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THE SPECTRA OF THREE O-TYPE STARS

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ABSTRACT

O-Type Stars. These stars show enhanced line spectra which can be terrestrially reproduced only under extreme conditions of excitation. Their spectra afford an opportunity on the one hand to test theories on the origin of spectra, and on the other hand to ascertain some of the physical conditions in the stellar atmospheres. Three typical absorption line O-type stars were selected for detailed investigation, viz.:—10 Lacertae (Oe5), 9 Sagittae (Oe5), B.D. 35° 3930 N (Oe). Spectra were secured with one, two and three-prism dispersion. The plates were measured and the wave lengths determined in I. A. Precautions were taken to eliminate systematic errors in the wave lengths.

Pickering Components to the Balmer Lines. If the Pickering lines are due to enhanced helium, Bohr's theory predicts the existence of *enhanced helium components* about 2A to the violet of the hydrogen lines. The evidence is given in detail for the existence of these components. In 10 Lacertae components with the predicted separations and the correct intensities were found for the lines $H\alpha$, $H\beta$, $H\gamma$, $H\delta$. In 9 Sagittae a component was found for $H\gamma$, and suspected for $H\beta$, $H\delta$. In B.D. 35° 3930N components were measured for $H\gamma$ and $H\delta$ and suspected for $H\beta$. In view of the fact that the components were found to four of the Balmer lines of 10 Lacertae, and to $H\gamma$ in the three stars, though they differ markedly in type, the evidence is satisfactory that Bohr's predicted components are present in stellar spectra. Their demonstrated existence makes it highly probable that the Pickering lines and 4686 are due to enhanced helium.

Rydberg Constant for Helium. The mean wave lengths (I.A.), for the enhanced helium lines in 10 Lacertae and 9 Sagittae are used to determine the value of the Rydberg constant, N_2, for helium. The position of the circle component of each line is computed on the assumptions of $\Delta\nu_H = 0{\cdot}36$ and certain simple intensity relations of Sommerfeld. From the resulting wave numbers of these circle components, using Sommerfeld's theory which takes account of the relativistic variation in mass of the electron, the value of N_2, the Rydberg constant, is computed for each line. The weighted mean value is,—$N_2 = 109722{\cdot}3 \pm 0{\cdot}44$.

Spectroscopic Universal Constants. On Bohr's theory the Rydberg constant can be computed from the mass of the electron m_o, its charge e, and the value of Planck's constant h. Reversing the process, the values of these universal and other related constants can be computed with a high degree of accuracy from the spectroscopic value of Rydberg's constant. Revised values of the atomic weights of hydrogen and helium are used, and a method due to L. Flamm is followed in carrying out these computations. Birge's recomputed value of N_1 for hydrogen is used. The resulting stellar values are: $N_\infty = 109737{\cdot}3$; the ratio of mass H atom / electron, $M_1 / m_o = 1840 \pm 20$; $e/m_o = (1{\cdot}762 \pm 0{\cdot}019) \times 10^7$e.m.u. Assuming Millikan's value of the electronic charge, $e = (4{\cdot}774 \pm 0{\cdot}005) \times 10^{-10}$ e.s.u., the additional results are obtained, $m_o = (9{\cdot}04 \pm 0{\cdot}10) \times 10^{-28}$ gms; the radius of the electron $a = (1{\cdot}869 \pm 0{\cdot}021) \times 10^{-13}$ cms; $h = (6{\cdot}567 \pm 0{\cdot}042) \times 10^{-27}$ ergs secs; and Wien's constant in the radiation law $C_2 = 1{\cdot}4353 \pm 0{\cdot}0093$ cms degrees. These values are compared with recomputed values from Paschen's value of N_2, and also with results from other methods of determination.

Wave Lengths in O-Type Stars. All the lines that could be detected on the various plates were measured. The mean wave lengths in I.A. are given for the lines in each of the three stars, together with the probable errors of the wave lengths, the intensities of the lines, and the identifications where possible.

Physical Interpretation of Stellar Spectra. After a brief review of theories on the origin of spectra for atoms with more than one electron, *Saha's ionization theory* of stellar spectra is discussed. Emphasis is placed on the fact that, when an arc or a spark spectrum is on the point of disappearing, a certain minimum number of atoms

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must be in a condition to absorb the arc or spark lines. The attainment of this minimum number is determined not only by the fraction of atoms once or twice ionized, but also by the density of radiation from the photosphere and the relative abundance of the element in question. The behaviour of barium and sodium in the sun, discussed by Russell, is explained when these additional factors are allowed for. The *causes of ionization* in stellar chromospheres are discussed. It is shown that while the density of radiation from the photosphere is incapable of giving rise to the hydrogen spectrum in A-type stars, electron collisions with hydrogen atoms will result in the observed effects. In view of some theoretical uncertainty in Saha's theory, a check hypothesis of *ionization by electron collisions* is formulated.

Physical Conditions in O-type Stars. From Saha's theory modified for the relative abundance of elements and independently from the electron collision hypothesis, the *temperature* of 9 Sagittæ is deduced from the disappearance of $Mg_{+}4481$ to be in the mean 18,500°K. The temperatures of the two other O-type stars are determined from this temperature and those of the early B's by the use of the intensity ratios of certain pairs of lines. The temperature of 10 Lacertae so deduced is 15,000° K, and of B.D. 35° 3930 N with considerable uncertainty as it is an extrapolation, is 22,000° K. Using these temperatures it is possible independently from the two ionization theories to determine the *relative abundance* of hydrogen and helium in these stars. It is found that while hydrogen has probably the same percentage abundance as in the first ten miles of the earth's crust, helium is probably 10^6 times more abundant. A simple physical interpretation of these facts is found when possible nuclear disintegration in deeper layers of the stars is considered.

Classification of Absorption Line O-type Stars. The Harvard class Oe5 is in practice distinguished from the classes Oe and Od by the absence of emission. This use of the presence or absence of an emission line as a criterion of type is shown to be inconsistent with the rest of the Harvard classification and physically unsound. Its use leads to the inclusion of the bulk of O-type stars in the class Oe5, though it is shown that from the intensities of absorption lines more than two-thirds of the stars so classified are earlier in type than Oe5. It is suggested that, as the absorption line O's form a continuous sequence with the B's the present Harvard symbols and their meanings be abandoned and the following decimal classification be substituted. Class Oo—Pickering lines disappeared; Class O5 (B.D. 35° 3930 N)—Ordinary helium disappeared; Class O7 (9 Sagittae)—Mg 4481 missing; Class O9—Si III pair 4552, 4567 on the point of appearing. The coordinates, Harvard class, line intensities and proposed classes of 33 absorption line O's whose spectra have been secured here are given. The exclusion of the Wolf Rayet stars from the tentative scheme is justified.

Introduction

From their spectra it is evident that the atmospheres of O-type stars are in a highly ionized condition. This is shown by the appearance of the ζ Puppis series in these stars, the lines of which were first discovered by E. C. Pickering[1*] in stellar spectra and were reproduced faintly in the laboratory under the most condensed discharge, by Fowler[2] some fifteen years later. The subsequent identification in O-type spectra of some enhanced carbon and oxygen lines produced under similar conditions of excitation pointed to a like conclusion, namely that the atoms in the atmospheres of these stars were highly ionized. The spectra of such stars, it may then be expected, will furnish a convenient method of studying the behaviour of ionized atoms, and also will give some information on the causes and the amount of ionization in the stellar atmospheres. It is with these aspects of O-type spectra that this paper is more particularly concerned.

The investigation had its origin in an attempt to discover in O-type stars the Pickering components to the Balmer lines. Shortly after Fowler's discovery of the Pickering lines and the two additional associated series in the laboratory, Dr. N. Bohr[3] on the basis of his atomic theory stated that all these lines were due, not to hydrogen atoms as had formerly been believed, but to helium atoms which had permanently lost one electron. If his theory were correct he predicted that there should appear, some 2A to the violet of the Balmer lines, enhanced helium components which would form with the lines already

* References will be found at the end of Part I.

found by Pickering, a single series. From laboratory experiments Paschen[4] showed in 1916 that these components existed. Clearly it was of importance, if only for the sake of completeness, to detect whether these components existed in stellar spectra; the more so as it would definitely show that the Pickering lines were due to enhanced helium. Preliminary experiments were initiated in 1920, and in 1921 sufficient evidence had been secured to justify an announcement[5] of the discovery of these components. The next step was to determine with every accuracy the wave lengths of all the enhanced helium lines in these O-type stars. This in its turn involved a determination of the wave lengths of other stellar lines. The final result has been, after several extensions of scope, a discussion, as complete as possible, of the physical problems of O-type stars.

The arrangement of the paper is briefly as follows: *Part* I contains the necessary introductory material on the selection of stars, the instruments used, the observational data and the method of wave length determination. Essentially *Part* II is an application of stellar data to the problems of atomic structure. The evidence is given in detail for the existence of Bohr's predicted components, the value of the Rydberg constant for helium is computed, and from it are deduced stellar spectroscopic values of the mass of the electron and related universal constants. On the other hand *Part* III is an application of physical theories to the interpretation of the observed O-type spectra. After complete tables of the wave lengths of the lines in the three O-type stars are given, there follows an application of the methods developed in the Appendix in order to ascertain from the stellar spectra some of the physical conditions in the chromospheres of O-type stars. A practical astronomical result is a tentative revision of the Harvard classification for absorption line Class O stars. At the close of the paper is an *Appendix* in which is discussed the physical interpretation of stellar spectra. Two methods are there developed, one a slight modification of Saha's well known theory and the other an independent hypothesis of ionization by electron collision, which give independently either the stellar temperature or the relative abundance of the element from the disappearance of arc or spark lines in the spectrum of the star.

In writing the paper an attempt has been made to give, as the occasion for their use arises, a brief explanation of the various recent developments on atomic structure and origin of spectra. It is hoped, by thus reducing the labour of looking up references, that the value of the paper will be somewhat increased. In conclusion I would like to acknowledge the assistance rendered by my father, Dr. J. S. Plaskett, in particular, for the use of spectra taken by him in the course of his radial velocity programme, for his suggestions on the classification of O-type stars and for his patience and helpful discussions during a somewhat protracted investigation. Acknowledgment should also be made to Prof. E. H. Archibald of the University of British Columbia for the data he made available on recent determinations of the atomic weights of hydrogen and helium.

