

1925PDAO.....3..189H

April 2, and 1924, April 7, were given half weight, as in the former a very narrow spectrum was secured and in the latter the focus was not of the best. Further, the plate of 1924, November 22, has not been used at all in the determination, owing to the fact that a few minutes' exposure of an A-type star was accidentally made on the same plate.

The spectrum contains numerous well-defined lines due principally to iron, titanium, manganese, chromium and calcium. On one plate, number 8541, there were measured 153 lines between $\lambda 3941$ and $\lambda 4600$. By selecting a plate of different exposure other lines could be measured and the total increased. In the list given in *Harvard Annals*, Volume 28, page 79, which lists the lines in stars of division c, there are 183 recorded between the same limits. The great majority, though not all, of the 153 measured in this star are found in that particular list.

Through the kindness of the Lick observers the data of their seven observations were furnished, and for convenience of reference these are tabulated here. All their plates were made with the three-prism Mills spectrograph and should be very reliable. The plates have assisted materially in fixing the period accurately, but otherwise have not been made use of in the determination. A systematic difference of -3.67 km. has been added to each velocity before the residuals as shown were obtained. Our own plates were measured on the spectrocomparator against the standard plate of Procyon, number 3375, and the velocities deduced are given in the table following. The tables list the phases of the observations as based upon the final elements, and the residuals from the final curve are given in the last column. The determination does not appear wholly satisfactory, as the probable error of a plate is ± 1.89 km. per sec., larger than one might expect from single-prism plates when the character of the spectrum is considered. In the final curve the way the observations fall above and below the curve around the γ -point is suggestive of the presence of the spectrum of the second component. The last observation was made on a fine-grained plate to see if the fainter spectrum could be recorded, but no certain evidence was obtained.

LICK OBSERVATIONS OF 45 AURIGAE

Plate Number	Date	Julian Date	Phase	Velocity	Lines	O-C
9503.....	1917 Nov. 1	2,421,534.036	6 291	+24.35	30	-2.14
10507.....	1919 Jan. 25	1,984.866	2.030	+ 3.60	32	-1.85
10901.....	Sept. 25	2,227.948	4.564	-19.43	31	+1.75
10934.....	Oct. 11	2,243.942	1.054	+31.86	30	+1.94
10992.....	Nov. 10	2,273.047	4.154	-27.64	31	-0.20
11030.....	Nov. 24	2,287.052	5.156	- 6.65	30	-0.58
11082.....	Dec. 29	2,422,322.935	2.032	+ 6.53	27	+1.19

VICTORIA OBSERVATIONS OF 45 AURIGAE

Plate Number	Date	Julian Date	Phase	Velocity	Wt.	O-C
8348.....	1922 Nov. 11	2,423,370.963	3.350	-31.2	1	-1.5
8495.....	1923 Jan. 10	3,430.865	4.741	-19.0	1	+2.0
8536.....	" 28	3,448.782	3.154	-29.6	1	-2.6
8541.....	" 31	3,451.754	6.126	+19.9	1	+0.6
8578.....	Feb. 24	3,475.780	4.147	-27.3	1	+3.7
8603.....	" 26	3,477.764	6.131	+23.0	1	+3.6
8637.....	Mar. 2	3,481.770	3.635	-32.4	1	0.0
8677.....	" 19	3,498.695	1.056	+24.4	1	-1.8
8722.....	" 26	3,505.728	1.588	+12.8	1	-1.4
8723.....	" 26	3,505.745	1.605	+11.9	1	-1.9
8762.....	" 30	3,509.771	5.631	+ 5.3	1	+0.3
8775.....	April 2	3,512.653	2.012	+ 6.6	$\frac{1}{2}$	+4.4
8776.....	" 2	3,512.666	2.025	+ 6.2	$\frac{1}{2}$	+4.2
8808.....	" 9	3,519.719	2.576	-18.7	1	-5.0
8809.....	" 9	3,519.732	2.589	-16.6	1	-2.3
9706.....	1924 Mar. 3	3,848.702	6.494	+28.8	1	+2.2
9721.....	" 10	3,855.721	.511	+32.9	1	+2.1
9742.....	" 14	3,859.712	4.502	-26.3	1	-0.3
9828.....	April 7	3,883.681	2.466	- 5.5	$\frac{1}{2}$	+5.2
9829.....	" 7	3,883.708	2.493	- 3.4	$\frac{1}{2}$	+8.1
10859.....	Oct. 10	4,069.020	5.768	+ 6.4	1	-2.6
10942.....	Nov. 22	4,112.908	4.147	-24.5		
11101.....	Dec. 23	4,143.918	2.651	-14.2	1	+1.5
11175.....	1925 Jan. 23	4,174.794	1.020	+25.3	1	-1.4
11214.....	Feb. 14	4,196.730	3.452	-33.5	1	-2.3
11226.....	" 24	4,205.734	5.955	+10.0	1	-4.4
11260.....	Mar. 6	4,216.730	3.948	-33.0	1	-0.5
11320.....	" 16	2,424,226.702	.918	+28.0	1	-0.2

From the first few plates it was seen that the period was in the neighbourhood of 6.5 days. Our later observations defined this more closely and with the aid of the Lick results a definite value of 6.5013 days was established. Preliminary elements were then adopted as follows.

PRELIMINARY ELEMENTS

$$\begin{aligned}
 P &= 6.5013 \text{ days} \\
 e &= .04 \\
 \omega &= 354^\circ \\
 K &= 32.3 \text{ km.} \\
 \gamma &= -0.7 \text{ km.} \\
 T &= \text{J.D. } 2,423,634.597
 \end{aligned}$$

The observations were grouped into 16 normal places and observation equations were built up according to the usual Lehman-Filhés form, connecting the residuals with the elements K , e , ω and T . In these equations

$$\begin{aligned}
 x &= \delta_1^m \gamma \\
 y &= \delta_2^m K \\
 z &= K \cdot \delta_1^m e \\
 u &= K \cdot \delta \omega \\
 v &= 31.292 \delta T
 \end{aligned}$$

NORMAL PLACES

	Mean Phase	Mean Velocity	Wt.	O-C			Mean Phase	Mean Velocity	Wt.	O-C	
				Prel.	Final					Prel.	Final
1	1.038	+24.85	2	-3.7	-1.66	9	4.502	-26.30	1	-0.8	-0.28
2	1.597	+12.40	2	-3.0	-1.92	10	4.741	-19.00	1	+1.6	+2.01
3	2.019	+ 6.40	1	+3.8	+4.47	11	5.631	+ 5.30	1	+0.5	+0.34
4	2.479	- 4.20	1	+7.0	+6.97	12	5.861	+ 8.20	2	-3.8	-3.64
5	2.605	-16.50	3	-1.8	-1.81	13	6.129	+21.40	2	+1.7	+2.29
6	3.252	-30.40	2	-2.5	-1.74	14	6.494	+28.80	1	+0.9	+2.23
7	3.543	-32.95	2	-2.0	-1.09	15	.511	+32.90	1	0.0	+2.14
8	4.047	-30.15	2	+0.8	+1.74	16	.918	+28.00	1	-2.3	-0.19

