form is such as to impose peculiar properties upon the resulting " stars." Conclusions from comparisons of the actual stars with this model should therefore be received with caution.

(3) The success of the homologous model in representing the observed mass-luminosity relation depends mainly (as Vogt points out) on the fact that this relation is insensitive to considerable changes in the model. The mean molecular weight, however, appears to be nearly the same in all stars—or else changes in the model counteract its variations.

(4) Professor Milne's discriminant C depends upon conditions in the outer layers of a star, but involves no property of the interior except its total mass.

(5) Stars of fixed composition should exhibit exact relations between mass, luminosity, effective temperature, etc. The conditions under which this can be proved are discussed, and the conclusions illustrated, by means of the " collapsed " and " point source " models.

(6) The relation between luminosity and effective temperature gives a much more searching test of theories of constitution than that between mass and luminosity. The observed data indicate that the stars must differ in composition—probably to a considerable extent.

(7) Until some physical theory of the energy-generation process becomes available, all theories of stellar constitution must involve a speculative element. Different investigations should be regarded as supplementary rather than antagonistic.

In conclusion it is a pleasure to express the writer's deep indebtedness to Professor Milne, Professor Atkinson, and Mr. Joy, who have courteously communicated important papers in manuscript or in proof.

Princeton University Observatory :
1931 *June* 12.

The Contours of Emission Bands in Novæ and Wolf-Rayet Stars.

By C. S. Beals, M.A., D.I.C., Ph.D.

Introduction.

In previous publications * the present writer has advanced a theory to account for the broad band emission of Wolf-Rayet stars. This theory attributes the appearance of emission bands to the continuous ejection of atoms from the surface of a star by radiation pressure. The observational evidence so far advanced in support of this theory has had to do mainly with the variation of band width with wave-length and with the appearance of absorption borders on the violet edges of emission bands. Certain points of similarity between Wolf-Rayet spectra and the spectra of novæ (where the hypothesis of violent

* *M.N.*, **90**, 202, 1929; *Pub. D.A.O.*, **4**, No. 17, 1930.

ejection of gases seems reasonably well established) have also been considered as evidence favouring the theory.

The present paper contains a discussion of the theory in its relation to band contours. On the basis of the ejection theory, it can be shown that, if certain reasonable assumptions be made concerning the mechanism of emission, it is possible to make predictions concerning the forms of emission band contours, which can be tested observationally. These predictions, which apply equally to novæ and Wolf-Rayet stars, are discussed in connection with spectro-photometric results on band contours obtained at this observatory. In developing the mathematical theory of band contours the writer has had the benefit of advice and criticism from Dr. R. O. Redman, whose assistance is gratefully acknowledged.

Theory of Band Contours.

If the assumptions be made (*a*) that radiation from each atom ejected contributes equally to the intensity of the emission band, and

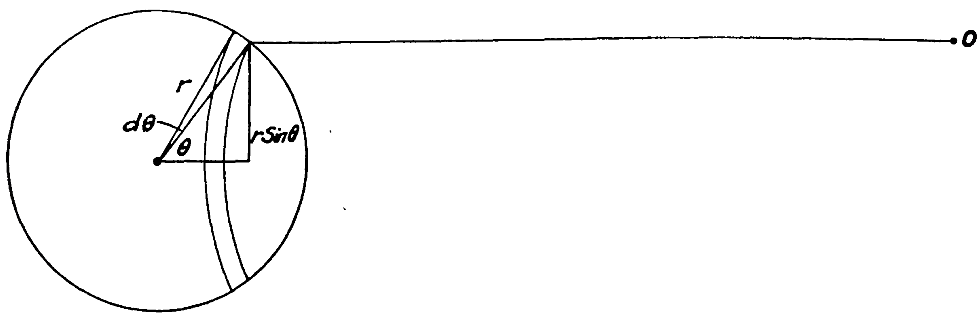


Fig. 1.

(*b*) that all atoms are ejected with the same velocity, we may derive an expression for the band contours as follows (see fig. 1) :—

Consider a shell of radius r surrounding the star. Let n be the number of atoms ejected per second from unit area of this shell. Let v be the velocity of ejection. If, now, we take a zone of radius $r \sin \theta$ and width $d\theta$, symmetrical with respect to the line of sight, the area of the zone will be $2\pi r^2 \sin \theta d\theta$; the number of atoms passing through it per second will be

$$N = 2\pi n r^2 \sin \theta d\theta,$$

and the energy which they will contribute to the band will be

$$dE = K \,.\, 2\pi n r^2 \sin \theta d\theta \qquad . \quad . \quad . \quad (1)$$

where K is some undetermined constant.

