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THE ASYMMETRY IN STELLAR MOTIONS AND THE EXISTENCE OF A VELOCITY-RESTRICTION IN SPACE[1]

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ABSTRACT

Observational evidence of a velocity-restriction in space.—The asymmetry in the distribution of stellar velocities, which has been found by several investigators, has been studied, and it is found that a very definite relation exists between group-motion and the internal motion for different classes of objects. This relation, which is expressed by a quadratic equation, holds for all classes of objects for which we have data regarding their velocities. This dependence of group-motion upon internal motion in the velocity-groups leads to the representation of stellar velocities as a product of two three-dimensional frequency-functions F_1 and F_2, symmetrical around two different centers. The sun's velocity relative to the center of F_1, which itself is a sum of several concentric ellipsoidal frequency-functions, is found to be $\alpha=261^\circ.6$, $\delta=+14^\circ.6$, $v=13.9$ km/sec. The center of the second function F_2 cannot be determined directly, but by assuming that the stars in group X, XI, and XII of Table I have ultimate velocities, the center of F_2 can be found. The sun's velocity relative to the center of the frequency-function F_2 is $\alpha=323^\circ$, $\delta=+60^\circ$, $v=300$ km/sec.

The function F_1 can be interpreted as representing the distribution law of the velocities in the local system of stars in the absence of an external effect. F_2 can be interpreted as an external velocity-restriction in a fundamental frame of reference, which seems to be the same for near and distant objects.

The consequences of the existence of two frequency-functions are first derived deductively; later an inductive study is made.

Physical aspects of the theory.—The present study indicates that the distribution of stellar motion is determined by two connections, one with the local system and another with a very large cosmical system. The physical nature of the second connection is uncertain. The action of the larger system is similar to that of a field of stars or dust. This field may also be an absolute property of space.

INTRODUCTION

The proper motions and radial velocities of the stars show a certain asymmetry in their distribution, which is generally explained as a reflection of the sun's motion relative to the stars in general. When the stellar velocities are referred to the centroid of a large number of stars, this asymmetry is nearly eliminated. We can thus determine the sun's velocity relative to the stars and we find its value to be about 20 km/sec. toward the apex $\alpha=270^\circ$, $\delta=+30^\circ$. Kapteyn found in 1904 that the proper motions of the stars indicated the existence of two interpenetrating star streams. Schwarzschild[2] showed that the distribution law of stellar motions could be

[1] *Contributions from the Mount Wilson Observatory*, No. 275.

[2] *Göttingen, Nachrichten*, p. 191, 1908.

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represented by an ellipsoidal frequency-function, and Charlier[1] found that the deviation from an ellipsoidal distribution-law could not be reduced by assuming two star streams in accordance with Kapteyn's suggestion. The deviations pointed to a skewness in the distribution together with an excess of stars of large linear velocities. It was noted by Adams and Kohlschütter[2] in 1913 that the large radial velocities for stars in the Northern Hemisphere, after correction for the sun's velocity, were overwhelmingly negative. B. Boss[3] computed the space-velocities of a number of stars of measured parallaxes and found that the high-velocity stars showed a marked tendency to move toward the region in the sky limited by galactic longitudes 140° to 340°. The same effect was independently found by Adams and Joy[4] from a study of the space-velocities of stars of high radial velocity. From an investigation of the space-velocities of 1300 stars of late spectral types the writer[5] found that about 100 stars of space-velocity larger than 100 km/sec. had their apices between galactic longitudes 143° and 334°, and *not a single one* between longitudes 334° and 143°. It was furthermore found that this asymmetry in the velocity-distribution, which remained after the velocities had been referred to the centroid of the stars, could be traced even among stars of ordinary speed, producing a difference between the most frequent velocity-vector and the mean velocity. The same skewness in the velocity-distribution was found by Boss, Raymond, and Wilson[6] from a study of the space-velocities of 520 stars of known parallaxes.

The line of asymmetry in the distribution of velocities is nearly opposite to the sun's motion relative to stars of ordinary speed. The sun's velocity is thus dependent upon the speed of the stars to which its motion is referred, although the apex of the solar motion is only slightly affected.[5] This latter circumstance makes it diffi-

[1] *Meddelanden från Lunds Observatorium,* Serie II, No. 9, 1913, pp. 92 ff.

[2] *Mt. Wilson Contr.,* No. 79; *Astrophysical Journal,* **39,** 341, 1914.

[3] *Popular Astronomy,* **26,** 686, 1918; *Publications of the American Astronomical Society,* 22d Meeting, 1918.

[4] *Mt. Wilson Contr.,* No. 163; *Astrophysical Journal,* **49,** 179, 1919.

[5] *Mt. Wilson Contr.,* No. 245; *Astrophysical Journal,* **56,** 265, 1922.

[6] *Astronomical Journal,* **35,** 26 (No. 820), 1923.

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cult to detect the phenomenon from a study of proper motions, which give only the direction of the sun's motion when the distances of the stars are not known. It makes the use of parallactic motion for computing mean parallaxes somewhat misleading, if the same solar velocity is used for stars of small and high velocity. The asymmetry seems to be the same for giant stars and dwarf stars and independent of spectral type, although it is most conspicuous among stars of high velocity. It has recently been found to exist in a very marked degree among stars of spectral types Me and variable stars of short period.

TENTATIVE EXPLANATION OF THE ASYMMETRY

The asymmetry in the distribution of stellar velocities seems thus to be a perfectly general phenomenon, dependent upon the speed alone, and to be independent of the physical nature of the objects concerned. It was first thought that it might be due to the excentric position of the sun in our local system of stars, whose center as defined by the B stars[1] is in galactic longitude 236°. Thus high speeds in a certain direction might be attained by stars falling toward the center of the local system. It was soon realized, however, that, if the stellar system has reached an approximately steady state, stars coming from the opposite side of the center should be equally numerous and have the same high speed, but in the opposite direction when passing our region in the local cluster. Furthermore, the effect in this case would vanish for stars moving perpendicularly to the direction toward the center.

Oort[2] suggested that there may be two or more distinct groups of stars having different group-motions and that the solar motion thus would come out differently as reference is to one or other of these groups.

It occurred to the writer that the asymmetry might be explained if for some reason high velocities referred to a certain reference-system in space were improbable. Thus if a star were moving toward the first quadrant of galactic longitudes (cf. the diagram in

[1] Charlier, "The Distance and the Distribution of the Stars of Spectral Type B," *Meddelanden från Lunds Observatorium*, Serie II, No. 14, 1916.

[2] *Bulletin of the Astronomical Institutes of the Netherlands*, No. 23, 1922.

