stellar radial velocities and the important conclusions derived from them.

The President further announced that Dr. Plaskett had been invited to deliver the George Darwin Lecture for 1930.

The President read the following resolutions of the Council, which were approved by the meeting :—

The Council desire to express their sorrow at the death of Major P. A. MacMahon, who was a Fellow of the Society for thirty-four years, during eighteen of which he served on the Council, being Vice-President for seven years and President from 1917 to 1919. They request the President to convey their sympathy to his widow, and to assure her of the high esteem in which Major MacMahon was held by the Society.

The Council desire to express their grief at the tragic death of Major P. H. Hepburn, who was a member of Council from 1922 to 1928, being Treasurer from 1927 to 1928. His legal knowledge was frequently of service to the Council. His chief astronomical study was the planet Saturn, and for many years he was director of the Saturn section of the British Astronomical Association. The Council request the President to convey to his widow and family their sincere sympathy in their sad bereavement.

The Motions and Distribution of Interstellar Matter.

By J. S. Plaskett and J. A. Pearce. (Plate 9.)

Introduction.

Some twenty-five years ago Hartmann * first observed, in the spectroscopic binary δ Orionis, that the H and K lines of calcium did not share in the velocity oscillations of the hydrogen and helium lines but remained relatively fixed in position, hence the name " stationary " calcium lines, which has served for their designation almost to the present time. For nearly twenty years, although many more examples of spectroscopic binaries with " stationary " calcium lines were found, it was generally believed that they were associated only with binaries, and it was not recognized that they were also present in other high temperature stars. The observational data in regard to these " stationary " lines was summarized by R. K. Young,† who reached the conclusion that they were due to the absorption of a surrounding cloud of ionized calcium, which was sometimes set into revolution in the same period, but with smaller amplitude, by the binary motion of the stars through the cloud.

* *Ap. J.*, **19**, 268, 1904. † *Pub. D.A.O.*, **1**, 219, 1920.

A new phase of the problem developed in a research of one of the authors, J. S. Plaskett,* on the O-type stars. It was conclusively shown that the "stationary" calcium lines, differentiated from the other star lines by their extreme narrowness and sharpness, were present not only in spectroscopic binaries but in all O-type stars, even including the Wolf-Rayet stars in which no other absorption lines were present. The velocity resulting from the calcium lines generally differed, and frequently markedly, from the velocity given by the other lines in the spectrum. These velocity differences were so great, up to 60 km. per second, in comparison with the probable errors of the determinations, frequently only about a kilometre per second, as to leave no possible doubt of the relative motion of the star and the absorbing material which produced the stationary lines. Further, when it appeared that the calcium velocities generally differed little from the reflex of the solar motion, the conclusion was hardly escapable that these high temperature stars were in motion, frequently in rapid motion, relative to or through clouds of ionized calcium whieh were, comparatively speaking, at rest with respect to the local cluster. As at that time the "stationary" lines had not been identified with any stars of type later than B3, it was supposed this interstellar material was ionized and rendered absorbing at H and K by the excitation of the stars of highest temperature.

A great stimulus to the problem of this interstellar matter was given by Eddington † in the 1926 Bakerian lecture of the Royal Society. He discussed theoretically the physical conditions present in diffuse matter in interstellar space and showed that it was probably uniformly distributed, except where condensations produced the diffuse nebulæ, and had a density of 10^{-24} gm./cm.^{3} and a temperature, defined by the molecular speed, of 10,000 or 12,000 degrees absolute.‡ At this temperature most of the calcium present would be doubly ionized, the sodium singly ionized, calcium and sodium being the only elements likely to be detected, but neither of these ionized atoms give any lines in the observable region. There would be, however, one part in 3000 of Ca^+, but only one part in 2,000,000 of Na and one in 2,000,000,000 of Ca. It was calculated that this quantity of Ca^+ would be sufficient to produce appreciable absorption of H and K in a depth of 100 parsecs, but the appearance of interstellar D lines was unexplained. It was pointed out that it was unnecessary to assume excitation from the high temperature stars, which would tend to decrease the amount of Ca^+ present by changing it into Ca^{++}.

On Eddington's hypothesis, therefore, the presence of this diffuse material in interstellar space should have two consequences as regards the stationary, or, as we prefer to call them, the "interstellar" Ca^+ lines H and K. First, these lines should be present in the spectra of

* *Pub. D.A.O.*, **2**, 335, 1924; *M.N.*, **84**, 80, 1924.

† *Proc. Roy. Soc.*, **111**, 424, 1926.

‡ Gerasimovic and Struve in *Ap. J.*, **69**, 7, 1929, have calculated that the density of the interstellar material is about 10^{-26} gm./cm.^{3}, one-hundredth of Eddington's value, and the temperature about 15,000°.

all classes of stars, and secondly, they should increase in absorption or strength with the increase in distance of the star. It would, therefore, be of considerable value to test observationally these two consequences of Eddington's hypothesis.

So far as the first consequence, the presence of these lines in all spectral types, is concerned, it will be seen later that, while "interstellar" lines appear in some B3 and B5 stars, they have not yet been detected in any later types. Detection in B8, B9, or A-type stars will be difficult owing to the presence of Ca^+ absorption in the stellar atmosphere itself, while in F and later types it will be impossible owing to the width and strength of the stellar Ca^+ lines H and K. Stellar H and K are plainly visible in some B3 and B5 stars, are strong and well marked in B8 and B9, with rapidly increasing strength and width in A stars. The detection and discrimination of "interstellar" from stellar calcium in stars later than B5 would require, first of all, relatively strong "interstellar" lines, this entailing on Eddington's hypothesis great distance and hence faint objects, difficult to observe spectroscopically. Secondly, in order to discriminate between "interstellar" and stellar calcium, the latter must be displaced by the Doppler effect by an amount equivalent to a velocity of about 100 km. per second before the two lines can be separated with the low dispersion necessary for faint stars. The number of B8, B9, or early A stars likely to have a peculiar velocity of 100 km. or greater, or of binaries of the same classes with a range of 200 km. or over, caught at the time of maximum displacement, is relatively small, and the chance of certainly finding "interstellar" calcium in later types than B5 correspondingly remote.

As regards the second necessary consequence of Eddington's hypothesis of uniform distribution of the interstellar material, that the absorption of this material and the strength of the "interstellar" calcium lines must increase with the distance of the stars, the position is a little more hopeful. Struve has already recognized this fact and presented two able papers in the *Astrophysical Journal* on the relation between the intensity of the "interstellar" calcium lines and the faintness and hence, presumably, the distance of the stars. In the first of these papers * he correlated the intensities of the "interstellar" K line, H on account of blending with $H\epsilon$ being unsuitable for intensity estimates, with the apparent magnitudes of B- and O-type stars observed at Yerkes, Mt. Hamilton, Mt. Wilson, and Victoria. From these estimates he showed that there was an increase in the strength of the "interstellar" K line as the stars became fainter until about magnitude 7, while for stars fainter than the 7th magnitude there was a decrease in intensity, this decrease being in opposition to Eddington's hypothesis. These results, however, are based on relatively few faint stars and, moreover, the material was heterogeneous and made up of spectra from four different observatories.

In Struve's second paper,† however, the objection of insufficiency of data for the fainter stars is removed by using the objective prism spectrograms of Harvard Observatory. The intensities of "inter-

* *Ap. J.*, **65**, 163, 1927. † *Ibid.*, **67**, 353, 1928.

stellar " K were estimated in 1718 stars of spectral types O to B3, ranging in magnitude down to about 10·5. The average intensities of K in varying magnitude groups of these 1718 stars increased linearly from intensity 1 for first magnitude stars to intensity 4·4 at magnitude 10·5, the plot showing almost a straight line over the whole range. The decrease at magnitude 7, given in the first paper, had entirely disappeared, and these results show a definite increase in strength of "interstellar" K with increase in apparent magnitude, thus generally confirming Eddington's hypothesis of uniform distribution of the interstellar material. There are, however, in these results some uncertain factors, and caution in their complete acceptance is desirable. Thus the material is decidedly not homogeneous, as the spectra were obtained by objective prism instruments with dispersions ranging from 36 to 320A per millimetre. The known dependence of the character of the lines of objective prism spectra on the conditions of focus, etc., and on the quality of the seeing, may easily produce great differences in spectra of the same star made with the same dispersion, to say nothing of the enormous differences inherent in spectra of 36A and 320A per millimetre dispersions. Such differences would seem to make the intercomparison of the intensities of such a narrow absorption line as K with different dispersions extremely uncertain. Indeed, Struve himself says that in the low dispersion spectra the two lines of *Si* III at 3924 and K at 3934 "are almost blended, and this caused considerable uncertainty in the estimates."

It would be desirable, therefore, that an independent determination of the manner in which the interstellar material is distributed, whether uniform or not, should be undertaken. It would also be desirable if this determination did not primarily or wholly depend upon estimates of the intensity of the "interstellar" K line. Such estimates are difficult and uncertain on account of the general weakness of the continuous spectrum in that region. It is proposed in this paper, therefore, to determine the distribution of the interstellar material by an entirely different method, a method based on a determination of the relative distances of the star and of the centre of absorption in the cloud. This method analyses the stellar and interstellar velocities with reference to a differential rotation of the galactic system.

