17	.267	...	...
0.6	3.98	.320	...	...
0.7	5.01	.388	...	...
0.8	6.30	.460	...	...
0.9	7.94	.545	...	...
1.0	10.0	.652	...	...
1.1	12.6	.762	...	...
1.2	16	.875	...	...
1.3	20	1.00	...	...
1.4	25	1.12	...	...
1.5	32	1.23	...	...
1.6	40	1.32	...	...
1.7	50	1.40	...	...
1.8	63	1.48	...	...
1.9	80	1.57	...	...
2.0	100	1.65	...	...
2.1	126	1.71	...	...
2.2	160	1.77	...	...
2.3	200	1.83	...	...
2.4	250	1.89	...	1.90
2.5	320	1.95	...	1.98
2.6	400	2.01	...	2.05
2.7	500	2.07	...	2.12
2.8	630	2.12	...	2.20
2.9	800	2.18	...	2.29
3.0	1000	2.23	...	2.38
3.1	1260	2.29	...	2.48
3.2	1600	2.34	...	2.58
3.3	2000	2.40	...	2.68
3.4	2500	2.44	...	2.79
3.5	3200	2.49	...	2.90
3.6	4000	2.54	...	3.02
3.7	5000	2.59	2.61	3.17
3.8	6300	2.64	2.67	3.32
3.9	8000	2.68	2.73	3.49
4.0	10000	2.72	2.80	3.67
4.1	12600	2.78	2.89	3.90
4.2	16000	2.82	2.98	4.20
4.3	20000	2.86	3.07	4.56
4.4	25000	2.90	3.16	5.00
4.5	32000	2.94	3.27	5.47
4.6	40000	2.98	3.38	6.0
4.7	50000	3.02	3.56	6.6
4.8	63000	3.06	3.63	7.4
4.9	80000	3.10	3.80	8.25
5.0	100000	3.14	4.00	9.2
5.1	126000	3.18	4.22	...
5.2	160000	3.22	4.50	...
5.3	200000	3.25	4.81	...

spectrum and to the fact that the comet possessed an appreciable radial velocity relative to the sun, the latter resulting in a Doppler displacement of the D-lines. Unsöld[12] and Korff[13] have mapped the experimental contours of the solar D-lines. From the known heliocentric radial velocity of the comet[14] and the work of Korff and Unsöld, it appears that

$$\frac{I_{0,2}}{I_{0,1}} \cong 0.96 \; .$$

The ratio Q_2/Q_1 of the observed total emission of the D-lines in the comet appears to be $0.9 \leqslant Q_2/Q_1 \leqslant 1.1$. The corresponding ratio of the quotient Q/I_0 for the lines D_2 and D_1 is

$$0.86 \leqslant \frac{\dfrac{Q_2}{I_{0,2}}}{\dfrac{Q_1}{I_{0,1}}} \leqslant 1.14 \; .$$

The corresponding possible limits of the density N of normal

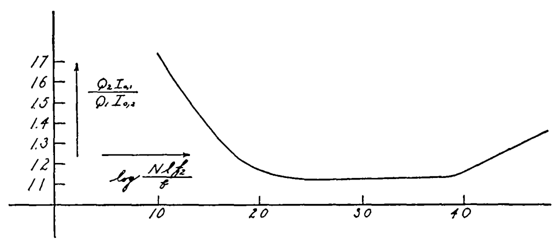


FIG. 3.—Relative intensity as function of optical density

sodium atoms are easily obtained from Table 1. They may also be obtained from Figure 3, which exhibits the dependence of

$$\frac{\dfrac{Q_2}{I_{0,2}}}{\dfrac{Q_1}{I_{0,1}}}$$

[12] *Zs. f. Phys.*, **46**, 772, 1928.

[13] *Ap. J.*, **76**, 291, 1932.

[14] The orbital elements for Comet Skjellerup (1927 k) have been given by A. C. D. Crommelin in the *M.N.R.A.S.*, **88**, 597, 1928, as

T = 1927 Dec. 18.18340 U.T. $\quad i = 85°6'22''.01$

$\omega = 47°11'\,13''.24$ $\quad \log q = 9.2462789$

$\Omega = 77°13'\,29''.62$

upon the optical density Nf_2l/b. The lower limit of the density is rather sharply defined, yielding $N > 170$ atoms/cm³; the upper limit is not quite so sharply defined, yielding $N < 22{,}000$ atoms/cm³. Consequently we may conclude, in the first approximation, that the density of normal sodium vapor lies between 10^2 and 2.5×10^4 atoms/cm³.

