NEBULOUS OBJECTS IN MESSIER 31 PROVISIONALLY IDENTIFIED AS GLOBULAR CLUSTERS¹

By EDWIN HUBBLE

ABSTRACT

One hundred and forty nebulous objects have been found in or close to the borders of Messier 31 which, from their numbers, their distribution, and the radial velocity of a typical example, are presumably associated with the spiral. From their forms, structure, colors, luminosities, and dimensions they are provisionally identified as globular clusters.

Absolute photographic magnitudes range from -4 to -7, the mean being -5.3. The luminosity function has a double maximum, which suggests a mixture of two homogeneous groups having most frequent magnitudes at -5.0 and -6.2. Diameters range from about 4 to 16 parsecs.

The number of objects per unit area decreases with distance from the nucleus of M $_{31}$, and occasional objects are found as far as $_{3.5}^{\circ}$ from the nucleus. The diameter of the spiral as derived from the distribution of these objects is probably of the order of $_{30,000}$ parsecs.

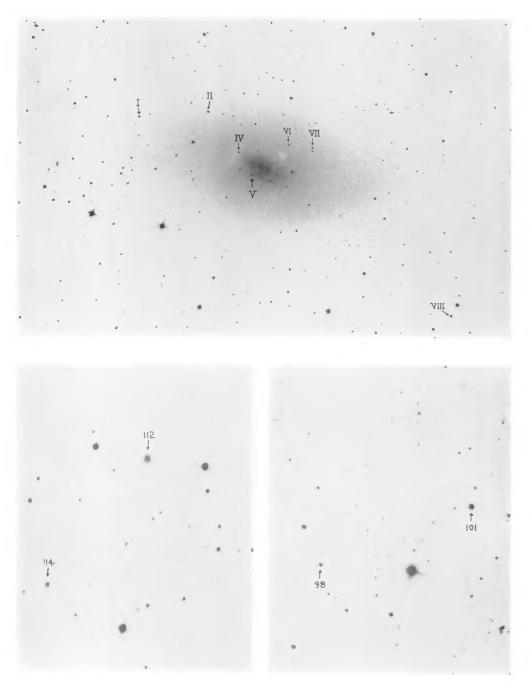
According to Shapley's distances and magnitudes for the clusters in our system, reduced to the conventional scale, *the objects in M* $_{31}$ are systematically fainter than the galactic globular clusters, by an amount varying from about 0.75 to 1.95 mag. according to the interpretation of the data. The ranges in absolute luminosity are of the same order, however, and the two groups overlap to a considerable extent.

The known globular clusters in the Magellanic Clouds are comparable with the brighter objects in M 31. Objects apparently similar to those in M 31 are found in N.G.C. 6822, M 33, M 81, and M 101.

The great spiral in Andromeda, Messier 31, lies in a region of the sky where the distribution of extra-galactic nebulae appears to be normal. Within the limits of M 31 the typical small nebulae are not numerous; in fact, they can be identified with reasonable certainty only in the outermost parts. Over the face of the spiral, however, are found many nebulous objects that approximate to the general appearance of extra-galactic nebulae but can be distinguished from the latter with some confidence on large-scale plates where the images are sharply defined.

The objects in question may be described as "nebulous stars." They are small, highly concentrated, round, and perfectly symmetrical. On the plates they resemble soft, hazy star images, but build up with increasing exposure in a manner perceptibly different from true stellar images. Visually they appear like small condensed nebulae. In the case of the brighter objects, the nebulous character is

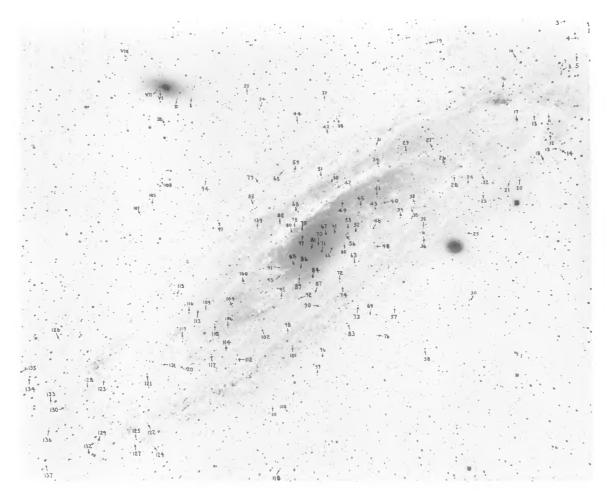
¹ Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington, No. 452.



(Above) N.G.C. 205, 100-inch reflector. North is to the right. Scale, 1 mm = 8".9. Object No. III is out of the field but is marked on Plate VIII.

(Below) Typical objects with 100-inch reflector. South is at the top. Scale, 1 mm = 3.7. Objects Nos. 101 and 112 represent the extreme range in degree of condensation.

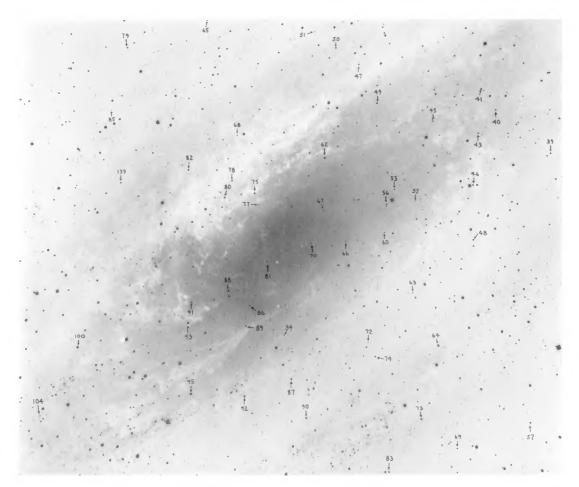
PLATE VIII



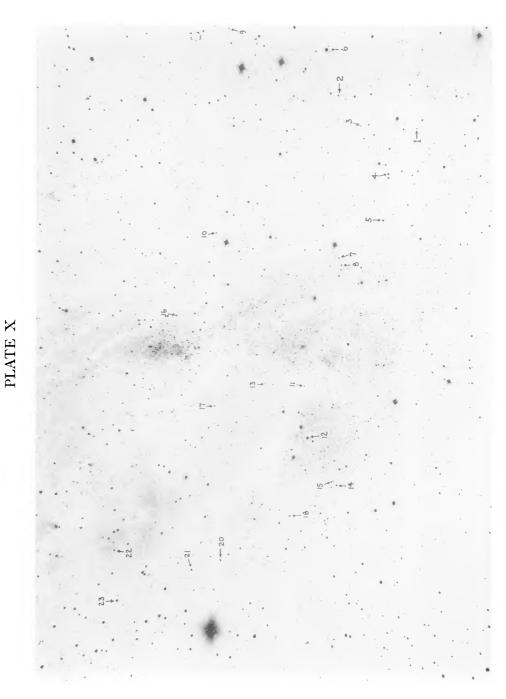
Messier 31 with the 24-inch Yerkes reflector. North is to the left. Scale, 1 mm = 44''. Objects Nos. 2, 6, and 9 are out of the field but are marked on Plate X. Objects Nos. IV and V, near the nucleus of N.G.C. 205, are omitted, but are marked on Plate VII. Objects Nos. 138, 139, and 140 are out of the fields of all plates. Their positions are indicated by the co-ordinates in Table I.

