THE RADIAL VELOCITIES OF LONG-PERIOD VARIABLE STARS[1]

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ABSTRACT

Radial velocity observations of 112 long-period variables and five irregular variables have been secured at Mount Wilson since 1919 in continuation of the program begun at Ann Arbor in 1913. The observing list consisted of Md variables having maximum magnitudes brighter than 9.0. Most of the stars were observed only near the time of maximum. The *instruments* employed were one-prism spectrographs having a dispersion at Hγ of 36 A per mm. They were attached to the 100-inch Hooker telescope or the 60-inch telescope.

Measurements of the spectrograms were made by the usual method of micrometer bisections of the lines. The *emission-line velocities* depend largely upon Hγ and Hδ, but several other lines were used when available (Table I). The *absorption-line velocities* are based primarily upon the low-temperature lines of several metals. The wave-lengths were revised and additional lines added from measurements of about 50 stellar spectrograms, the new list (Table II) being used throughout. Mount Wilson measurements of the emission-line velocities for 112 stars and of the absorption-line velocities for 43 stars are given in Table III.

Discussion of results for individual stars.—Velocities at different maxima are probably the same within errors of measurement. A slight variation of emission-line velocity with phase is shown by R Leonis, R Virginis, X Ophiuchi, χ Cygni, T Cephei, and possibly by other stars. The velocities appear to have algebraically low values for a month or two after maximum light. Stars measured at more than one observatory are listed in Table IV. Collected radial velocity data for 133 stars are given in Table V together with periods and new estimates of spectral type. Forty-seven stars have *radial velocities from both bright and dark lines.* The differences of these velocities, with other data of interest, are tabulated in Table VI.

Statistical studies.—The relative displacements of the bright lines are found to increase on the average with advancing spectral type and increasing period and range. The correlation with period is used for establishing an empirical correction to be applied to the bright-line velocities to reduce them to a dark-line basis, since the displacements of the dark lines rather than those of the bright lines appear to correspond to the true radial velocities. Curves showing the *relationship between period and relative displacement* for classes Me and Se are reproduced in Figure 2. The absorption-line velocities, either measured directly or found from the bright lines by use of the curves just mentioned, are made the basis for studies of the *apparent solar motion.* The speed of the sun is almost three times that usually found for K and M stars, but the position of the apex is nearly the same. The following values are representative: $A_0=281°$; $D_0=+34°$; $V_0=53$ km; $K=+1$ km; arithmetic mean residual, 31 km. Sixty-eight stars with residuals less than 25 km give $V_0=48$ km, and 65 with larger residuals, $V_0=65$ km. This increase in V_0 furnishes an excellent illustration of the well-known velocity asymmetry of high-speed stars. The average *residual radial velocity* is found to decrease with advancing spectral type and increasing period. Very high velocities are largely confined to stars of types M2e to M5e and to stars having periods in the neighborhood of 200 days.

A very brief *general discussion* of the properties of variables which show some degree of interdependence, and of the general evolutionary problems concerning these stars, is included.

[1] *Contributions from the Mount Wilson Observatory,* No. 264.

INTRODUCTION

The spectroscopic observations of long-period variable stars, made by the writer at Ann Arbor during the years 1913–16,[1] and at Mount Wilson since 1919, were planned with two principal objects in view. One was the measurement of the radial velocities of a considerable number of variables, upon which could be based a determination of the apparent solar motion and average peculiar motion for comparison with similar data relating to other stars. The possible binary character of these variables, and the interpretation of the relative displacement of bright and dark lines,[2] were, of course, to be considered in this connection. The second object was a study of the physical and chemical conditions which prevail in the long-period variables, with especial attention to changes which occur as the brightness varies.

The determination of radial velocities has thus far had the chief place on my observing program, and has now reached such a stage that it seems wise to collect and discuss the available data, as they are sufficiently numerous to yield fairly satisfactory results for several statistical inquiries. Should further observations of velocity be undertaken, the present discussion will serve to suggest how the work may most profitably be extended. The emission lines of many faint variables could still be observed with the Hooker telescope with exposure times of two or three hours. A few rather bright variables remain unobserved for the reason that during recent years the time of maximum brightness has nearly coincided with that of conjunction with the sun. Moreover, numerous stars already observed could profitably be made the subjects of further study. The total number of long-period variables now listed[3] is about 600, including 150 whose periods are not definitely known, but which are probably long, although some may be irregular. The spectra of 460 of these have been recorded: 415 are of class M, 385 having bright lines; 25 are of class N; and 16 of class S.

Prior to 1916 the radial velocities of five long-period variables of classes M and S had been published. The writer's observations

[1] *Publications of the Astronomical Observatory, University of Michigan*, **2**, 45, 1916.

[2] Previously recognized by other observers in the spectrum of *o* Ceti and of χ Cygni.

[3] *Harvard Annals*, **56**, 197 (Table IX), 1912.

at Ann Arbor increased the number to 43. Since 1916 Paddock has reported[1] the velocity of T Centauri, and the present investigation adds measurements of 89 variables, making 133 in all.

THE OBSERVATIONS

The observing program for the present investigation, as well as that for the previous work at Ann Arbor, was based on the list of variables in *Harvard Annals*, **56,** 197 (Table IX), 1912. It included Md variables having maximum magnitudes brighter than 9.0. There are 264 such objects in the Harvard list, of which 201 are north of declination $-30°$; over the whole sky 122 Md stars with maximum magnitudes of 9.0 or fainter are known.

The formation of the program for each night's observation has required much more attention than would ordinarily be necessary in the investigation of a group of stars of a certain spectral type. The faintness of these variables during the greater part of their light-cycles made it essential that nearly all of them be observed within a few weeks of the maximum phase, and after a few dozen of the brighter variables had been observed and eliminated from the program, the number available for observation on a particular night was often surprisingly small.

The predicted times of maximum in *Harvard Circulars*, Nos. 212, 220, 222, and 227, served as a general guide for the selection of stars for each night's work. As the light-curves are not uniform, however, the actual time of maximum is likely to deviate somewhat from the predicted time, and, moreover, the maximum brightness often differs very considerably from the average maximum value. Accordingly, in order to make spectroscopic work with the large reflectors as effective as possible, it was necessary to rely on a considerable number of current magnitude determinations. Usually it was not feasible for the writer to make these at Mount Wilson, but at the suggestion of Professor Bailey a very satisfactory arrangement was made by which predicted magnitudes of selected stars were sent each month from the Harvard College Observatory. These predictions were by Mr. Leon Campbell from current observations made at Cambridge and by members of the American

[1] *Lick Observatory Bulletins*, **9,** 68, 1917.

Association of Variable Star Observers, which is rendering highly important service by systematic observations of a large number of long-period variables. The photometric data in Table III were also supplied by Mr. Campbell. It is a pleasure to express my thanks to Professor Bailey, Professor Shapley, and especially to Mr. Campbell, for their kind co-operation, which greatly facilitated the present investigation.

Practically all of the radial-velocity observations in the present investigation were made with two single-prism spectrographs having camera lenses with focal lengths of 18 inches. One of these, which has been described by Mr. Adams,[1] is attached at the Cassegrain focus of the 60-inch reflector, and the other, which has nearly the same optical dimensions, is attached at the Cassegrain focus of the 100-inch Hooker reflector. The dispersion at various points in the spectrum is as follows: at Hβ, 56.6 A per mm; at Hγ, 36.1 A per mm; at Hδ, 28.2 A per mm. Nearly all of the photographs were on the Seed 30 emulsion. The slit-width was usually 0.05 or 0.06 mm. In connection with the 40-inch collimator lens this gave a satisfactory degree of purity.

With the Hooker telescope, spectrograms can usually be obtained in not more than half the time required with the 60-inch telescope. The importance of this gain in speed lies not so much in the reduction of the total exposure time on a long program as in the fact that most of the long-period variables are bright enough for observation during only a few weeks of each year, and must often be photographed, if at all, at large hour angles with exposure times not exceeding one or two hours.

As is well known, the color of the long-period variables is orange or red; the color-index of the Md stars is usually taken as 1.8 magnitudes. Compared with the visual brightness, the blue and violet light is very weak, and the continuous spectrum to the violet of λ 4500 is relatively difficult to photograph. On many plates only the bright hydrogen lines Hγ and Hδ are measurable, these lines usually being so strong that they can be photographed in 5 to 20 per cent of the time required for the adjacent continuous spectrum. The bright Hγ and Hδ lines of a tenth-magnitude star

[1] *Mt. Wilson Contr.*, No. 59; *Astrophysical Journal*, **35**, 163, 1912.

can ordinarily be photographed with the Hooker telescope in two hours or less. Stars fainter than the ninth magnitude, however, were seldom observed except for special reasons. Under average conditions an exposure of two hours on a star of visual magnitude 9.0 usually yields a plate with the bright hydrogen lines strong, and with the continuous spectrum sufficiently well recorded between λ 4500 and λ 5000 to allow a good estimate of the spectral type, but too weak in the region λ 4100 to λ 4500 for satisfactory measurement of the absorption lines.

I desire to express my appreciation of the efficient aid rendered in the observing by all the night assistants who have taken part in it. Those who have had the largest share in the work are Messrs. William Klemann and W. P. Hoge.

THE VELOCITY MEASUREMENTS

The velocity determinations have been carried out by the method of micrometer bisections of the lines on the spectrograms, with measuring machines of the usual type. All of the plates have been measured twice and a small number three or four times. Different measurements of the same plate usually agree well. Altogether about 800 plate measurements have been made, 60 per cent of them by the writer, and nearly all of the remainder by Miss Florence MacCreadie, Mr. T. S. Jacobsen, and Miss Cora G. Burwell. An interval of at least three months was allowed to elapse between two measurements of a plate by the same person.

Velocities from emission lines.—The velocities from the emission lines depend on the laboratory wave-lengths in Table I. The last two columns, which give, respectively, the numbers of plates upon which each line has been measured at Mount Wilson for the present investigation, and at Ann Arbor, show that Hγ and Hδ have been used for the velocity measurements far more frequently than any of the other lines. Hβ and Hζ have occasionally been omitted, even when visible on the plates, on account of poor focus. A spectrograph adjusted for the λ 3900 region and having optical parts more transparent in this region than those employed in the present investigation would give a greater relative frequency for the Hζ and Hη lines.

The bright lines offer very definite marks for the setting of the micrometer wire as they have been noticeably broadened in very few instances. The agreement between individual lines is generally good,[1] and the velocities for each plate are more accurate than one might expect from the small number of lines. Each bright line has as much weight for a velocity determination as four or five average absorption lines in the same stars.

Velocities from absorption lines.—A preliminary table for the absorption lines was formed by taking the available laboratory

TABLE I

LABORATORY WAVE-LENGTHS OF EMISSION LINES

I. A.		Number of Stellar Spectrograms	
		Mt. Wilson	Ann Arbor
3835.36	Hη	10	4
3889.05	Hζ	15	24
3905.51	*Si*	9	5
3970.08	Hε	5	0
4101.74	Hδ	316	115
4202.03	*Fe*	57	5
4307.91	*Fe*	41	0
4340.47	Hγ	345	113
4571.11	*Mg*	38	0
4861.33	Hβ	133	35

measurements of the low-temperature lines of *Ca*, *Cr*, *Fe*, *Mg*, *Mn*, *Sr*, *Ti*, and *V*. After about fifty stellar plates had been measured, the lines which had been used less than five times were rejected. The velocities, residuals, and probable errors were then computed for the remaining lines, and the list of wave-lengths was further revised as follows: (1) All lines showing a probable error for a single plate greater than 0.10 A were rejected; (2) wave-lengths of the remaining lines were corrected by amounts corresponding to the average residuals when these exceeded two and a half times their probable errors, otherwise the original laboratory values were retained; (3) lines not in the preliminary table, but measured on five or more plates were added if the probable error from a single plate was not in excess of 0.08 A.

[1] *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, 58, 195, 1923.

These rules obviously favor the original laboratory wave-lengths rather than those obtained by measurement of stellar spectrograms. This was thought desirable in order that the velocities might depend as directly as possible upon laboratory wave-lengths, and not be unnecessarily subject to systematic errors introduced through the use of wave-lengths derived from a limited number of stellar measurements. This "Second Revised Table of Absorption Lines," containing sixty-five lines from λ 4026 to λ 4580, is the basis of the

TABLE II

WAVE-LENGTHS OF ABSORPTION LINES—CLASS ME

(*Second Revised Table*)

I. A.	Element	I. A.	Element	I. A.	Element
4026.88	....	4147.68	*Fe*	4325.93	(*Fe*)
4030.85	(*Mn*)	4149.78	....	4330.02	*V*
4033.07	*Mn*	4179.62	(*V*)	4337.56	*Cr*
4034.58	(*Mn*)	4200.07	....	4344.43	(*Cr*)
4045.87	(*Fe*)	4206.70	*Fe*	4347.27	....
4054.86	....	4215.70	(*Sr*)	4375.93	*Fe*
4077.85	(*Sr*)	4226.82	(*Ca*)	4379.39	(*V*)
4090.50	*V*	4234.08	....	4383.64	(*Fe*)
4092.53	(*V*)	4254.35	*Cr*	4384.19	(*Fe, V*)
4105.03	....	4274.80	*Cr*	4389.44	(*Fe*)
4109.67	(*V*)	4282.62	(*Fe*)	4389.60	(*Fe, V*)
4111.76	*V*	4285.86	(*Ti*)	4395.22	*V*
4115.16	*V*	4287.49	(*Ti*)	4404.82	(*Fe*)
4116.58	(*V*)	4289.57	(*Cr*)	4408.24	(*V*)
4118.58	....	4291.47	*Fe*	4408.42	*Fe*
4121.65	....	4294.28	(*Fe*)	4412.22	....
4123.59	(*V*)	4296.04	....	4427.30	*Fe*
4128.00	*V*	4299.03	....	4455.39	....
4129.85	....	4300.79	....	4461.94	(*Fe?*)
4131.97	*V*	4302.63	....	4482.09	....
4134.40	(*V*)	4306.08	....	4580.29	....
4139.93	*Fe*	4307.76	(*Fe*)		

absorption-line velocities in Table III. It is reproduced in Table II. Lines having the chemical identification in parentheses are those for which slight changes from the laboratory values have been introduced as outlined above. Lines without identifications were derived directly from the stellar spectrograms.