The theory of a rotation of the galaxy was introduced by Lindblad * in a series of papers, and was developed by Oort,† who investigated its consequences on the proper motions and radial velocities of the stars, and showed the probability of such a differential rotation. Further evidence was given by one of the writers, J. S. Plaskett,‡ who showed that the differential radial motions that would be produced by galactic rotation were markedly present in the radial velocities of about 550 O and B stars. The rotational term $\bar{r}$A, determined from the radial motions of a group of stars, consists of a constant part A, which has a value of about ·017 km. per second per parsec, and the variable part $\bar{r}$,

* *Medd. Upsala*, Nos. 3, 4, 6, 13, 1925, 1926; *M.N.*, **87**, 553, 1927.
† *B.A.N.*, Nos. 120, 132, 133, 1927.
‡ *M.N.*, **88**, 395, 1928.

which is the mean distance in parsecs of the group of stars from which $\bar{r}$A is determined. It is the possibility of determining the mean distance of a group of stars, or of the centre of gravity of the absorption of the calcium clouds, which makes it possible to test the distribution of these interstellar clouds by the galactic rotation.* As a necessary preliminary, however, the rotation of the galaxy will be further tested by the radial velocities of the interstellar clouds, in order to confirm its applicability in obtaining mean distances.

I. *The Motions of the Interstellar Clouds.*

Observational Material.

The material for this discussion has been gradually accumulating for the past five years in the determination by the authors of the radial velocities of all B0 to B5 stars of visual magnitude 7·5 or brighter, and north of declination − 11°, which had not been previously investigated. There were about 450 stars in this programme, mostly fainter than 5·5 magnitude, and, in order that the velocities should be reliably determined, an average of about five plates per star was obtained. For many stars with variable velocities more plates were secured to enable an approximate value of the velocity of the system to be obtained. It is believed that the velocities, both of the stars and of the interstellar clouds in the line of sight from the stars, are reliable. The average probable error of the determination of the radial velocity of these B-type stars is ± 1·80 km. per second, while that of the interstellar clouds is slightly less, ± 1·74 km. per second.

It will be of interest to examine in more detail the probable errors of the radial velocities of both stars and interstellar clouds. The material selected comprised 235 stars, for which both stellar and interstellar velocities were determined, which were used in one group of solutions. In the table the column headed "Undetermined" refers generally, for the stars, to spectroscopic binaries with a γ velocity adopted from the mean of about eight observations, and, for the clouds, to interstellar velocities determined from only one or two measures. None were included of large range or otherwise uncertain, and the probable errors are not likely to exceed those of the preceding column. As both the dispersions used with these stars give a flat field and beautiful definition over the entire photographic region from below λ3800 to λ5000 or beyond, and as the number of lines measured per plate averages more than eight, it is believed that the velocities of

* Gerasimovic and Struve (*Ap. J.*, **69**, 9, 1929), in the last section of their paper on "Physical Properties of a Gaseous Substratum in the Galaxy," have discussed the galactic rotation of the interstellar matter from the Ca^+ velocities of 103 stars. The rA of the cloud of + 5·3 km. is compared with that of the stars of + 12·0 km. and the relation of 1 to 2·3 taken as evidence of approximately uniform distribution of the interstellar matter. The $\bar{r}$A for the stars instead of being directly computed was, however, merely interpolated from Plaskett's (*M.N.*, **88**, 395, 1928) values, and this with the somewhat heterogeneous data makes the conclusion not very convincing.

both stellar and interstellar lines are reliable, and that they form a much more accurate and homogeneous system than any previously used.

TABLE I.

Probable Errors of Radial Velocities.

Range of Errors.	0 to 1·5.	1·5 to 2·5.	2·5 to 3·5.	3·5 and greater.	Undetermined.
Nos. of Stellar velocities.	98 (42%)	42 (18%)	18 (8%)	18 (8%)	59 (25%)
Average Probable Error.	± 0·85	± 2·07	± 3·15	± 5·03	
Nos. of Interstellar Velocities.	100 (43%)	47 (20%)	24 (10%)	14 (6%)	50 (21%)
Average Probable Error.	± 0·91	± 2·10	± 2·95	± 4.40	

Average Probable Error, 176 Stellar Velocities = ± 1·80 km. per sec.
Average Probable Error, 185 Interstellar Velocities = ± 1·74 km. per sec.

Of the 500 or more B- and O-type stars observed here, nearly 300 had pronounced Ca^+ lines. In selecting from these 300 the stars which contained "interstellar" lines, two criteria were employed—the character of the H and K lines, and the relation or the difference between the stellar and the interstellar velocity. The characters of the "interstellar" lines are so different from those which arise in the stellar atmosphere that, in the great majority of cases, there is no possibility of mistakes. The "interstellar" lines are sharp, narrow, and strong, with sharply limited edges and no appearance of wings. They have a distinctive, clear-cut appearance, quite different from any other lines present in O- or B-type stars, and, indeed, are much more clear cut than lines in late-type spectra, the nearest approach in character being in emission hydrogen lines with strong absorption reversal. In all stars of types O to B2, when Ca^+ lines are present there can be no doubt that they are due to interstellar absorption, as no stellar Ca^+ lines have been seen in any of our plates of these types.

When, however, we come to types B3 and B5 the matter is not so simple, though the stellar can usually be told from the "interstellar" line by the sharp, clear-cut appearance of the latter. The only doubtful cases are the stars of these types which have strong and sharp stellar lines of helium, etc. These are probably "*c*" stars, super-giants, and at greater distances than the average. It is possible that stellar H and K might also be sharp in these sharp-lined stars, and could not be differentiated from the interstellar lines by the character or appearance alone. In these uncertain cases the second criterion, the relation of the velocities, must be used. If the difference between the velocities of stellar and interstellar lines is greater than would be expected from their probable errors, then the calcium lines are placed in the interstellar class.

By these criteria a critical examination was made of the 300 odd stars in which H and K were present, to determine whether the calcium lines were of stellar or interstellar origin. As previously stated there is no ambiguity about the O to B2 stars, and about 175 stars of these types were immediately included in the list of stars with interstellar velocities. In regard to the B3 and B5 stars, which may have stellar H and K, great pains were taken to see that only stars with "interstellar" lines were included. Both of the criteria mentioned above were used and all doubtful cases excluded, so that the 64 B3 and 12 B5 stars placed in the list undoubtedly have "interstellar" lines, and there can be no doubt then that these lines are present in some B3 and B5 stars. A further examination of the B3, B5 stars with the strongest interstellar lines showed that generally they were either Be stars or else absorption stars with unusually strong and well-defined lines—"*c*" stars. Both of these classes are generally believed to be considerably more luminous than the average, and hence at greater distance, thus accounting on Eddington's hypothesis for the strength of the "interstellar" lines.

In this way a list of 261 stars containing "interstellar" calcium was prepared. For 23 of these stars, consisting of 6 Wolf-Rayet stars, 3 novæ, and 14 spectroscopic binaries without reliable γ velocities, only the interstellar velocity was known with sufficient accuracy, thus leaving 238 stars with reliable stellar radial velocities (three of which were later rejected on account of abnormally high velocities) and with reliable velocities of the interstellar cloud in the line of sight of the star. Of these 238 velocities, 228 were obtained at this observatory, leaving only 10 obtained elsewhere, mostly spectroscopic binaries with orbital determinations. It was felt preferable, in order to have a reliable and homogeneous system of velocities, not to include more outside observations, many of which were known to have low weight. While the number of stars with interstellar velocities was thus considerably reduced, it seemed better not to include uncertain values which might distort the results of the analysis.

Analysis of the Interstellar Velocities.

The distribution of these 261 interstellar cloud velocities is represented in Plate 9, the position of each one being plotted on an Aithoff projection perpendicular to the galactic plane. It will be noted that the distribution is far from being symmetrical or uniform, most of the velocities being restricted to about 180° in galactic longitude (from 0° to 180°, with a few scattered stars between 340° and 0° and between 180° and 205°) and within ± 20° of the galactic equator. Any determination of the solar motion and of the galactic rotation from such an unsymmetrical distribution of velocities will be subject to considerable uncertainty.

The velocity of the Sun with respect to the interstellar clouds will be fairly well determined, as the apex is about longitude 22°, and we have nearly a hemisphere of velocities on one side. But the solar apex will be uncertain, especially in latitude, because of the restriction of the observations to the near neighbourhood of the galactic plane and their limitation to a hemisphere in longitude. Galactic rotation is rather more favourably situated as it is of double-wave form, depending on the sine of twice the longitude. Hence, with the centre of rotation assumed at 325°, the centre of the globular cluster system, there will be two positive maxima in the velocities at 10° and 190° longitude, and an intervening negative maximum at 100°. Here again we should expect the magnitude of the rotational term to be more closely determined than the direction to the centre.