The second approximation to the solution of the problem is achieved by evaluating the integral P (see Appen.). The results of

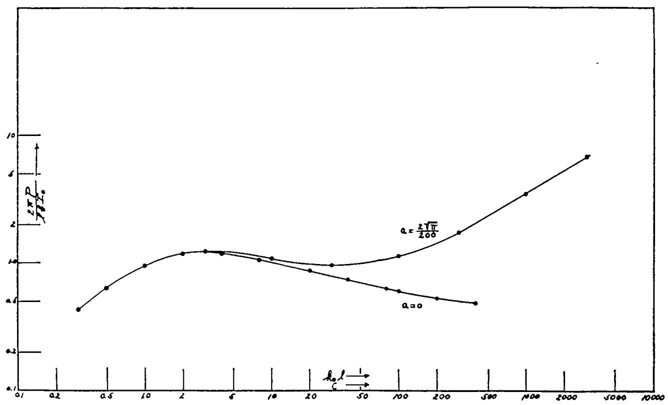


FIG. 4.—Curves of growth in the second approximation

these approximate calculations are collected in Figure 4. The quantity

$$k_0 l = C = Nfl \frac{2e^2}{\mu c} \frac{\pi^{3/2}}{b} = 2Nf \times 0.03 \; ,$$

which is plotted as abscissa, is found by introducing the values $e^2/\mu c = 8.45 \times 10^{-3}$, $b = 7.85 \times 10^9$, and $l = 5 \times 10^9$; it follows that $N = C_{D_2} \times 25 = C_{D_1} \times 50$; the ordinates are the values of $2\pi \; P/b \, I_0 \, l$. The lower curve is drawn for $a = 0$, and the upper one for $a/2 = \sqrt{\pi}/200 = 0.00887$, where, as before, $a = 2\omega_r/\omega_d$ $(ln 2)^{1/2} = 1.66 \; \omega_r/\omega_d$.

In the same manner in which Figure 3 was interpreted, we now consider the dependency of the ratio

$$\frac{\dfrac{P_2}{I_{0,\,2}}}{\dfrac{P_1}{I_{0,\,1}}}$$

upon C_{D_2}. For the temporary atmosphere of Comet Skjellerup, this ratio lies between 0.86 and 1.14, and we conclude from Figure 4 that the upper limit of the density of the normal sodium atoms lies in the neighborhood of 2500 atoms/cm^3 (instead of 22,000 as was determined in the first approximation) and that the lower limit may be as low as 50 atoms/cm^3 (instead of 170).

It must be borne in mind that the inequalities involving N should, strictly speaking, be expressed in the form of limits. Thus, the first approximation yields either $N = 170$ or $N = 22{,}000$, and the second approximation yields either $N = 50$ or $N = 2{,}500$. We must, therefore, find an independent method of approach which will indicate which limit is to be considered the solution to the problem. It is clear that if the lower limit is the appropriate one, then the effect of the cometary sodium upon the escaping radiation is negligible, and the second approximation adds little to the first.

IV. THE CALCULATED MONOCHROMATIC MAGNITUDE OF COMET SKJELLERUP

A knowledge of the approximate dimensions of the comet's atmosphere combined with the density of the normal sodium as determined in Section III, makes it possible to form an estimate of the magnitude of the cometary head in the light of the D-lines.

The energy of a single D-emission is $1/3 \times 10^{-11}$ ergs. The number of transitions in the entire cloud per second is equal to the volume of the cloud multiplied by the mean density, multiplied by the number of transitions per atom per second, or

$$\frac{\pi D^3 N}{6t} ,$$

where t is the time required by a sodium atom to execute an absorption and emission of a D-quantum. The total energy emitted per second by the cloud is $1/3 \times 10^{-11} \times \pi D^3 N/6t$ ergs. The luminous power output of the sun is 5×10^{32} ergs/sec. and its apparent magnitude is -26.7. The sun's apparent brightness, therefore, exceeded that of Comet Skjellerup's head by a factor

$$F = \frac{5 \times 10^{32}}{\frac{1}{3} \times 10^{-11} \times \frac{\pi D^3 N}{6t}} .$$

