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HARVARD COLLEGE OBSERVATORY

CIRCULAR 314

INVESTIGATIONS OF CEPHEID VARIABLES

II. THE PERIOD–LUMINOSITY RELATION FOR GALACTIC CEPHEIDS

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In the preceding Circular the classification is given for the spectra of seventy Cepheid variables on more than 1250 plates, one-fourth of which I made at Mount Wilson, nearly all the others being found in the Harvard collection. Variability of spectral class is recorded for every one of the seventy stars, and the interdependence of the median spectrum and the length of period is deduced and shown graphically. It is found that for all Cepheids with periods less than one day the average median spectral class is A6. For stars with periods longer than a day the following values are read from the period-spectrum curve:

Median Spectrum	Log Period	Median Spectrum	Log Period
F4	(0.16)	G2	0.79
F6	0.30	G4	1.04
F8	0.43	G6	1.38
G0	0.59	G8	(1.70)

The following discussion considers these new results in their bearing on the problem of the period-luminosity curve for galactic Cepheids and consequently on the use of Cepheids in the measurement of the distances of clusters, star clouds, and extra-galactic nebulae.

1. In a note on Cepheid variation and the period-luminosity relation,[1] it was pointed out several years ago that if we start with the general gravitational relation $P^2 \propto 1/\rho$, the total luminosity of a Cepheid variable,

$$L = \pi r^2 J \qquad (1)$$

can be written, with close approximation,

$$L \doteq k(\mu P^2)^{\frac{2}{3}} \phi(C.I.) \qquad (2)$$

where ρ, P, r, and μ are the mean density, period, radius, and mass, respectively, and the surface brightness, J, is taken as primarily a function of color index. (Equation (1) is of course rigorous; the approximation in (2) arises from writing $J = \phi$ (C. I.).)

It follows that if the differences in median color for typical Cepheids are ignored, that is, if $J = \phi$ (C.I.) = a constant, for periods between 3 and 15 days, the lumin-

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osity will increase throughout that range with the product of the mass and the square of the period; and since the range in the masses is believed to be small in comparison with the wide range in period, the absolute magnitude is given approximately by

$$M = a + b \log P$$

with a and b constants. This theoretical relation is of the same form as the observed relation for the longer period Cepheids in the Magellanic Clouds and globular clusters, and it amounts to a preliminary theoretical period-luminosity curve applicable to *all* Cepheids.[2] It would appear, therefore, that also among the galactic Cepheids, where direct observations are difficult because of insufficient data on parallaxes and absolute magnitudes, the intrinsic luminosity should necessarily increase with the periods as observed in the Magellanic Clouds. But the result obtained ten years ago was admittedly preliminary.

2. The variation in J with P need no longer be neglected, however, and with the data of the preceding paper, giving the relation of the median or mean spectral class to the period, it becomes possible, as shown below, to derive the period-luminosity curve for the galactic Cepheids as a direct consequence of the period-spectrum relation.

A determination of the actual absolute magnitude of a Cepheid from knowledge of its period and spectrum alone seems too uncertain, mainly because of our insufficient information concerning the masses, the darkening at the limb which affects the mean surface brightness, and the ratio of the specific heats, which may enter into a more rigorous form of the relation of period to mean density. The relative intrinsic luminosities, however, can be simply computed for typical Cepheids, since only the ratios of masses, periods, and surface brightness are involved, and the uncertain and undetermined factors cancel out or are of the second order. With the relative luminosities determined for a series of galactic Cepheids, the absolute luminosities follow from a determination of the mean parallax of the nearer stars.

3. Since the surface brightness of a star depends directly on the fourth power of the absolute effective temperature, T, which has been determined by several investigators for the various spectral types, we may write, for two Cepheids of different spectral class,

$$J/J_0 \propto T^4/T_0^4$$

Then, from equation (1) and (2),

$$\frac{L}{L_0} = k\,\frac{\mu^{\frac{2}{3}} P^{\frac{4}{3}} T^4}{\mu_0^{\frac{2}{3}} P_0^{\frac{4}{3}} T_0^4}$$

and we derive

$$M_0 - M = 10 \log \frac{T}{T_0} + \tfrac{10}{3} \log \frac{P}{P_0} + \tfrac{5}{3} \log \frac{\mu}{\mu_0} \qquad (3)$$

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where M is the absolute bolometric magnitude. Equation (3) is a definitive relation connecting the absolute magnitude, temperature, period, and mass of a Cepheid and involves no assumption beyond Stefan's law and the gravitational relation $P^2\rho$ = a constant. Our problem is to solve the equation for M in terms of P.

We could write the first term on the right side of the preceding equation as

$$10 \log \frac{T}{T_0} = 2.5 \log \frac{J}{J_0} = (s_0 - s) \tag{4}$$

where s is the surface brightness expressed in stellar magnitudes. Various kinds of evidence indicate that the difference in surface brightness from one Harvard class to the next is a little more than one visual magnitude. It will be best, however, not to use an assumed constant value of $(s_0 - s)$ but to deal directly with the temperatures of the Cepheids as indicated by their spectra, since the temperature scale is fairly well known over the range of spectrum here involved.