When all the measurements had been completed, the residuals were again formed and the process of correcting the table was repeated. This led to the list printed as Table IV in *Mt. Wilson Contribution* No. 265. A logical step would have been to re-reduce all the plates, using this final table, but this was not done because the resulting changes would probably have been too small to justify the additional labor. Table IV in *Contribution* No. 265 is, of course, the one recommended for future measurements of the spectra of long-period variables of class M, having dispersion comparable with that used in this investigation.[1]

Mount Wilson radial-velocity data.—Table III gives the data for the radial velocities of 111 long-period variables and six irregular variables observed at Mount Wilson. The first column contains the name of the star and the Harvard designation, of which the first four figures indicate the hours and minutes of right ascension for 1900, and the last two figures the degrees of declination, numbers in italics representing southern declinations. Dates of observation are given in the second column. Spectrograms on dates marked with an asterisk were made with the 60-inch telescope, all others with the 100-inch telescope. The column headed "Phase" gives the number of days before (−) or after (+) the nearest maximum. In the column "Absorption Velocity" are given the measured velocities in kilometers per second, and the number of lines. The last column, "Emission Velocity," gives the measured velocities and the particular lines used, the Greek letters referring to hydrogen lines in the Balmer series. The individual velocities are printed to the nearest kilometer only, but in forming the means the original values to a tenth of a kilometer were used.

[1] 36 A per mm at Hγ.

TABLE III

RADIAL-VELOCITY OBSERVATIONS OF LONG-PERIOD VARIABLES AT MOUNT WILSON

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
S Sculptoris *001032*......	1921 Nov. 12	7.9	−31	....		+11	γδ
	14	7.8	−29	....		+15	γδ
						+13.2	
X Androm. 001046......	1921 Nov. 13	9.5	−16	....		−16	βγ
	14	9.5	−15	....		−20	βγ
						−18.2	
T Androm. 001726......	1919 Sept. 3	9.0	+21	−90	8	−95	γδζ
	1921 Jan. 29*	8.5	−9	....		−95	γδ
				−90		−94.8	
T Cassiop. 001755......	1919 Aug. 29	9.1	−132	−15	19	−27	γδζ 3905
	31	9.0	−130	....		−26	δ
	Sept. 5	8.9	−125	(−9)	23		
	7	8.8	−123	....		−22	δ
	Oct. 3*	8.2	−97	....		−26	δ
	14	8.1	−86	−7	32	−22	γδ
	Nov. 9	8.3	−60	−13	15	−25	γδ 4202
				−11.0		−24.6	
Androm. 001838......	1919 Oct. 3*	7.5	−25	....		−44	βγδ
	15	7.5	−13	−6	11	−34	βγδ
	16	7.5	−12	−9	23	−31	βγδ
				−8.2		−36.3	
S Ceti *001909*......	1922 Nov. 6	9.0	−42	....		+20	γδ
U Cassiop. 004047......	1921 Nov. 12	8.2	−4	−45	21	−54	βγδ
	13	8.2	−3	....		−54	βγδ
	1922 Sept. 8	9.3	+19	....		−61	βγδ
						−56.6	
V Androm. 004435......	1923 Jan. 6	9.1	+16	....		+8	βγδ
	7	9.2	+17	....		+8	βγδ
						+8.0	
S Cassiop. 011272......	1922 Jan. 13*	8.9	+22	....		−54	βγ
Y Androm. 013338......	1920 Jan. 9	8.6	−1	....		−16	γδ
	16	8.6	+6	....		−17	γ
						−16.8	
U Persei 015354......	1922 Feb. 12	7.9	−2	+15	21	+9	βγδ
	13	7.9	−1	....		+11	γδ
						+9.9	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
R Arietis							
021024......	1921 Oct. 11	7.9	− 16	+114	24	+106	γδ
	12	7.9	− 15	+116	12	+101	βγδ
				+114.4		+103.5	
W Androm.							
021143*a*.....	1920 July 31	7.9	− 19	− 22	8	− 44	γδ
	31	7.9	− 19	− 33	10	− 47	γδ
	31	7.9	− 19	....		− 47	γδ
	1921 Oct. 12	8.5	+ 33	− 29	20	− 43	γδ 4202
				− 28.5		− 45.0	
R Ceti							
022000......	1919 Sept. 4	8.0	− 2	+ 46	23	+ 33	γδ
	6	8.0	0	+ 39	21	+ 30	γδ
	8*	8.1	+ 2	....		(+ 28)	γ
				+ 42.5		+ 31.6	
U Ceti							
022813......	1920 Dec. 27	7.6	+ 18	....		− 37	γδ
	29*	7.7	+ 20	....		− 45	βγδ
	1922 Nov. 6	7.4	− 6	− 27	18	− 35	γδ
	6	7.4	− 6	....		− 41	γδ
						− 39.4	
R Trianguli							
023133......	1919 Oct. 14	7.0	− 30	+ 68	45	+ 60	γδ
	15	7.0	− 29	+ 67	40	+ 59	γδ
	17	6.9	− 27	+ 66	39	+ 60	βγδ
				+ 66.7		+ 59.6	
R Persei							
032335......	1919 Oct. 14	8.6	− 18	− 82	10	− 90	βγδ
	16	8.5	− 16	− 74	16	− 88	βγδ
				− 78.2		− 89.1	
T Eridani							
035124......	1921 Sept. 22	8.9	+ 32	....		+ 36:	γδ
	23	8.9	+ 33	....		+ 33	βγδ
						+ 34	
W Eridani							
040725......	1921 Nov. 14	8.6	−36±	....		+ 14	γδ
	Dec. 15	8.5	− 5±	....		+ 9	γδ
						+ 11.4	
R Tauri							
042209......	1920 Oct. 27	8.7	− 13	....		+ 18	γδ
	28	8.7	− 12	....		+ 20	γδ
						+ 19.2	
T Camelop.							
043065......	1922 Aug. 9*	8.8	+ 64	....		− 19	βγ
X Camelop.							
043274......	1919 Sept. 9*	8.0	− 13	....		− 2	βγδ
	9*	8.0	− 13	....		− 4	βγδ
	Oct. 3*	7.8	+ 11	....		− 11	βγδ
						− 5.8	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity Absorption	Velocity Emission	
T Leporis *050022*......	1922 Feb. 12	8.1	+22	....	− 21	γδ
	1923 Feb. 6	8.6	+ 1	....	− 14	γδ
					− 18.	
V Orionis 050003......	1919 Oct. 15	8.8	+ 9	....	+ 13	βγδ
	16	8.8	+ 10	....	+ 17	βγδ
	17	8.8	+ 11	....	+ 12	βγδ
					+ 13.9	
R Aurigae 050953......	1917 Nov. 23*	8.1	+ 35	....	− 18	γδ
	1921 Sept. 20*	9.0	+ 20	....	− 13	γδ 4202
					− 15.1	
T Columbae *051533*......	1921 Nov. 12	8.0	+ 8	....	+ 52	γδ
	13	8.0	+ 9	....	+ 55	γδ
					+ 53.4	
U Orionis 054920......	1919 Sept. 4	8.0	+ 65	− 17 21	− 33	γδζη 4202, 3905
	1920 Sept. 2	7.0	+ 49	....	− 41	γδ 4202
	26	8.0	+ 73	....	− 39	γδ 4202
	Oct. 26	8.6	+103	....	− 36	γδ 4571, 4308, 4202
					− 37.2	
X Aurigae 060450......	1919 Dec. 12*	8.7	− 24	....	− 24	γ
	13*	8.6	− 23	....	− 28	γδ
					− 26.2	
X Gemin. 064030......	1920 Apr. 8	8.6	+ 25	....	+ 68	γδ
	10	8.7	+ 27	....	+ 66	γδ
					+ 67.2	
X Monocer. *065208*......	1919 Nov. 9	7.4	+ 32	+157 15	+152	γδ
	13*	7.5	+ 36	....	+146	γ
	14*	7.6	+ 37	....	+142:	γ
					+147.5	
R Gemin. 070122*a*.....	1919 Oct. 3*	7.4	− 27	....	− 58	βγδ
	14	7.0	− 16	....	− 54	βγδ
	15	7.0	− 15	....	− 56	βγδ
	17	7.0	− 13	....	− 56	βγδ
	1920 Sept. 28	7.2	− 30	....	− 52	βγ
	Dec. 27	8.3	+ 60	....	− 60	βγδ
	1921 Sept. 21	8.1	− 50	....	− 58	βγ
	Oct. 10*	7.2	− 31	....	− 62	βγδ
	Nov. 13	7.2	+ 3	....	− 59	βγδ
	Dec. 15	7.3	+ 35	....	− 55	βγδ
	1922 Nov. 6	6.6	+ 10	− 36 29	− 56	γδ
	1923 Jan. 6	7.6	+ 71	....	− 62	βγδ
					− 57.4	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity		
				Absorption	Emission	
R Can. Min. 070310......	1920 Mar. 3	8.0	− 10	....	+ 42	βγδ
	4	8.0	− 9	....	+ 34	βγδ
	7*	8.0	− 6	....	+ 32	βγ
	1921 Feb. 25	8.3	+ 20	....	+ 27	βγδ
	Mar. 26	8.4	+ 49	....	+ 26	βγ
	1922 Jan. 12	8.5	− 33	....	+ 33	βγ
	Dec. 8	8.1	− 42	....	+ 41	βγδ
					+ 33.3	
V Gemin. 071713......	1920 Feb. 7	8.5	− 18	....	+ 14	γδ
	Mar. 3	8.4	+ 7	+ 22 12	+ 7	βγδ
					+ 10.6	
S Can. Min. 072708......	1922 Apr. 13	9.6	− 72	....	+ 54	δ
	14	9.6	− 71	....	+ 56	δ
					+ 54.6	
Z Puppis *072820b*.....	1921 Feb. 26	7.9	+ 22	....	+ 13	γδ
	27	7.9	+ 23	....	+ 12	γδ
					+ 12.6	
T Gemin. 074323......	1921 Mar. 28	9.1	− 23	....	+ 15	βγδ
	Apr. 29	8.6	+ 9	....	+ 12	βγδ
	1923 Jan. 8	9.9	+ 65	....	+ 4	βγ
					+ 11.8	
R Cancri 081112......	1920 Mar. 4	7.0	− 36	+ 38 37	+ 23	γδ
	5	7.0	− 35	+ 38 32	+ 24	γδ
	5	7.0	− 35	+ 38 19	+ 26	γδ
	6*	6.9	− 34	+ 27 30	+ 12	γδ
	7*	6.9	− 33	....	+ 13	γδ
	7*	6.9	− 33	....	+ 16	γδ
	1921 Mar. 26	7.6	− 13	....	+ 17	γδ
	Apr. 26*	8.5	+ 18	....	+ 16	γδ
				(+ 32.1)	+ 18.4	
V Cancri 081617......	1919 Nov. 9	8.0	+ 10	....	− 6	βγδ
	13*	8.0	+ 14	....	− 17	βγδ
	14*	8.0	+ 15	....	− 13	βγδ
	1921 Apr. 27*	7.9	− 3	....	− 17	βγδ
	1922 Jan. 13*	7.9	− 12	....	− 16	βγδ
	Feb. 11	7.8	+ 17	....	− 11	βγδ
					− 13.5	
RT Hydrae *082405*......	1919 Dec. 13*	9.0	−100	....	+ 34:	γ
	1920 Mar. 5	8.0	− 17	+ 40 17	+ 36	γδ
					+ 35	
U Cancri 083019......	1921 Oct. 13	11.3	+ 44	....	+ 68:	γδ
	14	11.3	+ 45	....	+ 59	βγδ
					+ 61	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
X Urs. Maj. 083350......	1921 Mar. 28	9.2	0	....		−90	βγδ
S Hydrae 084803......	1920 Mar. 6*	8.0	−11	....		+62	βγδ
	1922 Apr. 14	8.2	−21	....		+70	βγδ
	1923 Jan. 5*	7.7	−3	....		+66	βγδ
						+66.1	
W Cancri 090425......	1921 Nov. 13	8.5	−24	....		+36	γδ
	14	8.4	−23	....		+33	γδ
						+34.7	
R Leo. Min. 093934......	1922 Feb. 12	7.8	−30	+8	47	+1	γδ
	Mar. 18	7.3	+4	....		−2	γδ
	18	7.3	+4	+12	19	−3	γδ
				+9.5		−1.5	
R Leonis 094211......	1920 Jan. 16	9.0	−59	....		+3	δ
	Apr. 8	6.4	+24	+12	19	−2	γδ 4202
	11*	6.5	+27	....		−4	γδ 4202
	11*	6.5	+27	....		−5	γδ 4202
	May 5*	7.4	+51	+16	14	−2	γδεζη 4571, 4308, 4202, 3905
	11*	7.6	+57	+16	10	−1	γδεζη 4571, 4308, 4202, 3905
	30	8.3	+76	....		+2	γδ 4571, 4308, 4202
	Dec. 26	5.8	−15	+14	47	0	γδ
	1921 Jan. 28	6.2	+18	+10	42	−2	γδ
	Feb. 25	6.9	+46	+11	31	+1	γδεζη 4202, 3905
	26	6.9	+47	....		+1	γεζηθι 3905
	Mar. 26	7.7	+75	....		−2	γδ 4571, 4308, 4202
	27	7.7	+76	+18	14	+3	γδεζη 4571, 4308, 4202, 3905
	Apr. 28	8.9	+108	....		+2	γδ 4571, 4308, 4202
	1922 Dec. 8	7.2	+71	....		+2	γδ 4202
				+13.8		0.0	
V Leonis 095421......	1920 June 1	9.0	−13	....		−29	γδ
	2	9.0	−12	....		−32	γδ
						−30.8	
Z Urs. Maj. 115158......	1920 Mar. 3	....	......	−53	34	−60	γδ
	4	....	......	−52	14	−57	γδ
				−52.6		−58.4	
R Comae 115919......	1919 June 9*	7.9	−7	....		−22	γδ
	10*	7.9	−6	....		−22	γδ
						−22.0	
T Urs. Maj. 123160......	1922 Jan. 13*	8.8	+8	....		−102	γδ