OBSERVATION EQUATIONS

1.....	1.000x	+ .904y	+ .389z	+ .499u	+ .537v	+3.7 = 0
2.....	1.000	+ .497	- .679	+ .885	+ .914	+3.0
3.....	1.000	+ .101	-1.002	+ .994	+ .994	-3.8
4.....	1.000	- .326	- .644	+ .926	+ .897	-7.0
5.....	1.000	- .433	- .450	+ .877	+ .843	+1.8
6.....	1.000	- .843	+ .648	+ .466	+ .435	+2.5
7.....	1.000	- .934	+ .939	+ .223	+ .209	+2.0
8.....	1.000	- .936	+ .856	- .224	- .203	-0.8
9.....	1.000	- .767	+ .211	- .595	- .556	+0.8
10.....	1.000	- .617	- .227	- .758	- .719	-1.6
11.....	1.000	+ .170	- .946	- .995	-1.010	-0.5
12.....	1.000	+ .394	- .695	- .939	- .970	+3.8
13.....	1.000	+ .631	- .219	- .811	- .852	-1.7
14.....	1.000	+ .885	+ .511	- .538	- .574	-0.9
15.....	1.000	+1.040	+ .998	- .025	- .023	0.0
16.....	1.000	+ .959	+ .602	+ .390	+ .422	+2.3

From these were formed the normal equations

$$\begin{aligned}
 25.000x - .428y + .631z + 2.228u + 2.100v + 19.700 &= 0 \\
 12.613 - 2.668 - 1.479 - 1.419 + 5.328 & \\
 10.991 - .367 - .256 + 7.712 & \\
 12.287 + 12.338 + 5.291 & \\
 12.405 + 5.513 &
 \end{aligned}$$

from which there resulted the corrections as follows:

$$\begin{aligned}
 \delta\gamma &= - 0.82 \text{ km.} \\
 \delta K &= - 0.56 \text{ km.} \\
 \delta e &= - 0.021 \\
 \delta\omega &= -23^\circ.40 \\
 \delta T &= - 0.431 \text{ days}
 \end{aligned}$$

The corrections for ω and T are large, but uncertainty in their determination is always present when the eccentricity is as low as in this case, resulting as it does in almost identical coefficients in the observation and normal equations for these two elements. Had either element been considered fixed, as is often done, the correction to the other would have been vanishingly small.

The sum of the squares of the residuals for the normal places was reduced from 184.5 to 156.6, or about 15 per cent, and satisfactory agreement existed between equation and ephemeris residuals.

FINAL ELEMENTS

$$\begin{aligned}
 P &= 6.5013 \text{ days} \\
 e &= .019 \pm .026 \\
 \omega &= 330^\circ.60 \pm 40^\circ.85 \\
 \gamma &= -1.52 \text{ km.} \pm 0.57 \text{ km.} \\
 K &= 31.74 \text{ km.} \pm 1.08 \text{ km.} \\
 T &= \text{J.D. } 2,423,634.166 \pm 0.741 \text{ day} \\
 a \sin i &= 2,837,000 \text{ km.}
 \end{aligned}$$

The graph shown (Fig. 1) represents the final elements with the observations as grouped.

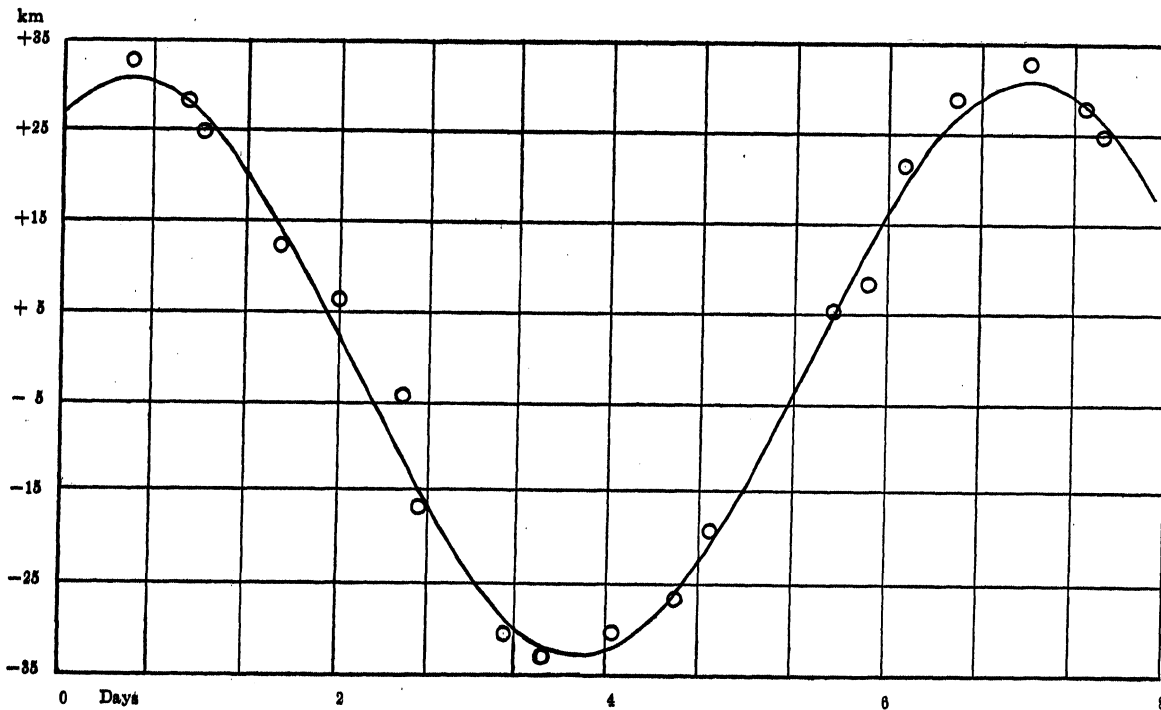


FIG. 1—Radial Velocity Curve of 45 Aurigae

In our absolute magnitude work we found from the ratios of six pairs of lines an absolute magnitude of +1.5 for this star corresponding to a prallax of $0''.016$.

In the foregoing the dates are given in Greenwich Mean Time as used prior to 1925.

Dominion Astrophysical Observatory,
Victoria, B.C.

March, 1925.

19 LEO MINORIS

Abstract.—Thirty-three spectrograms of this star obtained in 1923, 1924 and 1925 with the single-prism spectrograph serve to determine the orbital elements. Its spectrum is F4 and the plates were measured on the spectrocomparator against Procyon as a standard. The probable error of a plate is ± 1.2 km./sec. The period is 9.283 days and the eccentricity small.

The star 19 Leo Minoris, Boss 2665 ($1900 \omega = 9^h 51^m \cdot 6$, $\delta = +41^\circ 32'$), visual magnitude 5.19 and type F4, was noted to be a spectroscopic binary from the first two plates made in 1923. A total of 33 plates have been obtained with the single-prism spectrograph, and these serve as the basis for the determination of the orbital elements. The lines of the F4 spectrum are well defined and the plates were measured against Procyon on the spectrocomparator. As the star is bright the plates are all well exposed and, while two are slightly inferior to the rest, they have all been weighted equally in the solution. The probable error of a plate on the basis of the final elements is ± 1.2 km./sec. and is quite satisfactory.