If, now, we denote the radial velocity for an observer at O as U, then

$$U = -v \cos \theta.$$

Differentiating this expression and solving for $d\theta$ we get

$$d\theta = \frac{dU}{v \sin \theta} \qquad . \quad . \quad . \quad . \quad (2)$$

Substituting this value of $d\theta$ in (1) gives

$$dE = \frac{\text{const.}}{v} dU \quad . \quad . \quad . \quad . \quad (3)$$

which represents the energy in the velocity range from U to U + dU.

Equation (3) may also be written in the form

$$dE = \frac{\text{const.}}{v} d\lambda \quad . \quad . \quad . \quad . \quad (4)$$

since $dU = \frac{c}{\lambda_0} d\lambda$ from Doppler's principle.

Since dU may be treated as a constant, dE is also a constant and the contour will be perfectly flat on the top, will have a total width of $2v$, and will have perpendicular sides. This theory applies to any case of the ejection of atoms from a stellar surface in which the above conditions are fulfilled whether by a single outburst or by a continuous process.

A previous attempt by Menzel * to derive an expression for the band contour on the basis of similar assumptions led to an expression

$$I = I_0 \sin \theta$$

or

$$I = I_0 \sqrt{1 - \frac{c^2}{{v_0}^2}\left(\frac{\lambda_0}{\lambda} - 1\right)^2}.$$

Menzel apparently made the error of treating the differential coefficient $d\theta$ in (1) as a constant. Actually $d\theta = \frac{dU}{v \sin \theta}$ as in (2).

It is scarcely to be expected that under the actual conditions prevailing in the neighbourhood of novæ or Wolf-Rayet stars the assumptions on which equation (3) was derived will hold precisely. The first assumption, that each atom contributes equally to the intensity of the band would appear, to a first approximation, to be a reasonable one, and in order to avoid complications it will be retained. It is extremely unlikely, however, that all velocities of ejection will be equal, and the real problem is therefore to be able to compute the contour of a band produced by a known distribution of velocities in which there is a considerable spread. The problem is actually a simple one and may be worked out by a series of approximations, in which the frequency curve of velocities is divided into narrow strips of constant width, the velocity for each strip being taken as constant and equal to its mean value over the small section of the curve concerned. Each strip can then be treated separately on the basis of equation (3) stated in the form

$$dE = \text{const.} \frac{N}{v} dU,$$

in which N represents the ordinate of the frequency curve at the centre of the strip, v the velocity, and dU the width of the strip. The

* *P.A.S.P.*, **41**, 344, 1929.

band contour can then be computed by summation, starting with the larger velocities and building up the band from its outer edge to its centre.

Fig. 2 illustrates the result of such a process applied to a distribution of velocities corresponding to an ordinary error curve where the mean velocity of ejection is 20 km./sec. and the effective spread in velocities is ± 14 k./m sec. approximately. Any other distribution of velocities may be treated in a similar way, and an investigation of the various possibilities leads to the following general conclusions.

1. If suitable assumptions be made concerning the distribution of velocities, almost any observed symmetrical contour can be accounted for.

2. If the frequency distribution of velocities occurs within a range appreciably smaller than half the width of the band (so that there are