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption	Velocity: Emission
R Virginis 123307......	1920 May 1	9.1	− 41	....	− 26 γδ
	1922 Feb. 12	8.5	+ 32	− 34 22	− 43 βγδ
	May 15	9.0	− 32	....	− 24 γδ
					− 31.
S Urs. Maj. 123961......	1920 Apr. 8	8.5	− 31	....	− 4 βγ
	10	8.5	− 29	....	− 7 βγ
	June 3*	8.2	+ 25	....	− 7 βγ
	Dec. 29*	8.4	+ 25	....	− 10 βγδ
	1921 Jan. 28	8.8	+ 48	....	− 7 βγ
	1922 Feb. 11	8.5	− 28	....	+ 4 βγ
	Mar. 19	7.6	+ 8	....	− 4 βγδ
					− 5.0
U Can. Ven. 124238......	1920 Mar. 5	....	......	....	− 43 γδ
	10	....	......	....	− 44 γδ
					− 43.7
U Virginis 124606......	1920 Mar. 6*	9.1	− 37	....	− 61 γδ
	Apr. 8	8.2	− 4	− 42 16	− 56 βγδ
					− 58.1
RT Virginis 125705*a*.....	1920 Mar. 3	....	......	+ 21 24	
SW Virginis *130802*......	1920 Mar. 4	....	......	− 12 19	
V Virginis *132202*......	1921 Feb. 26	9.5	+ 7	....	+ 23 γδ
	27	9.6	+ 8	....	+ 27 γδ
					+ 25.0
R Hydrae *132422*......	1920 May 31	9.0	−162	....	− 18 4571, 4308
	July 8	7.9	−124	....	(− 22) δ
	1921 Jan. 30*	6.7	+ 82	....	(− 27) γδ
	30*	6.7	+ 82	....	− 18 γδ 4571, 4308, 4202
	30	6.7	+ 82	....	− 25 γδ 4571, 4308, 4202
	Feb. 25	7.7	+108	....	− 24 βγδ 4571, 4308, 4202
	Mar. 27	8.6	+138	....	− 19 βγ 4571, 4308, 4202
	Apr. 29	9.3	+171	....	− 20 4571, 4308, 4202
	May 26	9.6	+198	....	− 21 4571, 4308
	1922 Feb. 13	5.6	+ 52	....	− 21 γδ 4202
	1923 Jan. 4*	5.6	− 38	− 11 46	− 19 γδ
	5*	5.5	− 37	− 8 40	− 21 γδ
				− 9.3	− 21.4
S Virginis *132706*......	1920 Jan. 16	8.0	− 28	....	− 4 δ
	Mar. 4	7.7	+ 20	....	0 γδ
	6*	7.8	+ 22	....	− 10 γδ
	1922 Mar. 20*	7.9	+ 14	....	(− 8) δ
					− 4.7
T Centauri *133633*......	1921 Apr. 28	6.4	+ 20	+ 22 5	
	1922 July 12	6.3	+ 13	+ 27 9	+ 14 βγ
				+ 24.6	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
W Hydrae *134327*......	1921 Feb. 28	(7.3)	....	+ 41	19	+ 26	γδ 4202
	Mar. 28	(6.5±)	....	....		+ 24	γδ 4571, 4308, 4202
	May 24	7	....	....		+ 24	γδ 4571, 4308, 4202
	June 20	8	....	....		+ 25	βγ 4571, 4308, 4202
	1922 Apr. 13	7	....	+ 43	35	+ 29	γδ 4202
	May 15	7.2	....	+ 44	19	+ 27	βγδ 4571, 4308, 4202
	June 11	7.5±	....	....		(+ 39)	γ 4571
	15	7.5±	....	....		+ 28	γδ 4571, 4308, 4202
	1923 Jan. 8	7 ±	....	....		+ 31	γδ
				+ 42.3		+ 26.4	
R Can. Ven. 134440......	1922 Apr. 13	7.7	− 9	− 6	46	− 20	γδ
	14	7.7	− 8	....		− 24	γδ
	May 16	8.4	+ 24	− 12	40	− 25	γδ 4202
				− 8.9		− 23.1	
U Urs. Min. 141567......	1919 Aug. 6*	7.7	+ 9	....		− 44	γδ
	6*	7.7	+ 9	....		− 43	γδ
						− 43.3	
S Boötis 141954......	1919 July 9*	8.8	− 19	....		− 24	βγδ
	10*	8.8	− 18	....		− 26	βγδ
						− 25.0	
RS Virginis 142205......	1919 June 9*	7.8	+ 35	....		− 43	γδ
	10*	7.8	+ 36	....		− 37	γδ
						− 40.0	
R Camelop. 142584......	1919 June 9*	8.2	+ 7	....		− 50	βγ
	10*	8.2	+ 8	....		− 40	βγ
	July 8*	8.9	+ 36	....		− 41	βγ
	1922 May 17*	8.2	− 18	....		− 49	βγ
	1923 Feb. 5*	8.2	− 24	....		− 28	βγ
						− 42.9	
V Boötis 142539......	1919 June 9*	7.8	+ 7	....		− 47	γδ
Y Librae *150605*......	1921 May 25	9.2	+ 13	....		− 18	γδ
	June 21	10.2	+ 40	....		− 9	γδ
	1922 Mar. 18	9.6	+ 45	....		− 15	γδ
						− 15.0	
S Librae *151520*......	1922 Apr. 13	8.5	− 18	....		+288	βγδ
	14	8.5	− 17	....		+283	βγδ
	May 15	8.5	+ 14	....		+283	βγ
						+284.6	
S Serpentis 151714......	1920 July 30	7.8	− 6	....		0	γδ
	31	7.8	− 5	....		− 3	γδ
						− 1.9	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption	Velocity: Emission	
RS Librae *151822*......	1920 July 6	8.0	− 7	....	− 9	γδ
	1921 Feb. 26	7.4	0	....	− 18	γδ
					− 15.1	
RU Librae *152714*......	1921 July 21*	8.5	− 8	....	− 62	βγδ
	24	8.4	− 5	....	− 57	βγδ
					− 59.5	
S Urs. Min. 153378......	1920 Sept. 4*	8.3	− 30	....	− 55	γδ
	1921 Aug. 14*	8.3	− 12	....	− 50	γδ
	15*	8.3	− 11	....	− 53	γδ
					− 52.7	
R Serpentis 154615......	1920 May 1	7.6	+ 46	....	+ 3	γδ
	30	8.8	+ 75	....	+ 6	βγδ 4571, 4308, 4202
	June 1	8.8	+ 77	....	+ 4	βγδ 4571, 4308, 4202
	July 8	10.7	+114	....	+ 3	γδ 4571, 4308, 4202
	1922 Mar. 8	7.1	+ 12	+ 22 44	+ 7	γδ
	1923 Feb. 5*	7.0±	− 9	....	+ 10	γδ
					+ 5.3	
ST Herculis 154748......	1920 Mar. 3	....	....	− 33 40		
RR Librae *155018*......	1920 Apr. 10	9.2	+ 20	....	− 40	γδ
	1922 July 11	8.8	− 12	....	− 41	γδ
					− 40.6	
X Herculis 155947......	1920 Mar. 3	6.9	....	− 92 45		
	4	6.9	....	− 92 40		
	7*	6.9	....	− 91 38		
	1921 Apr. 26*	6.5	....	− 88 35		
				− 90.1		
R Herculis 160118......	1921 June 20	8.6	+ 11	....	− 40	βγδ
	22*	8.7	+ 13	....	− 44	γδ
					− 42.1	
U Serpentis 160210......	1920 Sept. 3	8.3	− 11	....	− 37	βγδ
	4	8.2	− 10	....	− 42	βγδ
					− 39.8	
RU Herculis 160625......	1920 July 6	8.8	− 18	....	− 33	γδ
	7	8.8	− 17	....	− 42	γδ
					− 37.6	
W Cor. Bor. 161138......	1921 Feb. 25	8.4	+ 1	....	+ 12	βγδ
	26	8.4	+ 2	....	+ 9	βγδ
					+ 10.5	
G Herculis 162542......	1920 May 1	5.2	....	+ 2 49		

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity		
				Absorption	Emission	
R Draconis 163266......	1919 July 9*	8.9	− 30	....	−143	γδ
	Aug. 2*	8.0	− 6	....	−151	γδ
	8*	8.0	0	−138 21	−143	βγδ
	Sept. 8*	8.8	+ 31	....	−143	γδ
	9*	8.8	+ 32	....	−142	γδ
	1922 Apr. 16*	7.9	− 8	....	−144	γδ
					−144.2	
RR Scorpii *165030*......	1920 July 31	6.6	− 3	....	− 47	γδ
	1921 Mar. 28	7.4	− 45	....	− 43	γδ
					− 45.1	
SS Ophiuchi *165202*......	1922 Aug. 9	8.7	+ 1	....	− 38	βγδ
	10	8.7	+ 2	....	− 48	βγδ
					− 42.9	
RS Herculis 171723......	1920 Sept. 3	9.5	+ 50	....	− 54	γδ
	4	9.5	+ 51	....	− 48	γδ
					− 50.8	
RY Herculis 175519......	1919 Aug. 11	9.1	− 18	− 39 15	− 50	γδζ
	12	9.1	− 17	− 39 11	− 50	γδ
				− 38.9	− 50.1	
T Herculis 180531......	1922 May 17*	8.1	− 2	....	−136	βγδ
W Lyrae 181136......	1919 Oct. 14	8.5	− 34	−174 44	−180	γδ
	15	8.5	− 33	....	−177	γδ
					−178.5	
RY Ophiuchi 181103......	1922 July 11	8.4	+ 6	....	− 71	βγδ
	12	8.4	+ 7	....	− 72	βγδ
					− 71.8	
X Ophiuchi 183308......	....	....	......	− 70.6	− 83.4	
AE Herculis 183922......	1922 July 7	9.5±	......	....	− 58	βγδ
	12	9.5±	......	....	− 62	βγδ
					− 60.4	
R Aquilae 190108......	1920 Oct. 26	5.9	− 4	+ 30 32	+ 22	γδ
	28	5.9	− 2	....	+ 23	γδ
	1921 Aug. 11	6.3	− 20	+ 33 38	+ 24	γδ
				+ 31.5	+ 22.8	
T Sagittarii *191017*......	1921 June 20	8.5±	+ 39	....	− 19	βγδ
	1922 July 13	9.1	+ 65	....	− 20	βγ
					− 19.4	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption	Velocity: Emission	
R Sagittarii *191019*......	1919 July 8*	7.7	0	....	− 50	βγ
	8*	7.7	0	....	− 54	βγδ
	Aug. 13	8.6	+ 36	....	− 52	γδ
					− 52.3	
R Cygni 193449......	1921 Mar. 26	7.2	+ 30	....	− 49	βγδ
	27	7.3	+ 31	....	− 47	βγδ
	Apr. 28	8.9	+ 63	....	− 45	βγδ 4308
	May 26	9.2	+ 91	....	− 44	βγδ 4571, 4308, 4202
	June 19	9.7	+115	....	− 44	βγδ 4571, 4308, 4202
	July 23	10.6	+149	....	− 42	βγ 4571, 4308, 4202
	1922 May 17*	6.8	+ 4	....	− 49	βγδ
	June 14*	8.0	+ 32	....	− 51	βγδ 4308
	July 11	8.9	+ 59	....	− 44	βγδ 4308
					− 46.2	
χ Cygni 194632......	1920 May 30	8.2	− 31	....	− 15	γδ
	June 1	8.1	− 29	....	− 17	γδ
	30*	5.2	− 1	− 1 29	− 21	βγδ
	July 29	5.8	+ 28	− 5 33	− 20	βγδ 4308, 4202
	Sept. 3	6.5	+ 64	0 18	− 18	βγδ 4571, 4308, 4202
	26	8.1	+ 87	....	− 19	βγδ 4571, 4308, 4202
	Oct. 26	9.6	+117	....	− 14	βγδ 4571, 4308, 4202
				− 2.1	− 17.8	
Z Cygni 195849......	1919 Aug. 6*	8.6	− 3	....	−168	γδ
	7*	8.6	− 2	....	−177	γ
	8*	8.6	− 1	....	−177	γδ
	10	8.7	+ 1	....	−173	γδ
					−173.1	
S Cygni 200357......	1920 June 2	8.9	− 2	....	− 34	βγδ
	1921 May 25	9.0	− 10	....	(− 27)	γδ
					− 33.7	
Z Aquilae *200906*......	1919 June 10*	8.8	+ 2	....	(− 1)	βγ
	Oct. 16	8.8	0	....	− 9	γδ
	17	8.8	+ 1	....	− 12	βγ
					− 10.3	
R Delphini 201008......	1919 June 9*	8.5	+ 9	....	− 56	γδ
	1921 Sept. 20*	8.7	− 7	....	− 54	γδ
					− 54.8	
V Aquarii 204102......	1919 Nov. 13*	....	......	....	− 49	γδ
	14*	....	......	....	− 57	γδ
					− 52.9	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption		Velocity: Emission	
T Aquarii 204405	1919 July 9*	7.8	− 16	....		− 58	βγδ
	9*	7.8	− 16	....		− 55	βγδ
	Aug. 6*	7.9	+ 12	....		− 54	βγδ
	11	8.0	+ 17	− 41	12	− 52	γδ
	12	8.0	+ 18	− 36	18	− 54	γδ
				− 38.2		− 54.9	
X Delphini 205017	1921 Oct. 12	8.7	− 9	− 58	21	− 64	γδ
	13	8.7	− 8	− 55	10	− 62	γδ
	13	8.7	− 8	....		− 63	γδ
				− 56.6		− 62.9	
T Cephei 210868	1917 Nov. 3*	7.6	− 74	....		− 29	δ
	Dec. 3*	6.8	− 44	− 9	12	− 21	γδ
	1918 Jan. 2*	5.8	− 14	− 8	16	− 26	βγδζη 3905
	Oct. 24*	7.2	−107	− 12	23	− 21	γδ
	1920 June 3*	8.0	+ 77	....		− 27	γδ 4571, 4308, 4202
	July 5*	8.9	+109	....		− 24	γ 4571, 4308, 4202
	28*	9.4	+132	....		− 19	4571, 4308, 4202
	Sept. 5*	10.1	+171	....		(− 19)	4571, 4308, 4202
				− 9.6		− 24.1	
R Equulei 210812	1921 June 21	9.4±	− 18	....		− 61	γδ
	July 24	9.0±	+ 15	....		− 63	γδ
						− 62.0	
RR Aquarii 210903	1922 Aug. 9	10.0	−34±	....		−187	βγδ
	10	10.0	−33±	....		−191	βγδ
	Sept. 8	9.2	− 4±	....		−195	βγδ
						−191.0	
T Pegasi 220412	1922 July 12	10.0	+ 42	....		− 22	γδ
	13	10.0	+ 43	....		− 26	γδ
						− 23.9	
RS Pegasi 220714	1921 July 24	9.5	+ 22	....		− 38	γδ
	25	9.5	+ 23	....		− 42	γδ
						− 40.0	
RT Aquarii 221722	1921 Aug. 12	9.5±	+ 14	....		− 43	γδ
S Lacertae 222439	1919 Aug. 29	8.3	− 14	− 53	16	− 63	γδ
	31	8.2	− 12	− 61	22	− 67	γδζ
	Sept. 4	8.0	− 8	− 58	40	− 65	γδζη 3905
	5	8.0	− 7	− 61	38	− 66	γδζ
	7	7.9	− 5	− 62	30	− 67	γδζη
				− 59.7		− 65.9	
R Lacertae 223841	1921 Sept. 22	10.0	+ 29	....		+ 10	γδ
	23	10.0	+ 30	....		+ 7	γδ
						+ 8.2	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption	Velocity: Emission
S Aquarii					
225120......	1919 Sept. 4	8.0	− 6	....	− 59 γδ
	9*	7.9	− 1	....	− 67 βγδ
	1922 Nov. 6	9.2	+ 29	....	− 74 γδ
					− 66.4
V Cassiop.					
230759......	1919 Aug. 6*	7.5	+ 2	....	− 53 γδ
	7*	7.5	+ 3	....	− 47 βγδ
	8*	7.6	+ 4	....	− 51 βγδ
	10	7.7	+ 6	− 27 40	− 45 γδζ
	12	7.9	+ 8	− 34 38	− 46 γδζη 4202
	1922 Oct. 10*	7.9	+ 8	− 31 16	− 43 βγδ
				− 30.5	− 47.6
R Aquarii					
233815......	....	....	....	− 19	− 33
V Ceti					
235209......	1921 July 25	8.8	− 4	....	+ 47 βγδ
	Aug. 11	8.8	+ 13	....	+ 40 βγδ
					+ 43.1
R Cassiop.					
235350......	1920 Sept. 27	8.6	+ 57	....	+ 6 γδ
	Oct. 28	9.2	+ 88	....	+ 5 γδ 4202
	1923 Jan. 4*	7.4	+ 5	....	+ 5 γδ
	8	7.5	+ 9	....	+ 13 γδ
					+ 6.3
SV Androm.					
235939......	1921 Sept. 21	9.4	− 6	....	− 97 γδ
	22	9.4	− 5	....	−100 γδ
	23	9.3	− 4	....	−100 γδ
					− 98.8