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Mt. Wilson Contr., No. 245, p. 20),[1] its velocity, referred to another origin in the third quadrant, might possibly be too great to be probable, whereas, if directed toward the third quadrant, its velocity referred to the origin mentioned would be smaller and in the realm of values occurring more frequently. Attention was then directed to the velocities of distant objects, and it was found that the translation of our local system, referred to the frame for which the "velocity-restriction" seems to exist, is the same as that derived from non-galactic nebulae and globular clusters, at least in so far as it was possible to determine this translation from observations now available.

To put these conceptions into mathematical form, we will assume that all the objects in our local system of stars form a unity, all parts of which have a common group-motion when no external "velocity-restriction" is acting. As an origin we shall use the sun, since all our measurements of proper motions and radial velocities are referred to it.[2] Since the collection of data is very heterogeneous with regard to speed, we assume the following general form of the frequency-function for the velocity-components referred to the element *dx dy dz* of velocity-space:

$$F_1=\frac{1}{\pi^{\frac{3}{2}}}\sum_{\nu} N_\nu l_\nu m_\nu n_\nu e^{-l_\nu^2(x-\alpha)^2-m_\nu^2(y-\beta)^2-n_\nu^2(z-\gamma)^2}\ . \tag{1}$$

This function represents a sum of concentric and coaxial ellipsoidal velocity-distributions. N_ν is the number of objects belonging to each one of the component frequency-functions; l_ν, m_ν, n_ν are inversely proportional to the dispersions (square root of the mean of the squares of the velocities) along the principal axes; and α, β, γ represent the velocity-components, relative to the sun, of the common center of the component frequency-functions. The co-ordinate system used has its origin in the sun and its axes along the common principal axes of the velocity-ellipsoids.

[1] *Astrophysical Journal*, **56**, 284, 1922.

[2] A short account of this investigation has been given in *Mt. Wilson Comm.*, No. 84; *Nat. Acad. Proceedings*, **9**, 312, 1923. The origin used there is one relative to which the sun has a velocity of 20 km/sec. toward the apex $\alpha=270^\circ$, $\delta=+30^\circ$.

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The additional velocity-restriction is represented by the frequency-function

$$F_2 = e^{-k^2[(x+x_0)^2+(y+y_0)^2+(z+z_0)^2]}\,, \tag{2}$$

which is supposed to "act" simultaneously with F_1 upon the stars within the same element of velocity-space.

This function corresponds to a spherical distribution-law. The velocity (x_0, y_0, z_0) is the velocity of the sun in the fundamental system of reference. The actual probability F of a certain velocity (x, y, z) can then be found by multiplication of the two probabilities F_1 and F_2.[1] The function F can be written in the following form:

$$\left.\begin{array}{lll} F=\sum_\nu A_\nu e^{-h_1^2(x-c_1)^2-h_2^2(y-c_2)^2-h_3^2(z-c_3)^2} & & \\ h_1^2=l_\nu^2+k^2 & c_1=-2k^2a_\nu^2(x_0+\alpha)+\alpha & a_\nu=\dfrac{1}{\sqrt{2}\,h_1} \\ h_2^2=m_\nu^2+k^2 & c_2=-2k^2b_\nu^2(y_0+\beta)+\beta & b_\nu=\dfrac{1}{\sqrt{2}\,h_2} \\ h_3^2=n_\nu^2+k^2 & c_3=-2k^2c_\nu^2(z_0+\gamma)+\gamma & c_\nu=\dfrac{1}{\sqrt{2}\,h_3} \end{array}\right\} \tag{3}$$

The function F represents a sum of coaxial ellipsoidal distribution-laws, whose centers are shifted in a direction nearly opposite to that of the translation of the adopted origin in our fundamental reference-system. The amount of the shift increases with the internal motion in the different groups. The center of the distribution represented by this function has two limits. For stars of small relative motions, for which l, m, and n are large compared with k, the limit is

$$c_1=\alpha\,, \qquad c_2=\beta\,, \qquad c_3=\gamma\,.$$

For stars of extremely large relative velocities the co-ordinates of the center approach the limit

$$c_1=-x_0\,, \qquad c_2=-y_0\,, \qquad c_3=-z_0\,,$$

[1] A simple illustration of the use of a product of two frequency-functions, both referred to the same velocity-element, $dxdydz$, is offered by individuals walking on shipboard in the presence of a strong head-wind. The two limits correspond to different classes of individuals with infinitely strong and infinitely weak connection with the ship, the latter group being statistically at rest in the air.

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which represents the center of the fundamental reference-system if referred to the sun as origin.

This shift in the center of the velocity-distribution with increasing internal velocities shows clearly for *all classes of celestial objects* for which we have data regarding their velocities.

COMPARISON BETWEEN THEORY AND OBSERVATION

In Table I are collected data that are representative of our present knowledge of stellar velocities. The constants of ellipsoidal frequency-functions for twelve different groups of objects are given in the eighth to the sixteenth columns. The quantities c_1, c_2, and c_3 are the co-ordinates of the centers of the velocity-groups referred to the sun as origin. The system of co-ordinates used is the galactic system with the x-axis toward the intersection of the galactic plane with the equator (in Aquila), the y-axis toward 90° galactic longitude, and the z-axis toward the north galactic pole, for which the adopted co-ordinates are

$$\alpha = 190.^\circ 6, \qquad \delta = +27.^\circ 2, \qquad 1900.0 \text{ (Kapteyn)}.$$

The equations for finding equatorial from galactic co-ordinates are then

$$\begin{aligned} \xi &= +0.1846\,x + 0.4494\,y - 0.8740\,z \\ \eta &= -0.9828\,x + 0.0844\,y - 0.1642\,z \\ \zeta &= + 0.8893\,y + 0.4573\,z \end{aligned}$$

The dispersions along the principal axes are denoted by a, b, and c. The quantity a is the dispersion along that principal axis which most nearly coincides with the x-axis, and b and c are the dispersions along the principal axes that are nearest to the y- and z-axes. The values given for the dispersions are in all cases corrected for the systematic effect of accidental errors in the radial velocities, proper motions, and parallaxes.