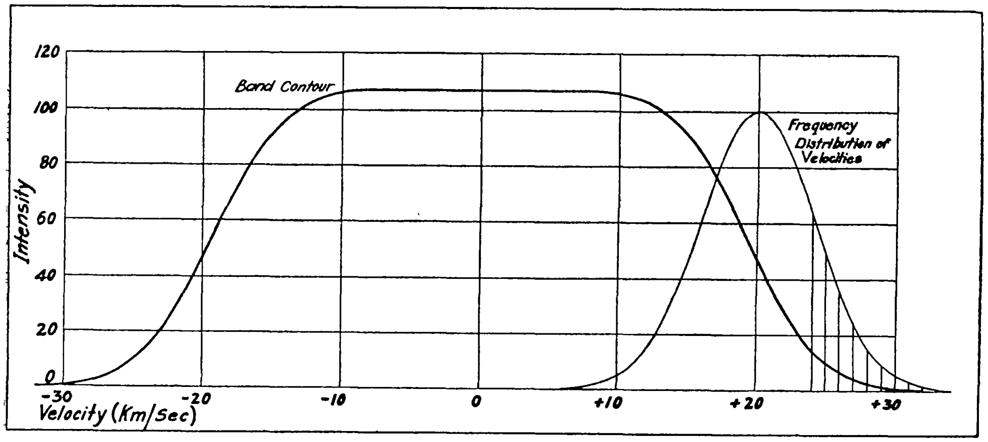


FIG. 2.

no velocities in the vicinity of zero), then the contour of the band will have a flat top (an example of this case is illustrated in fig. 2).

3. If the contour of the band be observed with sufficient accuracy, then it is possible to apply the converse process and determine the distribution of the velocities of the emitting atoms.

In a recent paper Miss Payne * has stated the physical consequences of equation (3) although she does not derive the formula. She has also called attention to conclusion (1) above.

It may be inferred from (2) that if the assumptions involved are justified, then one might reasonably expect to find flat-topped bands in the spectra of both novæ and Wolf-Rayet stars. The occurrence of other types of contours would also be expected, but would naturally be of less significance from the point of view of confirming the theory.

Evidence has already been presented by Miss Payne that flat-topped bands are present in the spectra of both Nova Pictoris and certain Wolf-Rayet stars. The present paper contains some additional evidence of the same sort derived from measures of the spectra of Nova Aquilæ, Nova Cygni, and three Wolf-Rayet stars.

* *H.B.*, **874**, 23, 1930.

Observational Data.

The observational data are represented by the seven graphs of figs. 3 and 4 and by Tables I. and II., which contain lists of the entries on the graphs. It should perhaps be mentioned that for the Wolf-Rayet stars having reasonably broad bands, the number of bands which are free from blends and which at the same time show reasonably strong contrast with the continuous spectrum are very few. The bands λ 4686, λ 4861, and λ 5695 appear to be the most promising, though λ 4686 probably has the band λ 4712 of neutral helium superposed on its less refrangible edge, while λ 4861 is probably a combination of $H\beta$ and λ 4859 of *He* II. As far as can be determined, λ 5695 is free from blends, though in the case of a band 100 A. wide it is naturally difficult to be certain. For the three bands of fig. 4 it seems reasonably certain that any blending effects which may be present are not serious enough to affect any conclusions which are drawn from the plotted contours. Descriptions of individual graphs are as follows :—

(a) *Novæ.*—Measurements of the band $H\beta$ in Nova Cygni and Nova Aquilæ are shown in the graphs of fig. 3. Unfortunately these plates were not standardized photometrically, and so the measures are given in terms of photographic density rather than absolute intensity. However, for the purpose of testing the particular point under discussion (the flatness of the top of the band), the density curves serve the purpose equally well.

The band of Nova Cygni is rather narrow, but its flat-topped character is well marked, and the evidence would appear even more convincing if a smaller scale of ordinates were employed. Three plates were measured, and each point on the graph is a normal from five observations.

The two graphs of the same band of Nova Aquilæ are plotted from measures of three plates each. Each point on the graph is a normal from six measures. All who are familiar with the details of the spectrum of this star will recall the complex structure shown by the bands at certain stages in its development. These finer details are obscured in these graphs and only the general outline of the band contour is represented. The two graphs leave little doubt of the general shape of the band as consisting of steeply sloping sides and a flat top. Indeed, at certain stages the bands of this star on a spectrogram approximate closely to rectangular strips of constant density, which is the form predicted by the simple theory of equation (3). The hump in the centre of the band observed on 1918 September 13–19 is real and not an effect of errors of measurement.

(b) *Wolf-Rayet Stars.*—The measures of Wolf-Rayet bands are represented by the graphs of fig. 4. With one exception they have been plotted from measures of standardized plates and so represent absolute intensities of the bands in terms of the neighbouring continuous spectrum. The standardization strips were impressed on the plate with the aid of a neutral tint wedge and an auxiliary spectrograph.