NOTES TO TABLE III

001755, T Cassiopeiae: On the plate of Aug. 29, 1919, the comparison spectrum is imperfect. On the plate of Sept. 5, 1919, the comparison spectrum is slightly shifted, owing to a change in the temperature of the spectrograph during the exposure. The definition of the stellar spectrum, however, is good and the measured difference of the velocities from the bright and the dark lines is considered reliable. The difference on this plate was found to be 15.5 km, which, when applied to the adopted velocity for the bright lines from the other plates, gives −9.1 km as the velocity from the absorption lines. Bright Hγ is very weak on the first plates, and increases in strength during the series. This is in harmony with the characteristic behavior as outlined in *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921. Hγ does not become a very conspicuous line on any of my plates, all of which, however, were taken well before maximum. In the notes to *Henry Draper Catalogue*, Miss Cannon has remarked upon the lack of strength of the bright Hγ and Hδ lines. The bright line λ 4202 is beginning to appear on the last two Mount Wilson plates. The presence of λ 3905 as a bright line on the first plate is interesting as showing its occurrence in considerable strength so long before maximum. The close bright companion to Hδ on the red side is well marked on most of the plates. Bright λ 4512 is especially strong in this star. The continuous spectrum from Hγ to λ 3900 is surprisingly strong compared with that from Hγ to Hβ.

004047, U Cassiopeiae: On the last plate the titanium bands are decidedly stronger than on the first two, and bright Hδ is stronger relatively to Hγ. These changes in the spectra of class S stars will be discussed more fully in another contribution.

022000, R Ceti: The plate of Sept. 3 is very poor.

065208, X Monocerotis: The bright hydrogen lines are less intense than in most variables. This star has been considered irregular (*Henry Draper Catalogue*, *Harvard Annals*, **92**, 308, 1918), and it was not included in the computations in this paper which relate to the period. A recent Harvard *Bulletin* (No. 787), however, gives the period as 155.3 days, and this value has been entered in Tables V and VI and the star has been added to the plot in Figure 2. The short period is in harmony with the star's high velocity and its spectrum.

074323, T Geminorum: The M-type bands are weaker on the second plate than on the other two. The third plate is poor and the velocity derived from it is assigned half-weight.

081112, R Cancri: The proper value for the mean velocity from the emission lines and especially that from the absorption lines is doubtful to the extent of a few kilometers, owing to the systematic divergence of the first three plates from the others. The discrepancy may be connected with the fact that the first three plates are strongly exposed, the bright hydrogen lines being much overexposed. In general, however, strong and weak exposures do not show any decided systematic differences, although there seems to be some obscure effect, which operates only occasionally, tending in a few instances to cause spectrograms made with the 100-inch telescope to yield algebraically larger velocities than spectrograms of the same star made with the 60-inch telescope. The adopted emission-line velocity for R Cancri is the simple mean of all the individual results; to obtain the absorption velocity, the average value, *absorption minus emission*, for the first four plates, 13.7 km, was added to the mean emission-line velocity.

115158, Z Urs. Maj.: The bright hydrogen lines are weak.

115919, R Com. Ber.: Not in the *Henry Draper Catalogue.*

123307, R Virginis: The apparent variation in velocity is probably larger than the errors of observation. Both the Ann Arbor and the Mount Wilson measurements give the largest negative velocity *after* light maximum. The star has a short period and considerable range in magnitude; hence, if the light variations are dependent on changes of a geometrical nature, we might expect to find in this star an unusually large range in apparent radial velocity.

124606, U Virginis: On the first plate Hβ is not seen, but on the second it is a strong bright line.

125705*a*, RT Virginis: No definite bright lines were seen on the plate of March 3, 1920, or on another, unlisted plate, taken four days later. The continuous spectrum from λ 4030 to λ 4227 is strong compared to that at longer wave-lengths. This relative strength of the continuous spectrum at short wave-lengths was noted by Miss Cannon. A remark in the *Henry Draper Catalogue* states that "The portion of the continuous spectrum between Hδ and Hε appears like a bright band, and the region between Hβ and Hγ is very faint."

130802, SW Virginis: The spectrum is much like that of RT Virginis (see preceding note) but an absorption line at λ 4535 is stronger. There appear to be absorption lines on either side of the position of Hγ. Possibly the narrow space between them should be interpreted as a bright Hγ. If so, it gives about the same velocity as the absorption lines. Remark in the *Henry Draper Catalogue:* "The brightest portion of the spectrum lies between Hε and 4226.9."

132422, R Hydrae: In forming the mean of the velocities from the emission lines, the last five plates were given unit weight; the weights of the others were, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, 2 2, 3, 2, respectively.

133633, T Centauri: Bright lines are seen on the plate of July 12, 1922, only, and are certainly stronger than on the other plates. The continuous spectrum also shows decided changes. On the first plate, which is too weak in the violet for measurement, the titanium bands are strongly marked in the region λ 4500 to λ 5000; the type is estimated as M4–5. The spectrum on the second plate is very similar to that of α Orionis and is of class M2. The absorption spectrum of the third plate more nearly resembles that of α Scorpii and is of class M7, having bright hydrogen lines superposed. Miss Cannon has noted in the remarks in the *Henry Draper Catalogue* that the spectrum varies from K2 having bright hydrogen lines to Ma having no bright lines. A comparison of the Mount Wilson measurements with those by Paddock[1] at Santiago, Chile, suggests that the radial velocity as well as the relative displacement of the bright lines may be variable. An extensive study of the variable spectrum of this star would be valuable.

134327, W Hydrae: The plate of June 11, 1922, was taken under poor conditions and is probably affected by some instrumental error. The velocity from this plate was not used in forming the mean. On the last plate the bright Hγ line is weak, showing that the phase is considerably before maximum. Bright Hδ is strong and the close companion line on the red side is visible. The velocity from this plate was given small weight in forming the mean. This variable has a very small magnitude range, but appears nevertheless to exhibit the changes in spectrum characteristic[2] of the long-period variables. From the behavior of the spectrum in 1921, it is estimated that maximum occurred about February 20. Combined with Chandler's date of maximum, February 27, 1889, the period is found to be 389 or 377 days accordingly as 30 or 31 periods are assumed to have elapsed in the interval. By comparing the spectrograms taken in 1922 with those of 1921 a period of approximately 384 days is found.

142584, R Camelopardalis: The last plate was taken with a wide slit; the velocity from it has half-weight.

150605, Y Librae: The second plate is given one-half weight as the velocities from Hγ and Hδ do not agree.

151822, RS Librae: The first plate is given one-half weight as the velocities from Hγ and Hδ do not agree.

154748, ST Herculis: Bright hydrogen lines are not seen in this spectrum.

155947, X Herculis: Bright hydrogen lines are not seen in this spectrum.

175519, RY Herculis: On the first plate the comparison spectrum is imperfect, but the stellar velocity appears not to be affected.

183308, X Ophiuchi: A double star. See discussion of the velocity in *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

194632, χ Cygni: Bright Hβ is stronger than usual for a star with absorption spectrum of class M6.

200357, S Cygni: The second plate is very poor.

204405, T Aquarii: On the last two plates the comparison spectrum is slightly imperfect. On these plates the continuous spectrum is underexposed and the absorption-line velocities have small weight.

210868, T Cephei: The velocity from the last plate, which was taken with the 7-inch camera, was not used in forming the mean.

222439, S Lacertae: The velocities from the first plate were given half-weight because the comparison spectrum is imperfect.

233815, R Aquarii: In addition to the usual M8e features, the spectrum contains lines characteristic of gaseous nebulae. The velocities are taken from *Mt. Wilson Contr.*, No. 206, p. 4; *Astrophysical Journal*, **53**, 378, 1921.

235350, R Cassiopeiae: The last plate is rather poor and the velocities measured by two observers are discordant. This plate is given one-half weight in forming the mean.

[1] *Lick Observatory Bulletin*, **9**, 69, 1917.

[2] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 1, 1921.

DISCUSSION OF RESULTS FOR INDIVIDUAL STARS

Velocities from emission lines.—An examination of the velocity measurements in Table III, as well as those obtained at Ann Arbor, appears to show that the ranges for individual stars can, in general, be accounted for by errors of observation. This may be inferred from the fact that the agreement of plates of a particular star taken on the same night or on successive nights is not decidedly better than that of plates separated by longer intervals. Moreover, the agreement of several plates of a star is in most instances about as good as could reasonably be expected from the number and internal agreement of the lines on each plate. Some apparent exceptions have been discussed in the notes to Table III, and the subject will be dealt with more fully in the following paragraphs.

In the present investigation and in the similar one previously carried out at Ann Arbor, the effort has been to determine the velocities of as many stars as possible rather than to secure extensive sets of observations of individual stars. Hence, most of the observed stars are represented by a small number of spectrograms taken near maximum. The desirability of testing the constancy of the velocity at different maxima and throughout the light-cycle has been borne in mind, however, and data for this purpose have been secured for a few stars.

The available measures show that velocities at different maxima are nearly the same. In a few instances the observed variations may exceed the errors of measurement, but the data are too meager to establish this as a fact. This question might better be studied with more powerful spectrographs as in numerous stars the bright $H\gamma$ and $H\delta$ lines could easily be photographed at maximum with a dispersion several times that which I have used.

A slight variation of velocity with phase is indicated for a number of stars, and as approximately the same behavior seems to be shown by those stars for which the data are most extensive, the effect is probably real. A study of X Ophiuchi has already been published.[1] The conclusion was that the velocities are not constant but have algebraically low values for a month or two after maximum light. The same statement seems to apply to several other stars.

[1] *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

The Mount Wilson observations of R Leonis, R Hydrae, X Ophiuchi, χ Cygni, and T Cephei are plotted in Figure 1. With

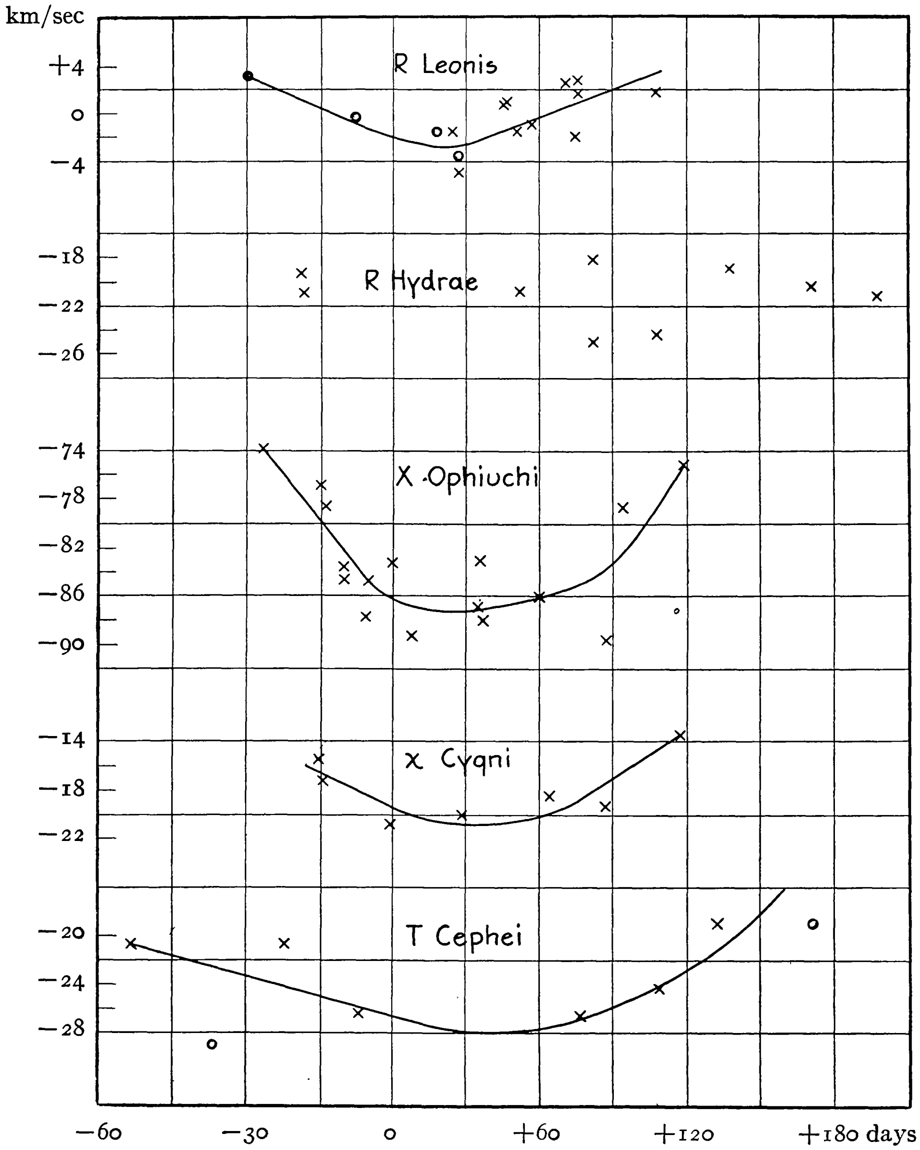

FIG. 1.—Variations in emission-line velocity. Circles represent observations of one-half weight.

the exception of R Hydrae, in which the effect is doubtful, they are better represented by a curve having a flat minimum a few weeks after maximum light than by a straight line. Several other stars observed over shorter intervals show changes in velocity which are in harmony with a similar hypothesis. One of the most striking examples, although based on very few observations, is R Virginis.[1] Another is the S-type star R Canis Minoris.