Hence, it seemed worth while to make a general solution of the 261 interstellar velocities for the solar motion and for the galactic rotation simultaneously. As the velocities are closely confined to the galactic plane, and as the rotational effect can best be referred to the galactic plane, galactic co-ordinates were used in the solution. The usual equations for determining the solar motion were used with the addition of Oort's equations for the galactic rotation. Thus we have the radial velocity V of the interstellar clouds:

$$\mathrm{V} = \mathrm{K} + \cos l \cos b \,.\, x + \sin l \cos b \,.\, y + \sin b \,.\, z + \bar{r}\mathrm{A} \sin 2(l - l_0) \cos^2 b$$

or

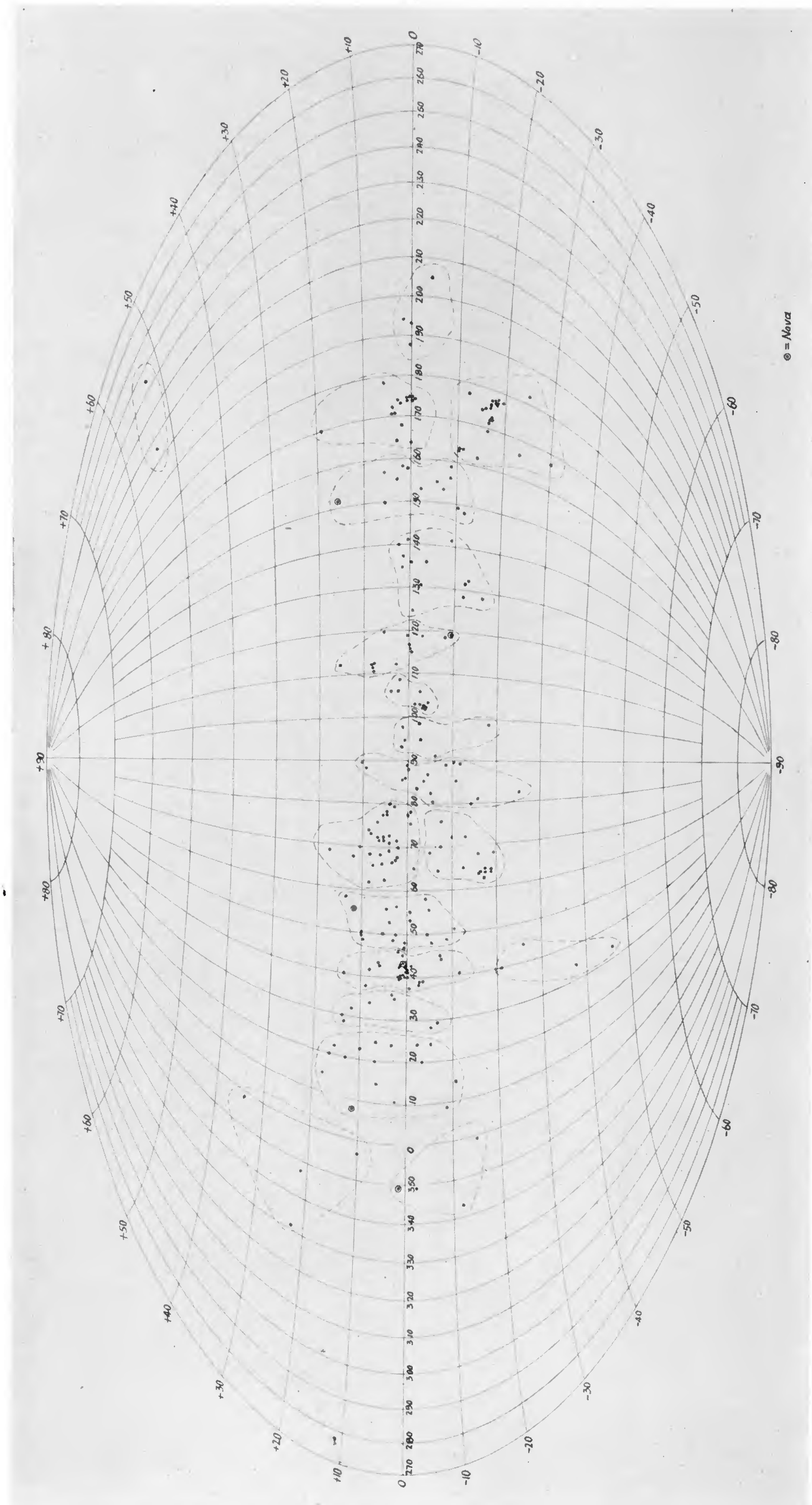
$$\mathrm{V} = \mathrm{K} + ax + by + cz + du + ev,$$

where K has the usual meaning of an average residual velocity, the coefficients a, b, c are obvious and refer to the solar motion, while d, e, u, v have the following values and refer to the galactic rotation, l_0 being the direction to the centre :—

$$d = \sin 2l \cos^2 b.$$
$$e = -\cos 2l \cos^2 b.$$
$$u = \bar{r}\mathrm{A} \cos 2l_0.$$
$$v = \bar{r}\mathrm{A} \sin 2l_0.$$

The usual plan of arranging the stars in a number of longitude groups was followed, and in doing this the plot of the distribution (Plate 9) was useful in arranging them in natural and compact groups. They were finally subdivided into 19 groups, the arrangement being indicated by dotted lines in Plate 9. No attempt was made to divide them into equal areas or equal numbers of stars, but, owing to the clustering of the B stars into groups, these natural divisions were followed as closely as possible. Some idea of the arrangement in longitude and latitude is given in Table II.

Distribution of Calcium Clouds with known Radial Velocities.



Galactic Co-ordinates.

TABLE II.

Arrangement of Groups.

Group.	No.	Region.	Range. Long.	Range. Lat.	$\bar{l}$.	$\bar{b}$.	$\bar{v}$.	$\bar{\rho}$.
1	4	Oph.	321 to 359	+10 to +35	345·9	+22·2	−12·68	+ 3·35
2	4	Scu.	343 ,, 360	+ 2 ,, −15	352·6	− 6·8	−12·20	+ 3·88
3	15	Aql., Her.	9 ,, 24	−11 ,, +18	18·9	+ 5·4	−13·40	+ 5·75
4	8	β Cyg.	28 ,, 36	− 8 ,, +14	31·7	+ 4·9	−13·14	+ 5·69
5	4	Peg.	34 ,, 44	−21 ,, −46	38·2	−32·4	− 6·40	+ 5·0
6	32	P Cyg.	37 ,, 46	−12 ,, +14	41·5	+ 0·6	−13·63	+ 4·15
7	20	α Cyg.	47 ,, 58	−11 ,, +13	51·6	+ 1·3	−10·83	+ 5·03
8	33	Cep.	58 ,, 80	− 1 ,, +18	71·0	+ 6·0	−14·96	− 2·32
9	17	Lac.	62 ,, 76	− 4 ,, −19	66·7	−13·1	−14·50	− 3·20
10	18	β Cas.	80 ,, 90	−25 ,, +10	85·8	− 5·2	−14·37	− 6·86
11	7	δ Cas.	91 ,, 99	−18 ,, + 2	95·7	− 3·4	−14·77	−10·53
12	11	h, χ, Per.	102 ,, 109	− 4 ,, + 4	104·2	− 1·3	−21·39	−18·94
13	15	Cam, Cas.	111 ,, 119	− 9 ,, +18	115·6	+ 2·5	− 3·51	− 4·39
14	13	Per, Aur.	124 ,, 141	−16 ,, + 2	131·7	− 4·9	+ 8·50	+ 0·95
15	14	ζ Tau.	148 ,, 159	−12 ,, +16	153·7	− 1·8	+12·04	− 0·49
16	16	Mon.	164 ,, 179	− 1 ,, +19	171·8	+ 2·8	+19·03	+ 3·22
17	24	ε Ori.	162 ,, 182	−20 ,, −10	172·5	−17·2	+19·28	+ 2·22
18	4	CMa.	187 ,, 205	− 4 ,, + 2	194·2	− 0·3	+31·33	+12·85
19	2	ρ Leo.	204 ,, 231	+53 ,, +54	217·6	+53·2	+13·40	+ 8·40

All the columns in this table are self-explanatory except the last two, where $\bar{v}$ is the mean radial velocity of the corresponding longitude group and $\bar{\rho}$ the mean residual or peculiar velocity, the radial velocity with the solar motion removed.

The solution of the normal equations based on observation equations formed from these groups resulted in the following values of the six unknowns :—

$$\begin{aligned} K &= + \ 0·55 \pm 2·70 \\ x &= - 17·87 \pm 1·52 \\ y &= - \ 8·67 \pm 4·51 \\ z &= - \ 1·26 \pm 4·08 \\ u &= + \ 4·71 \pm 1·47 \\ v &= - \ 5·57 \pm 2·29, \end{aligned}$$

from which it readily follows that

$$\begin{aligned} K &= \text{av. residual velocity} &&= + 0·55 \pm 2·70 \text{ km. per second.} \\ V &= \text{solar velocity} &&= 19·90 \pm 2·40 \quad \text{,,} \quad \text{,,} \\ L &= \text{longitude of apex} &&= 25°·9 \pm 11°·8 \\ B &= \text{latitude of apex} &&= + 3°·6 \pm 11°·8 \\ \bar{r}A &= \text{rotational term} &&= + 7·30 \pm 1·98 \quad \text{,,} \quad \text{,,} \\ l_0 &= \text{longitude of centre} &&= 335°·1 \pm 15°·5. \end{aligned}$$

The solar velocity as determined from the radial velocities of the interstellar clouds comes out practically exactly the same as determined from the naked-eye stars ; a useful and interesting result. The solar apex, however, is displaced about 20° to the eastward from its normal position which, in galactic co-ordinates, is L = 21°·8, B = + 20°·0. The greatest departure is in latitude, which was to be expected on account of the unsuitable distribution of the stars. The K term for the interstellar clouds is practically zero, whereas, as will be seen later, it is about + 4 km. per second from the radial velocities of the same stars.