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PART I—METHODS OF OBSERVATION

In this part of the paper are given details of the stars selected for observation, the instruments used and the observational data of the various plates. In the second section the method of wave length determination is explained.

SECTION 1—OBSERVATIONAL DATA

In view of the objects of the investigation three conditions determined the selection of stars for observation. First, the Pickering and Balmer lines had to be sharp enough to admit of the detection of components separated by less than 2A. Second, the stars had to be bright to permit the use of high dispersion. And finally, it was desirable that the spectra should show as large a range in type as possible. The first two of these conditions eliminated the Wolf Rayet stars, and restricted observations to the absorption line O's, Harvard types Od, Oe, Oe5. A preliminary examination in 1920 of the stars in

TABLE 1—O-TYPE STARS

Star	R.A. 1900		Dec. 1900		Vis. Mag.	Harv. Type	Remarks
	h	m	°	′			
10 Lacertae	22	34·8	+38	32	4·91	Oe5	Sharp lines. He strong. 4481 Mg_+ present. 4686 He_+ sharp. Plate, fig. 1.
9 Sagittae	19	47·9	18	25	6·29	Oe5	Lines more diffuse. 4481 Mg_+ disappeared. Fowler's N_+ lines strong. Plate, fig. 2.
B.D. 35° 3930N	19	59·8	35	45	7·23	Oe	North component of Σ 2624, sep. 2″ .He disappeared. He_+ strong.

Miss Cannon's list[6] and of 10 Lacertae, suggested by Frost,[7] led to the selection of B.D. 35° 3930N and 10 Lacertae. Before any further work was done the Director, Dr. J. S. Plaskett, had commenced in 1921 observing O-type stars for radial velocity, using a very complete list of stars sent him by Miss Cannon. He suggested the inclusion of 9 Sagittae, a star which showed well defined Pickering lines. The positions, magnitudes, Harvard types and remarks on the spectra of these three stars are given in Table 1. Observations were confined to them, as it was felt that more information could be secured from their detailed study than from the same number of observations scattered over a greater number of stars.

Spectra were made with the universal spectroscope[8] at the 108-foot focus of the 72-inch telescope. Four features in the design of the spectroscope may be noted,—(1) the mounting, (2) method of changing the dispersion, (3) the insertion of the comparison spectrum and (4) the temperature control. (1) All the optical parts of the spectroscope are enclosed in a braced aluminium box, so that they form a single unit. This box is flexibly supported at two points in a frame which is attached to the telescope. The result is a great reduction in flexure when the telescope turns through large hour angles as in a long exposure. (2) The prisms are carried on a minimum deviation link work inside the aluminium box; each prism is held in a cell, which cell can be accurately replaced on the link work by virtue of two dowel pins. So well is this feature of the apparatus designed that it is possible to change from one to two or three-prisms, alter the angle of minimum deviation when necessary and make a focus test within an hour. Throughout the work the medium focus camera of 28 inches focal length was used, the focal length of the collimator being 45 inches. The resulting linear dispersions in A.U. per mm. are given in Table 2.

TABLE 2—LINEAR DISPERSION.

Prisms	Linear Dispersion, A.U. per mm.			
	Hα	Hβ	Hγ	Hδ
I	122·90	45·16	29·34	23·22
II	66·10	23·42	14·34	10·92
III	45·33	16·56	9·26	6·62

(3) In order to insert the comparison spectrum two right angle prisms are held in position over the slit with a small aperture between them for the star image. The light from a 110 volt, 4 ampere D.C. iron arc is directed by a lens through several opal and ground glass screens on to the faces of these prisms, and thence on to the slit in the form of two limited patches on either side of the star image. The insertion of the opal glasses ensures that the collimator is filled. The advantage of this arrangement is that the comparison spectrum may be put on at intervals throughout the exposure without disturbing the slit or cutting off the light of the star. Comparisons were inserted at equal intervals at least three times, and upon occasion as many as ten times during the exposure. The only disadvantage of the arrangement is that the tips of the comparison lines are separated by approximately one-third of a mm. from the star spectrum, thus necessitating a correction for line curvature. (4) During the 1921 observations, over 80 per cent of the whole, the temperature in the spectroscope was kept constant, to about 1-20th degree centigrade by a Callendar Recorder.[9] In 1920 the more usual mercury thermostat arrangement was used and gave satisfactory results.

In Table 3 are given details of the various spectra which were secured of 10 Lacertae, 9 Sagittae, B.D. 35° 3930N. The first column contains the G.M.T. of the exposure, the second the slit width, units being thousandths of an inch, the third column gives the dispersion, the fourth the region in focus on the plate, the fifth the brand of plate used—Seed 30 (fast,) Seed 23 (fine grain), Ilford Panchromatic, Ilford Panchromatic hypersensitized—, and the sixth contains the duration of exposure in hours and fractions.

TABLE 3—OBSERVATIONAL DATA

Date G.M.T.	Slit Width	Disp.	Region	Plate	Exp.	Date G.M.T.	Slit Width	Disp.	Region	Plate	Exp.
											hrs.
10 Lacertae					hrs.	1921 Sept. 29·80	1·6	I*	0·39−0·67 μ	I.P.h.	1·00
1920 June 19·97	1·2	I	0·39−0·49 μ	S 30	0·40	Oct. 1·65	2·0	III	0·53−0·67	I.P.h.	1·97
July 31·91	1·5	I	0·39−0·49	S 23	1·00	*9 Sagittae*					
Aug. 30·85	1·8	I	0·39−0·49	S 30	0·13	1921 July 14·79	2·0	I*	0·39−0·49	S 30	1·00
1921 July 26·96	2·0	I	0·39−0·67	I.P.	1·70	" 21·89	2·0	I*	0·39−0·49	S 30	0·47
Aug. 11·92	2·0	I*	0·39−0·49	S 23	0·37	Aug. 9·73	2·0	II	0·41−0·49	S 30	2·10
" 12·98	2·0	III	0·43−0·49	S 30	1·42	" 10·86	2·0	I*	0·39−0·49	S 30	0·60
" 26·86	2·0	II	0·41−0·49	S 23	1·33	" 12·82	2·0	III	0·43−0·49	S 23	6·00
" 26·91	2·0	II	0·41−0·49	S 23	1·00	" 26·75	2·0	II	0·41−0·49	S 23	4·00
Sept. 2·82	2·0	II	0·49−0·67	I.P.	7·50	Sept. 23·72	2·0	II	0·39−0·45	S 23	5·00
" 8·95	1·5	I*	0·39−0·49	S 23	0·33	*B.D.35°3930 N*					
" 9·84	2·0	III	0·53−0·67	I.P.	9·50	1920 July 3·89	2·0	I	0·39−0·49	S 30	0·67
" 11·90	1·5	I*	0·39−0·67	I.P.	1·27	1921 Aug. 6·79	2·0	I	0·39−0·49	S 30	1·00
" 13·84	1·5	II	0·49−0·67	I.P.	9·73	" 11·79	2·0	I*	0·39−0·49	S 30	0·93
" 23·88	1·4	II	0·39−0·45	S 23	2·80	Sept. 8·77	2·0	I*	0·39−0·49	S 30	0·73
" 28·69	1·4	II	0·39−0·45	S 23	3·50						

*Spectra taken by J. S. Plaskett.

The following additional comments may be made. In 10 Lacertae the only difficulty lay in securing high dispersion spectra in the region 0·49 — 0·67 μ. Such spectra were essential in order to detect the component to Hα,—in view of the diffuseness of the lines and the small linear dispersion at Hα with I-Prism (see Table 2). With Ilford Panchromatic Plates and II or III-Prism dispersion, as Table 3 shows, exposures of the order of eight hours were necessary. The unavoidable flexure of the spectroscope in these long exposures resulted in spectra that were not of the best definition. Some experiments were tried with hypersensitizing the Panchromatic Plates with an ammonia bath.[10] Though the exposures in this way were reduced to less than one-third of their former value, the plates unfortunately showed reticulation and it was not until after several failures that a moderately satisfactory plate was obtained. In neither 9 Sagittae nor B. D. 35°3930N were spectra in the red secured, though two unsuccessful attempts were made in the case of 9 Sagittae. Spectra of B.D. 35°3930N were difficult to obtain, in so far as seeing conditions much above the average were required to separate it clearly from its southern component. A number of the I-Prism plates of the three stars were taken by Dr. J. S. Plaskett in the course of his radial velocity programme of O-type stars. These are denoted in the table by asterisks. Only three out of the ten 9 Sagittae plates of J. S. Plaskett were included,—namely those which, by inspection with an eyepiece, clearly showed the presence of the Pickering component to Hγ.

Section 2—Method of Wave Length Measurement

The plates, secured as detailed in Table 3, were measured on the Gaertner machine. In addition all II and III-Prism plates and all I-Prism panchromatic plates were remeasured after an interval of two weeks or longer on the Toepfer machine. The plates were measured red right and red left, iron arc comparison lines and stellar lines being measured as they occurred. Four settings were made on each line except in the case of the Pickering lines when as a rule, because of their diffuseness, eight settings were made. Each measure was reduced separately to determine the wave lengths. Three stages in this determination may be distinguished:—(1) the preliminary reductions, (2) the correction for curvature and (3) the correction for velocity. These may be considered in turn.

Preliminary Reductions. Using Burn's[11] wave lengths in I.A. of the lines in the iron arc comparison spectrum, the Hartmann constants were computed for each measure of a plate from three of these iron arc lines. From these constants the measures of comparison lines and stellar lines in mm. were converted into wave lengths. As the Hartmann formula does not hold accurately, the residuals (Burn's λ — computed λ) were formed for each comparison line and a curve of errors was drawn. From this curve the corrections for each stellar wave length were read off and applied.

Curvature Correction. Owing to the fact, as pointed out in sec. 1, that the tips of the comparison lines are separated by 0·3 mm. from the star spectrum, there must be applied to these computed wave lengths a correction for curavture. The curvature of the spectral lines results in a displacement to the red of the stellar lines relative to the comparison lines by an amount which is no simple function of the wave length. In order to determine the value of this correction the following procedure was adopted. A series of exposures

TABLE 4—CURVATURE CORRECTIONS.