About the time of maximum light the bright line λ 4202 becomes measurable, followed, as the light decreases, by λ 4308 and λ 4571. As these lines have been used together with the hydrogen lines in the velocity determinations, it is essential that the system of wave-lengths employed be relatively correct or apparent variations may be introduced. The wave-lengths used were the laboratory values indicated in Table I. The mean differences between the velocities from the hydrogen lines and those from the other bright lines have been computed and found to be small, showing that no large systematic errors exist.[2] For individual stars the displacements shown by the lines λλ4202, 4308, and 4571 correspond reasonably well to those of the hydrogen lines on the same spectrograms.

It is not certain that the slight changes in the positions of the bright lines are due to variations in radial motion, but no other explanation presents itself. If the well-known displacement of the bright lines toward the violet with respect to the absorption spectrum represents an outflow of incandescent gas, it is possible that on first coming into view the gas has a low velocity and is subject to acceleration while under observation, or is replaced by other gas having a higher outward velocity, the process being reversed as the incandescent gas begins to disappear. This would indicate the existence of agencies acting during a considerable fraction of the light-period, rather than a sudden outburst followed by a gradual resumption of the previous conditions. The observation of the hydrogen lines is subject to this interpretation, but for the other gases a somewhat different state of affairs appears to exist. The lines λλ 4202, 4308, and 4571 appear later in the light-cycle than the hydrogen lines, but when they first become measurable, they have nearly the same velocities as the hydrogen

[1] See the note concerning this star on p. 235.

[2] See Table II, *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 196, 1923. See also measurements of *o* Ceti by Adams and Joy, *Publications Astronomical Society of the Pacific*, **35**, 168, 1923.

lines on the same plates. Later they agree with the hydrogen lines in showing a tendency toward algebraically larger velocities, as illustrated by the mean curves in Figure 1. A detailed explanation of this behavior is impossible at present, but it seems justifiable to conclude that the appearances of bright lines at different times in the light-cycle do not correspond to separate outbursts of various elements, but are manifestations of successive phases of the same disturbance by which the bright hydrogen lines were produced shortly after minimum.

Velocities from absorption lines.—Not much can be said as to possible variations in the absorption-line velocities, owing to the small number of observations obtained for individual stars. Except for a few weeks near maximum, most of the stars are so faint that it is difficult to photograph the continuous spectrum. Moreover, when the stars are faint, the absorption spectrum may not yield a satisfactory velocity, even when normally exposed, because the lines are likely to be weak and indefinite and to be interfered with by the titanium-oxide bands.

Neither my observations nor the other available data present evidence of changes in velocity, but they do not prove that changes of a few kilometers may not occur. Apparent changes in absorption-line velocities indicated by Tables III and IV are probably due in most if not in all cases to errors of measurement. Observations of *o* Ceti at different maxima agree remarkably well, as shown in Table IV.

All the stars for which either the absorption or the emission-line velocity has been measured by more than one observatory are listed in Table IV. The agreement, on the whole, is very good. Hence, even though we admit the existence of slight variations, it appears that both the bright and dark-line velocities are definite and nearly constant quantities for which essentially the same values are found by different observers using different instruments and methods of reduction. The data are, therefore, suitable for a statistical study of the motions of these objects.

The data in Table IV are taken largely from Table III and from the *Publications of the Observatory, University of Michigan*, **2**, 50 ff., 1916. Professor Frost has had the kindness to send me for use in

TABLE IV

LONG-PERIOD VARIABLES OBSERVED AT MORE THAN ONE OBSERVATORY

DESIGNATION	NAME	OBSERVATORY	RADIAL VELOCITY				ADOPTED	
			A.	Wt.	E.	Wt.	A.	E.
001755........	T Cassiop.	Detroit	....	......	−27	0.5	....	−24.8
		Mt. Wilson	....	......	−25	6	....	....
001838........	R Androm.	Detroit	....	......	−36	1	....	−36.3
		Mt. Wilson	....	......	−36	3	....	....
021024........	R Arietis	Detroit	....	......	+101	3	....	+102.0
		Mt. Wilson	....	......	+104	2	....	....
021143........	W Androm.	Detroit	....	......	−44	1	....	−44.6
		Mt. Wilson	....	......	−45	3	....	....
021403........	*o* Ceti	Lick 1897–98	+62.3	14	+54	8	+64.3	+48.1
		Lick Nov. 1898	....	......	+44	10	....	....
		Lick Stebbins 3-pr	....	......	+44	8	....	....
		Lick Stebbins 1-pr	+66	1	+52	8	....	....
		Ottawa	+65.4	4	+46.1	14	....	....
		Yerkes 3-pr	+64.1	7	+44	7	....	....
		Yerkes 1-pr	+67.6	10	+49.6	27	....	....
		Bonn	+66.1	6	+51	8	....	....
		Detroit	+63.9	3	+52	4	....	....
		Cape	+63.4	10	....	......	....	....
		Mt. Wilson	+63.7	18	+46.7	18	....	....
050953........	R Aurigae	Detroit	....	......	−9	1	....	−13
		Mt. Wilson	....	......	−15	2	....	....
084803........	S Hydrae	Detroit	....	......	+77	1	....	+70
		Mt. Wilson.......	....	......	+66	2	....	....
093943........	R Leo. Min.	Detroit	....	......	−6	2	....	−3.1
		Mt. Wilson	....	......	−2	3	....	....
094211........	R Leonis	Yerkes	+18	2	−6.2	8	+15.0	−1.8
		Lick	+11	2	−7.	1	....	....
		Detroit	+26	1	0.0	7	....	....
		Mt. Wilson	+13.8	7	0.0	15	....	....
123160........	T Urs. Maj.	Detroit	....	......	−107	3	....	−106
		Mt. Wilson	....	......	−102	1	....	....
123307........	R Virginis	Detroit	....	......	−35	2	....	−33
		Mt. Wilson	....	......	−31	2	....	....
123961........	S Urs. Maj.	Detroit	....	......	−1	2	....	−4.1
		Mt. Wilson.......	....	......	−5	7	....	....
132422........	R Hydrae	Lick	−3	4	−26	2	−5.0	−22.8
		Detroit	+5	0.5	−26	3	....	....
		Mt. Wilson	−9	3	−21.4	12	....	....
133633........	T Centauri	D. O. Mills	+24	2	+28	1	+24	+21
		Mt. Wilson	+25	1	+14	1	....	....
134440........	R Can. Ven.	Detroit	....	......	−24	4	....	−23.6
		Mt. Wilson	....	......	−23	3	....	....
142539........	V Boötis	Detroit	....	......	−41	3	....	−42.
		Mt. Wilson	....	......	−47	1	....	....
154615........	R Serpentis	Yerkes	+30	1	+10	2	+27.	+6.9
		Detroit	+30	0.5	+8	4	....	....
		Mt. Wilson	+22	1	+5	7	....	....
180531........	T Herculis	Detroit	....	......	−130	4	....	−131.
		Mt. Wilson	....	......	−136	1	....	....

TABLE IV—*Continued*

Designation	Name	Observatory	Radial Velocity A.	Radial Velocity Wt.	Radial Velocity E.	Radial Velocity Wt.	Adopted A.	Adopted E.
181136........	W Lyrae	Detroit	....	......	−186	3	....	−183.
		Mt. Wilson	....	......	−178	2	....	......
183308........	X Ophiuchi	Detroit	....	......	− 86	3	....	− 83.8
		Mt. Wilson	....	......	− 83.4	16	....	....
193449........	R Cygni	Lick	....	......	− 34	1	....	− 45.0
		Mt. Wilson	....	......	− 46.2	9	....	....
194048........	RT Cygni	Yerkes	....	......	−125	6	....	−125.5
		Detroit	....	......	−127	4	....	....
194632........	χ Cygni	Potsdam 1901	+ 2.4	4	− 19.8	25	− 0.5	− 19.7
		Potsdam 1902	− 2.3	4	− 21.0	15	....	....
		Detroit	....	......	− 17	2	....	....
		Mt. Wilson	− 2.1	3	− 17.8	10	....	....
210868........	T Cephei	Yerkes	−14	1	− 30	2	−10.7	− 26.1
		Detroit	....	......	− 30	2	....	....
		Mt. Wilson	− 9.6	3	− 24.1	8	....	....
235350........	R Cassiop.	Detroit	....	......	+ 9.4	5	....	+ 8.2
		Mt. Wilson	....	......	+ 6.3	3	....	....

NOTES TO TABLE IV

021403, *o* Ceti: Lick, *Astrophysical Journal*, **9**, 32, 1899; *Lick Observatory Bulletin*, **2**, 93, 1902; Ottawa, *Journal Royal Astronomical Society of Canada*, **1**, 53, 1907; Bonn, *Astrophysical Journal*, **27**, 304, 1908; Cape, *Astrophysical Journal*, **48**, 265, 1918; Mount Wilson, unpublished measures by Mr. Joy, to whom I am indebted for permission to include them here.

094211, R Leonis: Lick, a slightly revised value of the emission-line velocity was communicated to me by Dr. Moore.

133633, T Centauri: D. O. Mills, *Lick Observatory Bulletin*, **9**, 68, 1917.

194632, χ Cygni: Potsdam, *Astrophysical Journal*, **18**, 198, 1903.

this connection unpublished measures of *o* Ceti, R Leonis, R Serpentis, RT Cygni, and T Cephei made at the Yerkes Observatory. The notes indicate data from other sources.

Collected radial-velocity data.—The adopted velocities are collected in Table V, which contains other data of value for statistical studies. The first four columns require no explanation. The fifth column gives my estimates of spectral type at maximum, on the new system adopted by the International Astronomical Union in 1922. The typical stars selected for the subdivisions Mo to M6 by Mr. Joy and the writer are as follows:

Mo	β Andromedae, H.D. 6860
M1	ν Virginis, H.D. 102212; α Scorpii, H.D. 148478
M2	α Ceti, H.D. 18884; α Orionis, H.D. 39801
M3	μ Geminorum, H.D. 44478
M4	ρ Persei, H.D. 19058
M5	α Herculis, H.D. 156014
M6	Boss 660, H.D. 18191

TABLE V

RADIAL VELOCITIES OF LONG-PERIOD VARIABLES

No.	Name	α 1900	δ 1900	Sp.	Period	Velocity Abs.	Velocity Em.
					days	km/sec	
1	S Sculptoris	0h 10m.3	−32°36′	M8e	366	(+28.4)	+13.2
2	X Androm.	0 10.9	+46 27	Se	346	(+1.)	−18.2
3	T Androm.	0 17.2	+26 26	M6e	281	−90.	−94.8
4	T Cassiop.	0 17.8	+55 14	M8e	444	−11.0	−24.8
5	R Androm.	0 18.8	+38 1	Se	411	−8.2	−36.3
6	S Ceti	0 19.0	−9 53	M5e	321	(+32.)	+20.
7	U Cassiop.	0 40.8	+47 43	Se	276	−45.	−56.6
8	V Androm.	0 44.6	+35 6	(M2e)	259	(+16.1)	+8.0
9	S Cassiop.	1 12.3	+72 5	Se	610	(−20.±)	−54.
10	R Piscium	1 25.5	+2 22	(M5e)	344	(−45.)	−59.
11	Y Androm.	1 33.8	+38 50	(M3e)	218	(−3.8)	−16.8
12	U Persei	1 53.0	+54 20	M6e	320	+15.	+9.9
13	R Arietis	2 10.4	+24 35	M3e	186	+114.4	+102.0
14	W Androm.	2 11.2	+43 50	M8e	395	−28.5	−44.6
15	o Ceti	2 14.3	−3 26	M6e	332	+64.3	+48.1
16	R Ceti	2 20.9	−0 38	M4e	167	+42.5	+31.6
17	U Ceti	2 28.9	−13 35	M3e	236	−27.	−39.4
18	R Trianguli	2 31.0	+33 50	M5e	267	+66.7	+59.6
19	R Persei	3 23.7	+35 20	M3e	210	−78.2	−89.1
20	T Eridani	3 51.0	−24 20	(M7e)	252	(+43.)	+34.
21	W Eridani	4 7.3	−25 24	M7e	374	(+27.1)	+11.4
22	R Tauri	4 22.8	+9 56	M5e	324	(+31.3)	+19.2
23	T Camelop.	4 30.3	+65 57	Se	370	(+3.)	−19.
24	X Camelop.	4 32.6	+74 56	M3e	142	(−2.3)	−5.8
25	T Leporis	5 0.6	−22 2	(M8e)	366	(−2.8)	−18.
26	V Orionis	5 0.8	+3 58	M3e	267	(+21.9)	+13.9
27	R Aurigae	5 9.2	+53 28	M7e	459	(+8.)	−13.
28	T Columbae	5 15.6	−33 49	M5e	225	(+66.4)	+53.4
29	U Orionis	5 49.9	+20 10	M8e	374	−17.	−37.2
30	X Aurigae	6 4.4	+50 15	(M2e)	163	(−20.2)	−26.2
31	V Monocer.	6 17.7	−2 9	(M6e)	332	(+29.)	+16.
32	X Gemin.	6 40.7	+30 23	M5e	262	(+75.2)	+67.2
33	X Monocer.	6 52.4	−8 56	M4e	155	+157.	+147.5
34	R Lyncis	6 53.0	+55 28	Se	379	(+34.)	+11.
35	R Gemin.	7 1.3	+22 52	Se	370	−36.	−57.4
36	R Can. Min.	7 3.2	+10 11	Se	338	(+51.)	+33.3
37	L2 Puppis	7 10.5	−44 29	(M5e)	140	+52.6	+51.
38	V Gemin.	7 17.6	+13 17	M4e	276	+22.	+10.6
39	S Can. Min.	7 27.2	+8 33	(M8e)	330	(+67.2)	+54.6
40	Z Puppis	7 28.3	−20 27	M7e	516	(+35.6)	+12.6
41	T Gemin.	7 43.3	+23 59	Se	288	(+25.)	+11.8
42	R Cancri	8 11.0	+12 2	M7e	362	+32.1	+18.4
43	V Cancri	8 16.0	+17 36	Se	272	(−2.)	−13.5
44	RT Hydrae	8 24.7	−5 59	M7e	irreg.	+40.	+35.
45	U Cancri	8 30.1	+19 14	(M2e)	305	(+71.)	+61.
46	X Urs. Maj.	8 33.7	+50 30	M4e	251	(−81.)	−90.
47	S Hydrae	8 48.4	+3 27	M4e	256	(+78.)	+70.
48	T Hydrae	8 50.8	−8 46	(M3e)	289	(−3.)	−12.
49	W Cancri	9 4.0	+25 39	(M7e)	385	(+51.3)	+34.7
50	R Leo. Min.	9 39.6	+34 58	M8e	372	+9.5	−3.1
51	R Leonis	9 42.2	+11 54	M8e	313	+15.0	−1.8
52	V Leonis	9 54.5	+21 44	M7e	273	(−22.7)	−30.8
53	R Urs. Maj.	10 37.6	+69 18	(M6e)	302	+34.	+23.
54	Z Urs. Maj.	11 51.3	+58 25	M6e	120	−52.6	−58.4
55	R Comae	11 59.1	+19 20	(M4e)	362	(−7.0)	−22.0