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PLATE IX



Central region of Messier 31 with 100-inch reflector. North is to the left. Scale, 1 mm = 14.7



Outer region of Messier 31, centered about 45' south preceding the nucleus, with the 100-inch reflector. North is at the top. Scale, 1 mm = 15''8.

more conspicuous in the telescope than on the photographic plate. Mr. M. L. Humason, who has examined several of the objects at the 135-foot focus of the 100-inch reflector, confirms this description and adds that the objects are appreciably more colored (larger colorindex) than certain small clusters of early-type stars that are known in the spiral.

The general features of the photographic images are indicated by typical examples shown in Plate VII. Their appearance approximates that of the most condensed of the globular nebulae, a type rarely found; but their numbers and their symmetrical distribution suggest that the objects are associated with the great spiral itself. This relationship is confirmed, even established, by a spectrogram of a typical example, No. 62 in Table I, obtained by Mr. Humason with a small-scale spectrograph on the 100-inch reflector. The spectral type is estimated as about F8 and the radial velocity as about -210 km/sec., with an uncertainty of the order of 100 km/sec. Within the uncertainties of the measures the object shares the unique velocity of M 31 and hence an actual association is presumed.

The observed characteristics of the objects appear to admit of but one interpretation. On the basis of structure, luminosity, diameters, and colors, the objects are provisionally identified as globular clusters. Their absolute magnitudes are systematically fainter by one or two magnitudes than the absolute magnitudes of the globular clusters in the galactic system derived from Shapley's distances; but the range in absolute luminosity for the two groups of objects is much the same, and there is a considerable overlap in luminosity.

OBSERVATIONAL DATA

The objects in M $_{31}$ are marked for identification on Plates VIII, IX, and X and are listed in Table I together with their positions, photographic magnitudes, diameters in seconds of arc, and rough indications of the degree of concentration. Positions are given in minutes of arc from the nucleus of M $_{31}$, X along the major axis (+ to the south preceding), Y along the minor axis (+ to the north preceding). The orientation of the major axis is assumed to be N. $_{36?7}$ E. in accordance with a previous determination.¹ The fourth

¹ Mt. Wilson Contr., No. 365; Astrophysical Journal, 69, 103, 1929.

TABLE I

NEBULOUS OBJECTS IN MESSIER 31

	X	Y	4 <i>Y</i>	D	m(pg)	d	Conc
I	+60'.6	+ 3'.4	13'.6	62'.1	17.4	6	m
2	58. 0	+7.8	31.2	65.9	17.1	б	c
3	57.9	+5.6	22.4	62.1	17.2	5	m
4	57.7	+ 2.2	8.8	58.4	16.2	Ğ	c
5	55.7	+ 0.1	0.4	55.7	16.7	6	m
6	59.I	+10.4	41.6	72.3	17.4	5	m
7	52.2	- o.2	0.8	52.2	17.0	5 6	m
8	52.0	- 0.6	2.4	52.1	17.2	5	m
9	54.7	+15.0	60.0	81.2	16.8	5 6	m
10	47.0	+ 5.3	21.2	51.6	17.1	6	d
11	45.8	- 5.4	21.6	50.7	18.0	5	m
12	44.I	- 8.I	32.4	54.7	15.0	IO	m
13	44.0	- 3.9	15.6	46.7	18.0	5	m
14	43.8	-11.5	46. 0	63.5	17.3	7	d
15	43.6	-11.1	44 · 4	62.2	17.2	5	m
16	42.I	+ 2.6	10.4	43.4	16.1	6	c
17	40.7	- 3.3	13.2	42.8	17.2	5	c
18	40.4	-11.4	45.6	60.9	17.2	7	m
19	36.2	+16.5	66. 0	75.3	16.0	6	c
20	35.5	-11.3	45.2	57.5	17.0	5	c
21	33 · 7	-10.7	42.8	54.5	17.0	5	c
22	30.6	- 7.0	28.0	41.5	17.3	7	d d
23	28.9	- 9.7	38.8	48.4	15.5	10	c
24	27.5	- 6.8	27.2	38.7	17.9	4	m
25	22.6	-13.3	53.2	57.8	18.2	4	m
26	26.5	— I.8	7.2	27.5	16.2	7	m
27	26.4	+ 1.5	6.0	27.I	16.3	7	m
28	26.5	- 5.7	22.8	35.0	18.2	4	m
29	22.4	+ 3.4	13.6	26.2	17.4	4	m
30	16.2	-23.I	92.4	93.8	16.8	5	m
31	18.9	+ 7.6	30.4	35.8	17.8	4	d
32	18.5	- 4.I	16.4	24.7	16.8	6	m
33	17.6	- 4.4	17.6	24.0	16.5	5	m
34	17.2	+ 4.5	18.0	24.9	16.8	5 6	m
35	17.2	-8.3	33.2	37 · 4	16.1	6	m
36	16.4	- 9.8	39.2	42.5	16.5	6	m
37	16.5	+19.0	76.0	77.8	17.1	5	m
38	16.4	+15.1	60.4	62.6	16.7	5 6	c
39	15.2	- 4.6	18.4	23.9	16.8		m
40	+14.1	- o.8	3.2	14.4	17.1	5	m

46

	X	Y	4 <i>Y</i>	D	m(pg)	d	Conc.
4 ¹ 4 ² 4 ³ 4 ⁴ 4 ⁵	+14'1 13.6 12.5 10.4 10.8	+ 0.6 +14.2 - 1.3 +18.5 + 0.8	2'4 56.8 5.2 74.0 3.2	14'3 58.4 13.5 74.7 11.3	16.7 15.3 17.1 15.5 16.8	5 8 5 9 4	C m m C C
46 47 48 49 50	10.6 9.3 8.8 8.9 8.8	$ \begin{array}{r} - & 3 \cdot 5 \\ + & 5 \cdot 8 \\ - & 6 \cdot 0 \\ + & 3 \cdot 4 \\ + & 7 \cdot 3 \end{array} $	14.0 23.2 24.0 13.6 29.2	17.6 25.0 25.6 16.3 30.5	16.8 17.5 16.9 16.3 16.8	5 4 6 5 5	m m c m
51 52 53 54 55	8.3 7.4 6.8 6.8 6.5	+ 8.7 - 2.4 - 1.1 + 24.6 + 27.6	34.8 9.6 4.4 98.4 110.4	35.8 12.1 8.1 98.6 110.5	18.0 17.5 16.9 17.6 16.3	4 5 4 3 7	m m m m
56 57 58 59 60	6.0 5.4 5.1 5.2 4.9	$ \begin{array}{r} - 1.5 \\ -16.3 \\ -25.2 \\ +12.2 \\ - 2.8 \\ \end{array} $	6.0 65.2 100.8 48.8 11.2	8.5 65.4 100.9 49.1 12.2	16.6 16.3 17.0 16.8 17.1	5 6 7 5 4	m m c m
61 62 63 64 65	4.6 4.7 4.2 3.7 3.8	$ \begin{array}{r} - & 0.2 \\ + & 2.6 \\ - & 6.5 \\ - & 9.7 \\ + & 12.8 \\ \end{array} $	0.8 10.4 26.0 38.8 51.2	4 · 7 10 · 5 26 · 3 39 · 0 51 · 3	16.6 15.4 17.5 15.2 16.9	5 8 5 8 5	C M M C M
66 67 68 69 70	2.9 2.8 I.3 I.I I.2	$ \begin{array}{r} - 1.9 \\ + 0.2 \\ + 6.6 \\ - 15.1 \\ - 0.9 \end{array} $	7.6 0.8 26:4 60.4 3.6	8.1 2.9 26.4 60.4 3.8	17.0 16.5 17.4 17.2 15.0	4 4 4 5 8	C C M M C
71 72 73 74 75	0.9 0.6 0.5 0.4 0.2	$ \begin{array}{r} - 2.3 \\ - 7.3 \\ - 12.6 \\ - 8.2 \\ + 3.3 \end{array} $	9.2 29.2 50.4 32.8 13.2	9.2 29.2 50.4 32.8 13.2	16.8 17.0 15.9 16.6 15.6	3 4 6 6 6	c m c m c
76 77 78 79 80	+ 0.1 - 0.4 - 0.8 - 1.3	$ \begin{array}{r} -18.5 \\ + 2.6 \\ + 4.6 \\ + 14.2 \\ + 4.1 \end{array} $	74.0 10.4 18.4 56.8 16.4	74.0 10.4 18.4 56.8 16.5	15.8 16.6 17.0 16.9 16.0	7 4 4 4 5	C C M M C