The rotational term $\bar{r}$A of + 7·3 km. per second is substantial, indicating, if the average value of A from the work of Oort and Plaskett of 0·017 km. per second per parsec be accepted, an average distance of the centre of gravity of the interstellar clouds of some 430 parsecs. The longitude of the centre of rotation, 335°·1, is ten degrees away from the usual value of 325°, but the difference is less than the probable error of the determination.

It will be noted that the probable errors of all the quantities are relatively high, and this has been ascribed by us to the uncertain determination of the solar apex from groups of stars very ill-suited for such a purpose. This general solution has, however, served a good purpose in indicating that the solar velocity with respect to the interstellar clouds is exactly the same as for the naked-eye stars, but it could scarcely be expected that stars so peculiarly distributed as these high-temperature stars, closely confined to the galaxy and extending practically over only some 200° in longitude, should give a determination of the apex of much weight. It seemed desirable, therefore, when the solar velocity agreed with the accepted value, to assume the usual solar motion and carry through a second solution for the two terms of the galactic rotation, the direction to the centre and the magnitude, and for the K term.

The equations thus became

$$K + du + ev = \rho,$$

where the terms on the left-hand side have the same meaning as before and ρ is the mean residual velocity, the radial velocity with the solar motion removed, or $\rho = V - V_0 \cos D$, and is tabulated in the last column of Table II. The results of this solution, using the same groups as before, were

$$\begin{aligned} K &= -0{\cdot}61 \pm 0{\cdot}57 \text{ km. per second.} \\ \bar{r}A &= +7{\cdot}90 \pm 0{\cdot}79 \quad ,, \quad ,, \\ l_0 &= 331^\circ{\cdot}7 \pm 5^\circ{\cdot}7. \end{aligned}$$

It will thus be seen that accepting the usual values for the solar apex has reduced the probable errors to about one-third of their former value and has given a strong value of the galactic rotation for the interstellar clouds, a positive value ten times the probable error, while the direction to the centre agrees with the direction to the centre of the globular cluster system within its probable error.* The comparison can be seen more directly by tabulating the two solutions.

TABLE III.

Rotation of the Galaxy from the Interstellar Clouds.

	1. General Solution.	2. Particular Solution.
V	= +19·90 ± 2·40 km. per second.	+ 20·00 Assumed.
L	= 25°·9 ± 11°·8	21°·8 Assumed.
B	= +3°·6 ± 11°·8	+20°·0 Assumed.
$\bar{r}$A	= +7·30 ± 1°·98	+ 7·90±0·79
K	= +0·55 ± 2·70	− 0·61±0·57
l_0	= 335°·1 ± 15·5	331°·7±5°·7

The remarkable way in which the motions of the interstellar clouds agree in magnitude and sign with the differential motions that would be produced by a rotation of the galactic system around a distant centre is best shown by a comparison of the observed motions with those computed on the assumption of such a galactic rotation.

TABLE IV.

Comparison of Observed and Computed Velocities.

				1. General Solution.			2. Particular Solution.		
No.	*.	$\bar{l}$.	$\bar{b}$.	$\bar{v}$.	v'.	$\bar{v}-v'$.	$\bar{\rho}$.	ρ'.	$\bar{\rho}-\rho'$.
1	4	346°	+22°	−12·7	−11·7	− 1·0	+ 3·4	+2·6	+ 0·8
2	4	353	− 7	−12·2	−11·7	− 0·5	+ 3·9	+4·6	− 0·7
3	15	19	+ 5	−13·4	−12·0	− 1·4	+ 5·8	+7·2	− 1·4
4	8	32	+ 5	−13·1	−12·6	− 0·5	+ 5·7	+6·2	− 0·5
5	4	38	−32	− 6·4	−11·0	+ 4·6	+ 5·0	+3·5	+ 1·5
6	32	41	+ 1	−13·6	−13·2	− 0·4	+ 4·2	+4·5	− 0·3
7	20	52	+ 1	−10·8	−14·1	+ 3·3	+ 5·0	+2·1	+ 2·9
8	33	71	+ 6	−15·0	−15·0	− 0·0	− 2·3	−3·1	+ 0·8
9	17	67	−13	−14·5	−14·2	− 0·3	− 3·2	−1·9	− 1·3
10	18	86	− 5	−14·4	−14·0	− 0·4	− 6·9	−6·4	− 0·5
11	7	96	− 3	−14·8	−12·6	− 2·2	−10·5	−7·9	− 2·6
12	11	104	− 1	−21·4	−10·6	−10·8	−18·9	−8·5	−10·4
13	15	116	+ 2	− 3·5	− 6·8	+ 3·3	− 4·4	−8·1	+ 3·7
14	13	132	− 5	+ 8·5	+ 0·8	+ 7·7	+ 1·0	−5·7	+ 6·7
15	14	156	− 2	+12·0	+12·4	− 0·4	− 0·5	−0·1	− 0·4
16	16	172	+ 3	+19·0	+20·9	− 1·9	+ 3·2	+4·5	− 1·3
17	24	173	−17	+19·3	+20·3	− 1·0	+ 2·2	+4·5	− 2·0
18	4	194	0	+31·3	+27·1	+ 4·2	+12·8	+7·2	+ 5·6
19	2	218	+54	+13·4	+12·4	+ 1·0	+ 8·4	+1·4	+ 7·0

* It should be noted that this determination of the direction to the centre of rotation is perpendicular to the direction of Stromberg's axis of asymmetry at galactic longitude 61°.

Comparing the columns $\bar{v}$ and v', observed and computed radial velocities, it is at once seen how closely the observed and computed velocities agree. Except for groups Nos. 12 and 14, which will be referred to later, and for Nos. 5 and 18, which have only four stars each, the agreement is remarkable and generally little exceeds the probable errors of the determination of the velocities. There is apparently little indication of any but very small group or peculiar motions of the clouds, such as have been found in the motions of the B stars themselves, except for the instances noted above.

If the residuals of the radial velocities of the general solution, column $\bar{v} - v'$, are compared with those of the residual or peculiar velocities of the particular solution, column $\bar{\rho} - \rho'$, we find that the principal difference is in the last group, which has probably been partly responsible for giving the discrepant position of the solar apex. The two stars in this group are 54° from the galactic plane, are probably relatively near, and should perhaps not have been used in the solutions.

In the second or particular solution, where the usual solar motion was assumed, and which forms a direct test of the reality of a galactic rotation, a simple comparison of the values of the computed residual or peculiar velocity ρ' with those of the observed $\bar{\rho}$ shows a remarkable correspondence. With the exception of groups 12, 14, 18, and 19, the average residual, $\bar{\rho} - \rho'$, is only 1·3 km. per second. Group 12 is composed of stars confined to the near neighbourhood of the h, χ clusters in Perseus, which are known to be much more distant than the average of the other groups. Similarly, for group 18 all the stars are of O-type, and this group probably is also more distant than the average. The rotational terms, the $\bar{r}$A's, for these two groups would be much greater than for the remaining groups, thus explaining the high residuals in these two cases. No. 19 has been explained above, and that leaves only No. 14, which has a positive residual of 6·7 km. This group is on the boundaries of Perseus and Auriga, where the interstellar lines are rather weak, and hence the distance of the group is probably smaller than the average. This smaller distance, however, could account for only part of the residual of 6·7 km. per second, possibly one-third of it, leaving a difference between the calculated rotational effect and the average velocity of the cloud in this region of 4 or 5 km. per second. This cannot be ascribed to accidental errors, as the interstellar velocities are well determined, the probable error of the mean of the 13 velocities of the group being $\pm$0·8 km. per second. There must hence be a real peculiar or group motion of recession of the interstellar material in this region of about 4 km. per second, but this need cause no surprise when there are known deviations of the same order in the velocities of different parts of the Orion Nebula which is probably a local condensation of the interstellar material.

With the exception of these four groups, whose deviations are thus accounted for, the agreement of the observed velocities with those that would be produced by a rotation of the galaxy is so remarkably close that there can be no reasonable doubt that *this diffuse interstellar matter partakes almost exactly of the differential motions given by a general* *galactic rotation.* The smaller differences observed may be reasonably accounted for by small peculiar motions in different parts of the cloud, or, possibly, by the method of grouping employed by which parts of the cloud at greatly different distances may be grouped together.

This important conclusion, which satisfactorily disposes of this first part of the discussion, that relating to the motions of the interstellar clouds, has also a direct bearing on the second part, the distribution of the clouds. For if the cloud follows generally the galactic rotation we may place considerable confidence in the values of the rotation term $\bar{r}A$ obtained, which, if the constant A is known accurately, should give accurately the mean distance of the cloud. The mean value of A, as given in the introduction, of ·017 * gives immediately, from $\bar{r}A = +7{\cdot}90$ for the cloud, the mean distance of 465 parsecs. But, even if A is not accurately known, the values of $\bar{r}A$ for different groupings of stars and clouds should give accurate ratios of the distances of these groupings. If, then, we can determine the rotational term from both the stellar and the interstellar velocities of the same group or groups of stars, we have a reliable method of determining the relative distance of stars and clouds.

II. *The Distribution of the Interstellar Clouds.*

Observational Material.

