

THE MAXIMUM MASS OF IDEAL WHITE DWARFS

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ABSTRACT

The theory of the *polytropic gas spheres* in conjunction with the equation of state of a *relativistically degenerate electron-gas* leads to a *unique value for the mass of a star* built on this model. This mass ($=0.91\odot$) is interpreted as representing the upper limit to the mass of an ideal white dwarf.

In a paper appearing in the *Philosophical Magazine*,¹ the author has considered the density of white dwarfs from the point of view of the theory of the polytropic gas spheres, in conjunction with the degenerate non-relativistic form of the Fermi-Dirac statistics. The expression obtained for the density was

$$\rho = 2.162 \times 10^6 \times \left(\frac{M}{\odot}\right)^2, \quad (1)$$

where M/\odot equals the mass of the star in units of the sun. This formula was found to give a much better agreement with facts than the theory of E. C. Stoner,² based also on Fermi-Dirac statistics but on uniform distribution of density in the star which is not quite justifiable.

In this note it is proposed to inquire as to what we are able to get when we use the relativistic form of the Fermi-Dirac statistics for the degenerate case (an approximation applicable if the number of electrons per cubic centimeter is $> 6 \times 10^{29}$). The pressure of such a gas is given by (which can be shown to be rigorously true)

$$P = \frac{1}{8} \left(\frac{3}{\pi}\right)^{\frac{1}{3}} \cdot hc \cdot n^{4/3}, \quad (2)$$

where h equals Planck's constant, c equals velocity of light; and as

$$n = \frac{\rho}{\mu H(1+f)}, \quad (3)$$

¹ 11, No. 70, 592, 1931.

² *Philosophical Magazine*, 7, 63, 1929.

μ equals the molecular weight, 2.5, for a fully ionized material, H equals the mass of hydrogen atom, and f equals the ratio of number of ions to number of electrons, a factor usually negligible. Or, putting in the numerical values,

$$P = K\rho^{4/3}, \quad (4)$$

where K equals 3.619×10^{14} . We can now immediately apply the theory of polytropic gas spheres for the equation of state given by (4), where for the exponent γ we have

$$\gamma = \frac{4}{3} \text{ or } 1 + \frac{1}{n} = \frac{4}{3} \text{ or } n = 3.$$

We have therefore the relation†

$$\left(\frac{GM}{M'}\right)^2 = \frac{(4K)^3}{4\pi G},$$

or

$$\begin{aligned} M &= 1.822 \times 10^{33}, \\ &= .91 \odot \text{ (nearly)}. \end{aligned} \quad (5)$$

As we have derived this mass for the star under ideal conditions of extreme degeneracy, we can regard 1.822×10^{33} as the maximum mass of an ideal white dwarf. This can be compared with the earlier estimate of Stoner²

$$M_{\max} = 2.2 \times 10^{33}, \quad (6)$$

based again on uniform density distribution. The “agreement” between the accurate working out, based on the theory of the polytropes, and the cruder form of the theory is rather surprising in view of the fact that in the corresponding non-relativistic case the deviations were rather serious.

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†A. S. Eddington, *Internal Constitution of Stars*, p. 83, eq. (57.3.)

²*Philosophical Magazine*, 9, 944, 1930.

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