TABLE I—Continued

TABLE I-Continued

	X	Y	4 <i>Y</i>	D	m (p g)	d	Conc.
81	-1'5	- 0:4	1.'6	2.2	15.5	4	с
82	2.0	+ 6.6	26.4	26.5	17.0	6	m
33	2.7	-14.0	56.0	56.I	16.8	6	d
34	3.I	- 4.2	16.8	17.1	17.0	5	m
85	3.6	+11.9	47.6	47.7	17.6	5	m
36	3.7	- I.8	7.2	8.1	15.9	5	с
87	4.2	- 6.5	26.0	26.3	15.8	7	m
38	4.2	- 0.4	I.6	4.5	16.5	5	с
89	4.5	- 2.6	10.4	11.3	16.0	5 6	с
0	4.8	- 8.8	35.2	35.5	15.7	6	с
91	6.3	+ 0.3	I.2	6.4	15.8	6	с
)2	6.9	- 5.8	23.2	24.2	15.7	7	С
93	7.I	- 0.6	2.4	7.5	15.4	8	С
94	7.7	+20.2	80.8	81.2	16.3	7	m
95	9.3	- 3.8	15.2	17.8	15.3	8	m
96	9.8	-16.3	65.2	65.9	17.3	4	m
97	IO.2	+13.2	52.8	53.8	16.1	8	m
98	11.9	- 9.4	37.6	39.4	16.6	6	с
99	I2.2	-18.0	72.0	73.0	15.8	6	с
	13.0	+ 2.0	8.0	15.3	15.7	7	с
01	12.6	-11.6	46.4	48.1	15.2	7	c
02	15.1	- 6.0	24.	28.4	15.8	7	c
03	I4.I	+24.4	97.6	98.6	17.I	5	m
04	17.0	+ 0.I	.4	17.0	17.I	4	m
05	17.7	+21.8	87.2	89.0	16.8	6	c
06	19.7	- 2.6	10.4	22.3	15.7	7	c
07	20.3	+21.8	87.2	89.5	17.3	6	m
o8	19.7	- 20.4	81.6	83.9	16.8	6	m
09	20.9	+ 2.0	8.0	22.4	17.0	4	m
10	21.1	- 0.8	3.2	21.3	17.0	4	m
II	21.4	- I7.7	70.8	74.0	17.1	5	m
12	21.6	- 7.7	30.8	38.1	15.7	10	d
13	21.8	+ 2.9	11.6	24.7	17.3	4	m
14	22.2	- 5.5	22.0	31.3	17.0	6	m
15	23.4	+ 6.8	27.2	35.9	15.9	7	с
16	23.5	+ 3.3	13.2	26.9	16.0	6	c
17	24.7	- 4.6	18.4	30.8	17.0	5 6	m
18	26.7	- 27.2	108.8	112.0	16.0		c
19	26.8	+ 0.4	I.6	26.8	17.0	5 8	m
20	- 30. I	- 2.9	11.6	32.3	17.1	8	d

	X	Y	4 <i>Y</i>	D	m(pg)	d	Conc.
121 122 123	-35.4 40.3 41.8	- 0.9 - 7.7 + 2.6	3'6 30.8 10.4	35'6 50.7 43.1	16.1 18.0 17.4	6 4 4	c m c
124 125	41.8 43.4	-11.5 - 8.8	46.0 35.2	62.2 55.9	17.7 15.8	4 5	c c
126 127 128 129 130	43 · 9 44 · 1 45 · 0 48 · 8 50 · 0	+12.9 - 9.1 + 3.2 - 5.3 + 2.7	51.6 36.4 12.8 21.2 10.8	67.7 57.2 46.8 53.2 51.2	17.5 17.7 17.7 17.0 16.3	4 5 5 4 5	m m c c
I 3 I	32.4 51.5 51.6 52.0 53.3	$ \begin{array}{r} - 1.0 \\ - 6.2 \\ + 4.8 \\ + 10.5 \\ + 11.5 \end{array} $	4.0 24.8 19.2 42.0 46.0	32.6 57.2 55.1 66.8 79.4	17.0 17.3 17.2 16.2 17.5	5 4 5 7 5	m c c m c
136 137 138 139 140	58.4 59.7	+ 1.5 - 3.0 - 14.9 + 8.2 - 51.0	6.0 12.0 59.6 32.8 204.0	54.6 59.6 84.4 33.3 213.3	16.9 17.0 17.0 17.2 15.5	5 6 5 7	m m d m c

TABLE I—Continued

column of Table I gives the Y co-ordinates quadrupled in order to correct for the tilt of the spiral, and the fifth column the distance from the nucleus as determined from the measured X and the corrected Y.

Photographic magnitudes for the objects brighter than about 17.2 were derived from extra-focal exposures, with the exception of several near the nucleus of M $_{31}$ and a few in the extreme north following end. Magnitudes for these exceptions and for the objects fainter than 17.2 were estimated on focal exposures, but closely conform to the scale established by the extra-focal measures. The magnitudes near the nucleus are slightly uncertain, since exposures of one minute or less were required to register the objects free from a luminous background, and the comparison with more distant objects assumes that the building-up of the images with increasing exposure is the same in both cases. With the exception of No. 12, 55' S. preceding the nucleus, No. 70, the object nearest the nucleus, is also the brightest of the list.

Diameters on hour exposures with fast plates range from about

10" to 4" as the magnitudes range from about 15.0 to 18.0; but these are minimum values, since the images were measured against a luminous background. Experience indicates that the background diminishes the apparent diameters, an effect previously observed¹ in the growth of the image of M 32, the brighter companion of M 31, and shown by the present data in the systematic increase in the sum $m+5 \log d$ with increasing distance from the nucleus.

The objects are all very similar and differ only in the degree of concentration toward the center. The luminosity fades from the center outward to undefined edges, and the rate of fading, i.e., the degree of concentration, ranges from that exhibited in No. 101 to that in No. 112 (see Pl. VII). The objects are roughly classed in three groups as "c," highly concentrated; "m," moderately concentrated; and "d," diffuse. The range is perhaps comparable to that exhibited by the globular clusters in the galactic system.