OBSERVATIONS OF 19 LEO MINORIS

Plate Number	Date	Julian Date	Phase	Velocity	Residual O-C
8680.....	1923 Mar. 19	2,423,498.773	.178	+ 6.2	+0.4
8702.....	" 24	3,503.779	5.184	-25.3	-0.2
8726.....	" 26	3,505.796	7.201	-11.7	-0.2
8779.....	April 2	3,512.717	4.839	-26.5	-1.3
8811.....	" 9	3,519.763	2.602	-11.7	+0.1
8866.....	" 23	3,533.725	7.281	- 9.9	+0.6
8943.....	May 7	3,547.694	2.684	-14.1	-1.4
8977.....	" 22	3,562.716	8.423	- 0.9	-1.3
9709.....	1924 Mar. 3	3,848.843	6.777	-15.3	+0.4
9725.....	" 10	3,855.839	4.490	-26.6	-1.8
9744.....	" 14	3,859.780	8.431	+ 0.1	-0.4
9767.....	" 17	3,862.793	2.161	- 7.6	-0.1
9785.....	" 22	3,867.740	7.108	-11.7	+0.7
9799.....	" 24	3,869.780	9.148	+ 8.0	+3.3
9813.....	" 28	3,873.795	3.880	-22.8	-0.4
9831.....	April 8	3,884.765	5.567	-24.4	-0.4
9838.....	" 14	3,890.737	2.256	- 6.6	+1.8
9850.....	" 21	3,897.758	9.277	+ 5.6	+0.5
9935.....	May 9	3,915.693	8.646	- 1.2	-3.5
9978.....	" 19	3,925.705	.092	+ 3.4	-2.1
10008.....	" 23	3,929.700	4.087	-26.0	-2.5
10055.....	June 2	3,939.705	4.809	-22.9	+2.4
10108.....	" 16	3,953.724	.262	+ 8.0	+2.5
10164.....	" 26	3,963.722	.977	- 0.8	-3.6
11105.....	Dec. 24	4,144.008	4.886	-27.3	-2.0
11141.....	1925 Jan. 9	4,160.955	3.267	-14.7	+3.5
11185.....	" 24	4,174.964	7.993	- 2.6	+0.6
11263.....	Mar. 6	4,216.775	3.389	-17.7	+1.4
11310.....	" 14	4,224.840	2.171	- 8.6	-1.0
11323.....	" 16	4,226.799	4.130	-23.1	+0.6
11346.....	" 22	4,232.912	.960	+ 1.7	-1.3
11388.....	" 30	4,240.851	8.899	+ 4.6	+0.9
11415.....	April 6	2,424,247.661	6.426	-16.7	+2.0

The period was quite early seen to be about 9.3 days, and a value of 9.283 was established from the first two years' observations. It has not been necessary to change this for the 1925 series. The preliminary elements adopted from graphical methods follow.

PRELIMINARY ELEMENTS

$$\begin{aligned}
 P &= 9.283 \text{ days} \\
 e &= .10 \\
 \omega &= 345^\circ \\
 \gamma &= -10.75 \text{ km.} \\
 K &= 16.0 \text{ km.} \\
 T &= \text{J.D. } 2,423,498.500
 \end{aligned}$$

The observations were grouped into 17 normal places and observation equations built up in the usual way connecting the elements with the residuals for the normal places.

NORMAL PLACES

	Mean Phase		Mean Velocity	Wt.	Residual O-C	
	Prel.	Final			Prel.	Final
1.....	8.994	8.899	+ 4.6	1	-0.2	+0.9
2.....	.079	9.267	+ 5.7	3	-0.8	+0.7
3.....	.315	.220	+ 7.1	2	+0.3	+1.9
4.....	1.064	.969	+ 0.4	2	-3.6	-2.5
5.....	2.291	2.196	- 7.6	3	+0.3	+0.3
6.....	2.738	2.643	-12.9	2	-0.4	-0.4
7.....	3.423	3.328	-16.2	2	+2.3	+2.5
8.....	4.127	4.032	-24.0	3	-1.2	-0.8
9.....	4.585	4.490	-26.6	1	-2.1	-1.8
10.....	4.940	4.845	-25.6	3	-0.5	-0.3
11.....	5.279	5.184	-25.3	1	-0.2	-0.2
12.....	5.662	5.567	-24.4	1	0.0	-0.4
13.....	6.521	6.426	-16.7	1	+3.3	+2.0
14.....	7.038	6.943	-13.5	2	+2.1	+0.5
15.....	7.336	7.241	-10.8	2	+1.7	+0.2
16.....	8.088	7.993	- 2.6	1	+1.2	+0.6
17.....	8.595	8.500	- 0.7	3	-2.3	-1.9

In the observation equations the following substitutions have been made in the Lehmann-Filhés coefficients.

$$\begin{aligned}
 x &= \delta\gamma \\
 y &= \delta K \\
 z &= K \cdot \delta e \\
 u &= K \cdot \delta\omega \\
 v &= 10.994 \cdot \delta T
 \end{aligned}$$

OBSERVATION EQUATIONS OF 19 LEO MINORIS

								Wt.
1.....	1.000x	+ .974y	+ .725z	- .506u	- .578v	+ 0.2	= 0	1
2.....	1.000	+1.078	+1.239	- .220	- .235	+ 0.8		3
3.....	1.000	+1.097	+ .966	- .026	000	- 0.3		2
4.....	1.000	+ .920	+ .064	+ .542	+ .644	+ 3.6		2
5.....	1.000	+ .181	- .998	+ .970	+ .961	- 0.3		3
6.....	1.000	- .107	- .759	+ .953	+ .893	+ 0.4		2
7.....	1.000	- .482	- .043	+ .789	+ .694	- 2.3		2
8.....	1.000	- .756	+ .681	+ .496	+ .427	+ 1.2		3
9.....	1.000	- .860	+ .949	+ .263	+ .234	+ 2.1		1
10.....	1.000	- .899	+ .996	+ .068	+ .076	+ 0.5		3
11.....	1.000	- .899	+ .903	- .120	- .077	+ 0.2		1
12.....	1.000	- .854	+ .640	- .335	- .259	0.0		1
13.....	1.000	- .302	- .853	- .943	- .890	- 2.1		2
14.....	1.000	- .109	-1.013	-1.004	- .989	- 1.7		2
15.....	1.000	+ .434	- .642	- .968	-1.052	- 1.2		1
16.....	1.000	+ .773	+ .132	- .763	- .866	+ 2.3		3
17.....	1.000	- .580	- .321	- .762	- .670	- 3.8		1

From these there resulted the following normal equations:—

$$\begin{array}{r}
 33.000x + 1.380y + 5.128z - .153u - .609v + 6.700 = 0 \\
 18.061 + 1.154 - 2.799 - 2.882 + 13.026 \\
 21.716 - .234 - .349 + 19.376 \\
 14.878 + 14.723 + 7.752 \\
 14.696 + 7.340
 \end{array}$$

which gave corrections as follows:—

$$\begin{array}{l}
 \delta\gamma = - .03 \text{ km.} \\
 \delta K = - .76 \text{ km.} \\
 \delta e = - .052 \\
 \delta\omega = + 6^\circ.09 \\
 \delta T = + .095 \text{ days}
 \end{array}$$

The sum of the squares of the residuals for the normal places was reduced from 91.2 to 56.6 and satisfactory agreement existed between equation and ephemeris residuals, the numerical average being 0.04 km.

The final elements, then, with their probable errors are the following:

FINAL ELEMENTS

$$\begin{array}{l}
 P = 9.283 \text{ days} \\
 e = .048 \pm .020 \\
 \omega = 351^\circ.09 \pm 14^\circ.95 \\
 \gamma = -10.78 \pm 0.26 \text{ km.} \\
 K = 15.24 \pm 0.35 \text{ km.} \\
 T = \text{J.D. } 2,423,498.595 \pm 0.383 \text{ day} \\
 a \sin i = 1,943,130 \text{ km.}
 \end{array}$$

The graph (Fig. 2) represents the final elements and the grouped observations are shown.