As the rotational term is to be determined for both stars and clouds, some revision of the data used in the first part is necessary. These referred to interstellar velocities only, and included a number of stars, such as the Wolf-Rayet and the novæ, for which no stellar velocity can be obtained. Also there are some spectroscopic binaries which give reliable interstellar velocities but uncertain stellar, and some constant velocity stars with average interstellar but abnormally high stellar velocities, whose inclusion would distort the mean peculiar velocity of any group to which they belonged. All such objects must be excluded from the revised list, which must only include stars which have reliable stellar and interstellar velocities. Altogether 26 stars were, for these reasons, excluded from the previous list, leaving 235 reliable stellar and interstellar velocities for the new discussion. While all the stars were combined into 19 longitude groups for the general galactic rotation, it would seem preferable to subdivide these into smaller groups, each composed of stars and clouds at approximately the same distances, for the determination of relative distances of stars and clouds by a comparison of their respective rotational terms.

To subdivide the O and B stars into groups according to distance is, however, a very difficult task, as so little is known about the distances of these high temperature stars. The distances are so great that trigonometric parallaxes are worthless, while the method of spectroscopic absolute magnitudes appears to provide little but group parallaxes. It is probable that the dispersion in absolute magnitude in the various groups is much greater than is generally believed, and that such group

* From Oort's value of ·019 and Plaskett's of ·0155 km. per sec. per parsec.

parallaxes do not give very certain information about distances. The mean or statistical parallaxes from proper motions provide information about mean distances, but again the dispersion in absolute magnitude may distort the results in such special circumstances as are considered here. As no better method seemed available, however, and as a first criterion, the 235 stars were divided into different magnitude groups, which should arrange them, roughly at any rate, in order of distance.

Arrangement and Solution of Magnitude Groups.

The number of magnitude divisions should not be too great, or the numbers of stars in the longitude subdivisions will be so small that the solutions will have low weight. Following the method adopted for solving for the galactic rotation from the radial velocities of 875 O to B5 stars, of which the results will be published later, groups including a magnitude difference of one magnitude were used. As there are very few stars brighter than 4·5 magnitude with interstellar lines, the first group comprised all stars brighter than 5·5 magnitude. Many stars in this group are so near, and the number 37 is so small, as to make the solution uncertain, so a second intermediate group of 45 stars from 5·00 to 5·99 inclusive was formed. The third group of 79 stars is from 5·50 to 6·49 magnitude inclusive, and the fourth group of 119 stars includes all 6·50 magnitude and fainter. A fifth group of 69 of the faintest, and probably most distant, of the stars of the fourth group from 7·00 to the faintest, 8·6 magnitude, was also formed. The data about these magnitude groups are combined in Table V.

Table V.

Magnitude Groups.

Group.	Mag. Limits.	No. Stars.	No. Longitude Sub-groups.	Mean Mag.
1	0 to 5·49	37	10	4·41
2	5·00 ,, 5·99	45	8	5·60
3	5·50 ,, 6·49	79	10	6·03
4	6·50 ,, 8·6	119	14	7·08
5	7·00 ,, 8·6	69	11	7·34

These five magnitude groups were each subdivided into longitude groups of the number shown in the fourth column, and each solved for the galactic rotation. The usual solar motion of 20 km. per sec. to the apex at $\alpha = 271°$, $\delta = +28°$ was assumed. Also the direction to the centre of rotation was assumed as 325°, the direction to the centre of the globular cluster system. Under these assumptions the equations reduce to $K + \bar{r}A \sin 2(l - 325°) \cos^2 b = \rho$. The results of $\bar{r}A$ and K for both stars and clouds with their probable errors are given in Table VI.

There are several interesting results of these solutions which may be discussed in turn. Consider the K term first. For the interstellar clouds it is practically zero, in the mean + 0·12, within the limits of the probable errors for each magnitude group, just as was found in the general solutions. For exactly the same stars which give a zero K term for the interstellar clouds, the stellar K term varies between + 1·4 and + 7·1 in the different magnitude groups. Obviously the K term has nothing to do with the properties of the space concerned but is inherent in the stars themselves. We may add that it has been clearly demonstrated, in the investigation referred to above, that this K term in the O and B stars is due to preponderating peculiar motions of recession of certain special groups of stars, particularly among the brighter stars in the southern hemisphere, which are not balanced by equal motions of approach. Hence the positive K term among the brighter stars, which disappears in the B-type stars at magnitude 6 and fainter. These results, including the galactic rotation for 875 stars, will be published later.

TABLE VI.

Solution of Magnitude Groups.

Group.	$\bar{m}$.	$\bar{r}$A.		K.	
		Stars.	Clouds.	Stars.	Clouds.
1	4·41	+ 1·81±2·83	+ 3·85±1·22	+7·12±1·42	+0·02±0·61
2	5·60	+10·26±2·12	+ 5·02±1·24	+3·99±0·96	+0·97±0·56
3	6·03	+13·86±1·75	+ 7·66±0·90	+1·67±0·77	+0·09±0·40
4	7·08	+16·58±2·20	+ 8·31±1·36	+1·39±1·46	−0·69±0·90
5	7·34	+20·49±2·32	+10·08±1·57	+2·34±0·86	+0·23±0·56

The rotational term $\bar{r}$A is, as will be noted, indeterminate for the small group of the brightest stars. This is partly due to the small numbers and the great part any large peculiar motions would have among so few stars, and perhaps, partly, to the fact that these stars are relatively near and the rotational term small. It should be noted, however, that $\bar{r}$A for the interstellar clouds in this group is fairly well determined, its probable error less than one-third the value, an indication of the smallness of the peculiar motions in the clouds, and that its magnitude, + 3·85 km. per second, is, considering the magnitude and distance, in line with the other values of $\bar{r}$A for the interstellar clouds. The great difference between the probable errors of star and cloud determination in all groups is undoubtedly mainly due to the large part the peculiar motions of the stars play in the case of small numbers and the smallness of such peculiar motions in the clouds.

Considering then the $\bar{r}$A terms in the remaining four groups, composed as already stated of the variable part $\bar{r}$, the average distance of the group of stars or clouds, and the constant part A previously taken as 0·017 kilometres per second, and corresponding to the rotational effect per parsec of distance, we find an increase in $\bar{r}$A and hence in $\bar{r}$, the mean distance, as the magnitude increases. This increase in distance corresponds closely to the increase in faintness at least for the two extreme groups, No. 2 and No. 5, where $\bar{r}A$ in No. 5 is twice that in No. 2, corresponding to a difference in brightness of four times, which is not greatly different from the ratio of magnitudes, 5·60 to 7·34. While the other ratios are not exact they correspond approximately, so that we can, from the galactic rotation, confirm the relation previously postulated that, generally speaking, the fainter B and O stars are farther away than the brighter. The above relation would place 5·0 magnitude O's and early B's at a distance of about 450, 6·0 magnitude at about 800, and 7·0 magnitude at about 1000 parsecs.

But the most important result of these solutions is undoubtedly the ratio of the rotational term $\bar{r}A$, for the stars of any magnitude group, to that for the interstellar clouds of the same group, the results for the clouds being based on the same stars as the results for the stars themselves. Omitting the first group, which is indeterminate, these ratios for the second, third, fourth, and fifth groups are respectively 2·04, 1·81, 1·99, 2·03, or in the mean 1·97. As A is a constant, the $\bar{r}$, the mean distance of the stars, is almost exactly twice that of the interstellar clouds. This is true for stars of the last four groups at approximately 600, 800, 1000, and 1200 parsecs away, and is the more remarkable when the relatively small numbers of stars in the groups and the effect any peculiar motions of stars or clouds might have on the results is considered. The only possible deduction from these results is that the effective distance at which the interstellar absorption is produced is half the distance of the corresponding stars. This surely means, when groups of stars at such greatly differing distances are involved, *that the interstellar matter is uniformly distributed*, and is a striking confirmation of Eddington's hypothesis.

Intensity of the "Interstellar" Lines.

In view of Struve's interesting results on the relation of the intensity of the interstellar lines to the apparent magnitude, and hence probably to the distance, of the stars, it seemed worth while to re-group these stars according to the relative intensity of the K line, and again solve for the galactic rotation and the distribution of the interstellar clouds.

The accurate estimation of the intensity of the K line, which is about the practical limit of visibility in the spectrum owing to absorption of the glass in the optical train, is no easy matter and appears to be beset with many pitfalls. While Struve's method of visual estimates is probably as satisfactory as can be expected from the character of the material, there is no assurance that one's mental scale would remain constant, and it seemed best to us to compare every intensity with some fixed standard. Fortunately such a standard was already available in one of the scales used by Young and Harper for estimating the relative intensities of the lines used in spectroscopic parallax work. One of these scales was especially suitable as it contained ten artificial lines, ranging in intensity from practically clear glass, called intensity 10, to a faint line just distinguishable from the continuous background, called intensity 1. The lines were hard and sharp, almost identical in appearance with the " interstellar " lines, which made the comparison more easy and accurate.