DISTRIBUTION OVER THE SPIRAL

Of the 140 objects listed in Table I, 130 are within the recognized boundaries of the spiral, 9 are in the immediate vicinity, and 1, a typical example, is well beyond the borders. Only 2 are fainter than magnitude 18.0. The list is believed to be reasonably complete, except for the extreme tips of the spiral, where few would be expected even if plates of the necessary quality were available. The nature of the very faint objects is somewhat uncertain, and further investigation may slightly revise the data for objects fainter than say 17.5.

The large number of objects within the borders of the spiral contrasts sharply with the number of extra-galactic nebulae to be expected on the basis of the normal distribution observed in the general vicinity. The normal distribution is represented by the equation²

$$\log N = 0.6 m - 9.5$$
,

where N is the number per square degree to the limiting photographic magnitude m. Since the spiral covers an area of the order of 1.4 square degrees (an ellipse about $160' \times 40'$), some twenty-eight

² Science, **75**, 25, 1931.

¹ Mt. Wilson Contr., No. 398; Astrophysical Journal, 71, 231, 1930.

nebulae may be expected to the limit 18.0. They should be distributed about as follows:

<15.5	Ι
15.5-16.0	I
16.0–16.5	2
16.5-17.0	4
17.0-17.5	7
17.5-18.0	14

Deviation from normal distribution might materially alter the numbers, but, in view of the relative infrequency of highly concentrated globular forms among nebulae in general, it seems improbable that any considerable number of extra-galactic nebulae are included in Table I. For the same reason it is also improbable that the objects represent a cluster of nebulae projected on the spiral.

The distribution of the objects with respect to the nucleus of M $_{31}$ is indicated in Figure 1. The co-ordinates of the points are the values of X and $_4Y$ listed in Table I; hence the distribution is that seen in a direction perpendicular to the plane of the spiral. The method of presentation neglects the scatter of the objects above and below the equatorial plane, but the uncertainty from this source should not seriously affect the general nature of the conclusions concerning the distribution. The brighter objects, magnitude 16.5 and brighter, are distinguished from the fainter in order to indicate separately the distribution of the two groups. The frequency distribution of magnitudes, as will appear later, suggests that the division may have some significance.

The distribution is roughly symmetrical about the nucleus, although the counts indicate an appreciable excess in the number of objects with negative Y over that for objects with positive Y. The same feature is exhibited in the distribution of novae and variable stars in M $_{31}$,^r and hence both the general distribution and the minor dissymmetry indicate an association of the objects with the spiral.²

¹ Mt. Wilson Contr., No. 376; Astrophysical Journal, 69, 103, 1929.

² Attempts have been made to account for the dissymmetry by various patterns of obscuration, but the procedure is arbitrary and the conclusions are uncertain. For instance, if we assume complete obscuration beyond the equatorial plane and ellipsoidal distribution of the objects, the observed distribution then indicates that the north-

The data are summarized in Tables II and III, which list the numbers of objects observed in successive zones concentric with the nu-

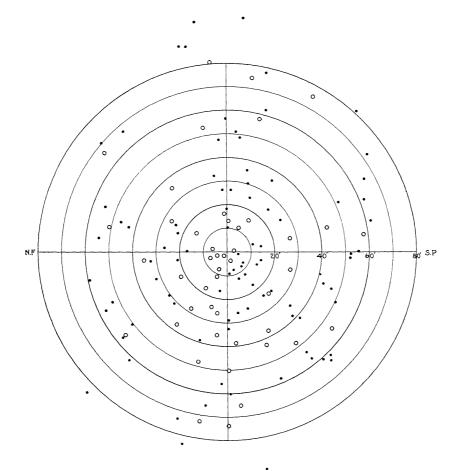


FIG. 1.—Distribution of nebulous objects in Messier 31. Horizontal line represents the major axis, the vertical line the minor axis. Y co-ordinates (distances from major axis) have been quadrupled to correct for the tilt of the spiral. Open circles indicate objects with magnitudes \geq 16.5; dots, magnitudes > 16.5. The apparently elliptical distribution is presumably due to the fact that regions more than about 65' from the minor axis have not been observed.

preceding half of the nebula is toward the observer, and it follows from the spectrographic rotation that the spiral is winding up rather than unwinding. The effects of obscuration both in M 31 and in our own system are obviously of great importance, but adequate discussion must be based on a wide range of data, much of which is as yet unorganized.

cleus of M $_{31}$, together with the numbers per unit area, and the mean distances from the nucleus, the magnitudes, the logarithms of the diameters, and the sums $m+_5 \log d$ for the objects in each zone. In

Distribution of Nebulous Objects with Respect to Nucleus of Messier 31 $(5^\prime\, {\rm Zones})$

7.007	NUMBERS			Area	DENSI-	D			m+
Zone	В	F	All	OF ZONE	N/AREA	D	m(pg)	LOG d	5 LOG <i>a</i>
o'- 5'	4	I	5	I	5.0	3.6	16.02	0.701	19.5
5-10	3	4	7	3	2.33	8.o	16.34	.680	19.74
0-15	3	7	10	5	2.00	12.3	16.59	. 698	20.0
5 - 20	4	4	8	7	I.14	17.0	16.40	.718	19.90
20 - 25	3	6	9	9	I.00	23.6	16.62	.717	20.2
25 – 30	5	8	13	II	1.17	26.8	16.75	.732	20.4
0-35	I	7	8	13	0.62	32.3	17.11	.732	20.7
5 -40	6	4	10	15	0.67	37.I	16.50	. 766	20.3
0 –45	2	3	5	17	0.29	42.7	16.90	. 740	20.6
.5 -50	2	4	6	19	0.32	47.8	16.80	.773	20.6
0-55	4	9	13	21	0.62	52.4	16.80	.741	20.5
5 -60	3		12	23	0.52	57.3	16.84	.718	20.4
o –65	õ	9 8	8	25	0.34	62.0	17.24	.733	20.9
5 -70	2	3	5	27	0.19	66.3	16.88	0.721	20.4

TUDDD III	TA	BLE	\mathbf{III}
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(10'2	Zones)
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7.000	Numbers			I AREA I	DENSI-	Đ		,	m+
Zone	В	F	All	OF ZONE	TY N'/AREA	D	$\overline{m}(pg)$	LOG d	5 LOG d
0'-10'	7	5	12	4	3.00	6.2	16.21	o.689	19.65
10-20	7	II	18	12	I.50	14.4	16.51	. 702	20.02
20 - 30	8	14	22	20	I.IO	25.5	16.70	. 726	20.33
30-40	7	II	18	28	0.64	36.2	16.77	.751	20.52
40-50	4	7	II	36	0.31	45.5	16.85	. 758	20.64
50-60	7	18	25	44	0.57	54.7	16.82	. 730	20.47
60 -70	2	II	12	52	0.25	63.7	17.10	. 728	20.74
70-80	4	4	8	60	(o.13)	73.9	(16.52)		20.37
80 - 90	I	5	6	68	(o.og)	84.9	(16.83)	(o.776)	20.71

addition to the total numbers, the numbers in each of the two groups, B (bright) and F (faint), referred to in the last paragraph, are given separately. The width of the zones is 5' in Table II and 10' in Table III. The results in Table II are exhibited graphically in Figures 2 and 3.