λ	I-Prism	II-Prism	III-Prism	λ	I-Prism	II-Prism	III-Prism
4000	−0·007A	−0·008A		5500	−0·024A	−0·030A	−0·025A
4300	·011	·013	−0·010A	5800	·028	·035	·028
4600	·014	·017	·014	6100	·031	·039	·032
4900	·018	·022	·018	6400	·035	·044	·036
5200	·021	·026	·021	6700	−0·038	−0·048	−0·039

was made with each of the three dispersions, the whole length of the slit being exposed to the light of the comparison arc. The curvature of some eight or more lines in the length of the spectrum from 3900—6700 were determined from measures of six points in each line. The curves were assumed, as is customary, to be parabolas, and a series of least-square solutions were run through to determine the constants for each line. From the constants the value of the correction in A.U. to be applied to a stellar line at the given wave length could readily be determined. As the value of these corrections were known for at least eight wave lengths through the spectrum, a curve could be drawn through these points for each dispersion. From these curves were read off the corrections to be applied to a stellar line at any wave length and for any dispersion. A shortened list of these corrections appears in Table 4. It is to be noted that even if these corrections are not absolutely accurate but are relatively accurate along the length of the spectrum, the resulting wave lengths, after the velocity correction is applied, will be correct.

Velocity Correction. The stellar wave lengths as so far obtained still differ from their true values by reason of the relative velocity of the star and the observer,—the Doppler effect. This causes a displacement, $\Delta\lambda$, in the stellar lines where $\Delta\lambda = \frac{v}{c} \cdot \lambda$, v being the relative velocity of star and observer and c the velocity of light. In order to eliminate this displacement certain standard velocity lines are selected, the value of $\Delta\lambda$ (star—laboratory) is determined and v is computed for each line. The weighted mean of these velocities is then used to recompute $\Delta\lambda$ for each stellar line. The correction is applied with the proper sign, and the resultant wave lengths are the true values in the star.

The selection of these velocity standards is most important as it is here that the chances of introducing systematic error are greatest. The following criteria were applied. (1) The lines must have good laboratory wave lengths, preferably determined from interferometer measures or based on Burn's tertiary iron arc standards. (2) The lines in the star must be sharp and readily measurable. (3) The lines must not be likely to be blended with other known lines. Following these criteria the list of lines in Table 5 was drawn up for each star. The first column contains the laboratory wave length in I.A., the second the source and the third the authority, the numbers in this column referring to references at the bottom of the table. The fourth column shows the number of measures in which the line was used as a velocity standard, and the fifth column is the mean weight assigned to the line.

TABLE 5—VELOCITY STANDARDS

λ (I.A.)	Origin	Authority	Used	Mean Wt.	λ (I.A.)	Origin	Authority	Used	Mean Wt.	λ (I.A.)	Origin	Authority	Used	Mean Wt.
10 Lacertae					4471·477	He	1	18	6·2	4340·467	H	4	10	5·1
3964·727	He	1	6	2·3	4481·195	Mg+	5	8	1·2	4387·928	He	1	5	2·1
4072·156	OII	2	5	1·0	4510·91	N+	3	7	1·3	4471·477	He	1	11	6·2
4075·869	OII	2	10	1·3	4514·865	N+	3	8	1·2	4510·91	N+	3	6	2·0
4097·327	N+	3	16	4·1	4713·143	He	1	10	3·1	4514·865	N+	3	7	1·6
4101·738	H	4	12	6·6	4861·326	H	4	5	5·0	4634·165	N+	3	3	1·0
4103·393	N+	3	12	2·2	4921·929	He	1	8	3·0	4640·649	N+	3	3	1·0
4119·222	OII	2	4	1·0	5015·675	He	1	5	2·8	4713·143	He	1	4	3·2
4120·812	He	1	16	2·2	5592·35	OIII	6	8	4·0	4921·929	He	1	1	3·0
4143·759	He	1	15	2·2	5875·618	He	1	8	6·6					
4319·647	OII	2	3	1·0	6562·793	H	4	3	6·7	*B.D. 35° 3930N*				
4340·467	H	4	13	6·5	6678·149	He	1	8	2·9	4101·738	H	4	2	4
4349·435	OII	2	10	1·5						4340·467	H	4	3	4
4366·906	OII	2	3	1·0	*9 Sagittae*					4634·165	N+	3	4	1
4387·928	He	1	20	3·9	4097·327	N+	3	4	4·2	4640·649	N+	3	4	1

Authorities
1— P. W. Merrill — Bureau of Standards, Scientific Paper, No. 302, 1917.
2— J. S. Clark — Astrophysical Journal, *40*, 332, 1914.
3— A. Fowler — Monthly Notices, R.A.S., *80*, 692, 1920.
4— W. E. Curtis — Proc. Royal Society A, *90*, 605; *96*, 147.
5— A. Fowler — Phil. Trans. Roy. Soc. A. *214*, 225, 1914.
6— A. Fowler — Monthly Notices, R.A.S., *77*, 511, 1917.

The following comments may be made on these velocity standards. No ionized helium lines were used as velocity standards as it was desired to determine their wave lengths independently of any previous determinations. The hydrogen lines were only used as standards when the Pickering components could be measured,—otherwise the measured Balmer line would be a blend. In *10 Lacertae* the He line 4026 was not used as a standard as it is blended with a Pickering line in the same position. The enhanced nitrogen lines 4379, 4523 were not included because they were poorly defined and difficult to measure. A number of oxygen lines are included as standards, but have uniformly been given low weight as compared with the stronger and better defined lines of other elements. In *9 Sagittae*, as the table shows, there are fewer velocity standards, and this has of necessity meant that the correction for velocity in the individual measures is not so good as it is in 10 Lacertae. Clearly, however, it is preferable to depend upon a few good lines of accurate wave length which are free from blends, than it is to use a greater number of lines of inferior quality. If this latter procedure be adopted the chances of systematic error are great. The use of few velocity standards will, at the worst, only introduce an accidental error which in the mean will be eliminated. For *B. D. 35° 3930N*, as well as for 9 Sagittae, the enhanced nitrogen lines 4634·165, 4640·649 appear as emission and have only been given unit weight in determining the velocity correction. On the whole it is felt that the lines selected form a homogeneous group, and that the mean stellar wave lengths of a number of measures which result from their use should be free from avoidable systematic error.

A by-product of the correction for velocity is of course the velocities of the stars themselves. These velocities, corrected to the sun[12], are given in Table 6. For 10 Lacertae the individual velocities are probably worthy of some confidence as they depend upon the use of a large number of velocity standards. Exceptions to this are the panchromatic plates with higher dispersion, in which only three or four velocity standards have been used. In these cases the velocity is of doubtful value. Omitting these cases, denoted by a P, it appears probable that 10 Lacertae has a constant radial velocity of −10·1 ± 0·4 km. In the case of 9 Sagittae the use of a few velocity standards for each

TABLE 6—RADIAL VELOCITIES OF O-TYPE STARS

Date G.M.T.	Disp.	Rad. Vel.	Date G.M.T.	Disp.	Rad. Vel.	Date G.M.T.	Disp.	Rad. Vel.
10 Lacertae			Sept. 9·84	III P	−19·0	Aug. 10·86	I	+ 3·6
1920 June 19·97	I	− 6·4	" 11·90	I	−11·1	" 12·82	III	+11·2
July 31·91	I	− 8·0	" 13·84	IIP	−10·4	" 26·75	II	+11·8
Aug. 30·85	I	−10·1	" 23·88	II	−14·4	Sept. 23·72	II	+ 4·8
1921 July 26·96	I	−11·1	" 28·69	II	−11·7			
Aug. 11·92	I	− 9·8	" 29·80	−I	− 9·2	*B.D. 35° 3930 N*		
" 12·98	III	− 8·1	Oct. 1·65	IIIP	−12·0	1920 July 3·89	I	+ 9
" 26·86	II	− 8·1	*9 Sagittae*			1921 Aug. 6·79	I	−19
" 26·91	II	−12·3	1921 July 14·39	I	+17·5	" 11·79	I	− 6
Sept. 2·82	IIP	− 2·2	" 21·89	I	+ 2·2	Sept. 8·77	I	− 1
" 8·95	I	−10·5	Aug. 9·73	II	+27·1			

measure makes it difficult to decide whether the star has a constant velocity, or is a spectroscopic binary. J. S. Plaskett, from his measures of a number of single-prism plates using a large number of velocity standards, has concluded that the star has a constant velocity. From Table 6 its mean velocity would be $+11 \cdot 2 \pm 2 \cdot 3$ km. Little can be concluded from the radial velocities for B. D. 35° 3930N. In view of the poor quality of the lines and the fact that as a rule only three velocity standards could be used, the range is no larger than would be given by a constant velocity star. From the measures the velocity would be -4 km.

In conclusion the following points on the method of wave length determination may be noted. The computations were carried through to the thousandth of an A.U. but after the application of the velocity correction the wave lengths were shortened to hundredths. Throughout wave lengths expressed in I.A. have been used so that the resulting stellar wave lengths will also be in this scale. The various computations have been checked several times, and every effort has been made to eliminate possible systematic error. As a consequence, some confidence is felt that the resulting mean wave lengths are as accurate as can be expected from stellar spectra with somewhat ill-defined lines.

References to the Introduction and Part I

[1] A brief history of the Pickering lines and their relation to Bohr's theory will be found under the title—The Pickering Series and Bohr's Atom.
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PART II—BOHR'S HELIUM LINES AND THE MASS OF THE ELECTRON

In this part of the paper are treated applications of astronomical measurement to theories of atomic structure and to the determination of the values of atomic constants. In the first section, after a theoretical introduction, the evidence is given in detail for the existence of Bohr's predicted Pickering components to the Balmer lines in O-type spectra. In the next section the mean wave lengths of the ionized helium lines in 10 Lacertæ and 9 Sagittæ are used to determine the value of N_2, Rydberg's constant for helium. From this value of N_2 and from Birge's recomputed value of N_1 for hydrogen, the spectroscopic values of the electron mass and other universal constants are determined in the final section.