Since $50 \leqslant N \leqslant 2500$, the limits of this factor are

$$10^{12}t \leqslant F \leqslant 5 \times 10^{13}t ,$$

and the monochromatic brightness of the cloud is $-26.7 + x$, where $2.5^x = F$. We must now determine t. This may be done by the application of Milne's method of finding the resultant momentum acquired by a sodium atom during the time t. The momentum acquired in falling toward the sun is mgt. The mean momentum acquired away from the sun in virtue of absorption (radiation pressure) is $h\nu/c$. The momenta toward and away from the sun are in the same ratio as the forces of gravitation and repulsion due to radiation pressure. Baade and Pauli[15] have determined this ratio and have shown it to be

$$Z = \frac{8\pi^2 e^2 h e^{-\frac{h\nu}{kT}}}{\mu m c G \lambda^3} = \frac{\text{radiation}}{\text{gravitation}} ,$$

where e is the electronic charge, μ is the electronic mass, m is the mass of the sodium atom, and T is the sun's temperature. Wurm[16] has shown that better agreement is secured when it is assumed that the ratio as determined by Baade and Pauli is too great by a factor of 2. We arrive, therefore, at the relationship

$$\frac{h\nu}{c} = \frac{Z}{2} mgt ,$$

from which it follows that $t = 0.5 \times 10^{-3}$ seconds. Consequently,

$$5 \times 10^8 \leqslant F \leqslant 2.5 \times 10^{10} ,$$

and therefore $22 \lesssim X \lesssim 26$. It would appear that the monochromatic magnitude of the comet's head was $-4.7 \lesssim M \lesssim -0.7$. (The comet was very nearly one astronomical unit distant from the earth at the time of observation.)

Reference to the description of the comet given in Section I indi-

[15] W. Baade and W. Pauli, *Naturwissenschaften*, **15**, 50, 1927.

[16] K. Wurm, *Zs.f. Ap.*, **10**, 287, 1935.

cates that the sodium magnitude of the comet was much nearer −0.7 than −4.7 and was probably in the neighborhood of magnitude −1. This would seem to indicate that the density of normal sodium atoms in the cloud was in the immediate neighborhood of the lower limit given in Section III, that is, in the immediate neighborhood of 50 or 100 atoms/cm³.

V. THE PROPORTION OF NORMAL TO IONIZED SODIUM ATOMS IN THE TEMPORARY ATMOSPHERE OF COMET SKJELLERUP

Owing to the high degree of ionization, the total number of sodium atoms per cm³ must have exceeded by far the density of normal sodium. An approximation to the degree of ionization prevalent in the comet's atmosphere can be ascertained from Saha's relation. The expression

$$\log \frac{x^2 p}{1 - x^2} = - 5048 \frac{I}{T} + \tfrac{5}{2} \log T - 6.5 + \log \frac{R^2}{4r^2} ,$$

where

x = the fraction of atoms ionized
p = the gas pressure
I = the ionization potential of sodium = 5.1 electron volts
T = the sun's temperature = 6000°
$(R^2/4r^2)$ = the factor of dilution of the sun's black-body radiation at the distance r of the comet from the sun,

leads to the value $Ni^2/N = 10^{15}$, where N_i = the number of ionized atoms per cm³, and N = the number of normal atoms per cm³. Since N appears to be in the neighborhood of 100 atoms per cm³, we have N_i approximately equal to 3×10^8. It follows that for every normal sodium atom present in the head of Comet Skjellerup there were approximately 3,000,000 ionized ones.

APPENDIX

The evaluation of the integral P given below is due to Mr. John Sweer, of Princeton University.

Evaluation of the integral

$$I = \int_0^\infty \frac{\{1 - e^{-\alpha(\nu)l}\}^2}{\alpha(\nu)} d\nu ,$$

which is identical with P when $I_0 = 1$.

$$\alpha(\nu) = k_0 \left\{ e^{-\omega^2} - \frac{a}{\sqrt{\pi}} (1 - 2\omega F(\omega)) \right\}$$

$$\mathrm{F}(\omega) = e^{-\omega^2} \int_0^\omega e^{y^2} dy$$

$a = 2\,\omega_r/\omega_d\,(ln\ 2)^{1/2}$ is taken to be $\leqq 0.02$.

CASE I

Taking $a = 0$ and writing $k_2 = C \times l$, we find

$$I = \frac{l}{C} \int_0^\infty \frac{\{1 - e^{Ce^{-\omega^2}}\}^2}{e^{-\omega^2}} d\nu = \frac{lb}{2\pi C} \int_{-\infty}^{+\infty} \frac{\{1 - e^{-Ce^{\omega^2}}\}^2}{e^{-\omega^2}} d\omega ,$$

where $b = \pi\omega_d/(ln\ 2)^{1/2}$, and where the lower limit is set at $-\infty$ because the integrand approaches zero very rapidly for values of $-\omega$ greater than approximately $100 \times \omega_d$. In order to carry out the above integration, let us first consider the function