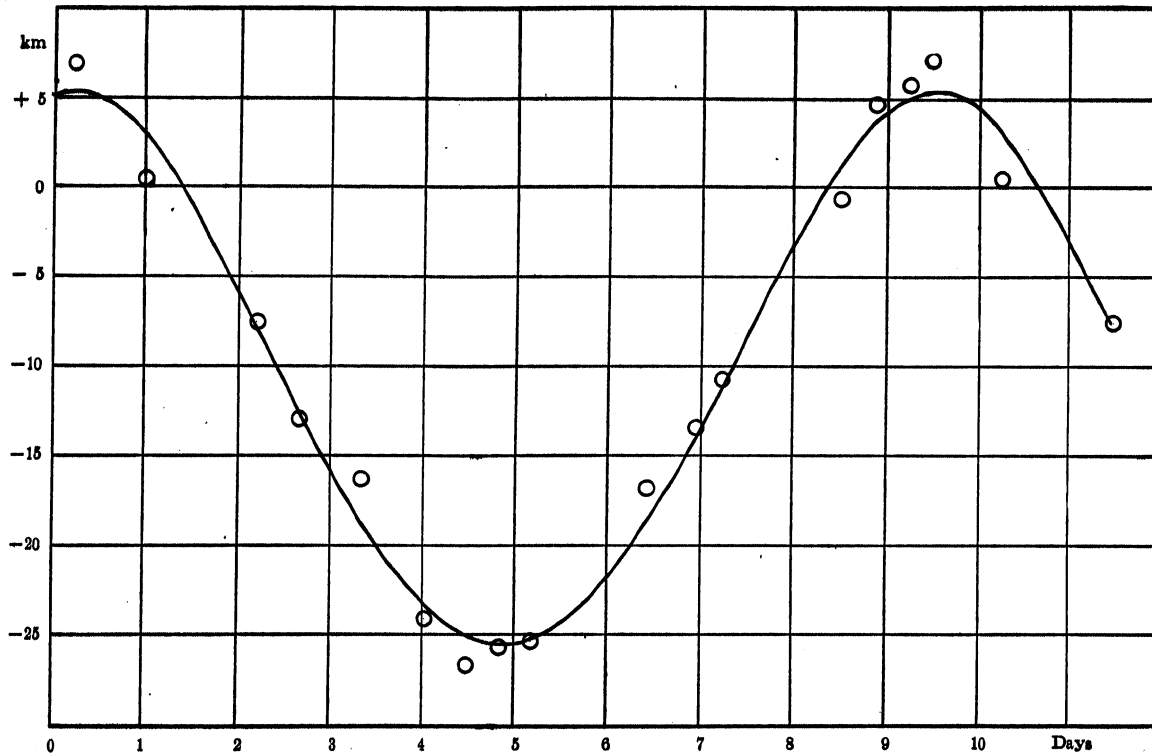


FIG. 2.—Radial Velocity Curve of 19 Leo Minoris

All times are expressed in Greenwich Mean Time as used prior to 1925.

Dominion Astrophysical Observatory,
Victoria, B.C.

April 10, 1925.

105 HERCULIS

Abstract.—Twenty-five spectrograms of this K-type star were obtained with the single-prism spectrograph since April, 1922, and upon these a solution of the orbit is based. The period adopted is 478 days, but this is uncertain to the extent of a few days, as the observations cover only two cycles. The eccentricity is .398 with semi-amplitude 16.07 km./sec.

The star 105 Herculis (1900 $\alpha = 18^{\text{h}} 15^{\text{m}}.1$, $\delta = +24^{\circ} 24'$) whose visual magnitude is 5.49 and type K5, was first observed here on April 17, 1922. A total of 25 plates has been secured with the single-prism spectrograph and these serve as the basis for the determination of the orbit of this spectroscopic binary. In our parallax work we gave the type as K2, but the best exposed plate, made probably since that type was assigned, shows it to be closer to K5 which Harvard gives for the star.

The plates as a whole are much inferior to our average run and the writer accepts the responsibility for such being the case. The star seems unusually red and it so happened that on many of the nights when the star was observed it was more or less hazy, resulting in a weakened spectrum in the violet. The plates were measured on the spectrocomparator against Arcturus as a standard, and the velocities deduced with the weights assigned are given in the table of observations. The phases are based on the final value of periastron passage with the period 478 days and the residuals are scaled from the curve shown which represents the final elements. The elements are not to be considered as definite for, owing to the fact that only two cycles of the period are covered, the period must be uncertain to the extent of a few days. Observations will be taken here in future years to improve the value of the period and by means of which it is hoped to lessen the probable errors of the elements. These are rather high due to the probable error of a plate being ± 2.0 km. per second and also to the small range in velocity variation.

OBSERVATIONS OF 105 HERCULIS

Plate Number	Date	Julian Date	Phase	Velocity	Wt.	O-C
7470.....	1922 April 17	2,423,162.001	99.35	- 2.7	3	-0.7
7622.....	May 28	3,203.867	141.22	- 7.8	3	-5.0
8254.....	Oct. 27	3,355.650	293.00	-21.2	2	-7.0
8284.....	Nov. 2	3,361.625	298.98	-11.4	2	+2.7
8308.....	" 8	3,367.572	304.92	-11.4	3	+3.4
8709.....	1923 Mar. 25	3,504.007	441.36	-34.3	3	-0.3
8875.....	April 23	3,533.971	471.32	-26.6	3	+3.5
8950.....	May 7	3,547.906	7.26	-21.8	2	+2.3
8971.....	" 19	3,559.942	19.29	-19.5	3	-0.6
9026.....	June 6	3,577.891	37.24	-11.6	2	-1.0
9084.....	" 25	3,596.846	56.20	- 7.3	3	-1.6
9190.....	July 20	3,621.827	81.18	- 4.3	3	-1.6
9304.....	Aug. 17	3,649.742	109.09	- 0.7	3	+1.3
9493.....	Oct. 8	3,701.613	160.96	- 3.4	3	+0.2
9941.....	1924 May 9	3,915.937	375.29	-22.0	3	+2.1
10065.....	June 2	3,939.950	399.30	-28.3	3	-0.2
10198.....	" 30	3,967.895	427.24	-38.8	1	-6.0
10242.....	July 7	3,974.780	434.13	-34.1	3	-0.5
10534.....	Aug. 25	4,023.699	5.05	-21.6	3	+3.6
10634.....	Sept. 8	4,037.729	19.08	-23.7	3	-5.2
10906.....	Oct. 27	4,086.592	67.94	+ 0.1	3	+4.0
10908.....	" 31	4,090.587	71.94	- 2.8	3	+0.6
10944.....	Nov. 24	4,114.558	95.91	+ 3.4	2	+5.4
11430.....	1925 April 6	4,247.978	229.33	- 8.2	3	-0.2
11503.....	" 13	2,424,254.981	236.33	- 8.6	2	-0.3

NORMAL PLACES

—	Mean Phase Final	Mean Velocity	Wt.	O-C	
				Prel.	Final
1.....	102.14	-0.42	.8	+2.40	+1.62
2.....	151.09	5.60	.6	-2.18	-2.50
3.....	232.13	8.36	.5	- .30	- .10
4.....	299.82	14.20	.7	- .17	+ .15
5.....	375.29	22.00	.3	+1.94	+2.12
6.....	399.30	28.30	.3	- .35	- .22
7.....	436.24	34.86	.7	-1.72	-1.18
8.....	471.32	26.60	.3	+2.09	+3.47
9.....	13.16	21.64	1.1	-1.26	- .57
10.....	37.24	11.60	.2	- .43	-1.01
11.....	56.20	7.30	.3	- .61	-1.59
12.....	73.69	-2.67	.9	+1.69	+ .67