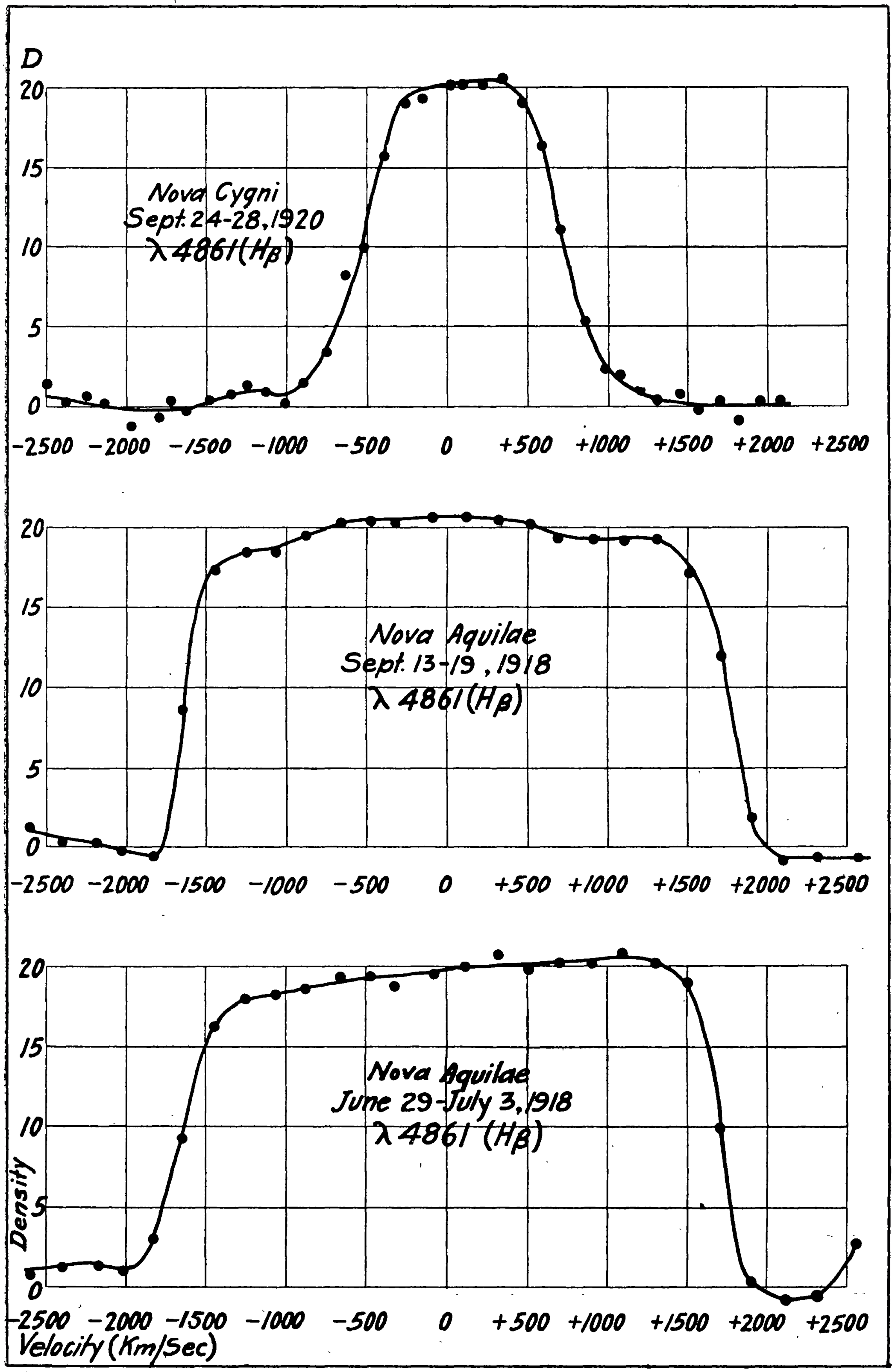

D
Density
Nova Cygni
Sept. 24-28, 1920
λ 4861 (Hβ)
Nova Aquilae
Sept. 13-19, 1918
λ 4861 (Hβ)
Nova Aquilae
June 29-July 3, 1918
λ 4861 (Hβ)
Velocity (Km/Sec)
-2500 -2000 -1500 -1000 -500 0 +500 +1000 +1500 +2000 +2500
0 5 10 15 20

FIG. 3.

The measures of density for both novæ and Wolf-Rayet stars were made with a Hartmann microphotometer.