4. Considering equation (3), we note that, since we are not making use of the data on parallaxes (except later to fix a zero point), both M and μ are unknown for any given Cepheid, but P and T can be obtained from observations. The mass-luminosity formula derived by Eddington gives the necessary additional information for a complete solution, but it is preferable in the present case to keep free from the theories underlying his formulae and to use instead only the *observed* mass-luminosity relation for a second approximation after provisional absolute magnitudes are derived from a preliminary solution.

As a first approximation, therefore, we take the masses of Cepheids as equal and (3) becomes

$$\begin{aligned} M_0 - M &= 10 \log T - 10 \log T_0 + \tfrac{10}{3} \log P - \tfrac{10}{3} \log P_0 \\ &= (10 \log T - 37.40) + (\tfrac{10}{3} \log P - 1.97) \end{aligned} \tag{5}$$

where T_0 has been set at 5,500°, corresponding to a Class G0 star, and the corresponding value of $\log P_0$ from the period-spectrum curve is 0.59.

TABLE I

Spectrum	T	Log P	10 Log T−37.40	10/3 Log P−1.97	M_0−M	M
A0	10,000°	−0.56:	+2.6	−3.8:	−1.2:	−0.7:
A5	8,500	−0.31	+1.9	−3.0	−1.1	−0.8
F0	7,400	−0.06	+1.3	−2.2	−0.9	−1.0
F5	6,500	+0.23	+0.7	−1.2	−0.5	−1.4
F7.5	6,000	+0.40	+0.36	−0.64	−0.3	−1.6
G0	5,500	+0.59	0.0	0.0	0.0	(−1.9)
G2.5	5,050	+0.85	−0.4	+0.9	+0.5	−2.4
G5	4,600	+1.22	−0.8	+2.1	+1.3	−3.2
G7.5	4,300	+1.62	−1.1	+3.4	+2.3	−4.2

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5. For the computation (Table I) of the relative period-luminosity curve (masses assumed equal) from equation (5), I have used the scale of effective temperatures shown in the first two columns of the table. The adopted temperatures are those deduced from colors, spectrophotometric data, and spectrum analysis on ionization principles. The logarithm of the period is derived from the data given in the preceding Circular, where the period-spectrum curve is shown on a small scale.

A plot of the computed $M_0 - M$, in the sixth column, against the logarithm of the period gives the preliminary period-luminosity curve for galactic Cepheids. To fix the zero point for an absolute period-luminosity curve I have assumed that for a Class G0 Cepheid the absolute bolometric magnitude is −1.9 and have then derived the values of the magnitudes in the last column of Table I.

The range of three and a half magnitudes in intrinsic bolometric luminosity, shown by the foregoing computation, indicates that the mass factor is not negligible for these variables. In Table II the relevant correction to the magnitude is computed. Corresponding to the provisional absolute magnitude of the first column, taken from Table I, the logarithm of the mass is read from the observed mass-luminosity relation as compiled by Eddington.[3] The computed correction, $\frac{3}{5} \log \frac{\mu}{\mu_0}$, gives the revised values of $M_0 - M$ in the fourth column of Table II, and the resulting absolute bolometric magnitudes of the fifth column.

TABLE II

M	Log μ	5/3 Log $\frac{\mu}{\mu_0}$	Revised $M_0 - M$	Revised M	Revised Log μ	Revised 5/3 Log $\frac{\mu}{\mu_0}$	Final $M_0 - M$	Final M
−0.7:	0.65	−0.4	−1.6:	−0.3:	0.59	−0.4	−1.6:	−0.3:
−0.8	0.67	−0.3	−1.4	−0.5	0.62	−0.4	−1.5	−0.4
−1.0	0.70	−0.3	−1.2	−0.7	0.65	−0.4	−1.3	−0.6
−1.4	0.76	−0.2	−0.7	−1.2	0.73	−0.2	−0.7	−1.2
−1.6	0.80	−0.1	−0.4	−1.5	0.78	−0.1	−0.4	−1.5
(−1.9)	0.86	0.0	0.0	(−1.9)	0.86	0.0	0.0	(−1.9)
−2.4	0.94	+0.1	+0.6	−2.5	0.96	+0.2	+0.7	−2.6
−3.2	1.13	+0.5	+1.8	−3.7	1.25	+0.7	+2.0	−3.9
−4.2	1.42	+0.9	+3.2	−5.1	1.77:	+1.5:	+3.8:	−5.7:

These improved values of the absolute magnitudes are used for a new determination of the mass factor and lead (through Columns 6, 7, 8, and 9, of Table II) to new values of the absolute magnitudes, which are considered final, since a third approximation would not introduce sensible alterations. It is to be noted that the mass-luminosity relation as presented by Eddington is probably subject to considerable modification as more data on masses become available; but a considerable change can be allowed without affecting either the legitimacy of the preceding com-

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putation or the final results. Nor will reasonable (and probable) alterations of the assumed zero point of the period-luminosity curve appreciably disturb the course of the computed curve.