PRELIMINARY ELEMENTS

$$\begin{aligned}
 P &= 478 \text{ days} & \gamma &= -14.30 \text{ km.} \\
 e &= .38 & K &= 15.35 \text{ km.} \\
 \omega &= 230^\circ & T &= \text{J.D. } 2,423,536.09
 \end{aligned}$$

From graphical methods the foregoing preliminary elements were adopted and then a set of observation equations according to the form of Lehmann-Filhés was built up connecting the five elements γ , K , e , ω and T with the residuals for the grouped observations. In these observation equations, weighted as above, the following substitutions were made for the sake of homogeneity.

$$\begin{aligned}
 x &= \delta\gamma & u &= -K.\delta\omega \\
 y &= \delta K & v &= .25495 \delta T \\
 z &= K.\delta e
 \end{aligned}$$

OBSERVATION EQUATIONS FOR 105 HERCULIS

1.....	1.000x	+ .748y	- .420z	- .415w	- .080v	-2.40	= 0
2.....	1.000	+ .709	- .961	+ .013	+ .139	+2.18	
3.....	1.000	+ .407	- .659	+ .470	+ .292	+ .30	
4.....	1.000	+ .018	+ .141	+ .676	+ .414	+ .17	
5.....	1.000	- .628	+1.106	+ .634	+ .628	-1.94	
6.....	1.000	- .889	+1.060	+ .475	+ .668	+ .35	
7.....	1.000	-1.227	- .232	- .105	+ .260	+1.72	
8.....	1.000	- .937	- .778	-1.010	-1.372	-2.09	
9.....	1.000	- .396	+ .750	-1.277	-1.735	+1.26	
10.....	1.000	+ .204	+1.420	-1.183	-1.185	+ .43	
11.....	1.000	+ .496	+ .940	- .962	- .693	+ .61	
12.....	1.000	+ .648	+ .340	- .740	- .376	-1.69	

These gave the following normal equations:—

$$\begin{aligned}
 6.700x &- .018y + .806z - 2.258u - 2.078v - .109 = 0 \\
 &3.136 &- .795 &- .074 &+ .326 &- 2.488 \\
 &&3.237 &- 1.229 &- 1.485 &- .043 \\
 &&&3.914 &+ 4.071 &+ .229 \\
 &&&&4.906 &- .755
 \end{aligned}$$

which resulted in the following corrections to the elements:—

$$\begin{aligned}\delta\gamma &= -\cdot06 \text{ km.} \\ \delta K &= +\cdot72 \text{ km.} \\ \delta e &= +\cdot018 \\ \delta\omega &= +4^\circ\cdot47 \\ \delta T &= +4\cdot56 \text{ days}\end{aligned}$$

The sum of the squares of the residuals for the normal places was reduced a small amount from 16.54 to 13.53 and good agreement existed between equation and ephemeris residuals, none being greater than .06.

The following, then, are the elements as determined for the present.

FINAL ELEMENTS

$$\begin{aligned}P &= 478 \text{ days} \\ e &= \cdot398 \pm \cdot038 \\ \omega &= 234^\circ\cdot47 \pm 5^\circ\cdot11 \\ \gamma &= -14\cdot36 \text{ km.} \pm 0\cdot41 \text{ km.} \\ K &= 16\cdot07 \text{ km.} \pm 0\cdot57 \text{ km.} \\ T &= \text{J.D. } 2,423,540\cdot65 \pm 4\cdot75 \\ a \sin i &= 96,904,000 \text{ km.}\end{aligned}$$

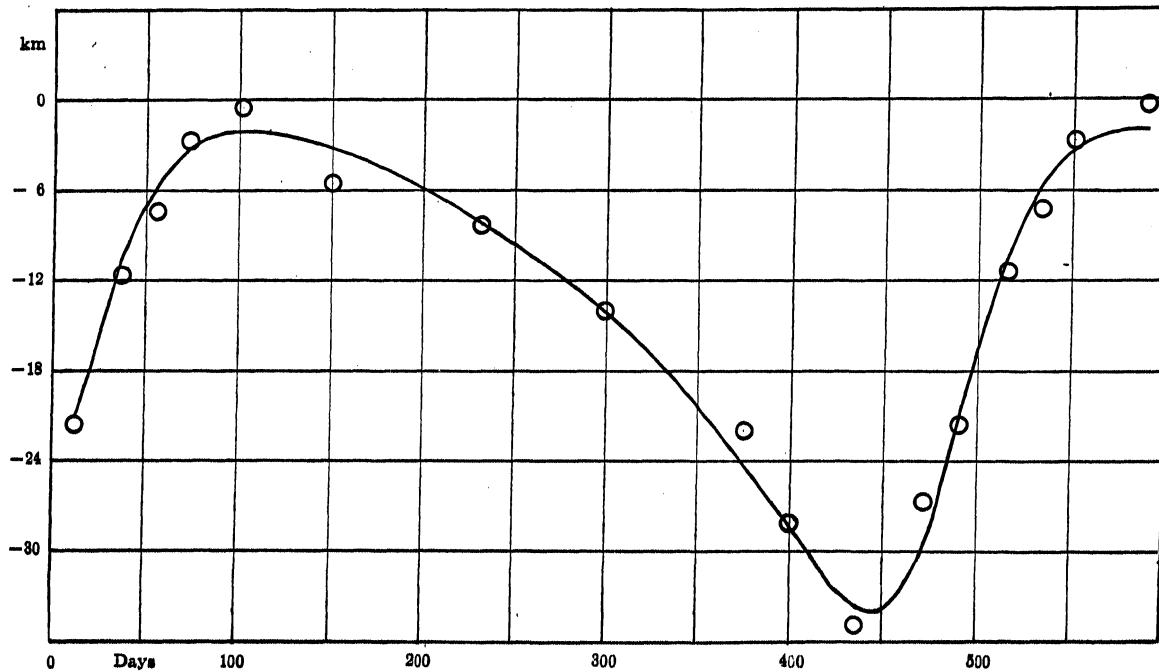


FIG. 3.—Radial Velocity Curve of 105 Herculis

In the foregoing the times are given in Greenwich Mean Time in the sense used prior to 1925.

Dominion Astrophysical Observatory,
Victoria, B.C.

April 24, 1925.

THE ORBITS OF FIVE SPECTROSCOPIC BINARIES

201

22 VULPECULAE

Abstract.—Twenty-seven spectrograms of this star whose visual magnitude is 5.4 and type G7 are made the basis of a determination of its spectroscopic orbit. The period is 251 days, the eccentricity .042 and the probable error of a plate ± 1.0 km. per sec.

Observations were begun on this star, whose right ascension is $20^{\text{h}} 11^{\text{m}} \cdot 2$ and declination $+23^{\circ} 12'$, in the year 1922 and twenty-seven spectrograms have been secured on which to base a determination of the orbital elements. The value obtained for the period was 251.0 days, and the observations, while not uniformly distributed over the period, are nevertheless sufficiently so to ensure a fair determination of the elements.

The spectrum is of type G7 and the plates have been measured on the spectro-comparator using α Boötis, No. 2702, as a standard plate. The table following contains the phases reckoned from the time of periastron, the velocities, the weights assigned the plates and the residual for each from the curve representing the final elements. Through a slip the weight 1 was used for the first plate, whereas it should have been 2, but any correction would be negligible.