TABLE V—*Continued*

No.	Name	α 1900	δ 1900	Sp.	Period	Velocity	
						Abs.	Em.
					days	km/sec	
56	R Corvi	12h14m4	−18°42′	(M6e)	318	(−22.)	− 34.
57	T Urs. Maj.	12 31.8	+60 2	(M6e)	257	(−98.)	−106.
58	R Virginis	12 33.4	+ 7 32	M6e	146	−34.	− 33.
59	S Urs. Maj.	12 39.6	+61 38	Se	226	(+2.)	− 4.1
60	U Can. Ven.	12 42.6	+38 55	(M8e)	....	(−27.)	− 43.7
61	U Virginis	12 46.0	+ 6 6	M5e	207	−42.	− 58.1
62	V Virginis	13 22.6	− 2 39	M6e	250	(+33.7)	+ 25.0
63	R Hydrae	13 24.2	−22 46	M8e	425	−5.0	− 22.8
64	S Virginis	13 27.8	− 6 41	M7e	377	(+11.3)	− 4.7
65	T Centauri	13 36.0	−33 6	M1e	90	+24.	+ 21.
66	W Hydrae	13 43.4	−27 52	M8e	384	+42.3	+ 26.4
67	R Can. Ven.	13 44.5	+40 2	M6e	333	−8.9	− 23.6
68	U Urs. Min.	14 15.1	+67 15	(M6e)	327	(−31.0)	− 43.3
69	S Boötis	14 19.5	+54 16	M4e	270	(−17.0)	− 25.0
70	RS Virginis	14 22.3	+ 5 8	(M6e)	355	(−25.6)	− 40.0
71	R Camelop.	14 25.1	+84 17	Se	270	(−32.)	− 42.9
72	V Boötis	14 25.7	+39 18	(M6e)	256	−31.	− 42.
73	R Boötis	14 32.8	+27 10	M4e	223	−46.	− 57.
74	Y Librae	15 6.4	− 5 38	(M5e)	272	(−6.9)	− 15.0
75	S Librae	15 15.7	−20 2	(M2e)	192	(+295.3)	+284.6
76	S Serpentis	15 17.0	+14 40	M5e	368	(+13.5)	− 1.9
77	S Cor. Bor.	15 17.3	+31 44	(M8e)	361	−1.0	− 22.
78	RS Librae	15 18.2	−22 33	M7e	219	(−2.1)	− 15.1
79	RU Librae	15 27.7	−14 59	(M4e)	314	(−48.2)	− 59.5
80	S Urs. Min.	15 33.4	+78 58	M7e	324	(−40.7)	− 52.7
81	R Serpentis	15 46.1	+15 26	M7e	357	+27.	+ 6.9
82	RR Librae	15 50.6	−18 1	(M5e)	277	(−32.3)	− 40.6
83	R Herculis	16 1.7	+18 38	M5e	318	(−30.6)	− 42.1
84	U Serpentis	16 2.5	+10 12	M4e	240	(−29.3)	− 39.8
85	RU Herculis	16 6.0	+25 20	M7e	486	(−15.3)	− 37.6
86	W Cor. Bor.	16 11.8	+38 3	M4e	244	(+20.1)	+ 10.5
87	U Herculis	16 21.4	+19 7	(M8e)	403	(−22.5)	− 40.4
88	W Herculis	16 31.7	+37 32	(M4e)	280	(−51.)	− 59.
89	R Draconis	16 32.4	+66 58	(M5e)	246	−138.	−144.2
90	S Herculis	16 47.4	+15 7	(M5e)	308	(−10.7)	− 21.3
91	RR Scorpii	16 50.2	−30 25	M7e	281	(−36.6)	− 45.1
92	SS Ophiuchi	16 52.7	− 2 36	(M6e)	230	(−30.3)	− 42.9
93	R Ophiuchi	17 2.0	−15 58	(M4e)	302	(−49.)	− 59.
94	Z Ophiuchi	17 14.5	+ 1 37	(M2e)	348	(−79.)	− 93.
95	RS Herculis	17 17.5	+23 1	(M6e)	223	(−37.8)	− 50.8
96	RY Herculis	17 55.4	+19 29	M4e	222	−38.9	− 50.1
97	T Herculis	18 5.3	+31 0	(M3e)	165	(−125.)	−131.
98	W Lyrae	18 11.5	+36 38	M5e	197	−174.	−183.
99	RY Ophiuchi	18 11.6	+ 3 40	M5e	153	(−67.2)	− 71.8
100	X Ophiuchi	18 33.6	+ 8 44	M6e	335	−70.6	− 83.8
101	AE Herculis	18 39.0	+22 54	M6e	....	(−48.)	− 60.4
102	R Aquilae	19 1.6	+ 8 5	M7e	355	+31.5	+ 22.8
103	T Sagittarii	19 10.5	−17 9	Se	381	(+4.)	− 19.4
104	R Sagittarii	19 10.8	−19 29	(M6e)	269	(−44.3)	− 52.3
105	RT Aquilae	19 33.3	+11 30	M8e	326	(−42.0)	− 54.2
106	R Cygni	19 34.1	+49 58	Se	426	(−15.)	− 45.0
107	RT Cygni	19 40.8	+48 32	(M3e)	190	(−115.0)	−125.5
108	χ Cygni	19 46.7	+32 40	M6e	406	−0.5	− 19.7
109	Z Cygni	19 58.6	+49 46	M5e	263	(−165.1)	−173.1
110	S Cygni	20 3.4	+57 42	(M2e)	323	(−21.7)	− 33.7

TABLE V—*Continued*

No.	Name	α 1900	δ 1900	Sp.	Period	Velocity	
						Abs.	Em.
					days	km/sec	
111	Z Aquilae	20h 9m.8	− 6° 27′	M3e	129	(−7.9)	− 10.3
112	R Delphini	20 10.1	+ 8 47	(M6e)	284	(−46.1)	− 54.8
113	V Aquarii	20 41.8	+ 2 4	M6e	246	(−43.4)	− 52.9
114	T Aquarii	20 44.7	− 5 31	M3e	203	−38.2	− 54.9
115	X Delphini	20 50.3	+17 16	M5e	281	−56.2	− 62.9
116	R Vulpeculae	20 59.9	+23 26	(M4e)	137	(−14.)	− 17.
117	T Cephei	21 8.2	+68 5	M7e	387	−10.7	− 26.1
118	R Equulei	21 8.4	+12 23	(M6e)	262	(−54.0)	− 62.0
119	RR Aquarii	21 9.8	− 3 19	M3e	180	(−182.0)	−191.0
120	W Cygni	21 32.2	+44 56	(M4e)	132	−27.	− 26.
121	T Pegasi	22 4.0	+12 3	(M6e)	374	(−8.2)	− 23.9
122	RS Pegasi	22 7.4	+14 4	(M6e)	436	(−20.0)	− 40.0
123	RT Aquarii	22 17.7	−22 34	M6e	241	(−33.)	− 43.
124	S Lacertae	22 24.6	+39 48	M5e	238	−59.7	− 65.9
125	R Lacertae	22 38.8	+41 51	(M6e)	300	(+18.1)	+ 8.2
126	S Aquarii	22 51.8	−20 53	(M6e)	280	(−57.9)	− 66.4
127	V Cassiop.	23 7.4	+59 8	M6e	230	−30.5	− 47.6
128	W Pegasi	23 14.8	+25 44	(M7e)	342	(−22.)	− 35.
129	S Pegasi	23 15.5	+ 8 2	(M6e)	318	(+5.)	− 7.
130	R Aquarii	23 38.6	−15 50	M7e+P	387	−19.	− 33.
131	V Ceti	23 52.8	− 9 31	(M3e)	261	(+51.1)	+ 43.1
132	R Cassiop.	23 53.3	+50 50	M7e	432	+30.	+ 8.2
133	SV Androm.	23 59.2	+39 33	M7e	298	(−89.1)	− 98.8

Two more subdivisions in continuation of this sequence, M7 and M8, were used, but no standard spectra not subject to variation are known to us. M9 and M10 are available for the spectra of certain stars at times other than maximum. Parentheses in the fifth column indicate that the classification is somewhat uncertain because, in most instances, of underexposure of the spectrograms. No star was included in this table unless emission lines of hydrogen had been measured in its spectrum.

The Harvard values of the period are given in the sixth column. Two stars marked as irregular, namely, X Monocerotis[1] and RT Hydrae, are included because their spectra are much like those of the periodic stars.

All the figures in the last column, and all those in the preceding column not in parentheses, are observed values and are taken from Tables III and IV or from the *Publications of the Observatory, University of Michigan*, 2, 50 ff, 1916. Slight corrections have been

[1] See note on p. 234.

made to some of the absorption-line velocities in the Ann Arbor list by using additional lines in the reductions. The absorption values in parentheses have been found from the emission velocities in a manner that will be described presently.

Relative displacement of bright and dark lines.—Spectrographic observations made at the Lick Observatory in 1897 and 1898[1] showed the effective centers of the bright Hγ and Hδ lines to be displaced toward the violet with respect to the absorption spectrum. Bright lines at λ 4308 and λ 4376, if identified with iron lines, showed corresponding displacements. These results were confirmed and extended by Stebbins in 1902.[2] Similar behavior was found for the bright lines of χ Cygni by Eberhard in 1901 and 1902.[3] It seemed probable, as Eberhard remarked, that this is typical of long-period variables having the same type of spectrum. The present investigation, including the Ann Arbor measurements, makes it certain that this is the case, and also indicates the same effect for three S-type stars. Moore has found the same phenomenon in the spectrum of one N-type variable, U Cygni.[4] The occurrence of this relative displacement in such diverse types of spectra is very interesting and should be held in mind in considering the general problems of long-period variables of the three spectral classes.

The relative displacements of the bright lines have been measured for 47 variables of classes M and S. The results, computed from data in Table V, together with the spectral class, the period, and the magnitude range are collected in Table VI. Omission of the decimal of a kilometer in the column A.—E. denotes considerable uncertainty, except for the stars S Coronae Borealis and R Aquarii, in which cases the record does not show the decimal. As a matter of fact, no value in Table VI is reliable to a tenth of a kilometer, but the decimal has been used when available to prevent the accumulation of errors in plotting and computing.

[1] *Astrophysical Journal*, **9**, 31, 1899.

[2] *Lick Observatory Bulletin*, **2**, 93, 1902.

[3] *Astrophysical Journal*, **18**, 198, 1903.

[4] *Lick Observatory Bulletin*, **10**, 166, 1922.

TABLE VI

MEASURED DISPLACEMENTS OF ABSORPTION AND EMISSION LINES

No.	Name	Spect.	Period	Mag. Range	A.−E.	Wt.
			days		km	
3	T Androm.	M6e	281	6.0	+ 4.8	1
4	T Cassiop.	M8e	444	5.8	+13.8	2
5	R Androm.	Se	411	8.4	+28.1	2
7	U Cassiop.	Se	276	8.0	+11.6	1
12	U Persei	M6e	320	3.9	+ 5.1	1
13	R Arietis	M3e	186	5.9	+12.4	2
14	W Androm.	M8e	395	7.5	+16.1	2
15	o Ceti	M6e	332	7.6	+16.2	2
16	R Ceti	M4e	167	6.2	+10.9	1
17	U Ceti	M3e	236	6.1	+12.4	1
18	R Trianguli	M5e	267	6.7	+ 7.1	2
19	R Persei	M3e	210	5.9	+10.9	1
29	U Orionis	M8e	374	7.0	+20.2	1
33	X Monocer.	M4e	155	2.0	+ 9.5	1
35	R Gemin.	Se	370	7.4	+21.4	1
37	L_2 Puppis	(M5e)	140	2.8	+ 1.6	2
38	V Gemin.	M4e	276	6.5	+11.4	1
42	R Cancri	M7e	362	5.3	+13.7	2
44	RT Hydrae	M7e	irreg.	1.8	+ 5.	1
50	R Leo. Min.	M8e	372	6.0	+12.6	2
51	R Leonis	M8e	313	5.9	+16.8	2
53	R Urs. Maj.	(M6e)	302	6.5	+11.	1
54	Z Urs. Maj.	M6e	120	1.5	+ 5.8	2
58	R Virginis	M6e	146	5.7	− 1.	1
61	U Virginis	M5e	207	6.0	+16.1	1
63	R Hydrae	M8e	425	5.8	+17.8	2
65	T Centauri	M1e	90	2.7	+ 3.	1
66	W Hydrae	M8e	384	1.3	+15.9	2
67	R Can. Ven.	M6e	333	6.6	+14.7	1
72	V Boötis	(M6e)	256	4.1	+11.	1
73	R Boötis	M4e	223	6.3	+11.	1
77	S Cor. Bor.	(M8e)	361	7.3	+21.	2
81	R Serpentis	M7e	357	7.4	+20.1	1
89	R Draconis	(M5e)	246	5.7	+ 6.2	1
96	RY Herculis	M4e	222	5.3	+11.2	2
98	W Lyrae	M5e	197	4.9	+ 9.	1
100	X Ophiuchi	M6e	335	5.2	+13.2	2
102	R Aquilae	M7e	355	>6.2	+ 8.7	2
108	χ Cygni	M6e	406	9.5	+19.2	2
114	T Aquarii	M3e	203	6.3	+16.7	1
115	X Delphini	M5e	281	>5.	+ 6.7	2
117	T Cephei	M7e	387	5.4	+15.4	1
120	W Cygni	(M4e)	132	1.7	− 1.	1
124	S Lacertae	M5e	238	4.5	+ 6.2	2
127	V Cassiop.	M6e	230	5.3	+17.1	2
130	R Aquarii	M7e	387	4.8	+14.	2
132	R Cassiop.	M7e	432	7.5	+21.8	2

STATISTICAL STUDIES

Relative displacement of bright lines.—Inspection of Table VI shows that the relative displacements (absorption *minus* emission-line velocity) for different stars differ by amounts greater than the errors of measurement. Studies were therefore made of the relationship of the displacement to spectral type, period, and range, respectively.

The mean displacements for each spectral type are shown in Table VII. The second column gives the average displacement (absorption *minus* emission) when each star is counted as one; the

TABLE VII

DISPLACEMENT OF EMISSION LINES AS RELATED TO SPECTRAL TYPE

Type	Equal Weights		Weights 1 and 2	
	Mean Displ.	No.	Mean Displ.	Wt.
M1e	3. km	1	3. km	1
M2e	....	0	....	0
M3e	13.1	4	13.0	5
M4e	8.7	6	9.2	7
M5e	9.0	7	7.3	10
M6e	10.6	11	11.8	16
M7e	14.1	7	15.7	10
M8e	16.8	8	16.2	14
Se	20.4	3	22.3	4

fourth column gives the displacements with weights 1 and 2, as indicated in Table VI, except that L_2 Puppis and S Coronae were given weight 1 because of some uncertainty in the determination of the spectral type. The table shows that the displacement increases with advancing spectral type, although class M3e is an exception, possibly because of the small number of stars included. The three stars of class Se have high values, but Figure 2 shows an influence depending on the period. The curve defined by the three S-type stars lies about 6 km above the corresponding portion of the curve for Me stars.