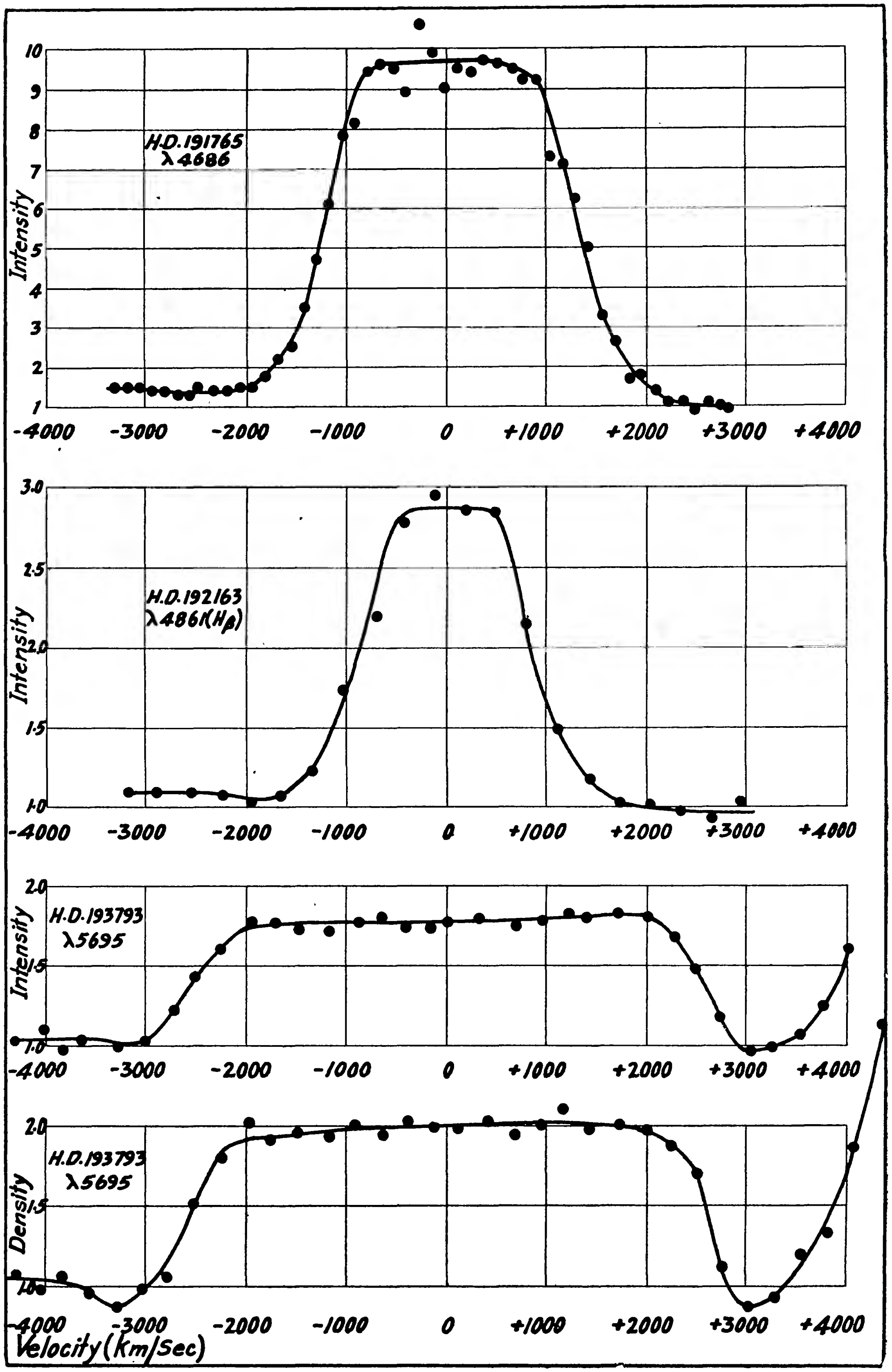
The graph of the band λ 4686 in the spectrum of the star H.D. 191765 has been plotted from measures of five plates, and each normal is the mean of five observations. At one section of the curve the points show a considerable scatter, and this is believed to be due, not to errors of measurement, but to irregularities in the emulsion of two of the plates. The curve as a whole is well determined and there seems no reasonable doubt that the contour is practically flat on the top.