OBSERVATIONS OF 22 VULPECULAE

Plate	Date	Julian Date	Phase	Velocity	Wt.	O-C
7852.....	1922 July 28	2,423,264.866	100.016	-24.1	1	+1.5
8009.....	Aug. 28	3,295.739	130.889	- 8.0	3	-0.4
8027.....	Sept. 1	3,299.802	134.952	- 2.9	3	+2.8
8052.....	" 7	3,305.815	140.965	- 5.8	3	-2.4
8078.....	" 10	3,308.766	143.916	- 2.4	2	-0.1
8136.....	" 16	3,314.798	149.948	- 3.6	2	-3.1
8213.....	Oct. 9	3,337.699	170.849	+ 4.3	3	+1.8
8255.....	" 27	3,355.697	190.847	+ 0.6	3	+1.8
8337.....	Nov. 11	3,370.571	205.721	- 6.8	3	+0.8
8379.....	" 14	3,373.650	208.800	-12.1	2	-2.5
8954.....	1923 May 7	3,547.989	132.139	- 6.7	2	+0.5
9086.....	June 25	3,596.908	181.058	+ 1.8	3	+0.3
9192.....	July 20	3,621.871	206.021	- 7.6	3	+0.3
9306.....	Aug. 17	3,649.798	233.948	-27.1	3	-0.7
9495.....	Oct. 8	3,701.667	34.817	-50.6	3	+0.4
9526.....	" 12	3,705.671	38.821	-49.6	2	+1.7
9539.....	" 22	3,715.594	48.744	-51.8	3	-1.1
10066.....	1924 June 2	3,939.964	22.114	-48.4	2	+0.4
10247.....	July 7	3,974.884	57.034	-50.7	3	-2.5
10331.....	" 21	3,988.844	70.994	-41.5	3	+1.0
10536.....	Aug. 25	4,023.758	105.908	-19.8	3	+2.1
10623.....	Sept. 6	4,035.753	117.903	-15.4	3	-0.8
10725.....	" 15	4,044.718	126.868	- 9.3	3	+0.7
10778.....	" 26	4,055.694	137.844	- 4.4	3	+0.1
10850.....	Oct. 9	4,068.723	150.873	- 1.6	3	-1.2
10910.....	" 31	4,090.670	172.820	+ 2.3	2	-0.2
10945.....	Nov. 24	2,424,114.600	196.750	- 2.1	2	+1.3

NORMAL PLACES OF 22 VULPECULAE

	Mean Phase	Mean Velocity	Wt.	Residual O-C	
				Prel.	Final
1.....	100.016	-24.10	1	+1.67	+1.52
2.....	105.908	-19.80	3	+2.30	+2.10
3.....	117.903	-15.40	3	-0.48	-0.76
4.....	129.694	- 8.16	8	+0.38	+0.08
5.....	137.920	- 4.37	9	+0.37	+0.07
6.....	148.621	- 2.40	7	-1.63	-1.63
7.....	172.837	+ 3.50	5	+0.73	+0.89
8.....	185.952	+ 1.20	6	+0.19	+0.64
9.....	202.133	- 4.92	5	0.00	+0.73
10.....	207.135	- 9.40	5	-1.89	-1.07
11.....	233.948	-27.10	3	-1.47	-0.70
12.....	22.114	-48.40	2	+0.26	+0.43
13.....	36.419	-50.20	6	+0.97	+1.06
14.....	52.889	-51.25	5	-1.94	-1.85
15.....	70.994	-41.50	3	+0.98	+1.04

The observations were grouped into 15 normal places on the basis of phase. A value of the period of 251.0 days seemed most satisfactory and while there are only four cycles covered by the observations and this value may be in error by as much as a day, it was not considered that any great improvement would result by including the period in the least-squares solution. Moreover, as the eccentricity was small it was necessary to consider either T or ω fixed, and the former was the element taken as fixed.

PRELIMINARY ELEMENTS

$$\begin{aligned}
 P &= 251.0 \text{ days} & \gamma &= -23.52 \text{ km.} \\
 e &= .05 & K &= 27.0 \text{ km.} \\
 \omega &= 120^\circ & T &= \text{J.D. } 2,423,415.850
 \end{aligned}$$

With these, observation equations were built up connecting the residuals with the four elements, e , ω , γ and K and for homogeneity the following substitutions were made:

$$\begin{aligned}
 x &= \delta\gamma & z &= K \delta e \\
 y &= \delta K & u &= -K. \delta\omega
 \end{aligned}$$

OBSERVATION EQUATIONS OF 22 VULPECULAE

1.....	1.000x	- .083y	+ .140z	- .955u	-1.67 = 0
2.....	1.000	+ .053	- .027	- .954	-2.30
3.....	1.000	+ .319	- .336	- .896	+0.48
4.....	1.000	+ .555	- .578	- .771	-0.38
5.....	1.000	+ .696	- .701	- .650	-0.37
6.....	1.000	+ .843	- .735	- .454	+1.63
7.....	1.000	+ .974	- .435	+ .090	-0.73
8.....	1.000	+ .909	+ .155	+ .402	-0.19
9.....	1.000	+ .689	+1.029	+ .743	.00
10.....	1.000	+ .593	+1.235	+ .829	+1.89
11.....	1.000	- .078	+ .811	+1.042	+1.47
12.....	1.000	+ .931	-1.186	+ .467	-0.26
13.....	1.000	-1.024	- .386	+ .097	-0.97
14.....	1.000	- .955	+ .280	- .323	+1.94
15.....	1.000	- .702	+ .698	- .693	-0.98

These resulted in the following normal equations:—

$$\begin{array}{rcccccc}
 7.100x & + & 1.932y & - & .540z & - & 1.047u & + & 1.031 & = & 0 \\
 & & 3.995 & - & .508 & - & .178 & + & .286 & & \\
 & & & & 3.322 & + & 1.935 & + & 1.498 & & \\
 & & & & & & 2.907 & + & 1.541 & &
 \end{array}$$

from which the following corrections were obtained:—

$$\begin{array}{l}
 \delta\gamma = - .23 \text{ km.} \\
 \delta K = - .01 \text{ km.} \\
 \delta e = - .008 \\
 \delta\omega = + 1^\circ.0
 \end{array}$$

The following, then, are the final elements obtained, with their probable errors attached.

FINAL ELEMENTS

- P = 251.0 days
- e = .042 ± .015
- ω = 121°.0 ± 0°.93
- γ = -23.75 km. ± 0.24 km.
- K = 26.99 km. ± 0.31 km.
- T = J.D. 2,423,415.850
- a sin i = 93,072,000 km.