The directions of the principal axes of the velocity-ellipsoids are given in the last three columns; L_1 is the galactic longitude of the axis along which the dispersion is equal to a, and B_1 the galactic latitude of the same axis. B_2 is the galactic latitude of the axis along which the dispersion is equal to b. The probable errors of

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TABLE I

Group	Objects	No.	V_0	α_0	δ_0	K	c_1	c_2	c_3	a	b	c	L_1	B_1	B_2
			km			km	km	km	km	km	km	km			
I......	Ursa Major group	22	18.5	127°8	+ 40.0	0	+ 12.6 ± 1	− 7.6 ± 1	− 11.2 ± 1	1 ± 1	1 ± 1	1 ± 1	....	....	....
II.....	Taurus group	39	45.6	271.8	− 6.9	0	− 44.7 ± 1	+ 8.0 ± 1	− 3.7 ± 1	1 ± 1	1 ± 1	1 ± 1	....	....	....
III....	B stars	284	22.1	286.2	+ 25.5	+ 4.3	− 19.8 ± 1	− 9.3 ± 1	− 2.6 ± 1	5.6 ± 1.0	5.6 ± 1.0	3.5 ± 0.5	....	(0°)	(0°)
IV.....	A stars (Central group)	230	14.2	271.7	+31.0	0	− 12.0 ± 6.1	− 5.6 ± 0.8	− 5.0 ± 0.5	20.2 ± 3	6.1 ± 1	5.4 ± 1	158° ± 22	− 6°8 ± 4.2	−22 ± 9
V......	Stars of moderate velocity of spectral class F, G, K, M	768	15.2	262.1	+21.9	0	− 13.4 ± 0.8	− 3.0 ± 0.8	− 6.6 ± 0.5	22.7 ± 0.9	16.6 ± 0.8	10.5 ± 0.6	165.4 ± 4.0	− 0.3 ± 1.4	+ 2.7 ± 2.4
VI.....		510	32.4	288.8	+46.0	0	− 22.3 ± 3.9	− 22.2 ± 2.6	− 7.8 ± 2.4	42.1 ± 2.5	22.1 ± 1.1	27.8 ± 1.9	155.3 ± 4.6	+ 1.1 ± 7.6	−12.7 ±12.5
VII....	Bright-line nebulae	101	30.2	288.9	+43.5	+ 1.8 ± 2.7	− 21.7 ± 4.4	− 19.9 ± 6.1	− 6.7 ± 7.7	37.4 ± 2	34.0 ± 2	34.0 ± 2	(163)	(−3)	....
VIII...	M1e to M6e	47	41.3	288.7	+30.3	0	− 35.3 ± 5.4	− 20.8 ± 4.0	− 5.1 ± 4.6	49.8 ± 6	30.9 ± 4	40.2 ± 5	164 ± 5	− 2 ± 3	+ 5 ± 3
IX.....	Short-period variables	26	109	306	+47	0	− 67 ± 20	− 85 ± 19	− 8 ± 16	69 ± 10	69 ± 10	69 ± 10	....	....	....
X.....	Stars of high velocity	21	268 ± 27	324	+58	0	−102 ± 46	−247 ± 22	− 17 ± 19	206 ± 80	82 ± 20	78 ± 20	167 ± 5	+ 5 ± 3	+ 3 ± 3
XI....	Globular clusters	16	266 ± 53	320	+64	0	− 90 ± 33	−247 ± 55	− 43 ± 47	113 ± 30	113 ± 30	113 ± 30	....	....	....
XII...	Non-galactic nebulae	43	401 ±102	304	+81	+689 ± 71	−138 ±144	−358 ± 97	−116 ± 78	359 ± 50	359 ± 50	359 ± 50	....	....	....

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these quantities are given below the numbers to which they refer, and in several cases are only estimates.

Referred to the principal axes and to the center whose galactic co-ordinates are c_1, c_2, and c_3, the frequency-function of the velocities for each group can be written

$$F=\frac{N}{(2\pi)^{\frac{3}{2}}abc}e^{-\frac{x_1^2}{2a^2}-\frac{y_1^2}{2b^2}-\frac{z_1^2}{2c^2}}.$$

The sun's velocity V_0 and the position of its apex as referred to the twelve different groups are given in the fourth to the sixth columns of Table I. These data are connected with the quantities c_1, c_2, and c_3 by the relations

$$V_0 \cos \alpha_0 \cos \delta_0 = -\xi$$
$$V_0 \sin \alpha_0 \cos \delta_0 = -\eta$$
$$V_0 \sin \delta_0 = -\zeta$$

where ξ, η, and ζ are the values of c_1, c_2, and c_3 expressed in equatorial co-ordinates. The systematic correction applied to the measured radial velocities is equal to $-K$, where K is the quantity given in the seventh column.

The elements for the Ursa Major group are those given by Bottlinger and Rasmuson.[1] For the Taurus group the elements computed by Boss[2] are given. The dispersion in these two groups has been assumed to be about 1 km/sec. For the B stars (B0 to B5) Gyllenberg's[3] elements for the solar motion have been used. The elements for the A stars are those for the Central group in *Contribution* No. 257,[4] referred to the sun and corrected for the systematic effect of accidental errors in the quantities involved.

For stars of spectral types F, G, K, M, a special computation was made, based upon the three velocity-components for stars of known proper motion, parallax, and radial velocity. In order to conform with the adopted frequency-function the distribution was represented by a sum of two ellipsoidal frequency-functions,

[1] *Meddelanden från Lunds Observatorium*, Serie II, No. 26, 1921.

[2] *Astronomical Journal*, **26**, 31 (No. 604), 1908.

[3] *Meddelanden från Lunds Observatorium*, Serie II, No. 13, 1915.

[4] *Astrophysical Journal*, **57**, 77, 1923.

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and the method devised in *Contributions* Nos. 251[1] and 257 was employed. One modification of the method was introduced, however. The equations of condition for determining the twenty constants involved are of the form

$$\frac{dn}{dN_1}\Delta N_1+\frac{dn}{dh_1}\Delta h_1+\ \ldots\ +\frac{dn}{dN_2}\Delta N_2+\frac{dn}{dh'_1}\Delta h'_1+\ \ldots\ =n_0-n_c\ ,$$

where N_1 and N_2 are the total number of stars in the two frequency-functions, the sum of which represents the actual distribution. These equations of condition were given additional weights inversely proportional to the square roots of the computed number (n_c) of velocity vectors terminating in the different parallelopipeds of the velocity-space.

The solar motion as based upon the bright-line nebulae was computed from the radial velocities measured by Campbell and Moore.[2] The direction of the axis of preferential motion was assumed to have the co-ordinates $\alpha=90°$, $\delta=+13°$. Six nebulae of excessively high velocities were omitted.

In group VIII are given the elements of the velocity-distribution for forty-seven long-period variable stars of spectral type M1e to M8e. The data are from an investigation by Mr. Merrill and the writer published as *Contribution* No. 268.[3]

Group IX is based upon the radial velocities of twenty-six variable stars of period less than one day. These velocities have all been determined by Messrs. Adams and Joy, and I am under great obligation to them for their courtesy in allowing me to use their data in advance of publication.