The next graph represents the band λ 4861 in the star H.D. 192163. Five plates were measured and each normal represents the mean of fifteen observations. The measures for this band were less consistent than those for the one just described, and this fact, combined with the narrowness of the top, makes the interpretation of the contour in terms of theory less certain. This band was, in fact, selected for measurement because it appeared to be a good example of absence of the flat-topped characteristic. It was somewhat of a surprise, therefore, when a final reduction of the observations gave some indications that a narrow section of the top of the band was flat.

The two lower graphs in the figure represent measures of the band λ 5695 in the spectrum of the star H.D. 193793. This band is approximately 100 A. in width, one of the widest appearing in any Wolf-Rayet star. Its origin is somewhat uncertain,* but it has been provisionally assigned to the atom of *C* III. Its great width would suggest the possibility of blending effects, but this idea is contradicted by the evenness of the contour and by the occurrence of other very wide bands in the spectrum of the same star. Also, in other stars where the same band appears in a narrower form there is no indication of other bands within 50 A. of the centre of λ 5695.

The upper graph of this star represents a series of measures made on two photometrically standardized plates. Each normal, on the average, represents the mean of four measures. Measurements of the two plates are in good agreement and clearly reveal the wide flat top of the band. In addition to the photometrically standardized plates, three other plates of this star were secured in the visible region before the equipment for standardizing plates was available. Since this band represents an important case, where the flat-topped characteristic appears to be established beyond possibility of doubt, it seemed worth while to combine density measures of these three plates with the first two. The density curve from measures of the five plates is shown by the lowest graph of fig. 4. Each normal of this graph is the mean of five measures. The second graph amply confirms the first in indicating a wide flat top to the band. It is of some interest to note that the second graph gives a clear indication of an absorption line on the violet edge of the band. The presence of another strong band to the red is indicated by the course of the graph to the right of the figure.

* A recent paper by Bowen (*Phys. Rev.*, **38**, 128, 1931) definitely identifies a line at 569 as due to *C* III.



H.D.191765
λ4686
Intensity
H.D.192163
λ4861(Hβ)
Intensity
H.D.193793
λ5695
Intensity
H.D.193793
λ5695
Density
-4000 -3000 -2000 -1000 0 +1000 +2000 +3000 +4000
Velocity (Km/Sec)

FIG. 4.

Table I.

Entries on Graphs.

Nova Bands.

Nova Aquilæ. $H\beta$. June 29–July 3, 1918.		*Nova Aquilæ.* $H\beta$. Sept. 13–19, 1918.		*Nova Cygni.* $H\beta$. Sept. 24–28, 1920.			
Velocity.*	Density.	Velocity.	Density.	Velocity.	Density.	Velocity.	Density.
−3170	0·5	−3170	0·7	−2744	−0·1	987	2·3
−2985	0·9	−2985	0·2	−2627	1·5	1085	2·0
−2787	0·9	−2787	0·7	−2504	1·4	1209	1·0
−2596	0·8	−2596	1·2	−2387	0·2	1314	0·4
−2405	1·2	−2405	0·3	−2257	0·6	1455	0·7
−2183	1·2	−2183	0·2	−2140	0·1	1573	−0·3
−2023	0·9	−2023	−0·2	−1973	−1·2	1708	−0·3
−1832	2·9	−1832	−0·6	−1807	−1·1	1813	−0·9
−1640	8·7	−1640	8·6	−1721	0·4	1955	0·4
−1449	16·2	−1449	17·3	−1622	−0·4	2084	0·4
−1252	17·9	−1252	18·4	−1492	0·4		
−1061	18·2	−1061	18·5	−1344	0·7		
− 870	18·5	− 870	19·5	−1246	1·5		
− 666	19·4	− 666	20·2	−1135	0·8		
− 475	19·3	− 475	20·3	−1011	0·1		
− 278	18·8	− 278	20·2	− 894	1·5		
− 80	19·8	− 80	20·7	− 759	3·3		
+ 117	19·9	+ 117	20·7	− 641	8·2		
+ 315	20·7	+ 315	20·4	− 524	9·9		
512	19·8	512	20·1	− 395	15·6		
709	20·1	709	18·7	− 259	18·9		
907	20·1	907	19·3	− 148	19·2		
1104	20·8	1104	19·3	− 19	20·1		
1307	20·2	1307	19·2	+ 99	20·1		
1505	18·9	1505	17·1	+ 228	20·1		
1702	9·7	1702	11·9	352	20·5		
1906	0·3	1906	1·8	463	18·7		
2109	−0·8	2109	−0·8	598	16·2		
2312	0·6	2312	−0·6	709	11·0		
2559	2·6	2559	−0·7	851	5·3		

* Velocities in these tables are given to the nearest kilometre per sec. for convenience in computation. The accuracy is actually much less and the error for a single observation may be as high as 20 km./sec.