Section 3—Evidence for the Pickering Components in O-Type Stars

Bohr[1]* formulated his theory for the purpose of explaining the spectrum of an element in terms of the structure of its atom. He considered the case of a single electron (charge —e, mass m_o) and a nucleus (charge E, mass M) revolving in circle orbits about their common centre of gravity. By assuming,—(1) that the combined angular momentum of electron and nucleus was only equal to $nh/2\pi$ where n = 1, 2, 3.. and h is Planck's constant; and by assuming,—(2) that losses of energy in the atomic system occasioned by the shrinkage of electron and nucleus to inner orbits appeared as monochromatic frequency, $\nu = \Delta W/h$ where ΔW is the loss of atomic energy, Bohr found that the formula for spectral emission was

$$\nu = \frac{2\pi^2 m_o e^4}{h^3 (1+m_o/M)} \cdot \left(\frac{E}{e}\right)^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right) = N\left(\frac{E}{e}\right)^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad \ldots . (1)$$

In this formula m is the quantum number of the initial orbits, n the quantum number of the orbits into which the electron and nucleus fall, and N, if the known values of m_o, e, h be substituted, turns out within the limits of error of m_o, e, h to be the Rydberg constant. For hydrogen, E = + e, (1) is then the formula for the Lyman series when n = 1, m = 2, 3, 4..., for the Balmer series when n = 2, m = 3, 4, 5..., and for the Paschen series when n = 3, m = 4, 5, 6.... The physical interpretation of the formula in the case of the Balmer series is that Hα is given by atoms in which the electron is falling from the three to the two quantum orbit, Hβ by atoms in which the electron is falling from the four to the two quantum orbit and so on. For ionized helium atoms (atoms which have permanently lost one of their two electrons) (1) then becomes, for n = 3, m = 4, 5, 6..., the formula for the complete 4686 series found by Fowler[2] in the laboratory. The case when n = 4 is $\nu = 4 N_2 \left(\frac{1}{4^2} - \frac{1}{m^2}\right)$, where m = 5, 6, 7..., is the complete Pickering series. Odd values of m give the lines 5411, 4541, 4200, etc., found by Pickering[3] in ζ Puppis, and even values give lines close to but not coincident with the Balmer lines, since N_2 (helium) = $N_1 \left[\frac{1 + m_o/m_1}{1 + m_o/4m_1}\right]$ approx. = $1 \cdot 000407\, N_1$ approx. where N_1 is the constant for hydrogen and m_1 is mass of hydrogen nucleus. Actually these additional Pickering lines should lie between 1 and 3 A to the violet of the Balmer lines.

*References at the end of Part II.

Evidently the discovery of these predicted Pickering components would furnish a most important verification of Bohr's theory. In the laboratory Evans [4] has been partially and Paschen[5] completely successful in isolating them. In stellar spectra the difficulty lies in the diffuseness of the Pickering lines, so that increase of dispersion only tends to weaken and widen without helping to separate them from the Balmer lines. Accordingly the most important factor in the detection of the components is the selection of stars with comparatively sharp Pickering lines; such in fact are the stars 10 Lacertæ, 9 Sagittæ and B.D. 35° 3930 N. The existence of the components in these stars will be most clearly shown by giving detailed tables of the individual wave lengths for each measure of the three stars.

The detailed measures for **10 Lacertae** are given in Table 7. The first column contains the date on which the plate was taken, and the second the dispersion which is indicated by the numerals I, II, III (see Table 2),—the letters T or G following these numerals indicate on which machine, Toepfer or Gaertner, the plate was measured. The succeeding columns are for Hα, its He+, component, a line of unknown origin at λ6558, the He+ line λ5411, Hβ, its He+ component, the He+ line λ 4686, the He+ line λ4541, Hγ, its He+ component, a Ti+ line λ4338, the He+ line λ4200, the N+ line λ4103, Hδ and its He+ component. In the columns themselves only units and fractions of an A.U. are given, the first three figures of the wave length being given at the head of each column. At the foot of each column is given the simple mean of the wave lengths, the probable error of the mean and the estimated intensity of the line. The following comments may be made on the individual lines in the table.

Hα. The detection of the Pickering component to Hα is made difficult for three reasons:—(1) the low linear dispersion, not greater than 45 A.U. to the mm. at Hα (see Table 2), (2) the flexure introduced by long exposures, and (3) the presence of a line of unknown origin at λ6558. Nevertheless on one each of the measures of the three high dispersion plates which could be used at Hα, a component has been set on with mean wave length 6560·04. When this component has not been seen, in two of the three cases the measures of Hα are shifted 0·6 A to the violet showing that a blend of Hα and its He+ component has been measured. It seems probable, therefore, that the predicted Pickering component to Hα is present.

He+ 5411. This line is not easy to measure as it is very ill-defined.

Hβ. A component to Hβ is readily seen on all the high dispersion spectra. Fig. 3 of the Plate is a reproduction of Hβ and its component from the III-prism spectrum of Aug. 12·98. In four out of the six cases, where Hβ has been measured, the wave length of the component has been determined. Its mean wave length, 4859·08, and its abnormally high intensity, compared with 5411 and 4541, point to a probable blend of He+ (Paschen's λ4859·34) and N+ (Fowler's λ 4858·82). Lines of enhanced nitrogen are very common in 10 Lacertæ as Table 13, Part III, shows.

TABLE 7—WAVE LENGTHS (I.A.) OF H, He+ AND OTHER LINES IN 10 LACERTAE

Date	Disp.	Hα 656–	He+ 656–	– 655–	He+ 541–	Hβ 486–	He+ 485–	He+ 468–	He+ 454–	Hγ 434–	He+ 433–	Ti+ 433–	He+ 420–	N+ 410–	H δ 410–	He+ 410–
1920June19·97	IG	.....	.....	.....	.....	1·31	.....	5·63	1·90	0·31	.....	7·81	0·28	3·45	1·66	0·15
July31·91	IG	.....	.....	.....	.....	.....	.....	.....	1·74	0·39	.....	7·64	9·62	.....	.....	.....
Aug.30·85	IG	.....	.....	.....	.....	.....	.....	.....	.....	0·58	9·12	.....	9·65	3·18	1·72	0·59
1921July26·96	IT	.....	.....	.....	2·16	.....	.....	5·70	1·69	.....	.....	.....	0·17	.....	.....	.....
	IG	.....	.....	.....	1·91	.....	.....	5·69	1·74	0·23	.....	7·88	9·99	.....	.....	.....
Aug.11·92	IG	.....	.....	.....	.....	.....	.....	.....	.....	0·32	.....	7·78	9·48	.....	.....	.....
Aug.12·98	IIIT	.....	.....	.....	.....	1·33	8·82	5·73	1·68	0·35	8·41	.....	.....	.....	.....	.....
	IIIG	.....	.....	.....	.....	1·35	8·90	5·63	1·59	0·32	8·50	.....	.....	.....	.....	.....
Aug.26·86	IIT	.....	.....	.....	.....	1·34	.....	5·69	1·73	0·39	8·90	.....	0·19	3·31	1·73	0·37
	IIG	.....	.....	.....	.....	.....	.....	5·72	1·82	0·41	9·05	.....	0·21	3·31	1·76	0·43
Aug.26·91	IIT	.....	.....	.....	.....	1·32	9·18	5·75	1·77	0·50	9·12	.....	0·01	.....	.....	.....
	IIG	.....	.....	.....	.....	1·30	9·40	5·73	1·72	0·42	8·88	.....	0·07	3·33	1·69	0·48
Sept. 2·82	IIT	2·66	.....	8·20	1·41	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	IIG	2·68	0·02	8·04	1·15	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Sept. 8·95	IG	.....	.....	.....	.....	.....	.....	5·77	1·50	.....	.....	.....	.....	3·31	1·72	0·27
Sept. 9·84	IIIT	2·46	0·09	8·63	1·62	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	IIIG	1·98	.....	8·16	1·43	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Sept.11·90	IT	.....	.....	.....	1·86	.....	.....	.....	1·32	.....	.....	.....	0·12	3·38	1·69	9·93
	IG	.....	.....	.....	1·46	.....	.....	.....	1·18	0·54	8·84	7·68	0·19	3·23	1·70	9·98
Sept.13·84	IIT	.....	.....	.....	1·61	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	IIG	.....	.....	.....	1·60	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Sept.23·88	IIT	.....	.....	.....	.....	.....	.....	.....	.....	0·16	8·49	.....	.....	.....	.....	.....
Sept.28·69	IIT	.....	.....	.....	.....	.....	.....	.....	.....	0·41	9·07	.....	0·19	3·36	1·74	0·61
	IIG	.....	.....	.....	.....	.....	.....	.....	.....	0·46	9·26	.....	0·17	3·39	1·73	0·61
Sept.29·80	IT	.....	.....	.....	1·73	.....	.....	.....	1·33	0·48	8·73	.....	0·27	3·39	1·70	9·70
	IG	.....	.....	.....	1·54	.....	.....	.....	.....	0·37	8·96	.....	0·34	3·26	1·77	0·11
Oct. 1·65	IIIT	2·00	.....	8·43	1·44	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	IIIG	2·62	0·01	8·04	1·72	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Means:—	......	2·59	0·04	8·25	1·62	1·32	9·08	5·70	1·62	0·42	8·87	7·76	0·06	3·32	1·72	0·27
Prob. Errors:—	......	±·04	±·02	±·06	±·04	±·01	±·09	±·01	±·04	±·02	±·05	±·03	±·04	±·01	±·01	±·06
Intensities:—	......	10	3	4	4	10	5	9	3	9	2	2	2	3	9	2

He+ 4686. This line is one of the best in the spectrum; it is very sharp and the measures are accordant. The mean wave length of 4685·70 may be compared with Fowler's[2] laboratory value of 4685·80, Frost's[6] 10 Lacertæ value of 4685·72, and Paschen's[5] wave length (the computed centre of gravity from his value of N_2) of 4685·74.

He+ 4541. This is the usual diffuse line of the Pickering series.