Group X consists of twenty-one stars of very high velocity, which in my previous study of velocities seemed to form a separate group. They are mostly of spectral class F and have space-velocities ranging from about 200 to 600 km.

For the globular clusters and non-galactic nebulae the radial velocities alone can be used. In most cases these have been determined at the Lowell Observatory by Professor Slipher, through

[1] *Astrophysical Journal*, **56**, 265, 1922.

[2] *Lick Observatory Publications*, **13**, Part IV, 1918.

[3] *Astrophysical Journal*, **59**, 148, 1924.

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whose courtesy several unpublished velocities could be included. The solar motion for the globular clusters has been computed by Lundmark.[1] The designation "non-galactic nebulae," as used by Hubble, has been adopted here; twenty-four of the forty-three objects are known to have spiral structure.

As seen in Table I, the grouping is based upon motion for groups I, II, and X, and upon other physical criteria for the other groups. For groups V, VI a statistical dissection into two velocity-groups has been performed. In several of the groups it was found that an asymmetrical distribution of velocities was noticeable, even after the velocities had been referred to the centroid chosen for the group. This asymmetry in the individual groups always presented itself as a tendency for the fastest moving stars in the group to move toward galactic longitudes 170° to 330°. A statistical dissection of the whole collection of velocities is out of the question, since we do not know the space-velocities for all the objects involved. Furthermore, the grouping on the basis of physical properties (apart from motion) is very convenient, and may give us valuable information concerning the motion of different classes of objects.

TABLE II

Groups	No.	V_0	α_0	δ_0	c_1	c_2	c_3
		km			km	km	km
IV, V.....	998	15.1	267°.1	+25°.3	−13.3 ± 0.8	− 4.3 ± 0.8	−5.8 ±0.5
VI, VII...	611	32.0	288.8	+46.0	−22.0 ± 3.5	−21.9 ± 2.5	−7.7 ±2.4
	a	b	c	L_1	B_1	B_2	
	km	km	km				
IV, V....	22.1 ± 0.9	13.1 ± 2.0	9.2 ± 0.6	164° ± 4	−1° ±1.5	+1° ±2	....
VI, VII...	39.8 ± 2	24.5 ± 2	30.9 ± 2	155 ± 5	+1 ±7	−13 ±12	....

The projection on the galactic plane of the different velocity-ellipsoids is shown in Figure 1 (p. 238). As several of the ellipsoids are nearly identical, groups IV and V were combined; and, similarly, groups VI and VII. The data for these combined groups are given in Table II.

[1] *Publications of the Astronomical Society of the Pacific*, **35**, 318, 1923.

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The shift of the center of the velocity-distribution with increasing internal motion in the different groups is very clearly seen in

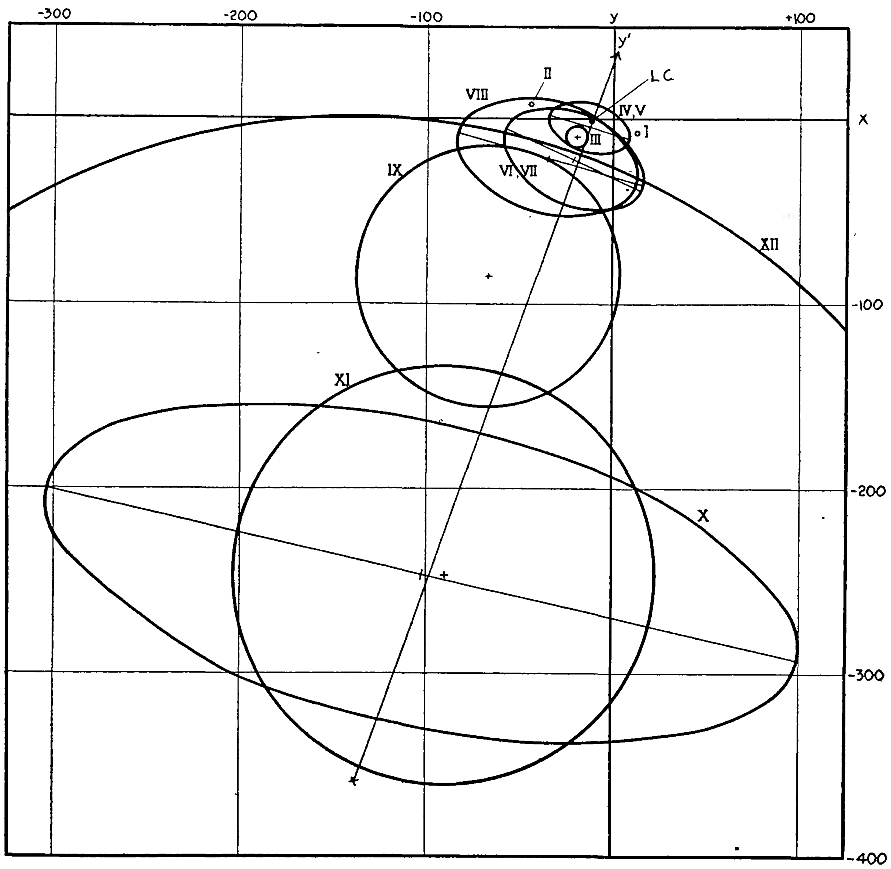

FIG 1.—Projection of the velocity-ellipsoids for different classes of stars on the galactic plane. The sun is at the origin and the vector from the origin to the center of the ellipses or circles represents the group-motion relative to the sun of the different groups of objects. The axes of the ellipses are equal to the dispersion in velocity within the groups. The systematic change in group-motion with increasing internal dispersion can be clearly seen. The point marked *L.C.* (limiting center) corresponds to the computed position of the center of the velocity-figure, where the dispersion vanishes. The line marked y' represents approximately the direction of translation of objects with small divergence in motion relative to objects of large divergence.

Figure 1. The direction of this shift in passing from slow- to fast-moving objects was found to be in galactic longitude $250.^{\circ}7 \pm 5^{\circ}$. The velocity-ellipsoids were then projected on a plane through the z-axis and through a line in galactic longitude $70.^{\circ}7-250.^{\circ}7$. This

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new projection is shown in Figure 2. From this diagram the galactic latitude of the line of displacement of the center was estimated to $-5^{\circ} \pm 3^{\circ}$.