In Figure 2 the density, i.e., the number of objects per unit area, is plotted against distance from the nucleus of M 31. The objects are conspicuously concentrated toward the nucleus, slightly more so for those in group B than for those in group F.

A few of the objects are found beyond the recognized borders of the spiral, i.e., more than 80' from the nucleus. In order further to investigate the extreme limits of the scatter, several 100-inch reflector plates were examined, which were centered in the vicinity of

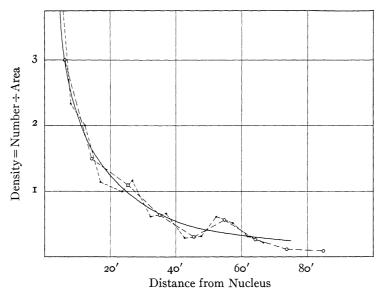


FIG. 2.—Distribution of nebulous objects with respect to the nucleus of Messier 31. Numbers of objects in successive zones divided by the areas of the zones are plotted against mean distances from the nucleus. Open circles, zones 10' in width; dots, 5' zones. Beyond 65' from the nucleus the data are incomplete.

M 31 but well beyond its borders. On four plates south following the major axis, centered about 1° from the axis, one typical object, No. 140 in Table I, and two probable and five possible examples were found. If No. 140 lies in the plane of the spiral, its distance from the nucleus is about 213'. The two probable examples,^T on the same assumption, would be at distances of 202' and 221', respectively. For purposes of comparison with the diameter of the galactic system as

¹ The positions of these objects are X = -0.3, Y = -55.2 and X = +5.4, Y = -50.4, with magnitudes estimated as 16.5 and 16.0, respectively. The latter appears on Pl. II. The position of No. 140 is 11.05 S. following B.D.+40°166 and 12.25 S. preceding B.D.+40°173.

outlined by the globular clusters, it seems possible that the diameter of M $_{31}$, as outlined by these objects, may be taken as of the order of 7°, or 100,000 light-years. As will appear later, the ratio of the diameters corresponds roughly to the systematic difference in the absolute magnitudes of the objects from which the diameters are derived.

On the north preceding side of the major axis the only plate available was centered on N.G.C. 205, the fainter companion of M 31. Eight objects were found in addition to a few distant ones listed in Table I. These new objects are concentrated about N.G.C. 205 and are probably associated with the nebula itself.¹ No similar concentration is found around the brighter companion, M 32. The most distant of the new objects, if associated with M 31 rather than with N.G.C. 205, would be about 180' from the nucleus of the spiral.

That the brighter objects in M $_{31}$ are more concentrated toward the center than the fainter ones is also apparent in Figure 3, where the mean magnitudes for successive zones are plotted against mean distances from the nucleus. The mean luminosity decreases by about 0.75 mag. as the distance increases from about 5' to 6o' and more. Very little of the change can be attributed to the difficulty of detecting faint objects in the nuclear region, since the influence of the luminous background is not serious beyond 5' from the nucleus. The

^r The objects that appear to be associated with N.G.C. 205 are marked for identification on Pls. I and II. Their positions with respect to the nucleus of the nebula, together with estimated diameters and magnitudes, are given below. The major axis of N.G.C. 205 is assumed to lie in position angle N. 9° W. X co-ordinates are positive south of the nucleus, and Y co-ordinates are positive west of the nucleus. Distances from the nucleus, D, are given with no correction for the tilt of the nebula. Co-ordinates of the nucleus with respect to the nucleus of M 31 are approximately X = -4.2, Y = +36.8.

	X	Y	D	m	d	Conc.
1 .	+4'.4	-0.8	4 · 5	16.8	5″	m
	+2.1	-1.4	2 . 6	16.7	5	m
	+0.8	-6.2	6 . 2	15.4	8	m
4	+0.8	-0.3	0.9	18.2±	4	m
5	+0.3	+0.3	0.5	18.0±	3	s
6	-0.8	-0.9	I.2	18.3±	4	d
7	-1.6	-0.9	1.8	17.6	4	m
	-7.4	+3.9	8.3	16.5	5	m

background does affect the determination of magnitudes, but its influence can be largely eliminated by using short exposures.

APPARENT DIAMETERS

The greatest effect of the background is on the estimation of diameters—the denser the background, the smaller the apparent diameter. Since diameters decrease with decreasing exposures, a direct determination of the quantitative relation is very uncertain. Some

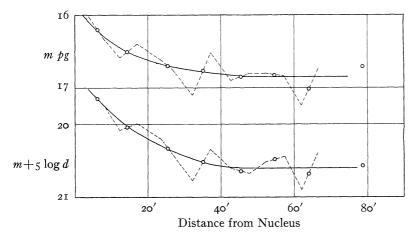


FIG. 3.—Dependence of magnitude and diameter upon distance from the nucleus of Messier 31. Above, mean magnitudes and distances of objects in successive zones concentric with the nucleus of Messier 31. Below, mean surface brightness $(m+5 \log d)$ and mean distance. Open circles refer to zones 10' in width; the broken lines to 5' zones.

notion may be obtained, however, by inspecting the values of m+ $5 \log d$ for successive zones listed in the last column of Tables II and III and plotted against distance from the nucleus of M 31 in Figure 3. These values of the surface brightness increase systematically out to about 30' from the nucleus, but beyond this limit are reasonably constant and have a mean value of 20.6. We may therefore suppose that the effect of the background becomes negligible at about this critical distance and that the outer objects are typical of the entire group. Mean diameters corresponding to various mean magnitudes may therefore be derived from the relation

$$m + 5 \log d = 20.6$$
.

The results are given in Table IV.

57

The linear scale at the distance of M $_{31}$ is about 1'' = 1.2 parsecs, hence the diameters appear to range from about 16 to 4 parsecs. The

TABLE IV

	Diameters		
m	$\log d$	d	D
15.0	I.I2	13".2	15.8 parsecs
15.5	I.02	IO.2	12.2
16.0	0.92	8.3	10.0
16.5	0.82	6.6	7.9
17.0	0.72	5.25	6.3
17.5	0.62	4.2	5.0
18.0	0.52	3.3	4.0

TABLE	v

FREQUENCY DISTRIBUTION OF MAGNITUDES	Frequency	DISTRIBUTION	OF	MAGNITUDES
--------------------------------------	-----------	--------------	----	------------

$m (pg) \qquad \qquad$	Numbers				Numbers		
	<i>r</i> <40′	r>40'	All	m (pg)	r<40'	r>40	All
15.00	I	I	2	16.7	I	2	3
15.1	0	0	0	16.8	7	6	13
15.2	I	I	2	16.9	2	3	5 18
15.3	I	I	2	17.0	II	7	18
15.4	2	0	2	17.1	5	5	10
15.5	I	3	4	17.2	I	7	8
5.6	I	0	I	17.3	I	5	6
5.7	5	0	5	17.4	2	3	5
5.8	3	3	6	17.5	3	2	5
5.9	2	I	3	17.6	0	2	2
6.0	3	2	5	17.7	o	3	3
.6. 1	2	2	4	17.8	I	0	c
6.2	I	2	3	17.9	I	0	I
6.3	2	4	6	18.0	I	3	4
6.4	0	0	0	18.1	0	0	c
6.5	3	I	4	18.2	I	I	2
6.6	5	0	5				
				Total	70	70	140

correlation of m and log d for objects more than 30' from the nucleus is shown in Figure 4.