Table II.

Entries on Graphs.

Wolf-Rayet Bands.

H.D. 191765. λ 4686.		H.D. 191765. λ 4686.		H.D. 191765. λ 4686.		H.D. 192163. λ 4861.	
Velocity.	Intensity.	Velocity.	Intensity.	Velocity.	Intensity.	Velocity.	Intensity.
−3314	1·5	− 659	9·6	2092	1·4	−3171	1·09
−3192	1·5	− 525	9·5	2227	1·1	−2893	1·08
−3065	1·5	− 397	8·9	2367	1·1	−2542	1·09
−2943	1·4	− 269	10·6	2495	0·9	−2246	1·07
−2815	1·4	− 141	9·9	2630	1·1	−1956	1·03
−2687	1·3	− 13	9·0	2764	1·0	−1653	1·07
−2566	1·3	+ 122	9·5	2828	0·9	−1345	1·22
−2438	1·5	+ 250	9·4			−1018	1·72
−2316	1·4	377	9·7			− 709	2·18
−2188	1·4	512	9·6			− 426	2·78
−2060	1·5	646	9·5			− 117	2·95
−1932	1·5	774	9·2			+ 204	2·86
−1811	1·8	902	9·2			+ 506	2·85
−1683	2·2	1036	7·3			802	2·14
−1555	2·5	1171	7·1			1123	1·49
−1427	3·5	1299	6·2			1431	1·17
−1299	4·7	1433	5·0			1734	1·02
−1171	6·1	1561	3·3			2042	1·01
−1043	7·8	1695	2·6			2332	0·97
− 915	8·1	1830	1·7			2653	0·92
− 787	9·4	1951	1·8			2937	1·02

H.D. 193793. λ 5695.		H.D. 193793. λ 5695.		H.D. 193793. λ 5695.		H.D. 193793. λ 5695.	
Velocity.	Intensity.	Velocity.	Intensity.	Velocity.	Density.	Velocity.	Density.
−4911	1·01	+ 326	1·79	−5722	−0·1	− 368	8·2
−4600	1·02	711	1·75	−5401	+0·2	− 116	7·9
−4317	1·02	969	1·78	−5106	0·1	+ 111	7·8
−4017	1·08	1221	1·83	−4833	0·8	+ 411	8·2
−3843	0·97	1400	1·80	−4596	0·1	705	7·5
−3643	1·04	1716	1·83	−4311	0·5	958	8·0
−3274	0·99	2011	1·80	−4043	−0·2	1184	8·8
−3011	1·02	2279	1·67	−3822	0·5	1437	7·8
−2732	1·23	2511	1·48	−3569	−0·3	1732	8·0
−2516	1·43	2758	1·17	−3301	−1·1	2006	7·7
−2258	1·59	3043	0·97	−3032	−0·2	2253	6·9
−1932	1·77	3264	0·99	−2779	0·4	2516	5·5
−1700	1·76	3537	1·06	−2516	4·1	2785	0·9
−1474	1·72	3780	1·06	−2243	6·4	3032	−1·1
−1174	1·71	4032	1·61	−1985	8·1	3280	−0·6
− 869	1·77			−1753	7·3	3559	1·6
− 637	1·79			−1479	7·6	3817	2·7
− 405	1·74			−1174	7·4	4085	6·9
− 137	1·74			− 895	8·0	4369	13·1
+ 11	1·77			− 637	7·5		

Discussion.