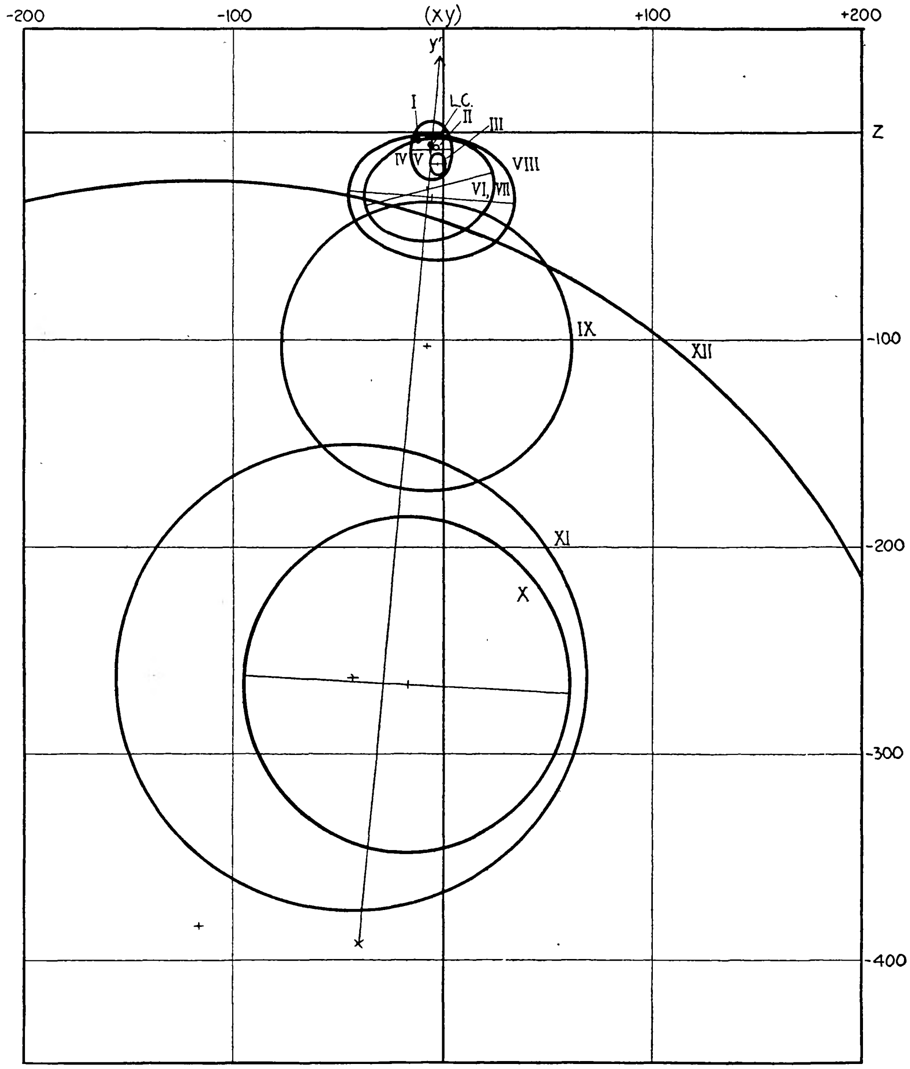

FIG. 2.—Intersection of the velocity-ellipsoids with a plane through the z-axis (toward the pole of the galaxy) and through a line in galactic longitude 70°7. This line is marked (xy).

By introducing a new system of co-ordinates we can confine the systematic displacement of the center of velocity-distributions to one axis, the displacements on the other two axes being much

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smaller and not systematic. The directions of the new axes (x' y' z') are

	L	B	α	δ
x'	340°7	0°0	271°5	−17°1
y'	70.7	+ 5.0	327.6	+61.2
z'	250.7	+85.0	188.9	+22.5

The x'-axis is approximately parallel to the direction of stream motion (major axes of the ellipsoids), the y'-axis is parallel to the systematic displacement of the centroids of the different velocity groups, when passing from groups of high to those of low dispersion, and the z'-axis points very nearly toward the north pole of the galaxy. The co-ordinates of the centers of the different velocity-ellipsoids referred to the sun and to the x'-, y'-, z'-axes are given in Table III.

TABLE III

Group	c'_1	c'_2	c'_3
	km/sec.	km/sec.	km/sec.
I	+14.4± 1	− 4.0± 1	−10.9± 1
II	−44.8± 1	− 7.5± 1	− 3.1± 1
III	−15.6± 1	− 15.5± 1	− 1.3± 1
IV, V	−11.1± 0.8	− 8.9± 0.8	− 5.0± 0.5
VI, VII	−13.5± 3.4	− 28.5± 2.6	− 5.2± 2.4
VIII	−26.4± 5.3	− 31.6± 4.2	− 2.4± 4.6
IX	−35 ± 20	−103 ± 19	+ 1 ±16
X	−15 ± 44	−267 ± 26	+ 6 ±19
XI	− 3 ± 36	−266 ± 53	−20 ±47
XII	−12 ±140	−392 ±103	−82 ±78
Mean of IV to XII	−11.6± 0.8	....	− 5.0± 0.5

Taking into account probable errors, we can regard the values of c'_1 and c'_3 in Table III as fairly constant. The outstanding discrepancies are the values for the Ursa Major and the Taurus groups. These deviations are confined to the x'-co-ordinate, however, and are probably connected with the higher mobility along this axis.

The systematic change in the group-motion is now confined to the y'-axis, and is measured by the quantity c'_2. According to our hypothesis of the existence of two frequency-functions, we may

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expect that the relation between c_2' and b will be that given in equations (3), namely,

$$\left.\begin{aligned} c_2' &= -pb^2+\beta' \\ p &= 2k^2(y_0'+\beta') \end{aligned}\right\} \qquad (4)$$

where

The accents here indicate that the co-ordinates in equations (3) are referred to the (x', y', z') system of co-ordinates, the axes of which nearly coincide with the mean position of the principal axes of the velocity-ellipsoids.

The quadratic relation expressed by equation (4) holds with remarkable accuracy for all the groups except the non-galactic nebulae. Among the other groups only the B stars show a deviation larger than can be accounted for by the errors in the constants b and c_2'.[1] Omitting groups III and XII, we find the following relation:

$$\left.\begin{aligned} c_2' = -&0.0323\,b^2 - 5.7 \\ \pm&0.0029 \quad \pm 0.7 \end{aligned}\right\} \qquad (5)$$

Figure 3 (p. 242) shows this parabola, its axis coinciding with the y'-axes, and the agreement with the individual values of c_2' and b. The points representing the non-galactic nebulae are not indicated on the diagram. The observed value of b for these objects is 359 km/sec., whereas $c'_2 = -392$ (Table III) by equation (5) would give $b = 109$ km/sec.

The discrepancy for the non-galactic nebulae is, however, confined to the dispersion. The solar motion as computed from these nebulae is about the same as from the fast-moving stars in group X and the clusters in group XI, if account is taken of the probable errors in the respective quantities.