FREQUENCY DISTRIBUTION OF MAGNITUDES

The data bearing on the frequency distribution of apparent magnitudes are summarized in Table IV, which gives the total numbers

of objects for each successive tenth of a magnitude, and, in addition, the corresponding numbers for the objects which are at distances less than and greater than 40', respectively, from the nucleus of M 31. The data for all objects together are exhibited in Figure 5. The ac-

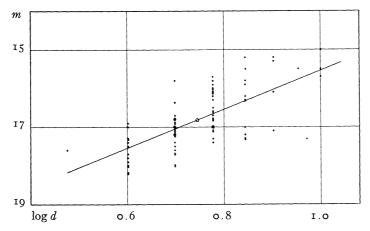


FIG. 4.—Luminosity-diameter relation for objects more than 30' from the nucleus of Messier 31.

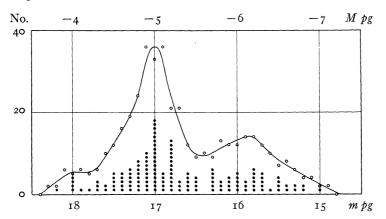


FIG. 5.—Frequency distribution of magnitudes of objects in Messier 31: dots, individual objects; circles, sums for three consecutive tenth magnitudes centered on the middle tenth.

tual counts are represented by the array of points, and the curve has been smoothed by plotting the sums of the counts for three consecutive tenths of a magnitude.

Two maxima are conspicuous, at about magnitudes 15.85 and 17.0, respectively, and the curve, as a whole, appears to be a combination of two symmetrical curves centered around the two maxi-

59

ma. The true maximum of the brighter curve, on this view, should be slightly brighter than that observed, and hence may be assigned the even tenth, 15.8. Since the distance modulus of M 31 is 22.0, the absolute magnitudes corresponding to the maxima are -5.0 and -6.2, respectively. The mean apparent magnitude of the entire group is about 16.7, corresponding to absolute magnitude -5.3.

The double maximum appears in the frequency-curves for each successive zone, although the numbers are so small that zones of considerable width must be used to render the phenomenon con-

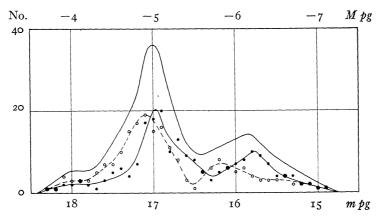


FIG. 6.—Frequency distribution of magnitudes of objects in the inner and in the outer regions of Messier 31: dots, objects less than 40' from the nucleus; circles, objects more than 40' from the nucleus. The top curve, representing all objects, is that shown in Fig. 5. The dots and circles represent sums for three consecutive tenth magnitudes centered on the middle tenth.

spicuous. The brighter component-curve is relatively more pronounced in the inner zones, as may be inferred from the dependence of mean magnitude upon distance from the nucleus (Fig. 3). This also appears in Figure 6, where the two curves for the zones o'-40' (70 objects) and the zone > 40' (70 objects) may be compared with one another and with the general curve copied from Figure 5. The three curves, all smoothed by the method described above, are similar in a general way, although the maxima for the outer zone appear to be displaced somewhat with respect to those for the inner zone. This feature may have some significance, since curves for the two subzones 10'-20' and 20'-40' are very similar to each other, while the curves for the subzones 40'-60' and > 60' are also similar.

For further investigation the objects are divided into the two groups, bright and faint, suggested by the general frequency-curve, with the division point at magnitude 16.5. A careful examination of the better plates indicates no conspicuous systematic difference in the character of the images. The brighter objects appear more condensed, but this may be a photographic effect.¹ The distribution of these two groups has already been discussed—both are concentrated toward the nucleus of M 31, although the concentration is more conspicuous for the brighter objects. Although the frequency-curve indicates the mingling of two groups of objects, the data available at present seem to offer no further information on the matter.

COLOR-INDICES

Photovisual magnitudes of sixty of the objects have been estimated on plates exposed through a color filter, but as extra-focal exposures were available for only three or four of the objects, the results are not very reliable. Color-indices range from +0.4 to +1.1mag., with a mean value of about 0.70 mag. The uncertainty of the mean is probably 0.2 mag., representing, for the most part, the possibility of systematic error in the photovisual scale as determined from a small number of extra-focal images.

The mean color-index, as it stands, corresponds to a mean spectral type of Go or slightly earlier. This is consistent with visual estimates of color for several of the brightest objects and with the type F8 indicated by the single spectrogram available. The range in the estimated color-indices corresponds to a spectral range from about Fo to Ko. This is similar to the range among the globular clusters in the galactic system, but the uncertainties in the estimates detract from the significance of the comparison.

COMPARISON WITH GLOBULAR CLUSTERS IN THE GALACTIC SYSTEM

Among known types of celestial bodies, the objects in M 31 find their closest analogy in globular clusters. The globular forms, symmetry, and spectral types are closely comparable, and no further discussion of these points is necessary. There remains, however, the comparison of luminosities and dimensions.

 ${}^{\rm r}$ As will appear later, the absolute luminosities of galactic globular clusters vary directly with the concentration.

60

1932ApJ....76...44H

Shapley's scale of magnitudes.—The most complete list of integrated photographic magnitudes and diameters of galactic globular clusters is found in Shapley's monograph on star clusters.¹ The estimates of apparent magnitudes were derived from comparisons with stars on focal exposures, and hence it is not surprising to find that they deviate widely from the usual Pogson system. Shapley characterizes his scale as "convenient but not conventional," and remarks

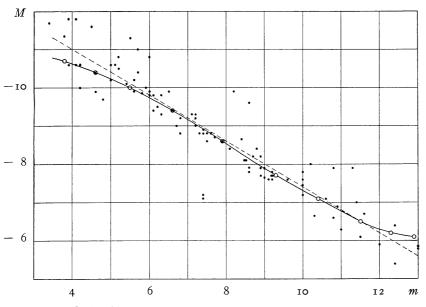


FIG. 7.—Relation between M and m on Shapley's scale of integrated magnitudes of globular clusters. The straight line represents the relation M=0.6 m-13.2. The circles and continuous curve represent Shapley's correlation between m and m-M.

that his estimates of apparent luminosity range through nearly 10 mags., which "would indicate a factor of 100 in the relative distances, rather than the factor of 10 which is actually found."

On Shapley's scale the apparent magnitudes show an approximately linear correlation with the absolute magnitudes. This correlation, which appears in Shapley's monograph² as a relation between m and m-M, is exhibited directly in Figure 7 in order to emphasize the relatively small scatter of the points about the correlation-curve. As m ranges from 3 to 12, M varies systematically from about -11.3

¹ Star Clusters, "Harvard Observatory Monographs," No. 2, 1930.

² Ibid., pp. 164–165, Fig. XI (2), and Table XI (IV).

to about -6. It is obvious that some sort of calibration is necessary before the clusters can be compared with other objects.