Hγ. Though not easy to see with a low power magnifier, the Pickering component has been seen and measured in 13 out of 17 measures of Hγ. Furthermore, in the cases in which it has not been measured, the mean wave length of Hγ – 4340·31, as compared with 4340·42 in cases where the component has been measured, shows that a blend of Hγ and its He+ component has probably been set on. On 5 I-prism plates a Ti+ line at 4338 has also been measured, but in only one of these measures have Hγ, its He+ component and the Ti+ line been seen. It is curious that the Ti+ line has never been measured on higher dispersion plates, and it is probable that the line is of low central intensitv and diffuse, so that increase of dispersion "washes it out."

He+ 4200. This line is not easy to measure as it is diffuse and ragged. From its mean wave length 4200·06 it is probable that it is composed almost entirely of N+ 4200·06.

Hδ. The He+ component has been measured most easily on I-prism plates in which the dispersion of 23 A to the mm. seems to be of the right order for separation, without widening the line too greatly. A reproduction from the I-prism plate of Sept. 11·90 of the group N+ 4103, Hδ 4102, He+ 4100 is given in fig. 4 of the Plate. No attempt was made to measure the He+ component unless the N+ line to the red of Hδ was clearly visible. The mean of 12 measures gives a wave length of 4100·27 which, when compared with Paschen's value 4100·05, would seem to indicate a blend. It is noteworthy, however, that in the plate of best definition, I-prism Sept. 11, the wave length 4099·96 is in close agreement with Paschen.

Summarizing, the evidence for 10 Lacertae definitely indicates the presence of components near the predicted and laboratory positions for Hβ, Hγ, Hδ. For Hα the evidence is not so good, but, in view of the low linear dispersion and the difficulty of measurement, there can be little doubt that the component is there.

The detailed measures for 9 Sagittae are given in Table 8. This table is of precisely the same form as that for 10 Lacertae, the successive columns after the date and disperion giving the wave lengths for the individual measures of Hβ, a N+ component, the Pickering line at λ4541, Hγ, its He+ component and measures in the neighbourhood of Hδ.

TABLE 8—WAVE LENGTHS (I.A.) OF H, He+, N+ IN 9 SAGITTAE

Date	Disp.	Hβ 468–	N+ 485–	He+ 454–	Hγ 434–	He+ 433–	He+ 420–	N+ 410–	Hδ 410–	He+ 410–
1921 July 14·79	I G	......	......	1·48	0·62	8·43	0·07	......	......	......
July 21·89	I G	......	......	2·07	0·30	8·52	0·30	......	......	......
Aug. 9·73	II T	0·83	8·83	1·62	0·49	8·24	9·69	......	......	......
	II G	......	......	1·65	0·54	8·61	9·85	......	......	......
Aug. 10·86	I G	......	......	2·06	......	......	0·40	......	......	......
Aug. 12·82	III T	......	......	1·52	0·66	9·36	......	......	......	......
	III G	......	......	......	0·61	9·15	......	......	......	......
Aug. 26·75	II T	0·66	8·23	1·87	0·40	8·45	9·86	......	......	......
	II G	0·74	8·41	1·84	0·36	8·85	0·07	3·19	1·84	0·77
Sept. 23·72	II T	......	......	......	0·20	8·64	0·20	3·66	2·20	1·03
	II G	......	......	......	0·30	8·86	0·24	3·73	2·11	0·70
Means:	......	0·74	8·49	1·76	0·45	8·71	0·08	3·53	2·05	0·83
Prob. Errors	......	±·03	±·12	±·05	±·03	±·07	±·05	±·12	±·07	±·07
Intensities	......	9	4	6	9	4	4	5	8	4

The means, probable errors of means and estimated intensities of the lines appear at the foot of each column. The following comments may be made on the table.

Hβ. The He+ component has not been measured separately, but the mean wave length of Hβ – 4860·74, displaced 0·6 A to the violet of its laboratory wave length, suggests very strongly that a blend of Hβ and its He+ component has been measured.

He+ 4541. The line is well defined and the large accidental error is probably a result of the enforced use of comparatively few velocity standards (see sec. 2).

Hγ. The He+ component is readily seen on high dispersion plates with a low power magnifier. Fig. 5 of the Plate is a reproduction of Hγ and its He+ component from the III-prism spectrum of Aug. 12·82. The component has been measured on all the plates with the exception of one, and the mean wave length λ4338·71 is in good accord with Paschen's laboratory value of λ4338·69.

He+ 4200. As in 10 Lacertae this line is probably composed largely of N+ 4200·06.

Hδ. N+ 4103, Hδ 4102, He+ 4100 all run into one another to form a wide fuzzy blend, as may be seen from the Plate, fig. 2. An attempt was made to measure the three components in the last three measures but without any marked success.

Summarizing, it is evident that in 9 Sagittae there is a component in the correct position for Hγ, probably a component to Hβ and doubtfully to Hδ.

The detailed measures for **B.D. 35° 3930 N** appear in Table 9 which is identical in form with Tables 7 and 8 for 10 Lacertae and 9 Sagittae. The probable errors of the means are not given, and the wave lengths are only carried to tenths of A.U. The

TABLE 9—WAVE LENGTHS (I.A.) OF H, He+, N+ IN B.D. 35° 3930 N

Date	Disp.	Hβ 468 –	N+ 485 –	He+ 468 –	He+ 454 –	Hγ 434 –	He+ 433 –	He+ 419 –	Hδ 410 –	He+ 410 –	He+ 402 –
1920 July 3·89	I G	0·8	7·7	6·1	0·8	0·5	8·5	9·8	1·8	0·2	5·5
1921 Aug. 6·79	I G	1·3	8·0	7·2	1·7	0·4	8·3	9·8	1·9	0·2	6·3
Aug. 11·79	I G	0·9	8·5	7·4	1·3	9·4	......	0·0	0·7	......	5·2
Sept. 8·77	I G	......	......	7·5	1·2	0·6	8·6	9·8	......	......	......
Means:—	......	1·0	8·1	7·0	1·2	0·5	8·5	9·8	1·8	0·2	5·7
Intensities:—	......	9	4	80 E	7	9	6	6	9	6	2

difficulty with this star, as already indicated in sec. 2, is that at the most there are only four velocity standards available, two of which are emission lines, and furthermore that the line character is exceedingly poor. The following comments may be made.

Hβ. It is possible that the measured wave length is a blend of Hβ and its He+ component, as the mean wave length is displaced 0·3 A to the violet of the laboratory position of Hβ.

He+ 4686. This line is a very strong emission band, sometimes as wide as 10 A. The measures of its centre are uncertain.

He+ 4541. This line was very ragged and the measures were difficult to make. It will be noted that it is comparable in intensity with the Balmer lines.

Hγ. This line and its Pickering component were measured separately in three out of four cases; the remaining case a blend of the two was measured.

He+ 4199. As the wave length shows, this line in B.D. 35° 3930 N is probably composed largely of He+.

Hδ. This line and its Pickering component have been measured separately twice, and on the third occasion a blend of the two was measured.

He+ 4025. B.D. 35° 3930 N shows possibly just a trace of ordinary He at 4471. Consequently this line will be composed almost entirely of He+. Its mean wave length 4025·7 is in good agreement with the centre of gravity of the line computed from Paschen's value of N_4, viz. λ4025·63.

Summarizing again, it is evident that, in B.D. 35° 3930 N there are components at the predicted positions and with the correct intensities for Hγ and Hδ, and there is possibly a suspicion of a component to Hβ.

Reviewing all the evidence there can be little doubt that the Pickering components to the Balmer lines, as predicted by Bohr, are present in O-type stars. The fact that they have been measured on so many plates of a given star puts out of court the possibility that plate flaws have been set on. On the other hand, the accordance of the intensities of the components and the members of the original Pickering series in the three stars makes it probable that the two sets of lines are related. Further if the components had only been measured in one star, or even in any number of stars of the same spectral type, it would be legitimate to suppose that the observed components were due to some other element, though the fact that they are obtained for four members of the Balmer series in 10 Lacertæ would make such a hypothesis doubtful. But when components have been measured in the predicted positions and with the correct intensities in stars which differ widely in type (see sec. 7 and 8), it is highly probable that such components are those predicted by Bohr and observed by Paschen in the laboratory.

TABLE 10—MEAN WAVE LENGTHS (I.A.) OF H AND He+

Stellar Wave Lengths						Laboratory Wave Lengths	
Hydrogen			Enhanced Helium			H	He+
λ	No. Me.	p. e.	λ	No. Me.	p. e.		
6562·59	3	±·04	6560·04	3	±·02	6562·793	6560·13
....	....	....	5411·62	14	·04	....	5411·55
4861·32	4	·01	4859·08†	4	·09	4861·326	4859·34
....	....	....	4685·70	10	·01	....	4685·74
....	....	....	4541·67*	22	·03	....	4541·61
4340·43*	23	·02	4338·80*	23	·04	4340·467	4338·69
....	....	....	4200·06*†	25	·03	....	4199·86
4101·72	12	·01	4100·27	12	·06	4101·738	4100·05

*Mean of all the 10 Lacertæ and 9 Sagittæ measures. Unstarred λλ's are means of 10 Lacertæ measures alone.
†Blends with enhanced nitrogen.

In Table 10 the mean stellar wave lengths of the hydrogen and enhanced helium lines are compared with the laboratory values of Curtis[7] for hydrogen and of Paschen[5] for ionized helium. The details of the table are self explanatory, but some comment is required on the method of forming the means. In view of the paucity of velocity standards and the poor character of the lines, the wave lengths of B.D. 35° 3930 N have not been included in the means. Of the eight enhanced helium lines in the table only three, denoted by asterisks, are means of the measures of both 10 Lacertæ and 9 Sagittæ; the remaining five are means from 10 Lacertæ alone. While a wave length of 4100 was available from 9 Sagittæ, it was not included since the measurement was, as previously noted, a matter of guess work. In forming the means in Table 10 each individual measure, as set forth in Tables 7 and 8 for 10 Lacertæ and 9 Sagittæ respectively, has been given unit weight. This actually results in giving more weight to II-and III-prism plates (since they are measured in duplicate), and more weight to the wave lengths of 10 Lacertæ than of 9 Sagittæ (since there are nearly twice as many measures of 10 Lacertæ available in the region common to both than there are of 9 Sagittæ). In short, the procedure adopted automatically assigns the weights that theory would prescribe, viz.—high dispersion plates and those with the greater number of velocity standards to be given the greater weight (compare sec. 2). Referring to the table it will be noted that, with the exception of Hα which is very difficult to measure, the wave lengths of the Balmer lines in the stars agree with Curtis' values almost within the probable errors of the mean stellar values. The discrepancies between the stellar wave lengths of the enhanced helium lines and Paschen's laboratory values of the same, in the final column of Table 10, are more marked, but are not extraordinary when the poor character of the stellar lines is considered.