The systematic shift in the center of the velocity-ellipsoids with increasing speed can be found from other but less complete data than those used in Table I. The Cepheids in our local system are stars of high luminosity and small peculiar motion. Using the radial velocities of twenty Cepheids and an assumed position of

[1] There is reason to believe that the splitting up into component frequency-functions is artificial and that all the objects in groups IV to X form one unit with an asymmetrical velocity-distribution. The Taurus and the Ursa Major group, however, as well as the B stars, can be regarded as real "moving clusters," with systematic motion differing somewhat from that of the majority of the stars.

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the apex of $\alpha=270°$, $\delta=+30°$, we find a solar velocity of only 14.5±2.0 km/sec. and an average residual radial velocity $\theta=10.6$ km/sec. From twenty-four stars of spectral type F4 to G8 and of extremely high luminosity (omitting Cepheids and stars of similar spectra), the author finds from their radial velocities a solar velocity of 14.2±1.9 and $\theta=9.1$ km/sec. This is corroborative evidence that the slow-moving objects give a smaller

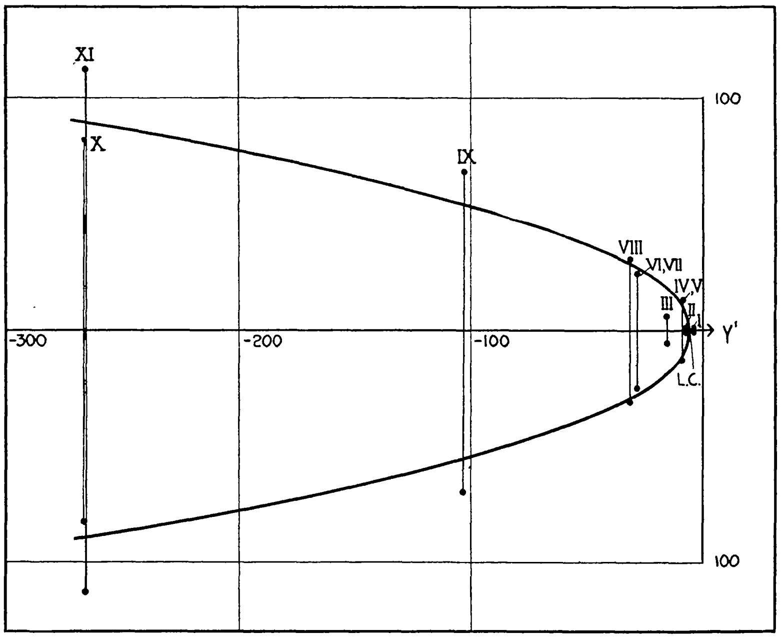

FIG. 3.—Relation between systematic change in group-motion relative to the sun (abscissae), and dispersion along an axis parallel to this systematic shift (ordinates). This relation is represented by a parabola with its axis along the line of systematic displacements.

velocity for the sun than stars of higher velocity. The B stars form an exception, as they give a solar motion of 22 km combined with a peculiar motion of only about 5 km. These stars seem to form a moving group of their own, for which the group-velocity along the y'-axis does not quite follow the simple law expressed by equation (5). In *Contribution* No. 267[1] it has been shown that the solar motion deduced from the radial velocities of the Me stars increases steadily with the residual radial velocities for the group of stars

[1] *Astrophysical Journal*, 59, 97, 1924.

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relative to which the sun's motion is referred. Using the approximate relations $c'_2 = V_0$ and $b = \sqrt{\pi/2}\,\theta$, we find new points which all fall almost exactly on the parabola in Figure 3.

In Table III are given the weighted means of c'_1 and c'_3. These values correspond to the quantities α and γ in equation (1), referred to the (x', y', z') system of co-ordinates. The corresponding value of β is that given in equation (5). These three quantities define one of the limits of the series of centers of the velocity-ellipsoids. According to equation (1), they give the common center of the velocity-ellipsoids in the absence of an external velocity-restriction.

The sun's velocity relative to this limiting point is

x' km	y' km	z' km	x km	y km	z km	α	δ	v km
+11.6	+5.7	+5.0	+12.7	+1.1	+5.5	261°6	+14°6	13.9
± 0.8	±0.7	±0.7	± 0.8	±0.7	±0.7			

The velocity of the sun is the same as that found from F and G stars of very great luminosity. This point, which we will call the "limiting center," coincides closely with the most frequent velocity found among stars of spectral type F, G, K, and M, as determined by the author,[1] and by Boss, Raymond, and Wilson.[2] The co-ordinates for this point as given by the author and referred to the sun are

$$x = -11.2, \quad y = -4.7, \quad z = -5.7 \text{ km/sec.}$$

The sun's velocity relative to this point, which can be found by reversing the signs, agrees well with the values given above.

The position of this limiting center is indicated by a dot in Figures 1, 2, and 3, and is marked *L.C.*

INDUCTIVE STUDY OF THE VELOCITY-DISTRIBUTION

The large amount of data collected in Table I makes it possible to refrain from using any hypothesis concerning the nature of the frequency-function, and to determine this function *a posteriori* from the observational data. We have studied different classes of celestial objects and determined their motions relative to the sun,

[1] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 292, 1922.

[2] *Astronomical Journal*, **35**, 26, 1923.

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using as reference frames "non-rotating" co-ordinate systems, i.e., systems in which the algebraic sums of the proper motions of a very large number of stars are zero, or, what amounts to the same, systems in which the proper motions of the stars in general are as small as possible.

The motions of the objects belonging to the Ursa Major group and the Taurus group, respectively, are very nearly the same, but the velocities differ more and more among themselves as we proceed to the later groups in Table I. At the same time the motion of the group as a whole increases systematically in a certain direction. For representation of the group-motion and of the divergence in motion of the individual members of the group we have used frequency-functions of the ellipsoidal type. This representation is known to be a close approximation for all the groups. The globular clusters are very unevenly distributed over the sky, however, and as we know only their radial velocities, we must make an assumption about the value of the K-term (systematic correction to the radial velocities) in order to find definite values of the constants. For the non-galactic nebulae the K-term can be directly determined, and, although of course the constants for these objects are rather uncertain, as is indicated by their probable errors, the residual velocities can be fairly well represented by an error-curve.

The constants of the velocity-ellipsoids have been determined for each class of objects. Referred to the principal axes, the frequency-function for each group can be written

$$\frac{Nh_1h_2h_3}{\pi^{\frac{3}{2}}}e^{-h_1^2(x''-c_1'')^2-h_2^2(y''-c_2'')^2-h_3^2(z''-c_3'')^2}$$

$$h_1^2=\frac{1}{2a^2}\,,\qquad h_2^2=\frac{1}{2b^2}\,,\qquad h_3^2=\frac{1}{2c^2}$$

N is here the number of stars in the group; x'', y'', and z'' are the velocity-components of a star referred to the sun and to the principal axes of the particular velocity-ellipsoid. We have one such frequency-function for each group, and, as the principal axes for the different groups are nearly parallel, the velocities can be referred to a common system of co-ordinates without changing the form of the frequency-function.