$$T(C) = \int_{-\infty}^{+\infty} \{1 - e^{-Ce^{-\omega^2}}\} d\omega .$$

We may now write

$$\int_0^C T(C) dC = \int_{-\infty}^{+\infty} \left\{ C + \frac{e^{-Ce^{-\omega^2}} - 1}{e^{-\omega^2}} \right\} d\omega ,$$

from which it follows that

$$\int_0^{2C} TdC - 2\int_0^C TdC = \int_{-\infty}^{+\infty} \left\{ 2C + \frac{e^{-2Ce^{-\omega^2}} - 1}{e^{-\omega^2}} - 2C - 2\,\frac{e^{-Ce^{-\omega^2}} - 1}{e^{-\omega^2}} \right\} d\omega$$

$$= \int_{-\infty}^{+\infty} \frac{\{1 - e^{-Ce^{-\omega^2}}\}^2}{e^{-\omega^2}} d\omega .$$

The function $T(C)$ has been tabulated.[9] It is equal to $\sqrt{\pi}$ times the product $C\cdot S(C)$, which is given on page 204 of the first reference in note 9 above. We may now put the integral in the form

$$I = \frac{l}{C}\frac{b}{2\pi}\left\{\int_0^{2C} TdC - 2\int_0^{C} TdC\right\}$$

$$= \frac{l}{C}\frac{b}{2\sqrt{\pi}}\left\}\int_0^{2C} C\cdot S(C)dC - 2\int_0^{C} C\cdot S(C)dC\right\}.$$

The values of the intensity function for $a = 0$ have been calculated from this formula by means of numerical integration of the tabulated function given in the first reference under note 9 above.

Case II

For $a \neq 0$ the integral becomes

$$I = \frac{lb}{2\pi}\int_{-\infty}^{+\infty} \frac{\left\{1 - e^{-C\left(e^{-\omega^2} - \frac{a}{\sqrt{\pi}}\{1-2\omega F(\omega)\}\right)}\right\}^2}{C\left\{e^{-\omega^2} - \frac{a}{\sqrt{\pi}}(1 - 2\omega F(\omega))\right\}} d\omega\ .$$

Numerical integration is again employed in this case, from $\omega = 0$ to that value $\underline{\omega}$ of ω for which $e^{-\omega^2}$ may be neglected, and for which a power series expansion of

$$e^{-C\left(e^{-\omega^2} - \frac{a}{\sqrt{\pi}}\{1-2\omega F(\omega)\}\right)}$$

may be assumed to be valid when fourth-degree terms of the argument are dropped. For these greater values of ω, we may write (putting $aC/\sqrt{\pi} = A$):

$$\frac{\pi I'}{bl} = \int_{\underline{\omega}}^{\infty} \frac{1 - 2e^{A(1-2\omega F)} + e^{2A(1-2\omega F)}}{-A(1-2\omega F)} d\omega$$

$$= \int_{\underline{\omega}}^{\infty} \{-A(1-2\omega F) - A^2(1-2\omega F)^2 - \tfrac{7}{12}A^3(1 - 2\omega F)^3 - \ldots\}d\omega$$

$$= \Big| -AF - A^2F - A^2\left\{\int F^2 d\omega - \omega F^2\right\}$$

$$- \tfrac{7}{12}A^3\left\{F - 2\omega F^2 + \tfrac{4}{3}\left(\omega^2 F + \frac{F^3}{3}\right) + \tfrac{2}{3}\right\}\int F^2 d\omega \Big|_{\underline{\omega}}^{\infty}\ .$$

The foregoing follows readily from the relation

$$1 - 2\omega F = \frac{dF}{d\omega}\ .$$

Upon the introduction of the semi-convergent series

$$F(\omega) = \frac{1}{2\omega}\left\{1 + \frac{1}{2\omega^2} + \frac{1\cdot 3}{(2\omega^2)^2} + \frac{1\cdot 3\cdot 5}{(2\omega^2)^3} + \frac{1\cdot 3\cdot 5\cdot 7}{(2\omega^2)^4} + \dots\right\} ,$$

we find

$$\frac{\pi I'}{bl} = \frac{1}{2\underline{\omega}}A + \frac{1}{4\underline{\omega}^3}(A - \tfrac{1}{3}A^2) + \frac{1}{8\underline{\omega}^5}(3A - \tfrac{6}{5}A^2 + \tfrac{7}{60}A^3) + \frac{1}{16\underline{\omega}^7}(15A - \tfrac{39}{7}A^2 + \tfrac{3}{4}A^3) .$$

This formula was employed to evaluate the integrals over the edges of the line.

Lowell Observatory
Flagstaff, Arizona
and
Princeton University
Princeton, New Jersey
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