Section 4—Value of the Rydberg Constant, N_2, for Helium

The Rydberg constant N in the formula $\nu = N\left(\frac{E}{e}\right)^2\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$, as was shown in the early part of sec. 3, has the value

$$N = \frac{2\pi^2 m_o e^4}{h^3(1+m_o/M)} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It was also pointed out in sec. 3, that if the known values of m_o, e, h were substituted, the values of the computed and the spectroscopically determined constants agreed within the limits of the errors of m_o, e, h. Now clearly this process may be reversed, and the value of N, derived from spectroscopic observations, may be used to compute the values of m_o, e, h. In this section it is proposed to use the mean stellar wave lengths of the enhanced helium lines, as given in Table 10, to compute spectroscopically the value of N_2, the Rydberg constant for helium. In the following section this value of N_2 will be used to compute the universal constants m_o, h and so on.

In determining the value of N spectroscopically, the simple Bohr formula for the frequency of the lines, has to be modified to allow for the relativistic variation in mass of the electron. The complete theory is due to Sommerfeld[8], and only the necessary formulæ will be summarized here. Removing the restriction that the electron and nucleus must revolve in circular orbits, and "quantizing" with respect to radial as well as angular momentum, Sommerfeld has found that the 1 quantum orbit will be a circle, the 2 quantum orbit a circle or an ellipse of eccentricity $\sqrt{3/2}$, the 3 quantum orbit a circle or one of two ellipses of eccentricities $\sqrt{5/3}$, $\sqrt{8/3}$, and so on. Furthermore, when the variation in mass with speed of the electron is considered, its kinetic energy is no longer $\frac{1}{2} m_o v^2$ but $m_o c^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)$, where $\beta = v/c$, and c is the velocity of light; also the perinuclei

45171—2½

of the electron ellipses are no longer fixed, but show a slight forward progression. Taking these various factors into consideration, Sommerfeld has derived a somewhat complex expression for the energy of an atomic state, an expression which, however, on expanding and retaining the first two powers of the small quantity $\alpha^2 = (2\pi e^2/hc)^2$ (the square of the ratio-electron velocity in one quantum orbit of H atom: velocity of light), results in the more simple form for the frequency

$$\nu = (n, n') - (m, m') \text{ where } (n, n') = N\left(\frac{E}{e}\right)^2 \left[\frac{1}{(n+n')^2} + \frac{\alpha^2}{(n+n')^4}\left(\frac{E}{e}\right)^2\left(\frac{1}{4} + \frac{n'}{n}\right)\right] \ldots (3)$$

Here n, m are the angular quantum numbers, n′, m′ the radial quantum numbers, and the other symbols have their earlier significance. By virtue of the fraction n′/n there will be several frequencies, slightly differing from each other, corresponding to a given quantum sum n+n′; spectroscopically this means that the main lines specified by Bohr's simple theory will actually show a fine structure. The main structure of a line is determined by the quantum number of the orbit into which the electron falls. Thus the main structure of a Balmer line, where the electron falls into the 2 quantum orbit, is a doublet of separation, from (3), $(1, 1) - (2, 0) = N\left(\frac{E}{e}\right)^4 \frac{\alpha^2}{2^4}$, and since for H, E = e, this becomes $N\frac{\alpha^2}{2^4}$. This latter is an important constant in fine structure theory to which the symbol $\Delta\nu_H$, the separation of the H doublet, is applied. Similarly the main structure of a line in the 4686 series is a triplet of separation from (3) of

$$\left.\begin{array}{l} (2, 1) - (3, 0) \quad N\left(\frac{\cdot}{e}\right)^4 \cdot \frac{\alpha^2}{3^4} \cdot \frac{1}{2} = \frac{N\alpha^2}{3^4} \cdot \frac{2^4}{2} \cdot \\ (1, 2) - (2, 1) = N\left(\frac{E}{e}\right)^4 \cdot \frac{\alpha^2}{3^4} \cdot \frac{3}{2} = \frac{N\alpha^2}{3^4} \cdot 2^4 \cdot \frac{3}{2} \cdot \end{array}\right\} \text{for He+ atoms E = 2e}$$

Now, furthermore, each of these main components is itself complex, the complexity arising from the quantum number of the orbit from which the electron has fallen. Thus each component of the Hα doublet is a triplet, and each component of the 4686 triplet is itself quadruple, so that in all there are 12 components to 4686. In the case of ionized helium series, where the separations of components are magnified by 16, Sommerfeld's predictions have been completely verified by Paschen.[5] In the case of the Balmer series a smaller separation, the large Doppler effect of the light atoms, and large Stark effects make the measurements and their theoretical interpretation difficult. Undoubtedly, however, the lines are doublets and the separation is of the right order.[9]

Confining attention to circle orbits, i.e. n′ = 0, (3) becomes

$$\nu = N\left(\frac{E}{e}\right)^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\left[1 + \frac{\alpha^2}{4}\left(\frac{E}{e}\right)^2\left(\frac{1}{n^2} + \frac{1}{m^2}\right)\right] \ldots\ldots\ldots (4)$$

a formula which was first developed by Bohr.[10] This formula shows clearly a second effect, in addition to the fine structure, of the introduction of relativity mechanics—namely, that the lines are all shifted slightly to the violet of the positions given by the simple formula (1). It is this formula which will be used to evaluate the Rydberg constant N_2 for helium. Curtis[7] has also used (4) in determining the value of N_1 for hydrogen by including all his lines in a least-squares solution and solving for N_1 and α. This procedure is inadmissible since the formula refers only to circle compenents, whereas the measured wave lengths refer to the optical centres of gravity of the lines. To derive the position of the circle components from the measure of wave lengths, the value of α must be known. Here it has been assumed that $\Delta\nu_H = 0{\cdot}36$ cms $^{-1}$, a value taken from

Paschen's[8] measurements of the structure of the enhanced helium lines and in fair agreement with the various measures[9] of the separation of the hydrogen doublet. From this, taking $N = 1 \cdot 097 \times 10^5$ $\alpha^2 = 5 \cdot 25 \times 10^{-5}$. The procedure will then be to compute, by means of this value of α^2, the positions of the circle components and from their frequencies, by the use of (4), the value of N_2, for helium.

The various steps of the process are given in Table 11. The first two columns contain the wave lengths in air of the enhanced helium lines selected for determining N_2, and their probable errors, carried to three places of decimals. The component to Hα at λ6560·04 has been eliminated, because the wave length is an indication rather of the presence of the component than of its position (compare sec. 3). The component to Hβ and the line at 4200 have also been eliminated because they are almost certainly blended with enhanced nitrogen lines (compare sec. 3). In the third column of the table the wave lengths in air are converted to their vacuum values by the Bureau of Standards Tables,[11] and these values in column four are turned into wave numbers by the formula $\nu = 10^8/\lambda$. These wave numbers refer, of course, to the optical centre of gravity of the various lines; the next step is to determine from these, by means of formula (3), the positions of the circle components. In order to do this, it is necessary, in addition to knowing the separation of the components, also to know their relative intensities. In view of our ignorance of electrical conditions in the stellar photospheres, the simple intensity relations suggested by Sommerfeld[12] have been used, namely:—in a doublet 2: 1, in a triplet 3 : 2 : 1, in a quadruplet 4: 3 : 2 : 1 and so on in the order of circle component and ellipse components where the intensity of the component decreases as the eccentricity of the orbit increases. This in effect states that circle orbits are most likely, and, the

TABLE 11—COMPUTATION OF N_2

λ air (I.A.)	p.e.	λ vac.	ν	ν (⊙ compon.)	N_2	p.e.	Wt.
4685·704	±·010	4687·011	21335·56	21335·21	109722·9	±0·2	44
5411·617	·045	5413·117	18473·64	18473·42	109720·4	0·9	3
4541·674	·032	4542·943	22012·16	22011·94	109720·6	0·8	5
4338·802	·043	4340·018	23041·38	23041·16	109719·4	1·1	3
4100·270	·058	4101·423	24381·78	24381·56	109716·6	1·6	1

more eccentric the elliptical orbit, the less likely its corresponding frequency is to occur,—a very probable assumption. A diagrammatic representation of the components of 4686 with their correct relative separations (using $\Delta\ \nu_H = 0 \cdot 36$) and intensities is given in fig. 6. Paschen's notation is used, where the Roman numerals refer to the main components, I being due to the circle orbit, and where the letters refer to the superposed structure of these components, a referring to the circle component. The main separations are from (3)

$$\text{IIa—Ia} = (2,1) - (3,0) = 16\frac{N\alpha^2}{3^4} \cdot \frac{1}{2} = 8\left(\frac{2}{3}\right)^4 \Delta\nu_H = 0 \cdot 57 \text{ cms.}^{-1}$$

$$\text{IIIa—Ia} = (1,2) - (3,0) = 16\frac{N\alpha^2}{3^4} \cdot 2 = 32\left(\frac{2}{3}\right)^4 \Delta\nu_H = 2 \cdot 28 \text{ cms.}^{-1}$$

$$\text{Ia—Ib} = (4,0) - (3,1) = 16\frac{N\alpha^2}{4^4} \cdot \frac{1}{3} = \frac{16}{3}\left(\frac{2}{4}\right)^4 \Delta\nu_H = 0 \cdot 12 \text{ cms.}^{-1} \text{ etc.}$$