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Thus the frequency-function for the velocities of the whole collection of objects can be written

$$F=\sum_{\nu} A_{\nu} e^{-h_1^2(x'-c_1')^2-h_2^2(y'-c_2')^2-h_3^2(z'-c_3')^2} . \tag{6}$$

The values of h_1, h_2, and h_3 vary considerably for the different groups. Using the axes defined on page 240 we find that h_1 in general is smaller than h_2 and h_3, and h_2 nearly equal to h_3. This indicates a preferential motion along the x'-axis. The x'-co-ordinate of the center, c_1', is fairly constant, but deviates somewhat from the mean for the Ursa Major and the Taurus group; c_2' shows a very large variation (from 0 to -400 km); and c_3' is very nearly constant. The large variation in c_2' is found to be correlated with the dispersion of the velocities in the group, and the following empirical relation has been found to hold with a very high degree of accuracy,

$$\left.\begin{aligned} c_2' &= -pb^2+\beta' \\ \text{where} \qquad p &= +0.0323 \pm 0.0029 \text{ sec./km} \\ \beta' &= -5.7 \pm 0.7 \text{ km/sec.} \end{aligned}\right\} \tag{7}$$

It is only for the B stars and the non-galactic nebulae that this relation does not hold within the uncertainties of the quantities involved.

Equation (7) can be written

$$c_2' = -\frac{p}{2h_2^2}+\beta' .$$

Inserting this value of c_2' in equation (6) we find, after a simple transformation,

$$F=e^{-p(y'-\beta')}\sum_{\nu} B_{\nu} e^{-h_1^2(x'-c_1')^2-h_2^2(y'-\beta')^2-h_3^2(z'-c'_3)^2} .$$

As the variation in c_1' and c_3' is fairly small, we will replace them by their mean values α' and γ'.

Thus

$$F=e^{-p(y'-\beta')}\sum_{\nu} B_{\nu} e^{-h_1^2(x'-\alpha')^2-h_2^2(y'-\beta')^2-h_3^2(z'-\gamma')^2} . \tag{8}$$

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The first part of the frequency-function F is thus found to be constant for all the groups, and another part is found to correspond to a symmetrical distribution around a mean velocity whose components are α', β', γ'. The frequency-function F is proportional to the probability of the occurrence of a velocity-vector, the components of which are x', y', and z'. That this probability can be expressed as a product of two functions of these simple types immediately suggests that there are two independent causes which contribute to the actual frequency-function. It is the existence of the factor $e^{-p(y'-\beta')}$, or rather the fact that p is not zero, that produces the systematic change in group-motion along the y'-axis. The frequency-function $e^{-p(y'-\beta')}$ alone represents an increase in probability of velocity-components along the negative y'-axis. This increase in probability is exponential, and should give an infinitely large probability for $y'=-\infty$. This of course would be counteracted by the fact that $e^{-h_2^2(y'-\beta')^2}$ becomes zero for $y'=-\infty$. The function outside the summation sign, however, can easily be transformed into an ordinary frequency-function.[1]

By subtracting and adding the expression

$$k_1^2(x'-\alpha')^2+k_2^2(y'-\beta')^2+k_3^2(z'-\gamma')^2$$

to the exponents in equation (8), we find

$$\left.\begin{aligned}
F&=F_1F_2\\
F_1&=\sum_\nu C_\nu e^{-l_\nu^2(x'-\alpha')^2-m_\nu^2(y'-\beta')^2-n_\nu^2(z'-\gamma')^2}\\
F_2&=e^{-k_1^2(x'+x_0')^2-k_2^2(y'+y_0')^2-k_3^2(z'+z_0')^2}\\
l_\nu^2&=h_1^2-k_1^2\,,\qquad m_\nu^2=h_2^2-k_2^2\,,\qquad n_\nu^2=h_3^2-k_3^2\\
x_0'&=-\alpha'\,,\qquad y_0'=\frac{p}{2k_2^2}-\beta'\,,\qquad z_0'=-\gamma'
\end{aligned}\right\}\qquad(9)$$

The two frequency-functions F_1 and F_2, whose product represents the distribution law for all the cosmic velocities studied with a fair degree of approximation, are both symmetrical functions, but the centers of symmetry are different. On account of the particular choice of axes used the line joining these centers is parallel to the

[1] Although this transformation facilitates the physical interpretation of the distribution-law, it is not essential for the description of this law.

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y'-axis. The velocity-components x'_0, y'_0, z'_0 represent the sun's velocity relative to the center of the velocity-distribution F_2, and $-\alpha'$, $-\beta'$, $-\gamma'$ the sun's velocity relative to the center of the distribution represented by the function F_1.

In this way we have found that the assumption of the existence of two frequency-functions made in the beginning follows naturally from the observed facts that the systematic shift of the centers of the velocity-distributions along the principal axes is related to the dispersion by equations of the form given in equation (7). The inductive study shows us further that the function F_2 is not necessarily spherical but can equally well be ellipsoidal.

In the distribution-function F in equation (9) the velocities are referred to the axes given on page 240. For a group of stars of small relative velocities, i.e., one for which l, m, and n are large compared with k_1, k_2, and k_3, the function F_1 alone determines the velocity-distribution, and the limiting center of the distribution has the co-ordinates α', β', γ' relative to the sun. For stars of excessive velocities, for which l, m, and n are small compared with the k's, the function F_2 defines the velocity-distribution. The limiting center of the distribution F_2 has the co-ordinates $-x'_0$, $-y'_0$, $-z'_0$. In the particular system of co-ordinates used we have further

$$\left.\begin{aligned}\alpha'&=-11.6\pm0.8 \text{ km/sec.}\\ \beta'&=-\ \ 5.7\pm0.7\\ \gamma'&=-\ \ 5.0\pm0.7\end{aligned}\right\}\qquad(10)$$

The function F_2 produces a "velocity-restriction" in a co-ordinate system relative to which the sun has a velocity, whose components are x'_0, y'_0, z'_0.