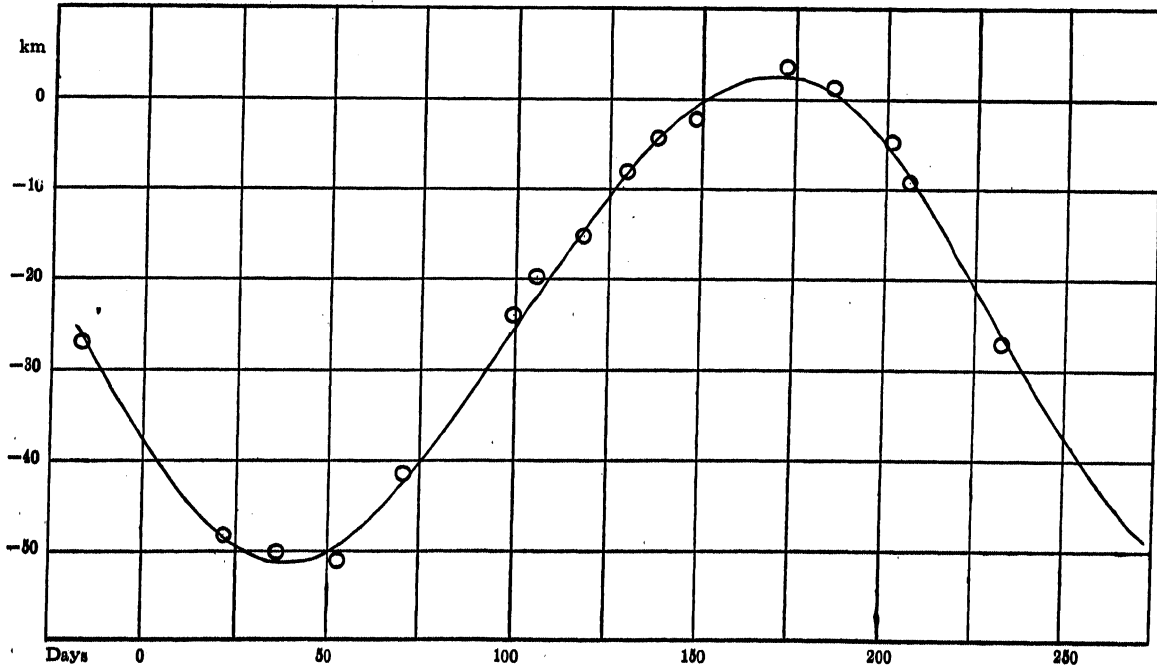


FIG. 4.—Radial Velocity Curve of 22 Vulpeculae

The curve (Fig. 4) accompanying represents the final elements, and on it are shown the observations as grouped.

Dominion Astrophysical Observatory,
Victoria, B.C.

Dec. 8, 1924.

π CEPHEI

Abstract.—Twenty-three spectrograms taken with the single-prism spectrograph and measured on the spectrocomparator against the sky as a standard, were made the basis of the determination of the elements. Five observations taken 25 years ago at the Lick Observatory were made use of also and fixed the period as 556.2 days. The semi-amplitude is 23.02 km. and the eccentricity 0.281.

The star π Cephei (1900 $\alpha = 23^{\text{h}} 04^{\text{m}} \cdot 7$, $\delta = +74^{\circ} 51'$, visual magnitude 4.56 and type G5) was announced¹ as a spectroscopic binary from the Lick Observatory from five plates taken in the years 1899 and 1900. The results of four plates made at the Bonn Observatory in the years 1909 and 1912 have since been published² as well as two of Hnatek³ at Vienna in 1913. For convenience of reference all are given in the table of observations.

The star is a long period visual binary listed as number 12196 in Burnham's catalogue. While the separation at present is less than a second of arc, yet the yellow component to which the present discussion relates is over two magnitudes brighter than the fainter and hence the measures should not be vitiated to any appreciable degree by the light from the fainter star.

The star has been observed here with the single-prism instrument since the beginning of 1922. The measures have been made on the comparator against the sky as a standard and in many cases against Arcturus also, the two results agreeing closely. Plate 8030, about which there was some uncertainty as to the focus of the instrument was not used in the solution, though its residual is less than that of another plate which was used.

It was felt that the Lick plates would, if used, add considerable weight to the solution, particularly as they were of three-prism dispersion. There seems to be systematic difference between ourselves and the Lick Observatory as judged from other results, and after adding -1.5 km. to their values they were incorporated with our own. Such a proceeding is open to question, but a determination from our own results alone would be very little different to that here given. The observations are not distributed over the curve as uniformly as desired, and the elements, while regarded as closely approximate, are not to be treated as the best that can be secured. There is a suspicion of a small secondary oscillation, but the measures are not sufficiently numerous or refined to decide. While the probable error of a plate, ± 1.58 km. per sec., is reasonably low for single-prism dispersion, one would expect from the character of the lines for measurement a probable error not exceeding ± 1.0 km. The star will be kept on our list to secure plates occasionally.

The table of observations follows. No use was made of the Bonn and Vienna single-prism results, though the residuals from our curve are tabulated and are exceedingly small for the Bonn Observatory.

¹ Ap. J. Vol. 14 p. 138

² A. N. 4780.

A. N. 197, p. 187

THE ORBITS OF FIVE SPECTROSCOPIC BINARIES

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OBSERVATIONS OF π CEPHEI

Observatory Plate No.	Date	Julian Date	Phase	Velocity	O-C
Lick III.....	1899 Aug. 8	2,414,875.9	193.37	-33.0	+0.6
".....	" 23	4,890.9	208.37	-36.0	-1.4
".....	" 29	4,896.9	214.37	-37.0	-2.0
".....	1900 Oct. 7	5,300.9	62.17	-5.0	+1.0
".....	Dec. 24	5,378.9	140.17	-23.0	+4.0
Bonn I.....	1909 Oct. 24	8,604.32	28.39	+3.1	-1.6
".....	1912 Sept. 26	9,672.39	540.26	+8.9	-0.4
".....	Oct. 9	9,685.34	553.21	+11.0	+1.2
".....	Oct. 11	2,419,687.36	555.23	+8.8	-1.0
Vienna I.....	1913 Oct. 31	2,420,072.332	384.00	-20.3	+7.9
".....	Nov. 21	0,093.308	404.98	-28.5	-3.5
Victoria—					
6975.....	1922 Jan. 6	2,423,061.594	36.07	+3.8	+1.4
8030.....	Sept. 1	3,299.869	274.34	-30.8	+4.9
8258.....	Oct. 27	3,355.772	330.24	-37.2	-3.3
8286.....	Nov. 2	3,361.672	336.14	-34.3	-0.9
8307.....	" 5	3,364.772	339.24	-32.5	+0.7
8311.....	" 8	3,367.664	342.14	-27.6	+5.4
8433.....	" 27	3,386.687	361.16	-28.4	+2.4
8497.....	1923 Jan. 13	3,433.594	408.07	-24.8	-0.1
9312.....	Aug. 17	3,649.969	68.24	-10.6	-2.0
9501.....	Oct. 8	3,701.848	120.12	-18.8	+4.2
9566.....	" 26	3,719.772	138.04	-25.6	+0.9
10250.....	1924 July 7	3,974.959	393.23	-27.7	-1.0
10334.....	" 21	3,988.961	407.23	-26.0	-1.3
10571.....	Aug. 29	4,027.895	446.17	-20.0	-3.3
10610.....	Sept. 3	4,032.925	451.20	-13.6	+1.9
10627.....	" 6	4,035.909	454.18	-12.3	+1.9
10636.....	" 8	4,037.876	456.15	-15.4	-1.4
10727.....	" 15	4,044.821	463.09	-10.8	+1.0
10853.....	Oct. 9	4,068.832	487.10	-5.8	-1.3
10989.....	Nov. 28	4,118.723	537.00	+7.5	-1.3
11030.....	Dec. 8	4,128.716	546.99	+11.0	+1.3
11092.....	" 23	4,143.638	5.71	+10.7	+1.4
11197.....	1925 Feb. 9	2,424,191.590	53.66	-7.8	-3.6

All the plates were considered of equal weight and some occurring at or about the same phase were grouped as shown in the table of normal places. The mean phases there listed are based on the final elements.