The straight line in Figure 7 appears to be a fair first approximation to the correlation between m and M, but Shapley, using various refinements, arrives at the non-linear form indicated by the open circles (derived from his relation between m and m-M). If we assume no correlation between M and m on the conventional scale or, what amounts to the same thing, since distances, and hence the values of M, are based on the luminosity of stars, no correlation between the absolute magnitudes of clusters and of their brightest stars, then the correlation-curve in Figure 7 indicates the deviation of Shapley's scale of cluster magnitudes from Pogson's scale. Shapley's correlation-curve indicates a systematic deviation of 4.6 mag. in the interval 3.1-12.2. The deviation seems large, but a smaller value would leave a correlation between m and M to be explained away, as well as one between M and distance from the observer.¹

The order of the deviation appears to be confirmed by Holetschek's visual magnitudes of forty-four clusters as corrected by Hopmann.² These data conform approximately to the Pogson scale —at least they represent a deliberate attempt in that direction by an observer of wide experience in measuring the luminosity of comets. They are in fair agreement with other less extensive lists of visual magnitudes of clusters, and for the fainter objects the analogy with nebulae, for which Holetschek's measures are known to be of the proper order, offers circumstantial evidence of considerable weight.

A plot of Shapley's photographic magnitudes against the differences Holetschek-Shapley is shown in Figure 8. An approximately linear correlation is suggested, but the precise slope is difficult to

¹ Since the correlation holds for clusters more than 45° from the direction of the center of the galactic system as well as for clusters less than 45° from the center, the relation is between M and distance, not from the center of the system, but from the observer. The relation is in the sense that the nearer clusters are the brighter.

² Annalen der Wiener Sternwarte, 20, 1907. The revision by Hopmann (Astronomische Nachrichten, 214, 425, 1921) consists in photometric measures of the comparison stars to replace the BD magnitudes used by Holetschek. As not every comparison star was measured, the corrections are provisional. The corrections probably become appreciable between magnitudes 6 and 7, say 6.5, reach about +0.3 at 8.5, +0.6 at 9.0, and +1.1 at 9.5, beyond which they are presumed to be about constant at the value last mentioned.

63

determine. The full line bisects the angle made by the regressioncurves derived from the data when the two discordant points, $m_s = 7.3, H-S = 4.0$ (N.G.C. 5897) and $m_s = 7.5, H-S = 3.1$ (N.G.C. 6681), are omitted. It represents the relation

$$H - S = 4.23 - 0.446m_s$$

This formula indicates a deviation of about 4 mag. over the range 3.1-12.2, as compared with 5.3 for the linear correlation in Figure 7 or with the value 4.6 given by Shapley's curve.

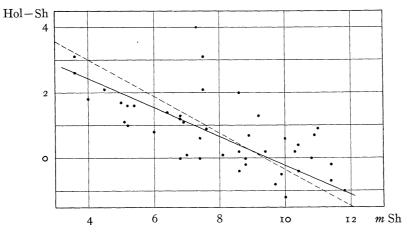


FIG. 8.—Relation between Shapley's photographic and Holetschek's visual magnitudes of globular clusters. The full line, Hol-Sh=4.23-0.446 Sh, represents the correlation when the two most discordant points are omitted. The broken line, Hol-Sh=5.25-0.56 Sh, represents the correlation when all data are included.

When the discordant points are included, i.e., when all the data are used, the same procedure leads to the relation

$$H-S=5.25-0.56m_s$$
.

The slope of this line closely approximates that of the straight line in Figure 7, the deviations being 5.1 for the former and 5.3 for the latter. The unsymmetrical distribution of the residuals detracts from the value of the comparison and, in fact, suggests that further analysis of the material is scarcely profitable. It is clear, however, that Holetschek's data confirm the general order of the correction to Shapley's scale derived on the assumption that, on the conventional scale, M is independent of m.

Zero-point of Shapley's scale.—Figure 8 further indicates that the zero-point, i.e., the point at which Shapley's scale coincides with the conventional scale, is probably in the vicinity of $m_s = 11$, since in that region the differences Shapley—Holetschek approximate the color-indices of clusters in general. This conclusion is consistent with fragmentary evidence from a few extra-focal images of faint clusters and also with photo-electric-cell measures made by Professor Stebbins with the large reflectors at Mount Wilson.

The measures by Stebbins are unpublished, but he very kindly permits me to quote some of the results, which include determinations of colors and of photo-electric-cell magnitudes (practically equivalent to photographic magnitudes) through apertures of different dimensions. The accuracy is of a higher order than that possible with the methods usually applied to the measurement of clusters. By plotting magnitudes against diameters of apertures for individual objects it is possible to derive total magnitudes, diameters, and magnitudes for arbitrarily selected diameters. Results are at present available for only five objects—N.G.C. 2419, 6760, 6779, 6864, and 7006. The diameters appear to be greater than those estimated by Shapley. The mean values of the magnitudes are as follows:

Shapley	10.2
Stebbins	•
Stebbins	10.7†

* Total magnitude. † Magnitude corresponding to Shapley's diameter.

The zero-point is determined by the correction 0.3 mag. required at $m_s = 10.2$. Inspection of Shapley's correlation in Figure 7 indicates that at $m_s = 10.8$ the curve is 0.3 mag. lower than at $m_s = 10.2$. Hence the zero-point may be taken at 10.8, which is in good agreement with the value 11.0 suggested by Holetschek's data. The corresponding corrections to Shapley's magnitudes are in Table VI.

The zero-point may be somewhat uncertain, since it is derived from a limited number of objects; but it is based on the most accurate measures available and is consistent with data from other sources.

Comparison of absolute magnitudes.—On the basis of the corrections in Table VI, the mean M of the ninety-three globular clusters

in Shapley's catalogue is -6.77. The frequency distribution of the magnitudes is shown in Figure 9, the curve being smoothed by the method used for the curves relating to the objects in M 31. The two exceptionally bright clusters are N.G.C. 6356 and 6864, noted by Shapley as peculiar.

If these two isolated clusters are omitted, the range is from -5.4 to -7.6, as compared with -4 to -7 for the objects in M 31. Two maxima are suggested, at about -5.75 and -6.95, as compared

TABLE VI	
Corrections to Shapley's	Scale

Shapley m	Corr.	M _{pg}	Shapley m	Corr.	M _{∲g}
3	+4.0	7.0	8	+1.6	9.6
4	3.8	7.8	9	+1.0	10.0
5	3.4	8.4	10	+0.4	10.4
6	2.8	8.8	II	-0.I	10.9
7	+2.2	9.2	12	-o.5	11.5

with those at -5 and -6.2 in M $_{31}$; but the relative numbers in the groups are reversed. In the galactic system the brighter group dominates the distribution and the fainter group is inconspicuous.^I In M $_{31}$ the fainter group dominates, and only one-quarter of the total number can be assigned to the brighter group.

The most favorable agreement between the absolute magnitudes of the objects in the two systems is obtained by accepting the reality of the double maxima; the galactic globular clusters are then systematically brighter than the objects in M 31 by about 0.75 mag. The least favorable agreement follows on comparing the most fre-

¹ The fainter secondary group of globular clusters consists almost entirely of the looser clusters—Shapley's classes IX–XII. Among these classes the double maximum is conspicuous, and it is the fainter group which appears in Fig. 9, the brighter being lost among the more concentrated clusters. This suggests the possibility that the objects in M 31 may be analogous to the looser clusters in the galactic system.