The values of k_1, k_2, k_3 and of y'_0, which is a function of k_2, cannot be directly determined. It is only the *direction* of the vector on which y'_0 is situated that is given. It is further obvious that y'_0 must be larger than 267 km/sec. As the direction of the sun's velocity relative to fast-moving objects and relative to distant objects appears to be the same, we wish to define y'_0 as the sun's velocity relative to the most distant objects, whose velocities are known. The value of y'_0 for the globular clusters is +266 km

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(probable errors less than 100), and for the non-galactic nebulae $+392\pm 103$ km. As we have not yet found any class of objects with higher velocity and larger group-motion, we may regard $+300$ km as a probable value of y'_0. When this is combined with the previously given values of y'_0 and z'_0, we find that the sun's velocity is 300 km/sec. in the direction

$$L=68^\circ, B=+6^\circ;\ \alpha=323^\circ, \delta=+60^\circ,$$

with an uncertainty probably less than 10°.

It is rather unexpected to find a very pronounced stream-motion among the fast-moving stars in group X. On account of the small number of objects in this group, the existence of stream-motion can hardly be regarded as established in this case. The elongation of the velocity-ellipsoid is mainly due to two stars, Numbers 289 and 560 in *Cincinnati Publications*, No. **18**. The highest velocity known, on the other hand, is that of the two stars A. Oe. 14318 and 14320, which have a common motion of about 900 km/sec. toward the apex $L=259^\circ$, $B=-17^\circ$. These two stars were omitted in the present study, but their inclusion would increase the dispersion along the y'-axis. Regarding them as a single star, we thus find the following elements for group X:

$$c_1=-100 \qquad c'_1=-8 \qquad a=205$$
$$c_2=-278 \qquad c'_2=-299 \qquad b=130$$
$$c_3=-30 \qquad c'_3=-4 \qquad c=96$$

The dispersion is increased from 82 to 130 km, and the group-motion c'_2 from -266 to -299 km. The constants in the relation between displacement and dispersion have been recomputed, using the new elements for group X. Further, the Taurus group, the Ursa Major group, and the B stars have been omitted, as they form separate moving groups. The new relation is

$$c'_2=-0.0311b^2-6.2\ ,$$

which differs only slightly from equation (5). The velocity-ellipsoid for group X is now less elongated, however.

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Using a value of $y'_0=300$ km/sec., we can determine the corresponding value of k_2 from equation (4). We thus find

$$k_2=0.00741 \text{ sec./km}$$

The corresponding values of k_1 and k_3 are arbitrary, but must be smaller than h_1 and h_3 for group X, which, if we use the revised values for the dispersion, are 0.00345 and 0.00737, respectively. It is possible that the three k's are all alike, but we have not enough data to determine their values. The values we adopt for k_1 and k_3 do not, however, affect the values of x'_0 and z'_0, which have been previously given.

PHYSICAL ASPECTS OF THE ASYMMETRY

The motions of the brighter stars have been well explained by Jeans[1] through the gravitational action of the known stars in the local or "Kapteyn" universe. As a general relationship exists between the mean stellar velocity, on the one hand, and the density and the dimensions of the gravitating system, on the other, we can readily see that, in order to account for the existence of stars of very high velocity through gravitational action of the system as a whole, we must include in our study a system of much larger dimensions.

The direction of the asymmetry is nearly in the galactic plane and nearly perpendicular to the direction toward the center of the system of globular clusters as determined by Shapley.[2] This center is in the direction of about 325° galactic longitude, whereas the longitude of the translation of our local system relative to fast-moving stars and distant objects is about 71°. The velocity of translation is about 300 km/sec., and it is possible that the local system moves around the center of the larger system with this velocity. If there exists any appreciable interaction by encounters between stars belonging to the two systems, the ultimate effect would be an increased dispersion perpendicular to the relative motion and a retardation of the systematic motion. As the sum-total of the energy must remain constant, an increase in dis-

[1] *Monthly Notices*, **82**, 122, 1922.

[2] *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, **48**, 170, 1918.

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persion must correspond to a definite decrease in group-motion until the latter becomes zero.[1] A disintegration of the local system would then occur, the stars escaping through the fan-shaped rear part of the moving system. Among the fast-moving stars we should then expect to find a mixture of field-stars and stars belonging to the local system. The stars of very high velocity can readily be detected by their large proper motion (even for fairly distant objects) combined with very large radial velocities. It is very doubtful, however, whether the number per unit volume is large enough to produce an appreciable asymmetry in the velocity-distribution, without using a time-scale almost inconceivably great.

The interaction between the larger universe and the smaller system may not necessarily be due to encounters between stars. Stars, like the sun, are probably surrounded by dust and emit corpuscles, and this system of fine particles must statistically be at rest with reference to the system from which it has originated. The system of small particles which has originated from the larger system may have an effect upon the internal motions in the smaller system, but the effect would probably be altogether too small to account for the observed phenomenon.

Another possibility is that the connection between the larger and the smaller systems is not of a gravitational or material nature, but represents a velocity-restriction in a world-frame that coincides with the large system of clusters and spirals. The restriction may then be either an effect of the matter in this frame, or an absolute property of the frame itself. It is a remarkable fact that even the most distant clusters and spirals have velocities that are small compared with that of light. The small velocities are ordinarily supposed to be due to initial conditions, in general the assumption being made that the relative initial velocities were small or zero, or that an exchange of kinetic energy has taken place and produced a distribution-law of velocities relative to the system as a whole.

[1] It is interesting to note that the mean square velocity, $a^2+b^2+c^2$, for a star in group X is of the same order of magnitude as the mean square velocity of the first groups in Table I if their motion is referred to the centroid of group X. The ratio of the squares is 55:71. The mean square velocity of a star belonging to any of the groups I to XI can thus be made nearly the same if referred to the larger frame. The mean kinetic energy varies considerably, however.

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But we are far from sure that all stars, clusters, and nebulae have some time or other been more intimately connected than they are now. The fact that their velocities are limited, and that the velocity system is practically the same for objects several hundred thousand light-years apart, may be an effect of a velocity-restriction in a frame that can be statistically determined by their present velocities. The question whether it is a property of gravitating matter in space, of space itself, or of the ether is probably intimately connected with the similar question concerning the origin of inertia. In Einstein's elliptical space-time we have a fundamental system of space-co-ordinates in which all the matter in the universe is permanently at rest in a statistical sense. The result of the present study may indicate that it is relative to this world-frame that the velocity-restriction occurs.

These hypotheses as to the physical significance of the asymmetry are suggested only to arouse discussion about the remarkable distribution of cosmic velocities. In both it is supposed that the individual stars are connected both with the local system and with a world-frame. The nature of the first of these connections must almost of necessity be one of origin, combined with the effect of encounters, especially since we have found that the symmetry in the velocity-distribution can be restored by the introduction of the second connection. The nature of the second connection is very uncertain. The principal result of this study is that we have found reason to believe that an additional connection exists, and that this connection is with a cosmical system of enormous dimensions.

Mount Wilson Observatory
November 1923

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