The main difficulty in the estimates is the great variation in the intensity of the continuous background in star spectra of various degrees of exposure, and the difficulty of matching the intensity of the lines in star backgrounds of varying strength with the artificial lines in a background of constant strength. Fortunately we had a large number of spectra of varying exposure for each of a considerable number of stars, and it was found that there was a continuous increase in the estimated intensity of the K line with the increase in the strength of the continuous background. A careful comparison and matching of intensities with the standard for spectra of different density enabled us to obtain a fairly definite and constant relation between the intensity and the strength of the continuous spectrum. This strength was defined by the wave-length to which the continuous spectrum extended,

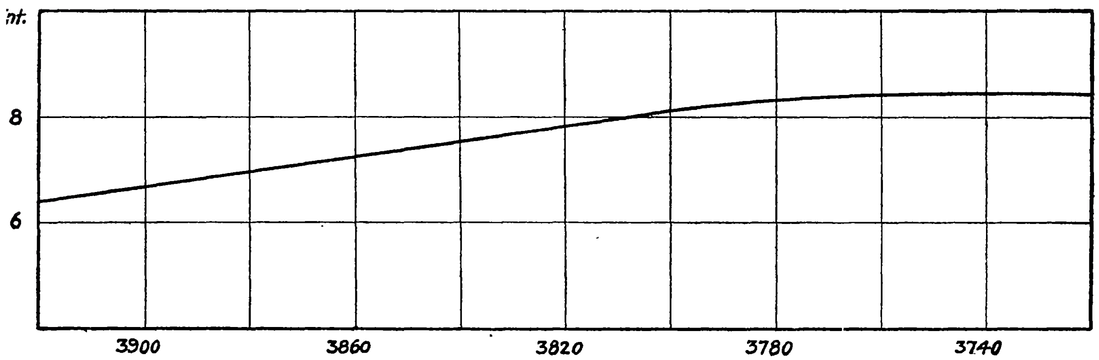


Fig.1 Differences in Estimated Intensity of K for Different Extensions of the Continuous Spectrum.

which varied between about λ 3700 and λ 3920. No reliable estimates of the intensity of K could be made unless the continuous spectrum extended at least to λ 3920.

It was found that the estimated intensity of K increased slowly with the intensity of the continuous background, with the extension into the violet. The amount of the increase is represented by the curve in fig. 1, which is a mean curve based upon the measures of numerous spectra of several stars. It is an inclined straight line, the intensity increasing about 0·3 unit for each 10A of the extension of the spectrum into the violet until about λ 3800, when it curves towards the horizontal.

In estimating the intensity of the K line in some 600 spectra of about 230 stars, the standard scale was placed in contact with the spectrum to be estimated and shifted along under an eye-piece until the K line was adjacent to the artificial lines, when it was easy to select the artificial most closely matching the star line. The number of this artificial line, with the wave-length to which the spectrum extended, was noted. After the intensity of K had thus been estimated on the 600 plates, all these estimates were reduced by means of the curve in fig. 1 to the intensity of a spectrum in which the background extended to λ 3860, the average extension of the continuous background, and thus made directly comparable with each other. The individual corrected estimates were averaged for each star, and these means were then used for regrouping the stars into intensity instead of magnitude groups.

Before giving the details and solutions of these groups it will be worth while to attempt to translate the arbitrary scale of estimates from 1 to 10 into magnitude differences. Unfortunately, we have no record of how the scale was formed, although even this would be of little use, as the light used was probably daylight of quite a different wave-length to that of the K line and with a different characteristic curve. Again, we have sensitometer patches on each spectrum plate, but these have the same drawback of being made with an incandescent lamp with maximum effect around λ 4700 instead of λ 3930. Consequently, a series of exposures was made on the star 9 Cam., with strong interstellar lines, and a comparison of these by the microphotometer with one another and with sensitometer patches, made with both violet light and light from the usual source, showed the following relation between the successive numbers of the artificial scale. These relations are, of course, only approximate, but give some idea of the intensity of the absorption corresponding to the numbers of the scale. The magnitude differences between the continuous background of the standard and the corresponding numbers of the lines are here given:

Line .	10	9	8	7	6	5	4	3	2	1
Mag. Diff.	·90	·72	·55	·43	·33	·28	·23	·20	·17	·14

It will be noted that the steps are much greater for the strong than for the weak lines, a result of the way in which the scale was made. A comparison of the intensity of the interstellar line K in 9 Cam. with that of the continuous background by means of successive exposures of different lengths showed a difference of intensity of nearly one magnitude. These results agree approximately with those of Struve for the intensity of K, but the ratio for the steps most frequently used here is slightly greater than that assumed by him.

Arrangement in "Intensity" Groups.

The intensities of the "interstellar" K line varied from 1·6 in the O8 star ι Orionis to 9·5 in H.D. 14134, a B2 star of 6·66 magnitude in the h, χ clusters in Perseus. There are only 14 stars with intensities less than 4·4, whose mean apparent magnitude is 4·85 and mean type is B1·6. The number of stars out of the 235 used in the last grouping for which intensities of the "interstellar" K line could be reliably estimated was 226, of which 122, or 54 per cent., had intensities greater than 7·0.

The stars were arranged in the order of intensity of K and divided first of all into four intensity groups, Nos. 1, 2, 4, 5, of the following table, which gives all the useful data about the groups. Then, owing to the uncertainty attaching to the results from the nearer stars, in this case presumably those with low K intensities, all stars with intensities of K less than 4·4, where there is a natural break in the run of the intensities, were discarded, and the remaining 28 stars, too small a number to solve separately, were combined with those of group 2 to form group 3 of 90 stars.

TABLE VII.

Arrangement in Intensity of K Groups.

Group.	No. Stars.	No. Long. Groups.	Range of Intensity.	Mean Int.	Mean Mag.	Mean Type.
1	42	9	0 to 5·9	4·72	5·77	B2·2
2	62	10	6·0 „ 6·9	6·50	6·45	B1·2
3	90	11	4·4 „ 6·9	6·08	6·35	B1·6
4	79	11	7·0 „ 7·9	7·46	6·73	B0·9
5	43	9	8·0 „ 9·5	8·42	6·56	B0·4

It is interesting to note that, although there is a change of the mean intensity of K from 4·72 to 8·42, the increase of mean apparent magnitude is only from 5·77 to 6·56, and of temperature from B2·2 to B0·4. Also, in the four last groups, with a change in intensity of K from 6·08 to 8·42 there is a change of less than 0·4 magnitude in the mean magnitude. In the hope of providing some explanation, the stars were divided in the four main groups 1, 2, 4, 5 into three type divisions, O, B0 to B2, and B3 to B5, and the number of the stars of each type, with the mean magnitude and type, are given in Table VIII.

TABLE VIII.

Arrangement in Types.

Group.	Intensity Range.	O Stars.			B0 to B2 Stars.			B3, B5 Stars.		
		No.	Type.	Mag.	No.	Type.	Mag.	No.	Type.	Mag.
1	0 to 5·9	3	O7·8	4·47	14	B1·1	5·37	25	B3·3	6·38
2	6·0 „ 6·9	15	O6·7	6·39	25	B1·3	6·38	22	B3·1	6·58
4	7·0 „ 7·9	22	O6·8	6·78	38	B1·3	6·48	19	B3·1	6·49
5	8·0 „ 9·5	14	O7·1	6·08	22	B1·2	6·69	7	B3·3	6·48

But this does not give much additional information except that the mean magnitudes of all types, especially of the three last groups, are very similar.

There is, however, a very considerable difference in the average distances of these groups, as will appear from the solutions below, and the only possible explanation seems to be a larger dispersion in absolute magnitude in any one type than is generally believed. It will be noted that the mean magnitude of group 5 of Table VII. is less than that of group 4, and yet, as the solutions show, the former is at nearly twice the distance. This cannot be accounted for by the change in type of half a division, and must be due to some other cause. On examining the stars of group 5, it was found that the smaller mean apparent magnitude was due not to the smaller magnitude of all the stars, but to the presence of a number of relatively bright stars. Thus 11 of the stars have a mean apparent magnitude of 5·61 as compared with 6·90 for the remaining 32 stars. These brighter stars include such well-known super-giants as H.D. 1337 and B.D. 6° 1309, and many with sharp, strong, stellar lines, all probably being stars of great intrinsic luminosity.

The presence of a number of super-giant stars in increasing proportions with increase in the intensity of the K line and in distance, together with the advance in spectral type from B1·6 to B0·4 for groups 2 to 5, and consequent increase in absolute magnitude, may account for the nearly constant apparent magnitude in these groups. In any event there seems to be no doubt of the presence of stars of very great luminosity, instanced by several B3 and B5 stars brighter than 6th magnitude in the last group, combined with the usual dispersion in absolute magnitude.

Solution of Intensity Groups.

After the 226 stars, of which there were plates with K lines measured for radial velocity and estimated for intensity, had been arranged into intensity groups as in the above table (Table VII.), each intensity group was divided into the number of longitude groups shown in the third column of the table.

These groups were then solved for the galactic rotation and the K term for both stellar and cloud velocities, separately, the usual solar motion and the longitude of the centre of rotation being assumed as in the previous magnitude groups. The results of these solutions, with their probable errors, are as shown in the following table :—

TABLE IX.

Solutions for the Intensity Groups.

Group.	Mean Int.	No. Stars.	$\bar{r}A$.		K.	
			Stars.	Clouds.	Stars.	Clouds.
1	4·72	42	+ 3·64±3·25	+ 4·97±0·77	+6·12±2·14	+1·15±0·50
2	6·50	62	+12·12±1·88	+ 4·93±0·92	+4·66±1·27	+0·10±0·62
3	6·08	90	+10·22±1·72	+ 5·03±0·85	+5·24±1·17	+0·10±0·58
4	7·46	79	+14·53±2·93	+ 6·91±1·06	−0·14±1·95	−0·21±0·70
5	8·42	43	+27·52±2·46	+13·72±1·16	+3·18±1·74	−1·17±0·82

Again, the K term for the interstellar clouds is negligibly small, the mean of the five values being 0, while it ranges up to 6·1 for the stars. When it is remembered that, for the magnitude groupings as well as for the general solutions, the value was of the same order, there can be no doubt that there is no K term for the interstellar clouds, though it has a high positive value for the stars.