The stars were then divided into three nearly equal groups according to the size of the displacements, omitting the three stars of class S. The resulting mean displacements and types are given in Table VIII. The correspondence between advancing type and increasing displacement is again shown.

If the stars with known periods are divided into three groups according to the period, we find the results given in the first portion of Table IX. If the same stars are grouped according to the size of the displacement, we obtain the figures in the second portion of Table IX. In both cases the period increases with the displacement.

TABLE VIII

MEAN DISPLACEMENTS AND MEAN TYPES BY GROUPS

Group	No.	Mean Displ.	Mean Type
1	15	5.6 km	M 5.0
2	15	12.5	M 5.5
3	14	17.8	M 6.9

TABLE IX

MEAN DISPLACEMENT AND MEAN PERIOD BY GROUPS

	Group	No.	Mean Period	Mean Displ.
			days	km
Arranged by period	1	15	180	9.2
	2	15	297	10.3
	3	15	394	17.9
Arranged by displacement	1	15	219	5.7
	2	15	314	12.7
	3	15	356	19.0

TABLE X

MEAN DISPLACEMENT AND MEAN RANGE BY GROUPS

	Group	No.	Mean Range	Mean Displ.
			mag.	km
Arranged by range	1	15	3.6	9.4
	2	15	5.8	12.2
	3	15	7.5	17.5
Arranged by displacement	1	15	4.1	5.5
	2	15	5.7	12.7
	3	15	6.7	19.0

Proceeding in the same way for displacement and magnitude range, we have the figures in Table X, which show that the displacement increases with the range.

It thus appears that the displacement increases, on the average, with spectral type, with period, and with magnitude range. The accordance of individual stars for the period relationship is fair, but for spectral type and especially for range it is poor, the plotted results being scattering. We might expect the displacements to be closely correlated with spectral type, but this is not the case, although a statistical relationship is undoubtedly present.

The stars with ranges of less than five magnitudes have in general small displacements, and those with ranges above seven magnitudes have large displacements, but for the numerous stars having ranges between five and seven magnitudes the dispersion in displacement is large. One star, W Hydrae, deserves special mention as the outstanding exception to the rule that stars with small ranges have small displacements. Its period and spectroscopic behavior are typical of stars of type M8e, but it has the remarkably small range of 1.3 magnitudes. It is barely possible that, like X Ophiuchi, it is a double star. R Aquarii also has a magnitude range somewhat smaller than is typical of its period and spectral type. This may be due to the influence of the portion of the star connected with the emission of the nebular lines.

The best accordance shown by individual stars is in the case of the period relationship. Even here the deviations of some stars appear to exceed the errors of observation, although more material is perhaps necessary to establish this beyond doubt. In any event the period relationship offers the best method available of reducing the measured bright-line velocity of a star to a dark-line basis.

We must now face the question whether the displacements of the bright lines, or of the dark lines, or neither, yield the true radial velocity. In the case of one star, X Ophiuchi, a direct answer is available in favor of the absorption lines. The evidence is fully stated in another paper[1] and need not be repeated here. In R Aquarii the velocities from the nebular lines more nearly coincide with those from the absorption lines than with those from the bright lines connected with the M-type spectrum.[2] In both of these stars the Me spectrum seems to be a normal one. Statistical investigations of the motions also favor the absorption lines as yielding

[1] *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

[2] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 375, 1921.

essentially the true radial velocities. A solution for the sun's motion with respect to 83 Me variables, using emission-line velocities, gave a K term of -11.7 km, but a solution from the dark-line velocities of 133 stars (119 of class Me and 14 of class Se) gave a K term of $+3.9$ km; omitting one very high velocity star, S Librae, the K term came out -0.2 km. The K term for 76 of the slower moving variables (using absorption-line velocities) was $+1.3$ km. Another selection of 68 slow-moving variables gave a K term of $+1.1$ km. It seems clear therefore that for a study of the motions of these stars the absorption-line velocities are to be preferred to those from the emission lines.

In order to reduce the emission-line velocities to an absorption-line basis, the measured displacements were plotted against the periods; a curve drawn to represent the resulting points is shown in Figure 2. The observations seem to demand a maximum near 220 days, but it is uncertain what physical meaning should be attached to this feature. From the appearance of the plot we might infer that there exists a group of stars with periods in the neighborhood of 200 days, which have larger displacements than called for by the general progression shown by stars of longer and shorter periods. We might accordingly continue the general run of the curve across the interval from 140 to 260 days, but for the purpose of making an empirical determination of the displacement corresponding to a given period, it seemed best to draw the curve as indicated. For those stars for which emission lines only have been measured, the hypothetical dark-line velocities were obtained by applying the displacements read from the curve. The S-type stars were treated separately; only 3 were available as standards and these are marked by the letter S in Figure 2. The dark-line velocities determined in this manner are given in the next to the last column in Table V, *in parentheses* to distinguish them from values depending directly upon measurement.

In the earlier work at Ann Arbor it was noticed that two stars with especially short periods, namely L_2 Puppis and W Cygni, showed practically zero displacements. From a study of the Ann Arbor data Ludendorff was led to state[1] that "the magnitude of

[1] *Astronomische Nachrichten*, **212**, 483, 1921.

the displacement of the hydrogen emission lines depends on the period of the star." In spite of the meager material at his disposal, this conclusion is essentially correct, being substantiated by the more extensive data reported in the present paper. It has seemed best, however, not to assume a linear relationship between displacement and period, as was done by Heiskanen and Ludendorff,[1] to reduce emission-line velocities to a dark-line basis.

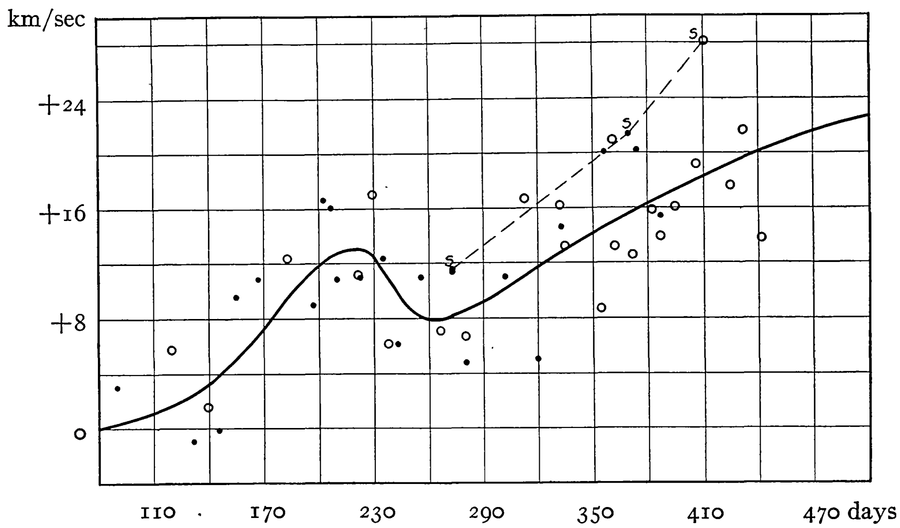

FIG. 2.—Relative displacement of emission lines (Abs. − Em). Observations represented by dots have one-half the weight of those represented by circles.

Group-motion of long-period variables.—The highly specialized character of the light variations and of the spectra of long-period variables leads us to examine their motions to ascertain whether they differ in any way from the average motions of other stars. We may consider first the common or group-motion of the variables, and second their random or individual motions.

The Ann Arbor data[2] strongly suggested a group-motion in a general direction opposite to that in which the sun is moving with respect to other stars. Heiskanen and Ludendorff[3] emphasized the fact that the result depended to a considerable extent on the

[1] *Ibid.*, **213**, 297, 1921.

[2] *Publications of the Observatory, University of Michigan*, **2**, 63, 1916.

[3] *Astronomische Nachrichten*, **213**, 297, 1921.

influence of a few rapidly moving stars, and thought that no real stream-motion existed. With the additional velocity determinations now available, we are in a position to discuss this question to much better advantage than before.

The following computations have been based on the absorption-line velocities as tabulated in the next to the last column in Table V. The positions and velocities of the 133 stars included in this table are charted in Figure 3. The deficiency of observed stars in the

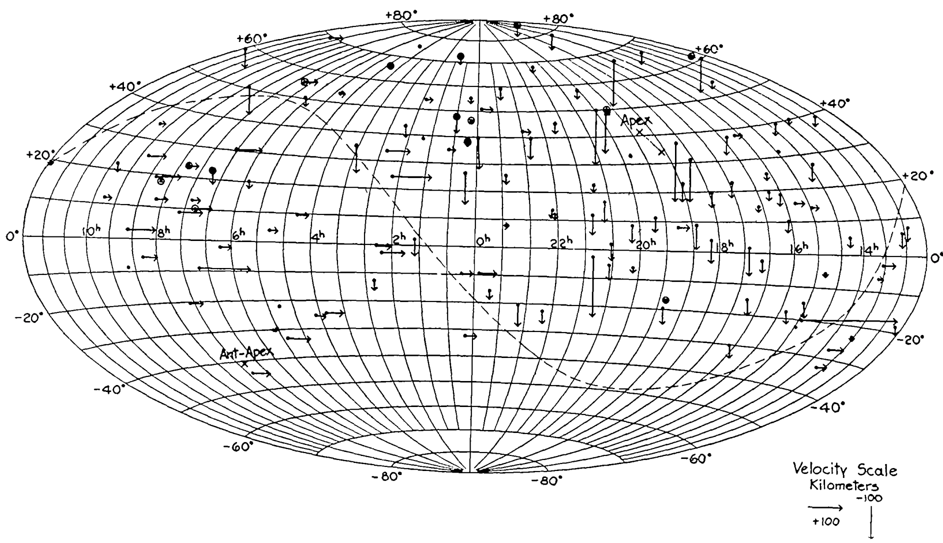

FIG. 3.—Chart of long-period variables whose radial velocities have been measured. Class Se stars are indicated by circles. The position of the solar apex for the 133 stars is marked by the cross just below the word "apex"; another cross, to the right and below, indicates the computed position when the most rapidly moving star, S Librae, is omitted. The dotted curve is the great circle whose poles are the apex and ant-apex.

southern hemisphere is, of course, an unsatisfactory feature. It is hoped that before long data will be available for additional southern stars. Other irregularities in the chart correspond more or less closely to the actual distribution of variables in the sky. As has been frequently pointed out, this is by no means uniform, a decided lack of stars existing near right ascension eleven hours.

In the first place, a least-squares solution for the solar motion with respect to all the 133 variables was made with the following results:

$$A_0 = 287\overset{\circ}{.}1,\ D_0 = +41\overset{\circ}{.}0,\ V_0 = 55.0 \text{ km/sec.},\ K = +3.9 \text{ km/sec.}$$

The position of the apex is about the same as that given by several investigators for stars of classes K and M, but the speed of the sun is nearly three times as great as that usually found. As the variables have high random motions,[1] we may recognize in the facts above a phase of the dependence of solar motion upon the speed of the stars, as brought out by Boss,[2] Adams and Joy,[3] and Strömberg.[4]

In computations concerning the motions of the stars the questions of grouping the material and of rejecting apparently exceptional stars are often troublesome, especially when, as in the present investigation, the total number of stars treated is small. If the stars are divided into strictly homogeneous groups, the number in each group may become so small that the results are untrustworthy. For this reason I have avoided the use of more than two groups in solutions for the solar motion. Several methods of grouping are possible, but the only one used here is that based on the random motion of the individual stars as a criterion of selection. The following computations are those which seemed the most suggestive, but they do not constitute an exhaustive treatment of the data. Other investigators may find further computations profitable.

One star, S Librae, has such a high velocity that it stands quite by itself in the plot in Figure 4. A solution for the solar motion with this star omitted is shown in Table XI. The large effect of this star is, of course, due to the fact that the determining factors are the *squares* of the residuals. Some other system of solution would probably be preferable. The arithmetic mean residual for the 132 stars has not been computed, but it is estimated to be between 30 and 31 kilometers.

The small value of the K term gives us confidence that the method of reducing the emission-line velocities to a dark-line basis is reasonably accurate.

In order to bring out the possible dependence of the computed solar motion upon the peculiar velocities of the stars involved, the variables were divided into two groups according to the numerical size of their residual velocities, which were found by applying

[1] The arithmetic mean residual from this solution is 33.4 km.

[2] *Astronomical Journal*, **35**, 26, 1923, and references there given to earlier papers.

[3] *Mt. Wilson Contr.*, No. 163; *Astrophysical Journal*, **49**, 179, 1919.

[4] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 265, 1922.

to the absorption-line velocities a correction for the solar motion computed from the 133 variables themselves, as given on page 252. Sixty-eight stars were found to have residual velocities equal to or less than 25 km, and 65 to have residual velocities exceeding this figure. Separate solutions for the solar motion were based on these

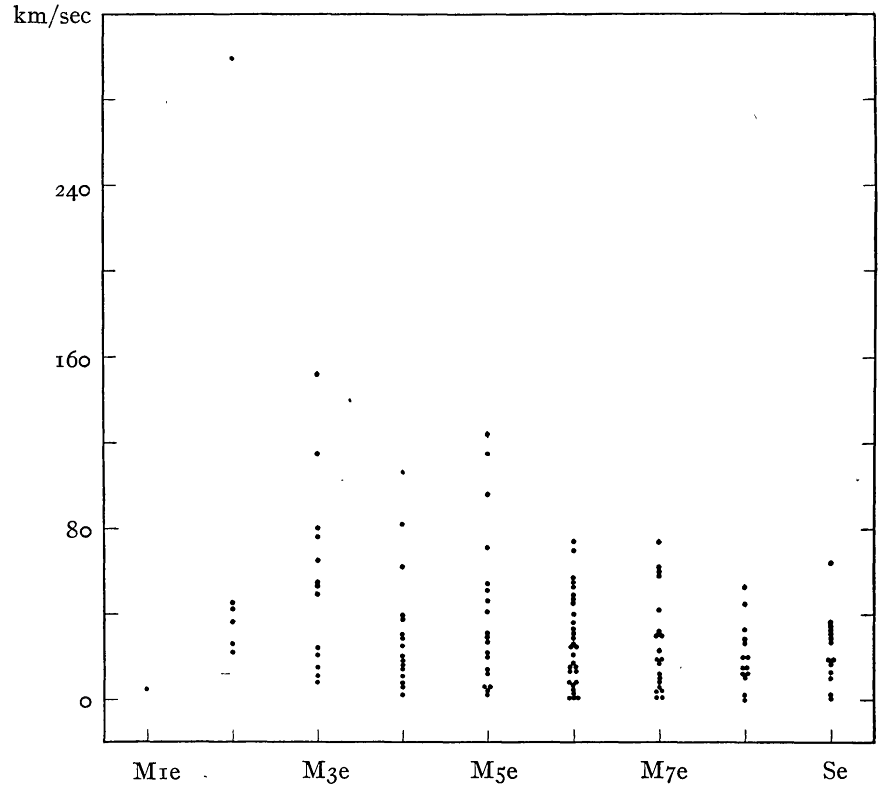

FIG. 4.—Residual radial velocity and spectral type. Spectroscopically class Se is not a continuation of the sequence M1e–M8e; it is placed at the end of the diagram for convenience and to show the correspondence in velocity to classes M7e and M8e.

two groups. The resulting data are shown in Table XI. A previous solution based on the emission-line velocities of 83 stars[1] is included for comparison. The solution based on 76 stars with low residuals will be described presently.