NORMAL PLACES FOR π CEPHEI

	Phase	Velocity	Wt.	O-C			Phase	Velocity	Wt.	O-C	
				Prel.	Final					Prel.	Final
1	36.07	+3.8	1	-0.3	+1.4	10	377.20	-28.0	2	+0.7	+1.1
2	57.92	-6.9	2	-2.4	-1.9	11	407.65	-25.4	2	-1.3	-0.7
3	68.24	-10.6	1	-2.2	-2.0	12	448.67	-16.8	2	-1.2	-1.0
4	129.08	-22.2	2	+4.5	+2.8	13	455.17	-13.8	2	+0.1	+0.2
5	140.17	-24.5	1	+4.3	+2.5	14	463.09	-10.8	1	+1.0	+1.0
6	193.37	-34.5	1	+0.5	-0.9	15	487.10	-5.8	1	-1.0	-1.3
7	211.37	-38.0	2	-2.0	-3.2	16	537.00	+7.5	1	-1.8	-1.3
8	333.19	-35.8	2	-2.5	-2.1	17	546.99	+11.0	1	+0.2	+1.3
9	340.69	-30.0	2	+2.7	+3.1	18	5.71	+10.7	1	-0.3	+1.4

Graphical methods in use here gave the following preliminary values of the elements.

$$\begin{aligned} P &= 556.2 \text{ days} \\ e &= .30 \\ \omega &= 15^\circ \\ K &= 24.0 \text{ km.} \\ \gamma &= -19.75 \text{ km.} \\ T &= \text{J.D. } 2,414,138.0 \end{aligned}$$

In the observation equations which were built up the following substitutions were made.

$$\begin{aligned} x &= \delta\gamma \\ y &= \delta K \\ z &= K.\delta e \\ u &= -K.\delta\omega \\ v &= [9.49460] \delta T \end{aligned}$$

OBSERVATION EQUATIONS FOR π CEPHEI

								Wt.
1.....	1.000x	+ .996y	+ .086z	+ .786u	+1.124v	+0.3	= 0	1
2.....	1.000	+ .634	- .867	+1.017	+1.290	+2.4		2
3.....	1.000	+ .472	-1.111	+1.061	+1.253	+2.2		1
4.....	1.000	- .289	- .624	+ .893	+ .653	+4.5		2
5.....	1.000	- .378	- .393	+ .822	+ .550	-4.3		1
6.....	1.000	- .635	+ .520	+ .458	+ .220	-0.5		1
7.....	1.000	- .677	+ .725	+ .332	+ .139	-2.0		2
8.....	1.000	- .566	+ .697	- .438	- .262	+2.5		2
9.....	1.000	- .540	+ .625	- .480	- .286	+2.7		2
10.....	1.000	- .374	+ .175	- .670	- .420	-0.7		2
11.....	1.000	- .180	- .307	- .805	- .558	+1.3		2
12.....	1.000	+ .175	- .952	- .915	- .786	+1.2		2
13.....	1.000	+ .245	-1.031	- .921	- .825	-0.1		2
14.....	1.000	+ .332	-1.107	- .921	- .873	-1.0		1
15.....	1.000	+ .624	-1.125	- .865	- .988	+1.0		1
16.....	1.000	+1.212	+ .385	- .308	- .591	+1.8		1
17.....	1.000	+1.272	+ .760	- .111	- .305	-0.2		1
18.....	1.000	+1.282	+1.010	- .207	+ .218	+0.3		1

There resulted the following normal equations:—

$$\begin{aligned} 27.000x + 2.033y - 4.093z - 2.845u - 1.502v + 2.400 &= 0 \\ 10.629 - 3.002 + .945 + 1.145 + 9.334 & \\ 15.176 + 1.097 + .262 + 1.180 & \\ 14.669 + 13.491 - 6.200 & \\ 13.505 - 1.913 & \end{aligned}$$

from which the following corrections resulted:

$$\begin{aligned} \delta\gamma &= + .12 \text{ km.} \\ \delta K &= - .98 \text{ km.} \\ \delta e &= - .019 \\ \delta\omega &= -9^\circ.30 \\ \delta T &= -11.67 \text{ days} \end{aligned}$$

so that the final elements, with their probable errors attached are as follows:—

FINAL ELEMENTS

$$\begin{aligned}
 P &= 556.2 \text{ days} \\
 e &= .281 \pm .020 \\
 \omega &= 5^\circ.70 \pm 4^\circ.21 \\
 K &= 23.02 \text{ km.} \pm 0.59 \text{ km.} \\
 \gamma &= -19.63 \text{ km.} \pm 0.37 \text{ km.} \\
 T &= \text{J.D. } 2,414,126.33 \pm 5.83 \text{ days} \\
 a \sin i &= 168,970,000 \text{ km.}
 \end{aligned}$$

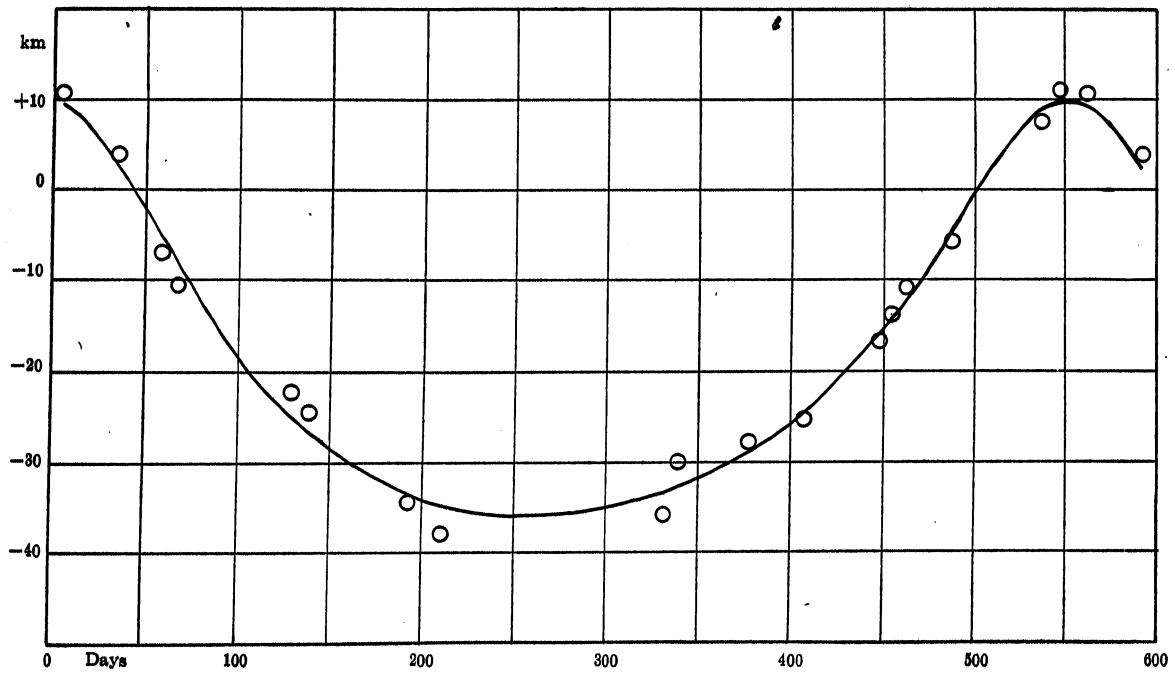


FIG. 5.—Radial Velocity Curve of π Cephei

The curve shown (Fig. 5) represents the final elements.

Dominion Astrophysical Observatory,
Victoria. B.C.

Feb. 27, 1925.