The value of $\bar{r}A$ is indeterminate for the stars of the first group, owing probably, as previously, to large peculiar motions in the velocities of these nearer stars. The $\bar{r}A$ for the clouds is, however, reliable, probable error less than $\frac{1}{6}$ value, as any peculiar motions here are small, and is practically identical for groups 1, 2, 3, indicating possibly some uncertainties in the estimates of the intensities of K in those stars in which it is weaker, and some mixing up of nearer stars with more distant ones. It was to avoid this difficulty that group 3 was formed of groups 1 and 2, omitting 14 stars with intensities of K less than 4·4 which were relatively close to us and hence would give uncertain values. Moreover, in this combined group there are about twice as many stars as in group 1, and one and a half times as many as in group 2.

In order to show how closely the residual velocities of stars and clouds follow those that would be produced by a rotation of the galaxy around a distant centre at $\lambda = 325°$, the resulting residual velocities for both stars and clouds were computed for the various values of $\bar{r}A$ and K in the last three magnitude groups (Table VI.) and the last three intensity groups (Table IX.). The groups of low magnitude and low intensity are not included on account of the relative nearness of stars and clouds in both groupings, and the uncertainty, at least for the stars, of $\bar{r}A$.

In order to enable the comparisons to be more readily made, the corresponding magnitude and intensity groups are placed adjacent to one another—in one table (Table X.) for the stars, and in another (Table XI.) for the clouds—giving in each the number of the longitude group, the number of stars, and the mean longitude, as well as the comparison of residual velocities. As the mean longitudes are by no means the same in the different groupings, those which most nearly agree are placed in the same line so as to facilitate comparison between the observed and computed velocities. It will be found that such comparisons are worth while, in order to show how closely the stars and clouds follow the double wave-form of the galactic rotation, and how small are the differences, considering the effect of peculiar motions in such small numbers of stars, and considering also the possible effects of group motions in the B stars, such group motions being probable in stars which are arranged into groups in position.

Let us compare first of all the computed with the observed stellar residual velocities, the velocities with the solar motion removed (Table X.). It will be noted immediately, as the eye glances down the columns, the general swing from positive velocities to negative and back again to positive with the progression in longitude, obviously due to the double wave-term of the galactic rotation, in each of the six magnitude and intensity groups. Generally speaking, the differences between observed and computed velocities are relatively small. The larger differences, except around galactic longitude 100°, where the very distant Perseus clusters distort the results, are mostly in the longitude sub-groups where the numbers of stars are small, and consequently where the peculiar velocities play a relatively large part.

Table X.

Comparison of Residual Stellar Velocities in Magnitude and Intensity Groups.

Magnitude 5·5 to 6·5.

$\bar{r}A = +13{\cdot}86$. $K = +1{\cdot}67$.

Group.	No.	Mean Long.	Residual Velocities. Observed.	Residual Velocities. Computed.
		°		
1	3	351	+22·8	+12·2
2	5	21	+ 9·6	+14·5
3	11	33	+10·4	+11·2
4	11	48	+ 4·6	+ 5·2
5	16	67	− 2·6	− 4·0
6	9	81	−13·7	− 9·2
7	3	134	+ 8·0	− 3·5
8	8	158	+ 7·8	+ 7·6
9	11	175	+ 9·9	+13·4
10	2	192	+29·1	+15·5

Magnitude 6·5 to 8·6.

$\bar{r}A = +16{\cdot}58$. $K = +1{\cdot}39$.

Group.	No.	Mean Long.	Observed.	Computed.
		°		
1	2	351	+36·6	+14·2
2	8	18	+ 1·6	+17·6
3	16	41	+11·8	+ 9·7
4	11	51	+ 3·8	+ 4·0
5	11	66	− 1·3	− 4·5
6	11	76	− 7·8	− 7·8
7	14	91	−13·6	−14·1
8	8	103	−40·0	−14·8
9	13	113	− 5·9	−13·2
10	8	134	+ 3·1	− 4·5
11	5	153	+ 5·7	+ 6·5
12	8	173	+19·3	+15·2
13	3	175	+ 8·3	+14·7
14	1	231	+17·0	+ 2·3

Magnitude 7·0 to 8·6.

$\bar{r}A = +20{\cdot}49$. $K = +2{\cdot}34$.

Group.	No.	Mean Long.	Observed.	Computed.
		°		
1	15	40	+12·5	+12·5
2	7	51	+ 3·3	+ 5·4
3	4	66	− 0·6	− 5·6
4	3	76	−14·0	−11·3
5	12	85	−16·4	−15·3
6	4	104	−38·6	−17·9
7	7	113	− 3·7	−15·9
8	5	136	− 0·7	− 3·6
9	3	155	+ 7·6	+ 9·1
10	8	172	+17·9	+19·1
11	1	231	+17·0	+ 4·0

Intensity 4·4 to 6·9.

$\bar{r}A = +10{\cdot}22$. $K = +4{\cdot}24$.

Group.	No.	Mean Long.	Residual Velocities. Observed.	Residual Velocities. Computed.
		°		
1	2	345	+36·2	+10·8
2	2	17	+ 5·9	+14·0
3	14	38	+10·4	+ 9·9
4	11	61	+ 6·4	+ 2·1
5	13	75	− 7·3	− 2·4
6	8	89	− 6·8	− 5·3
7	8	131	− 2·1	− 5·0
8	6	131	+ 4·5	− 0·5
9	8	156	+ 7·0	+ 8·0
10	13	174	+14·4	+12·6
11	3	205	− 2·6	+ 9·8

Intensity 7·0 to 7·9.

$\bar{r}A = +14{\cdot}53$. $K = -0{\cdot}14$.

Group.	No.	Mean Long.	Observed.	Computed.
		°		
1	8	12	− 5·5	+14·3
2	12	38	+ 8·9	+ 8·0
3	11	51	+ 7·2	+ 1·7
4	10	65	− 1·5	− 5·3
5	4	75	− 7·8	− 9·5
6	8	89	−20·2	−13·7
7	3	110	−23·9	−13·9
8	3	139	+ 1·7	− 3·2
9	9	160	+10·6	+ 7·2
10	10	174	+17·2	+12·2
11	1	192	+44·6	+14·4

Intensity 8·0 to 9·5.

$\bar{r}A = +27{\cdot}52$. $K = +3{\cdot}18$.

Group.	No.	Mean Long.	Observed.	Computed.
		°		
1	2	358	+35·7	+24·3
2	2	17	+12·2	+29·5
3	6	34	+21·5	+21·3
4	2	47	+11·8	+10·4
5	12	70	− 3·4	−10·4
6	4	97	−26·8	−23·4
7	9	107	−28·4	−23·5
8	4	141	− 4·1	− 0·9
9	2	169	+13·6	+16·0

Table XI.

Comparison of Residual Cloud Velocities in Magnitude and Intensity Groups.

Magnitude 5·5 to 6·5.

$\bar{r}A = 7{\cdot}66.$ $K = +0{\cdot}09.$

Group.	No.	Mean Long. (°)	Residual Velocities. Observed.	Residual Velocities. Computed.
1	3	351	+ 5·0	+5·9
2	5	21	+ 9·0	+7·2
3	11	33	+ 4·7	+5·3
4	11	48	+ 5·4	+2·1
5	16	67	− 2·4	−3·0
6	9	81	− 7·2	−5·9
7	3	134	− 1·7	−2·8
8	8	158	− 1·1	+3·4
9	11	175	+ 5·6	+6·6
10	2	192	+12·4	+7·7

Magnitude 6·5 to 8·6.

$\bar{r}A = +8{\cdot}31.$ $K = -0{\cdot}69.$

Group.	No.	Mean Long. (°)	Residual Velocities. Observed.	Residual Velocities. Computed.
1	2	351	+ 1·1	+6·7
2	8	18	+ 4·2	+7·4
3	16	41	+ 6·2	+3·4
4	11	51	+ 4·7	+0·6
5	11	66	− 4·4	−3·7
6	11	76	− 3·2	−6·2
7	14	91	− 7·2	−8·5
8	8	103	−23·1	−8·8
9	13	113	− 6·2	−8·0
10	8	134	+ 0·3	−3·7
11	5	153	− 1·6	+1·8
12	8	173	+ 3·9	+6·2
13	3	175	− 3·2	+5·9
14	1	231	+14·2	−0·2

Magnitude 7·0 to 8·6.

$\bar{r}A = +10{\cdot}08.$ $K = +0{\cdot}23.$

Group.	No.	Mean Long. (°)	Residual Velocities. Observed.	Residual Velocities. Computed.
1	15	40	+ 6·4	+5·2
2	7	51	+ 6·0	+1·7
3	4	66	− 1·0	−3·7
4	3	76	− 3·4	−6·5
5	12	85	− 8·5	−8·5
6	4	104	−21·8	−9·8
7	7	113	− 8·2	−8·8
8	5	136	+ 1·8	−2·6
9	3	155	− 0·8	+3·1
10	8	172	+ 2·7	+8·0
11	1	231	+14·2	+1·1

Intensity 4·4 to 6·9.