In general, the luminosity of clusters decreases with the concentration:

Class	No.	M
I–IV.	27	-7.14 (-7.02, omitting N.G.C. 6356 and 6864)
V–VIII.	34	6.94
IV–XII.	32	-6.24

quent magnitudes, the double maxima being disregarded. The systematic difference is then about 1.95 mag. The difference in the mean magnitudes of all objects is intermediate—about 1.5 mag.

The two groups, however, overlap to a very considerable extent, and the brighter objects in M $_{31}$ are strictly comparable with the most frequent types of globular clusters. The systematic differences, which range from about 0.75 to 2 mag. according to the interpretation of the data, are sufficiently small to suggest that the objects in M $_{31}$ should be provisionally classed with the clusters.

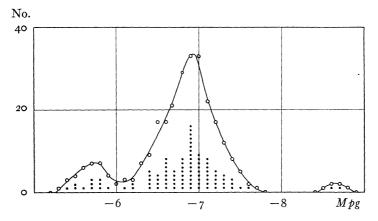


FIG. 9.—Frequency distribution of absolute photographic magnitudes among globular clusters in the galactic system: dots, individual clusters; open circles, sums for three consecutive tenth magnitudes centered on the middle tenth. The two very bright isolated clusters are N.G.C. 6356 and 6864.

Comparison of diameters.—A comparison of diameters is difficult since the criteria used in the estimations are necessarily rather arbitrary. Shapley's estimates of apparent diameters of clusters vary directly with the corresponding absolute diameters, but not in the same manner that his apparent luminosities vary with the absolute luminosities. His apparent magnitudes and the logarithms of his diameters approximate a linear relation, with a considerable scatter introduced by the looser clusters, but the slope of the correlationcurve does not conform to the relation $m+5 \log d = \text{Const.}$ The mean value of the constant for all clusters is 20.97 (clusters) as against 20.6 for the objects in M 31. For the calculation, d is expressed in seconds of arc and the magnitudes are reduced to the conventional scale. The value of the constant for the galactic globular clusters

ranges, however, from about 19.4 for m = 11.5 to about 22.3 for m = 8. Around m = 9.5 it approximates the value observed in M 31.

The absolute diameters as derived from the apparent diameters and distances range from about 6 to 50 parsecs as compared with a range from about 4 to 16 parsecs for the objects observed in M $_{31}$. There is thus some overlap, regardless of uncertainties in the criteria employed.

COMPARISON WITH GLOBULAR CLUSTERS IN THE MAGELLANIC CLOUDS

A comparison with globular clusters in the Magellanic Clouds is perhaps more significant, since the distance of M 31 is determined in terms of the distance of the Small Cloud and since, moreover, Shapley's magnitudes of the clusters are in the region where the scale corrections are relatively unimportant. Shapley reports two clusters in the Small Cloud,¹ but regards the fainter as somewhat uncertain. His magnitudes are 11.3 and 10.2, corresponding to conventional magnitudes 11.1 and 10.5. Since m - M = 17.3 for the Cloud, the absolute magnitudes of the clusters are -6.2 and -6.8, the mean being -6.5. Since the diameters are 0.9 and 1.4, respectively, corresponding to 6".4 and 9".9 at the distance of M 31, these clusters are strictly comparable with the brighter objects in M 31. It is uncertain, of course, whether the comparison should be made with the brighter objects in M 31 or with the mean of all. In the latter case the discrepancy would be 1.2 mag., but it is reasonable to suppose that no brighter clusters are associated with the Cloud and hence that any others that may be found would decrease the discrepancy.

Eight clusters are reported in the Large Cloud.² These are listed in Table VII with Shapley's magnitudes, diameters, and, in addition, the revised absolute magnitudes and apparent diameters corresponding to the distance of M $_{31}$. Shapley suggests the possibility that of the two brightest clusters the brighter may be a foreground object and the fainter, a cluster of a type other than globular.

The revised absolute magnitudes range from -7.5 to -6.4 with a mean value of -6.8, and hence are comparable with those of the brighter objects in M 31. The brightest cluster, which Shapley sug-

¹ Op. cit., p. 187, Table XIII (I). ² Ibid.

gests may be a field object, is above the upper limit of the objects in M $_{31}$ by 0.5 mag. The diameters also are consistent, averaging somewhat less than those in M $_{31}$ for objects of the same luminosity. As in the case of the Small Cloud, there is the uncertainty as to whether the comparison should be with the brighter objects in M $_{31}$ or with the mean of all.

The general conclusion appears to be that the analogy between the objects in M $_{31}$ and the known globular clusters is too close to be

NCC	Shai	PLEY	16	d at Distance of M 31
N.G.C.	d	m	М	
1783	ı.'4	10.I	-6.6	8."8
и8́об	0.9	10.6	6.4	$5 \cdot 7$
1831	I.3	10.0	6.7	8.2
1835	I.2	9.8	6.8	7.6
1846	I.2	10.4	6.5	7.6
1856	2.I	8.8	7.2	13.3
1866	2.2	8.o	$7 \cdot 5$	13.9
1978	Ι.Ο	IO.2	-6.6	6.3
Mean			-6.8	

TABLE VII Globular Clusters in Large Magellanic Cloud

ignored and that, provisionally at least, they should both be included in a single class. The significance of the double maximum in the frequency distribution, i.e., the possibility of subclasses whose relative richness varies from system to system, is a matter for further investigation. The present comparisons involve several approximations and, of course, assume the validity of Shapley's scale of distances. More reliable results will be derived when the scale of cluster magnitudes and effects of obscuration in both the galactic system and M 31 have been determined.

SIMILAR OBJECTS IN OTHER NEBULAE

The subject is further complicated by the examination of other nebulae. In M $_{33}$ some twelve or fifteen objects may be of the type under discussion, but they average about 1.5 mag. fainter than those in M $_{31}$, although the latter nebula is slightly more distant. Several brighter objects in M $_{33}$ were examined as possibly of the type

69

under discussion, but in each case a negative color-index differentiated them, and on the best plates it was generally possible to detect a sharp star image on a nebulous background.

N.G.C. 6822 may be analogous to M 33. A few objects found in N.G.C. 6822 were discussed in a former investigation of that system,¹ and on the basis of luminosities and dimensions were assumed to be field nebulae. The new data from M 31 and M 33 appear to reopen the question of their status.

In M 101, on the other hand, a half-dozen apparently typical objects are found, the brightest being about 1 mag. fainter than the brightest in M 31. The difference is consistent with the relative apparent luminosities of variables and brightest stars in the two nebulae. Other bright knots in M 101 have been identified as patches of emission nebulosity surrounding early-type stars. These are common features of late-type spirals and irregular nebulae and generally exhibit an appreciable lack of symmetry.

Among the numerous other conspicuous nebulae for which suitable plates are available, the suspected objects have been identified with confidence only in M 81, although a few questionable cases have been reserved for further investigation. It seems improbable, however, that the existence of these objects can seriously affect the validity of the apparent photographic magnitudes of the brightest stars as a statistical criterion of nebular distances.

CARNEGIE INSTITUTION OF WASHINGTON MOUNT WILSON OBSERVATORY February 1932

¹ Mt. Wilson Contr., No. 304; Astrophysical Journal, 62, 409, 1925.