It is desired to determine the distance in wave numbers of the component Ia, the circle component, from the optical centre of gravity of the line. It can readily be computed, and as reference to fig. 6 will show, that Centre of Gravity of 4686 = Position of circle component + 0·35. Hence if the wave number of the centre of gravity of 4685 is 21 335·56, the position of its circle component in wave numbers is 21 335·56 − 0·35 = 21 335·21. Similarly it may be shown for the Pickering lines, considering only the main

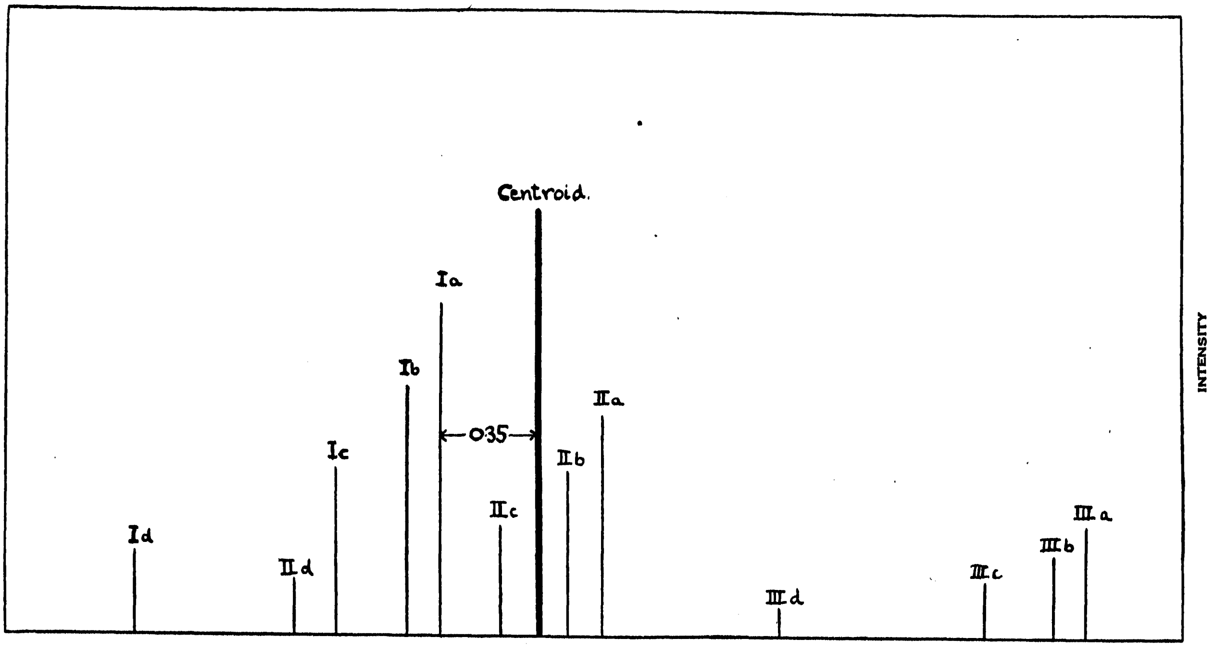

FIG. 6—Components of 4686

quartette structure and neglecting the superposed fine structure, which is greatly below the errors of measurement, that the positions of the circle components may be derived from the optical centres of gravity by subtracting 0·22. The positions of the circle components of the five enhanced helium lines as thus computed are given in the fifth column of Table 11.

The last step in the computation of N_2 is quite simple. For the ionized helium atom where $E = 2e$ and $\alpha^2 = 5\cdot25 \times 10^{-5}$ equation (4) becomes

$$\nu = 4N_2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\left[1 + 5\cdot25 \times 10^{-5}\left(\frac{1}{n^2} + \frac{1}{m^2}\right)\right]$$

For any of the lines in Table 11, ν the position of the circle component, and n, m the quantum numbers of the final and commencement orbits respectively of the electron are known, and hence the value of N_2 may be computed. Thus for 4686, n = 3, m = 4 and $N_2 = 109\ 722\cdot9$. It is interesting here to note that if, instead of using circle components and the relativistic equation (4), the original simple Bohr formula (1) of sec. 3 had been employed, the resultant value of N_2 for 4686 would have been 109 725·7, an error of nearly 3 in the units place. In a similar way the relativistic formula was used for the remaining four enhanced helium lines; the resultant values of N_2 are given in the sixth column and their probable errors, carried through from the original probable errors of the lines, are given in the following column.

In view of the character of the lines from which the wave lengths have been determined, it is evident that the five values of N_2 given in Table 11 are not of equal weight. The best and least arbitrary method of weighting these values would be inversely as the squares of their *true* probable errors. The true probable error, r, of any quantity is given by $r^2 = r^2_1 + r_2^2$ where r_1 is the accidental probable error and r_2 the systematic probable error. As pointed out in sec. 2 every care was taken to avoid systematic errors in the wave lengths; there is, however, one irremovable cause of systematic error, namely the possibility that the enhanced helium lines are blended with unsuspected lines of unknown origin. The magnitude of the possible systematic error introduced in this way will clearly vary with the character of the line. Thus the maximum error to which 4686 is open is less than 0·2 A, since a line farther out than0·5 A would be readily detected so sharp is 4686; on the other hand a fuzzy line like 4541 might have a systematic error as great as 0·5 A. It has, therefore, been assumed that the systematic probable error of a wave length is equal to its accidental error as given in the second column of Table 11; this in effect uses the accidental probable error as a measure of the character of the line, of which the probable systematic error in turn is a function. Now lines that have been measured in two stars are likely to have a smaller systematic error than those measured in one star, since, if the unsuspected line of unknown origin occurs in one star, it is not probable that it will occur with the same intensity, if at all, in a second star of different type. In calculating the "true" probable errors then the following formula has been used

$$r^2 = r_1^2 + \frac{r_2^2}{n} = r_1^2\left(1 + \frac{1}{n}\right)$$

since r_2 is assumed equal r_1. For lines, like 4686, measured in one star, $n = 1$; for lines like 4542 measured both in 10 Lacertæ and 9 Sagittæ, $n = 2$. The result of this is to give lines measured in two stars a lower relative "true" probable error than those measured in one star. Thus the ratio of errors of 4541: 4686 is, for accidental probable errors, 3·2 and for "true" probable errors, 2·8. The "true" probable errors of the wave lengths as thus computed are carried through, and the "true" probable errors of the values of N_2 are determined. The weights are then assigned inversely as the squares of these errors; these weights appear in the final column of Table 11. It might be thought that 4686 is thus assigned too high a weight, but in view of its sharp symmetrical character, in contradistinction to the fuzzy ill-defined lines of the Pickering series, it is evident that 4686 is of more value than the other four lines together. It may be noted that a more arbitrary method of weighting, earlier tried, led to closely the same weighted mean as that given by the weights finally adopted.

The resultant weighted mean of the five values of N_2 in Table 11 is

$$N_2 = \mathbf{109\,722{\cdot}3 \pm 0{\cdot}44}$$

This may be compared with Paschen's value of $N_2 = 109\,722{\cdot}14 \pm {\cdot}04$. These are in agreement within the probable error of the stellar value. They differ in the much greater probable error of the stellar value, scarcely a surprising result when the fuzzyness of the stellar lines and the smaller linear dispersion are remembered.

SECTION 5—THE ELECTRON MASS AND RELATED PHYSICAL CONSTANTS

From the theory given in sec. 3, it will be recalled that the relations between the Rydberg constants for hydrogen and helium, and the various electronic and atomic constants are given by

$$N_1 = \frac{2\pi^2 m_o e^4}{h^3(1+m_o/m_1)} \quad . \; . \; . \; (5) \text{ for hydrogen where } m_1 = \text{mass of H nuclues.}$$

$$N_2 = \frac{2\pi^2 m_o e^4}{h^3(1+m_o/m_2)} \quad . \; . \; . \; (6) \text{ for helium where } m_2 = \text{mass of He nucleus.}$$

$$N\infty = \frac{2\pi^2 m_o e^4}{h^3} \quad . \; . \; . \; (7) \text{ for an atom with nucleus of infinite mass.}$$

From these three equations may be derived immediately the result

$$N_1(1 + m_o/m_1) = N_2(1 + m_o/m_2) = N\infty \quad . \; . \; (8)$$

from which, if the spectroscopic values of N_1, N_2 and the mass ratio m_1/m_2 of the hydrogen and helium nuclei are known, the value of the electron mass may be determined. Paschen[5] in his original work assumed m_1/m_2 to be the ratio of the atomic weights of hydrogen and helium, thereby neglecting the masses of the electrons. Flamm[13], in a recomputation of Paschen's work, arranged (8) in a form to admit of the use of atomic weights in place of masses of the nuclei.

Flamm's method is used here and a brief summary may therefore be given. If M_1, M_2 are the masses of a hydrogen and helium atom respectively, then the masses of the nuclei are $m_1 = M_1 - m_o$; $m_2 = M_2 - 2\,m_o$ since there is 1 electron in the normal hydrogen atom and 2 electrons in the normal helium atom. If these values of m_1 and m_2 be placed in (8) a quadratic equation in $1/m_o$ results. Solving for $1/m_o$ the expression under the radical is

$$\left(\frac{M_2 - M_1}{2}\right)^2 - M_1^2\,\frac{N_1}{N_2}\left(1 - \frac{N_1}{N_2}\right) \quad ,$$

which may be expanded by the binomial theory to one or two powers of the very small quantity $1 - \frac{N_1}{N_2}$ (=approx. 0·000405). Furthermore, if L is the number of atoms in a gram atom, then $LM_1 = A_1$ where A_1 is the chemical atomic weight in grams of hydrogen. We have $M_1 = A_1/L$, $M_2 = A_2/L$, with the final resulting expression

$$\frac{1}{Lm_o} = \frac{A_2 - A_1}{A_1 A_2} \cdot \frac{N_2}{N_2 - N_1} + \frac{1}{A_2}\left[1 - \frac{A_1}{A_2 - A_1} \cdot \frac{N_1}{N_2} - \left(\frac{A_1}{A_2 - A_1}\right)^3 \left(\frac{N_1}{N_2}\right)^2 \left(1 - \frac{N_1}{N_2}\right)\right] \quad . \; . \; . \; (9)$$

Similarly it may be shown that

$$N\infty = N_2 + \frac{A_1(N_2 - N_1)}{A_2 - 2A_1}\left[1 - \frac{A_1}{A_2 - A_1} \cdot \frac{N_1}{N_2} - \left(\frac{A_1}{A_2 - A_1}\right)^3 \left(\frac{N_1}{N_2}\right)^2 \left(1 - \frac{N_1}{N_2}\right)\right] \quad . \; . \; . \; (10)$$

The expressions have been cast in this particular form, to avoid the production of large probable errors in determining the unknowns from quantities already subject to some uncertainty. They differ slightly in form, though they lead to the same results, from the expressions finally given by Flamm. The reason for this difference is another mode of expansion of the radical, and a further series of approximations introduced by Flamm.