The main conclusions to be drawn from these observations appear to be reasonably obvious. The measurements of the bands of Nova Aquilæ and Nova Cygni, combined with the results of Miss Payne for Nova Pictoris, definitely establish the fact that flat-topped bands are a characteristic feature of the spectra of these two novæ at certain stages of their development. The agreement of the observed contours with the theoretical ideas developed in the first part of this paper provides additional confirmation, if such be necessary, of the ejection theory of novæ, which has held the field for a considerable number of years. The existence of flat-topped bands in Wolf-Rayet spectra also would appear to be in agreement with the theory that the emission bands in these stars are due to the continuous ejection of atoms from their surfaces. The fact that many, or even the majority, of Wolf-Rayet bands may not exhibit this characteristic is not a serious objection since the theory is able to account for any contour ordinarily observed.

If the theory be accepted, then it follows from the observed contours that the frequency distribution of velocities in each of the three Wolf-Rayet stars examined has a very considerable spread. The data of the present paper are not considered to be sufficiently extensive for a precise determination of the frequency curves of velocities for the ejected atoms. It is possible, however, from a study of the graphs to make a rough estimate of the maximum and minimum velocities for a given star. The maximum velocity, of course, corresponds to the extreme edge of the band. The minimum velocity corresponds to the edge of the flat portion of the top of the band. For the three stars of fig. 4 these values are approximately as follows :—

H.D. 191765	λ 4686	V. max.	=	2400 km./sec.
		V. min.	=	700 ,,
H.D. 192163	λ 4861	V. max.	=	1800 ,,
		V. min.	=	400 ,,
H.D. 193793	λ 5695	V. max.	=	3200 ,,
		V. min.	=	2000 ,,

It will be observed that the minimum velocity for the star H.D. 192163 is 400 km./sec., a figure considerably less than the velocity of escape at the surface of the sun. There are almost undoubtedly other stars for which the minimum velocity is much less, perhaps even less than 100 km./sec. We have no knowledge of the value of gravity or the velocity of escape at the surface of a Wolf-Rayet star. It will presumably be of the order of a few hundred kilometres per second. Whatever its value, it seems entirely probable that there will be some stars from which considerable numbers of atoms will be shot off at velocities approximately equal to or less than the parabolic velocity of escape. Such atoms may come to rest at considerable distances from the star and remain in equilibrium under the action of whatever forces are operative in the case of the atoms of planetary nebulæ. If conditions were sufficiently favourable, considerable accumulations of atoms might occur. Such a process offers a possible mechanism for the formation of planetary nebulæ. The existence of Wolf-Rayet stars as the nuclei of planetaries would appear to lend some probability to such a suggestion, though many such nuclei do not show the Wolf-Rayet characteristic.

A similar idea concerning the origin of planetaries was advanced by Menzel * in 1929. At that time the writer considered that the magnitude of the velocities involved was too great to render accumulation possible. The present work, by demonstrating that comparatively low velocities may occur even in a star having fairly wide bands, has made this difficulty less serious. It would appear, therefore, that this hypothesis, which suggests that the origin of planetary nebulæ may be due to the continuous ejection of atoms from the surface of a Wolf-Rayet star, while not without its difficulties, may still be worthy of serious consideration.

Dominion Astrophysical Observatory,
Victoria, B.C., Canada:
1931 *June* 4.

Selective Absorption. By R. v. d. R. Woolley, Ph.D.

In the quantum theory, as originally given by Einstein and Bohr, the absorption of light takes place as the result of the capture of a quantum by an atom, and in capturing the quantum one of the electrons in the atom jumps from a certain energy level A to a higher level B. If E_A, E_B are the energies of the two levels and ν the frequency of the quantum, then $E_B - E_A = h\nu$ where h is a universal constant. The atom has a certain discrete set of energy levels, and it is therefore only able to absorb a certain discrete set of frequencies, the frequencies characteristic of the spectrum. Absorption lines, however, are observed to have a finite width. This width can be made greater than the width due to the Doppler effect of the motions of the absorbing particles, and to the effect of pressure. An example of this is afforded by the B band of molecular oxygen, which appears sharp when observed with an air path of 10 metres at atmospheric pressure and room temperature. In the solar spectrum, where the absorption is due to a column of the earth's atmosphere, the lines appear much broader. It is clear that in this case the broadening cannot be attributed to anything except abundance of the absorbing atoms. The atom performs a certain amount of absorption at a frequency a small distance away from the central frequency of the line.

We are therefore led to the notion that the energy levels are not ideally sharp. This leads to the question, of great importance in the theory of the formation of absorption lines in solar and stellar

* Menzel, *loc. cit.*