$\bar{r}A = +5{\cdot}03.$ $K = +0{\cdot}10.$

Group.	No.	Mean Long. (°)	Residual Velocities. Observed.	Residual Velocities. Computed.
1	2	345	− 1·2	+ 3·3
2	2	17	+ 4·9	+ 4·9
3	14	38	+ 5·7	+ 2·9
4	11	61	− 0·4	− 1·0
5	13	75	− 0·4	− 3·2
6	8	89	− 6·0	− 0·6
7	8	113	− 6·9	− 4·4
8	6	131	+ 2·7	− 2·2
9	8	156	− 1·7	+ 2·0
10	13	174	+ 4·9	+ 4·1
11	3	205	+ 3·4	+ 0·2

Intensity 7·0 to 7·9.

$\bar{r}A = +6{\cdot}91.$ $K = -0{\cdot}21.$

Group.	No.	Mean Long. (°)	Residual Velocities. Observed.	Residual Velocities. Computed.
1	8	12	+ 4·6	+ 6·7
2	12	38	+ 6·3	+ 3·7
3	11	51	+ 4·8	+ 0·6
4	10	65	− 0·9	− 2·6
5	4	75	− 2·4	− 4·7
6	8	89	−10·6	− 6·7
7	3	110	− 9·7	− 6·7
8	3	139	− 0·3	− 1·7
9	9	160	+ 0·2	+ 3·3
10	10	174	+ 2·7	+ 5·6
11	1	192	+13·0	+ 6·7

Intensity 8·0 to 9·5.

$\bar{r}A = +13{\cdot}72.$ $K = -1{\cdot}17.$

Group.	No.	Mean Long. (°)	Residual Velocities. Observed.	Residual Velocities. Computed.
1	2	358	+ 8·2	+ 9·4
2	2	17	+ 6·1	+11·9
3	6	34	+ 7·5	+ 7·9
4	2	47	+ 6·6	+ 2·4
5	12	70	− 5·4	− 8·0
6	4	97	−15·5	−14·3
7	9	107	−17·7	−14·5
8	4	141	+ 1·2	− 3·2
9	2	169	+ 1·9	+ 5·3

The high negative velocities of the stars in the neighbourhood of the h, χ clusters in Perseus give high negative differences in the second and third magnitude groups—note sub-group 8 in second, and sub-group 6 in third (Table X.). That this is due to the much greater distance of this group than the average of the other groups is obvious when we compare the above magnitude groups with the group of highest intensities, 8·0 to 9·5. It will be noted that the differences above of some − 20 km. have in this case practically disappeared (difference 3·4 in sub-group 6 and 4·9 in 7). This appears to show clearly that the method of grouping by the intensity of the "interstellar" K line arranges the stars much more closely in order of distance than grouping by magnitudes. The average distance of the stars in this intensity group $\bar{r}$A/A is about 1600 parsecs, and, as the residuals in Perseus are still negative, the Perseus group is indicated to be about 2000 parsecs distant, which does not differ materially from the slightly larger estimates from other sources.

It will be noted that the differences between observed and computed are generally smaller in the intensity than in the magnitude groups. This is an indication that the grouping according to intensity arranges the stars into groups of more nearly equal distances than the grouping in magnitudes, in which, therefore, the dispersion in absolute magnitudes must be considerable. This is particularly noteworthy in the group of only 43 stars of intensity 8·0 to 9·5, where the correspondence is remarkably close, indicating that the stars of the group are all at approximately the same distance of about 1600 parsecs. Nobody who compares the observed and computed velocities in this group can fail to be convinced of the reality of a rotation of the galaxy around a distant centre at longitude 325°.

The same is true of all the groups of Table X., which show markedly that, if the distribution of velocities with longitude is not due to a rotation of the galaxy, the cause is something which produces a velocity distribution almost identical with that which would be produced by a rotation of the galaxy. The same thing is shown, though even more convincingly, by a comparison of the residual velocities of the interstellar clouds in Table XI., where the peculiar motions of the clouds are much less than those of the stars and the agreement is correspondingly closer than with the stars. Any large deviations here are probably due to the large dispersion in distance in the groups caused by the uncertainty of the methods employed for defining the distances. It seems to us that no one can compare the observed and computed velocities in Tables X. and XI., and also in Table IV., without being convinced that the motions of the O and early B stars, and also, more particularly, of the interstellar clouds, confirm in a remarkable degree the hypothesis of a rotation of the galaxy.

Turning again to the results of the solutions of the intensity groups (Table IX.) we at once obtain as before from the $\bar{r}$A of groups 3, 4, 5 that the average distances of the stars in these groups are 600, 850, 1600 parsecs; the first two somewhat similar to the magnitude groups, but the last considerably further away than the group of stars fainter than 7·0 magnitude. As indicated above, this shows that intensity estimates of the K line are more critical in differentiating distance than magnitude differences.

Finally, comparing the $\bar{r}$A for stars and clouds in the three major groups 3, 4, 5 of Table IX., the remarkable relation is again found that its value for the stars is almost exactly double that for the clouds. The exact ratios are 2·03, 2·10, 2·01—in the mean 2·05 as compared with the mean of 1·97 for the magnitude groups. This gives a final mean for magnitude and intensity groups of 2·01, or the average distance of the stars is practically exactly twice that of the clouds, or the centre of gravity of the absorbing matter producing the "interstellar" calcium lines is exactly half the distance of the stars in whose spectra these lines appear. When we consider that this relation holds for star groups of distances varying between 600 and 1600 parsecs, there can be no possible doubt that *Eddington's hypothesis of uniform distribution of the interstellar matter is fully confirmed.*

Conclusions.

The conclusions that may be legitimately drawn from this investigation may be briefly summarized.

1. The general solution simultaneously for the solar motion, the K term, and the galactic rotation of the interstellar clouds from the Ca^+ velocities of 261 stars resulted in a solar velocity of 19·9 km. per second, the same as the usual accepted velocity of 20 km. per second, to an apex about 20° away from the normal apex; this discrepancy being probably due to the unsymmetrical and unsuitable distribution of the stars. The K term is zero within its probable error, while the rotational term of + 7·3 km., equivalent to a mean distance of the clouds of over 400 parsecs, is directed to a centre at galactic longitude 335°·1, 10° away from its usual position of 325°.

2. As the stars were poorly distributed for the determination of the solar apex, a second solution assuming the usual solar motion was carried through for the K term and the galactic rotation. The K term is again zero, the rotational term + 7·90±0·79 km. per second, and the galactic centre at longitude 331°·7±5°·7. A comparison of the observed mean velocities of the groups with those computed from the above values showed such remarkable correspondence as strongly to confirm the theory of a rotation of the galaxy.

3. The 235 stars, for which both stellar and interstellar velocities were known, were divided into five magnitude groups, and consequently approximately into five groups at different distances, and again solved for the K term and the galactic rotation. This solution has the interesting and important result that for all except the group of brightest and nearest stars the rotational term for the stars is almost exactly twice that for the clouds, or the stars are at twice the mean distance of the clouds.

4. The intensity of the "interstellar" K line was estimated in the spectra of 226 stars, for which there were both stellar and cloud velocities.

The stars were again arranged into five groups according to the intensity of K, and consequently, on Eddington's theory, according to the distance. A solution for the K term and the galactic rotation again showed for the three groups with the strongest K lines, the remarkable relation that the rotational term for the stars is almost exactly twice that for the clouds.

5. A comparison of the mean observed velocities of both stars and clouds in the longitude divisions of three magnitude and three intensity groups showed such a close agreement with the computed velocities as to leave no reasonable doubt of the reality of differential motions of these high temperature stars and the intervening clouds, corresponding almost exactly with those that would be produced by a rotation of the galactic system around a distant massive centre in the same direction as the centre of the globular cluster system.

6. The relations found in 3 and 4 above, that the rotational term for the stars is almost exactly twice that for the clouds in each of four different magnitude groups and three different intensity groups, can only mean that the average distance of each of seven different groupings of stars is exactly twice that of the average distance of the corresponding groups of clouds. When it is considered that this relation holds for groups of stars of very different distances distributed over a considerable portion of the galactic plane, the only possible conclusion that can be drawn from these relations provides a strong confirmation of Eddington's hypothesis of a uniform distribution of the interstellar material.

The Shortt Clocks at the Royal Observatory, Greenwich, with Special Reference to the Effect of the Variation of Arc. By J. Jackson, M.A., D.Sc., and W. Bowyer.

(*Communicated by the Astronomer Royal.*)

Introduction and Summary.—The performance of the two clocks, Shortt 3 and Shortt 11, up till the end of 1928 has been given in two papers published in the *Monthly Notices* for 1928 March and 1929 January. Shortt 3 continued to follow pretty closely the formula based on the performance in 1927 up to 1929 May, but afterwards it showed rather sudden changes of rate of the order of $0^{s}{\cdot}02$ per day, lasting for about a week. These irregularities took place at intervals up till the end of October, when the clock case was opened. The irregularities in the clock rate were accompanied by irregularities in the arc, and an examination of the residuals indicated that changes in the arc produced the theoretical effect calculated for a simple pendulum. Re-examination of the residuals from the formula in 1928 showed that they could be practically explained by the observed variations in the arc. The principal change in 1928 was a decrease in the semi-amplitude by about 0·1 mm. or 18″ in the middle of July. It then seemed worth while to reconsider the performance of the clock from the time of its