The increase in V_0 in going from the 68 slow-moving stars to the 65 rapid ones is very marked, and seems to be a fine illustration of the velocity asymmetry of high-speed stars.

[1] These stars were not selected for small velocity but included all that were available when the solution was made.

The 68 slow-moving stars give a value of V_0 much greater than that usually found, and, on the face of it, this indicates a strong group-motion prevailing among the variables independently of their high speed. These figures may be somewhat misleading, however, as a little consideration will show that the two solutions lead of necessity to results much like those based on all the stars. As far as the 68 stars are concerned, they have been selected in such a way that their motions must nearly equal that called for by the general solution, and hence they are not likely to yield very different results. On the other hand, the group of 65 rapidly moving stars will contain the high residuals which, since the method of solution is that of least *squares*, counted most heavily in the general

TABLE XI

SOLUTIONS FOR SOLAR MOTION

	All	Omitting S Librae	Low Residuals		High Residuals	Emission-Line Velocities
Number....	133	132	68	76	65	83
A_0........	287°.1	280°.7	283°.5	263°.9	289°.2	274°.4
D_0........	+41.0	+33.2	+35.6	+34.4	+46.2	+44.0
V_0........	55.0 km	52.5 km	47.9 km	29.0 km	64.6 km	55.9 km
K.........	+ 3.9	− 0.2	+ 1.1	+ 1.3	+ 7.9	−11.7
Arith. Mean Resid.}	33.4	....	11.1	13.3	56.8	30.9

solution, and hence will tend to reproduce the same figures. We must therefore exercise caution in interpreting similarities in the results of the three solutions. The same reasoning, however, leads us to rely on *differences* in the figures as probably having a real meaning. The increase in V_0 with speed, for example, is scarcely to be explained except as a physical fact.

Another procedure was adopted to test the systematic motions of the slowly moving variables. The residuals from the ordinary solar motion, $A_0 = 270°$, $D_0 = +30°$, $V_0 = 20$ km, were first computed. Seventy-six stars having residuals less than 30 km were then made the basis of a new solution. The results are in the fifth column of Table XI. Although the mean residual, regardless of sign, 13.3 km, is not greater than that for stars of classes F, G, K, and M, the solar motion nevertheless is increased from 20 to 29 km. Moreover, reasoning similar to that outlined above shows that the method of selection tends to make this difference unduly small. Hence the conclusion seems to be justified that long-period variables have a general group-motion, largest, it is true, for the high-speed stars, but not zero for the slow ones.

Random motions.—It is well known that the average random motion varies for groups of stars of different spectral types or different absolute magnitudes. The general rule is that it increases with advancing spectral type, and decreases with increasing luminosity. The residual motions of the individual variables have been computed from several solutions for the group-motion, and have been grouped and tabulated in various ways in an attempt to bring out possible systematic relationships to other quantities.

TABLE XII

SPECTRAL TYPE AND RESIDUAL VELOCITY

Type	Number	Arith. Mean Residual
		km
M1e	1	5
M2e	6	78
M3e	13	56
M4e	16	32
M5e	19	40
M6e	30	28
M7e	20	25
M8e	14	21
Se	14	24

First, the residuals from the ordinary solar motion were found, assuming $A_0 = 270°.0$, $D_0 = +30°.0$, $V_0 = 20.0$ km. The arithmetic mean residual is 35.5 km. This is the largest value found for any group of stars selected on the basis of spectral type.[1] It may be reduced to 35.3 km by applying an arbitrary correction of +3.6 km to all the residuals. For a solar motion based on the 133 variables themselves the arithmetic mean residual velocity is 33.4 km. By omitting one star this would be reduced about 2.5 km.

The relationship of residual motion to spectral type was next considered. The mean residual velocities for each type from the solution for 133 stars were found with the results shown in Table XII.

[1] The corresponding value for 102 planetary nebulae is 36 km (*Publications of the Lick Observatory*, **13**, 168, 1918). Is the agreement accidental?

Here we find a strong tendency for the random motion to decrease with advancing type, the velocity of the Se stars corresponding to that of classes M7e and M8e. The residuals from the other solutions have also been grouped according to spectral type, the results being exhibited in Table XIII.

The most rapidly moving stars show an evident tendency to favor the earlier spectral classes. Among the slowly moving stars one could not expect to find a decided progression with type, but in the case of the 68 stars selected by residuals from the solution based on the variables themselves, there appears to be a trace of it. In the case of the 76 stars selected on the basis of the residuals from the ordinary solar motion, the selection has little or nothing

TABLE XIII

MEAN SPECTRAL TYPE AND RESIDUAL VELOCITY BY GROUPS

Types	All			High Residuals			Low Residuals					
							68 Stars			76 Stars		
	Mean Type	No.	Mean Resid.	Mean Type	No.	Mean Resid.	Mean Type	No.	Mean Resid.	Mean Type	No.	Mean Resid
			km			km			km			km
M1e–M4e.	3.5	36	47	3.1	20	74	3.4	16	14	3.1	17	10
M5e–M6e.	5.6	49	32	5.6	25	53	5.7	24	11	5.7	23	15
M7e–M8e.	7.4	34	24	7.4	13	46	7.4	21	10	7.4	26	15
Se......	.....	14	24	.....	7	40	.....	7	11	.....	10	12

to do with characteristic properties of variable stars and the mean residuals probably have no significance. The correlation of high speed with early spectral type is evidenced, however, by the *numbers* of stars involved in the various groups. Among the high residuals the ratio of the number of stars in classes M1 to M4 to those in classes M7 and M8 is 1.5, while the same ratio for the stars with low residuals is 0.8 in one case and 0.7 in the other. If the stars are divided into several groups according to size of the residual velocity, the mean spectral type of each group tends to decrease with increasing velocity. Both methods of grouping exhibit irregularities in the correlation between type and velocity, however, showing that the relationship is of a general statistical nature and not binding on individual stars.

The relationship between light-period and velocity was next examined. The stars involved in each solution were arranged in order of period and divided into equal or nearly equal groups. In the main solution for 133 stars, seven groups were formed; in the solution for 68 and 65 stars, five groups each. The mean periods and the mean residuals for these groups are tabulated in Table XIV.

On the whole there is a decided tendency toward slower motion with increasing period. This appears most clearly among the stars with high residuals. Among the stars with low residuals the total range of velocity is so small that a strong progression cannot be expected. The fact that long-period stars tend to have small residuals is indicated, however, by the fact that the last two groups have a larger mean period than the corresponding groups of the stars with high residuals.

TABLE XIV

PERIOD AND RESIDUAL VELOCITY

Group	All		High Residuals		Low Residuals	
	Mean Period	Mean Residual	Mean Period	Mean Residual	Mean Period	Mean Residual
	days	km	days	km	days	km
1	155	64	174	90	171	13
2	227	27	254	62	242	12
3	260	37	293	50	291	10
4	289	36	328	38	350	7
5	323	21	380	48	428	15
6	359	31	....	....	....	....
7	418	24	....	....	....	....

The correlation between period and velocity is incomplete in that stars of low velocity may have any length of period. It seems clear, however, that very high velocities are largely confined to stars having short periods.[1]

Heiskanen and Ludendorff, in a study based on 44 variables,[2] called attention to the probable dependence of velocity upon period, and pointed out that at least the dispersion of the radial velocities

[1] Averaging a little over 200 days. The stars with the very shortest periods, 90–150 days, do not appear, from present data, to have extremely high velocities.

[2] *Astronomische Nachrichten*, 213, 297, 1921.

decreases with increasing period. This is confirmed by the more extensive data of the present article.

That the apparently fainter variables are moving more rapidly than the brighter ones, as indicated by the Ann Arbor measurements, appears to have been an accidental result due to high velocities of comparatively few stars, as the present data do not afford much evidence of such a relationship.

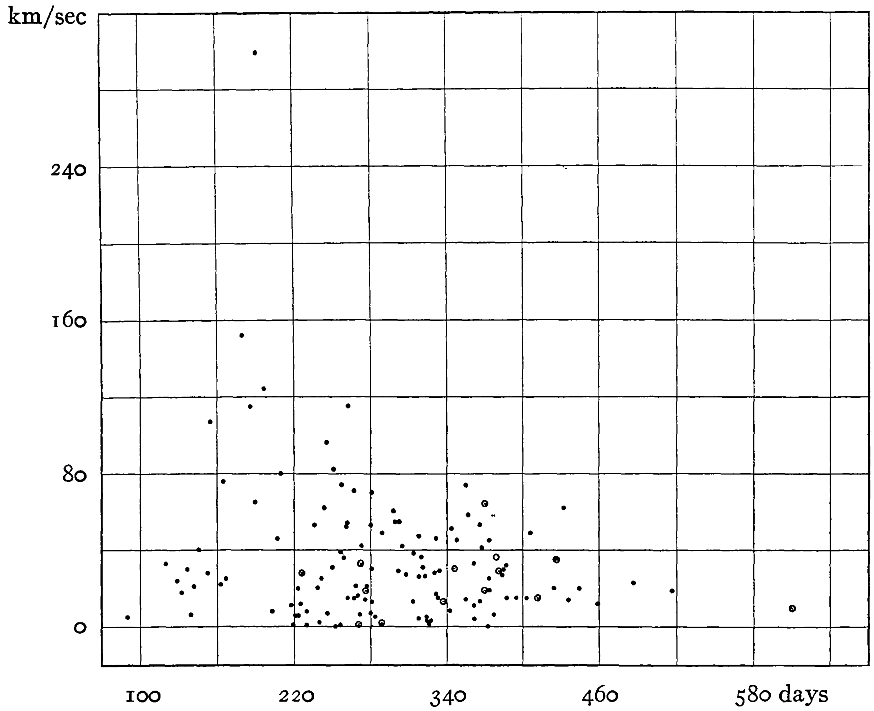

FIG. 5.—Residual radial velocity and period. Class Se stars are indicated by circles.

It is a pleasure to acknowledge the able assistance of Miss Cora G. Burwell in making the computations described in the foregoing pages.

GENERAL DISCUSSION

The main purpose of this article, which is to present observational results concerning the radial velocities of long-period variables, has been accomplished in the preceding pages, which contain also some computations and tabulations exhibiting certain statistical relationships. Some features of the data are very puzzling and seem quite out of harmony with the more general facts of stellar motions and with plausible hypotheses as to the place of the variable stars in stellar evolution. No attempt is made to explain these phenomena and to co-ordinate them with other astronomical facts, and the article is concluded with only a brief recapitulation of the general results and a cursory examination of some of the problems involved.

The very extensive photometric and spectroscopic data concerning long-period variables of class Me gathered by many observers, but chiefly by those of the Harvard College Observatory, have shown that the average values of the following characteristics vary together as indicated:

1. Advancing spectral type[1]
2. Increasing period
3. Increasing magnitude range

The spectrographic observations discussed in this paper enable us to add to the list:

4. Increasing displacement of bright lines relative to absorption lines
5. Decreasing random velocity

It is not easy to decide which of these relationships arise directly from physical causes, and which are incidental. The correlation of magnitude-range with relative displacement of the bright lines and with random velocity appears to have little direct significance. The relative displacement of the bright lines seems to be most clearly connected with the period, while the random velocity is correlated about equally well with type and with period. But in all these correlations neglected factors are evidently present, which in some stars are more potent than the general statistical tendencies and cause a considerable spread in values from individual stars.

One might expect the relative displacement of the bright lines to vary more directly with type than with period, but this is not the case, and it is really not surprising in view of the fact that S- and

[1] The correlation of the subdivisions of class Me with period becomes quite clear when the S-type stars are removed from the list. See *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **46**, 472, 1922.

N-type stars exhibit similar displacements and that the S stars also show a progression with period. Some fundamental property such as mass or density probably influences both period and displacement.

The decrease of average random velocity with advancing type and increasing period is difficult to understand if we make the natural assumption that the Me stars continue the main sequence of types from B to M. According to this view, there is a great increase in velocity in passing from the ordinary M stars to those of classes M3e to M5e and then a decrease for classes M6e to M8e. This is so improbable that one can scarcely avoid the conclusion that we have to deal with something more complex than a single evolutionary sequence of identical objects passing from M8e to K. It may well be that individual stars differ in certain properties; e.g., mass or chemical composition, or that their cosmical environments are not sufficiently uniform to cause all stars to pursue precisely the same train of behavior.

The periods and the random velocities of the S-type stars are nearly the same as those of types M7e and M8e, but their spectra certainly do not belong at the end of the sequence M1e-M8e. Judging from its general appearance, the typical S spectrum more nearly corresponds to that of Class M1 or M2.

From twenty-five stars of class N, Moore[1] found a mean residual velocity of 18.0 km, which is decidedly less than the corresponding value for class Me, and somewhat less than that for class Se. The solar motion from the N stars does not resemble that from the Me stars, so that we may say that radial velocity data do not suggest a close connection between variables of classes M and N.

The average residual radial velocity of the Me stars is nearly the same as that of 102 planetary nebulae.[2] Whether or not this fact has any physical significance is a question for the future.

If the Me variables form a part of the main evolutionary sequence, they apparently represent either an initial or a final stage; their high velocities, however, make it difficult to consider

[1] *Lick Observatory Bulletins*, **10**, 160, 1922.

[2] *Publications of the Lick Observatory*, **13**, 168, 1918.

them very young stars just beginning their visible careers, while, on the other hand, it is very improbable that they are aged and highly condensed objects about to sink into obscurity.

In spite of our very considerable store of data concerning these objects, it seems plain that we cannot cope with the fascinating problems which they present without securing many more facts of various kinds.

The discussion of the velocity determinations may profitably be extended by using the proper motions which, chiefly through Wilson's work,[1] are now available for about 90 of the 133 stars having measured radial velocities. It is the intention of Dr. Strömberg and the writer to do this in another contribution.

MOUNT WILSON OBSERVATORY
June 1923

[1] *Astronomical Journal*, **34**, 183, 1923.