In Table III the final bolometric magnitudes of the last column of Table II are changed to visual and photographic values (the latter with zero point independently adjusted) and compared with observation. The observed visual period-luminosity curve is taken without change from Mount Wilson Contribution 151, and the observed photographic values are from Harvard Circular 280. The former involves mainly the Cepheids in the Magellanic Clouds and in globular clusters; the latter is altogether based on the Cepheids in the Small Magellanic Cloud. The agreement of the values computed by way of the spectrum-period curve of galactic Cepheids with the observed values is surprisingly good.

TABLE III

Spectrum	Log P	Bolometric Magnitude	Computed Vis. M	Observed Vis. M	$O-C_1$	Computed Pg. M	Observed Pg. M	$O-C_2$
A0	−0.56	−0.3:	−0.1:	−0.3	−0.2:	−0.3:	−0.2:	+0.1:
A5	−0.31	−0.4	−0.3	−0.3	0.0	−0.3	−0.3:	0.0
F0	−0.06	−0.6	−0.6	−0.6	0.0	−0.4	−0.4	0.0
F5	+0.23	−1.2	−1.2	−1.0	+0.2	−0.8	−0.4	−0.1
F7.5	+0.40	−1.5	−1.5	−1.4	+0.1	−1.0	−1.1	−0.1
G0	+0.59	(−1.9)	(−1.8)	−1.8	(0.0)	(−1.2)	−1.4	(−0.2)
G2.5	+0.85	−2.6	−2.4	−2.6	−0.2	−1.7	−1.8	−0.1
G5	+1.22	−3.9	−3.6	−3.9	−0.3	−2.8	−2.6	+0.2
G7.5	+1.62	−5.7:	−5.2:	−5.4	−0.2:	−4.3:	−3.5:	+0.8:

It should be noted incidentally that the computed values for spectral classes A0 and G7.5 involve extrapolations and are of low weight.

6. The systematically negative residuals, $O-C_1$, in Table III for the stars of longer period would suggest that the observed values may be too bright. This is now known to be the case. In reducing from photographic to visual magnitudes for the original construction of the period-luminosity curve in Mount Wilson Contribution 151, the color correction used for the longer period Cepheids was too great. This may be verified by considering the spectrum-period curve in the preceding paper. As a consequence of the erroneous color correction, the visual period-luminosity curve gives the absolute magnitudes too bright for periods greater than ten days, and the application of a suitable correction would therefore reduce the corresponding negative residuals, $O-C_1$, in Table III. In fact, this necessary correction somewhat overdoes it, changing systematically the signs of the residuals, and it then appears that a still more satisfactory agreement of both visual and photographic observation with theory will be obtained when we introduce Edding-

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ton's formulation of the pulsation hypothesis and substitute $P' = P\,(3\gamma-4)^{-\frac{1}{2}}$ for P in the foregoing discussion.

In a later note the visual period-luminosity curve will be completely revised and compared with the theoretical curve for galactic Cepheids computed with account taken of γ, the ratio of specific heats.

7. In summary, it is found that the relative period-luminosity curve computed from spectroscopic observations of galactic Cepheids is in close agreement with the visual and photographic period-luminosity curves observed in clusters and star clouds. This conclusion is reached independently of data on proper motions, radial velocities, or trigonometric or spectroscopic parallaxes; it is without assumption as to the theory of Cepheid variation. Good accordance is attained without considering the part played by the ratio of specific heats, γ, in the gravitational relation between period and mean density. But by assuming the relation $P^2\rho(3\gamma-4)$ = constant, proposed by Eddington, and revising the visual period-luminosity curve in accordance with the new data on spectra, a better accordance is obtained with both visual and photographic observations.

A closer accordance of the generalized theory used above with the observations cannot legitimately be sought until more refined work is done with the spectrum-period relation and the observed period-luminosity curves.

Finally, attention should be called again to the possibility of using the periods of galactic Cepheids in the search for spectroscopic criteria of absolute magnitudes among the stars of high luminosity. Also, since the spectra of Cepheids change throughout their periods, it may be possible to find criteria of mass; for instance, Class G0 may represent the minimum luminosity for one Cepheid and the maximum luminosity for another of longer period and greater mass, and absolute magnitude −2.5 may represent the minimum brightness for one variable and the maximum brightness for another of earlier median spectral type and smaller mass.

[1] Mt. W. Contr. 154, 4, 1917.

[2] Subsequently Eddington, Seares, and others have calculated period-luminosity curves on the basis of the pulsation theory of Cepheids; M.N., **79**, 2, 1918; Mt. W. Contr. 226, 40 ff, 1921. Cf. also Shapley, Mt. W. Contr. 190, 7, 1920.

[3] The Internal Constitution of the Stars, 153, 1926.

May 1, 1927

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