In order now to determine the values of $N\infty$ and $1/m_o$, the values of N_1, N_2, A_1, A_2 are required. By far the best determination of N_1, the Rydberg constant for hydrogen, is that which has resulted from Birge's[14] recomputation, using the method of sec. 4 with probably more accurate intensity relations, of the measures by Curtis[7] and Paschen[5] of the Balmer lines. The resulting value is $N_1 = 109\ 677 \cdot 7 \pm 0.2$. The stellar value of N_2 as determined in the previous section is $N_2 = 109\ 722 \cdot 3 + 0 \cdot 44$. Flamm[13], in carrying through his calculations used, of course, Paschen's[5] value of N_1 and N_2. For the atomic weights of hydrogen and helium Flamm used the following:— $A_1 = 1 \cdot 0007 \pm \cdot 00013$ (Morley and Noyes), $A_2 = 4 \cdot 002 \pm \cdot 0017$ (W. Heuse). Since that time (1917), there have been redeterminations of the atomic weight of hydrogen by T. S. Taylor[15], Burt and Edgar[16], and of helium by Taylor. Taylor's values have been recomputed by P. A. Guye[17]. The whole matter has been considered by the International Committee [18] who adopt the values 1·0078 for hydrogen and 4·000 for helium. The probable errors of these weighted means may be readily determined, and are found to be $A_1 = 1 \cdot 0078 \pm 0 \cdot 00003$: $A_2 = 4 \cdot 000 \pm 0 \cdot 0007$. Summarizing, in computing the stellar values of the universal constants the following have been used,—

$N_1 = 109\ 677 \cdot 7 \pm 0 \cdot 2$ (Birge) : $N_2 = 109\ 722 \cdot 3 \pm 0 \cdot 44$ (stellar)

$A_1 = 1 \cdot 0078 \pm 0 \cdot 00003$: $A_2 = 4 \cdot 000 \pm 0 \cdot 0007$

With these values, carrying through the probable errors, the following immediately result from equations (10) and (9)

$$N_\infty = \mathbf{109\ 737 \cdot 3 \pm 0 \cdot 47}$$

And $1/Lm_o = 1826 \pm 20$. Since $A_1 = LM$, it follows that $M_1/m_o = A_1/Lm_o$ and we have

$$M_1/m_o = \mathbf{1840 \pm 20}$$

Further since the electrochemical equivalent of silver [19] is 0·00111827 gms/coulombs it follows that for the hydrogen atom $e/M_1 = 9647 \cdot 0/1 \cdot 0078$ in e.m. units. Therefore the ratio charge to the mass of the electron is given by $\frac{M_1}{m_o} \times \frac{e}{M_1}$, namely

$$e/m_o = (\mathbf{1 \cdot 762 \pm 0 \cdot 019}) \times \mathbf{10^7}\ \textbf{e.m.u.}$$

To make any further progress the value of the charge carried by the electron must be known. In his original work Paschen[5] derived the value of e from his value of $\Delta\nu_H = N\alpha^2/2^4$, where $\alpha = 2\pi e^2/hc$. The value of $\Delta\nu_H$, though very accurately determined by Paschen, has a large percentage error which results in an absurdly large probable error for the electron charge. In his revision of Paschen's work, Flamm, therefore, used Millikan's[20] very accurate value of the electron charge, $e = (4 \cdot 774 \pm 0 \cdot 005) \times 10^{-10}$ e.s.u. This procedure has also been adopted here, and results immediately in the values of a number of other constants. Thus from the value of e/m_o above, $m_o = (9 \cdot 04 \pm 0 \cdot 10) \times 10^{-28}$ gms. Assuming that the charge on the electron is spread uniformly over the surface and that the mass is entirely electromagnetic, the relation[21] between the radius, charge and mass of the electron is given by $m_o = 2e^2/3ac^2$ where c is the velocity of light. This gives $a = (1 \cdot 869 \pm 0 \cdot 021) \times 10^{-13}$ cms. More important than these constants, however, is the fact that the value of Planck's radiation constant, h, may be computed if e is known. From (7) it follows that

$$h = e \sqrt[3]{\frac{2\pi^2 m_o c}{N_\infty \cdot c}}$$ where $c = 2 \cdot 9986 \times 10^{10}$ cms. per sec. is the velocity of light.

Inserting the necessary values there results

$$h = (\mathbf{6 \cdot 567 \pm 0 \cdot 042}) \times \mathbf{10^{-27}}\ \textbf{ergs. secs.}$$

Finally the constant c_2, in the Planck radiation law, is related to h in the following manner,

$c_2 = ch/k$ where c is the velocity of light and k[22] is the Boltzmann gas constant for one molecule, $k = (1 \cdot 372 \pm 0 \cdot 0014) \times 10^{-16}$ ergs/degrees. Substituting the necessary values

$$c_2 = 1 \cdot 4353 \pm 0 \cdot 0093 \text{ cms. degrees}$$

Flamm's revision of Paschen's work has been repeated, using equations (9) and (10), Birge's value of N_1 in place of Paschen's and the more recent values of the atomic weights. Paschen's values, thus revised, of these various universal constants are compared in Table 12 with what may be called the stellar values, derived immediately above. The first column of the table contains the symbol for the constant, the second the stellar spectroscopic value, the third Paschen's spectroscopic value and in the fourth column are given results from other methods. It will be noted that the stellar values are not only in good

TABLE 12—SPECTROSCOPIC UNIVERSAL CONSTANTS

Constant	Stellar Value	Paschen Value	Other Determinations
N_1	....	....	109 677·7 ± 0·2—Birge[14] (Curtis and Paschen)
N_2	109 722·3 ± ·44	109 722·14 ± ·04	
N_∞	109 737·3 ± ·47	109 737·11 ± ·08	109 736·9 ± 0·2—Birge[14]
M_1/m_o	1840 ± 20	1847·1 ± 8·3	
e/m_o	$1 \cdot 762 \pm \cdot 019 \times 10^7$	$1 \cdot 768 \pm \cdot 008 \times 10^7$	β rays: 1·763 Bucherer[23], 1·769 Malassez[24]. Zeeman Effect: 1·7636 Fortrat[25], 1·771 Gmelin[26].
e	....	....	$4 \cdot 774 \pm \cdot 005 \times 10^{-10}$ Millikan[20].
m_o	$9 \cdot 04 \pm \cdot 10 \times 10^{-28}$	$9 \cdot 01 \pm \cdot 04 \times 10^{-28}$	
a	$1 \cdot 869 \pm \cdot 021 \times 10^{-13}$	$1 \cdot 875 \pm \cdot 009 \times 10^{-13}$	
h	$6 \cdot 567 \pm \cdot 042 \times 10^{-27}$	$6 \cdot 547 \pm \cdot 019 \times 10^{-27}$	6·5543—Birge[22], mean of 7 independent determinations.
c_2	1·4353 ± ·0093	1·4309 ± ·0044	1·4320—W. W. Coblentz[27]

agreement with Paschen's spectroscopic values, which was to be expected since the two values of N_2 are identical within the errors of observation, but are also very accordant with determinations by other methods. It was not to be expected that there would be any startling changes, and these stellar values, weighted according to their unavoidably large probable errors, will serve, in addition to determinations by many other methods, to fix with certainty the values of these important universal constants. It is of interest, however, to note that these "stellar" determinations *are* in agreement with the terrestrial values, in so far as it shows that *the implicit assumption of identical atomic structure, identical electrons and identical laws of radiation on the earth and in the stars, is in some measure justified.*

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PART III—WAVE LENGTHS AND PHYSICAL CONDITIONS IN O-TYPE STARS

In Part II of this paper the O-type spectra were used to confirm Bohr's theory of spectral emission and to determine "stellar" values of atomic constants. In Part III, on the other hand, the O-type spectra are interpreted by physical theories so that some knowledge is acquired of the physical conditions in this class of stars and a possibly more rational classification of O-type stars thus becomes possible. In the first section are given the wave lengths of all the lines measured in the three O-type stars, with the estimated intensities of the lines and their probable origins. In the second section, from these observed spectra the temperatures and the relative abundance of hydrogen and helium in these stars are determined. The two methods (one of which is a slightly modified form of Saha's theory) which make this physical interpretation possible are fully explained and developed in the Appendix to the paper, entitled "Physical Interpretation of Stellar Spectra." Finally in the third section from the line intensities and from the probable stellar temperatures, a modification of the Harvard classification of O-type stars is suggested.

Section 6—Wave Lengths in O-Type Stars

Tables. In this section are given tables of the wave lengths, estimated intensities and identifications where possible of the lines in the three O-type stars, 10 Lacertæ, 9 Sagittæ and B.D. 35° 3930 N. Following the tables are some notes on the presence and absence of various elements together with a graphical comparison of the line intensities in the Harvard class B with the line intensities in the three O-type stars. The tables are arranged in the following manner. In the first column is given the stellar wave length (I.A.), a mean as a rule of at least four measures. No line is included unless it was measured on at least two plates; in this way chances of plate flaws appearing as lines are largely eliminated. In the second column of each table are given the probable errors of these mean wave lengths. A modification of Burns'[1]‡ notation is used which has the advantage of indicating more fully the value of a given wave length than the probable error alone. The notation is,—

A indicates the mean of 6 or more measures and a probable error of the mean 0·00 – 0·01.
B indicates the mean of 4 or more measures and a probable error of the mean 0·01 – 0·04.
C indicates the mean of 4 or more measures and a probable error of the mean 0·05 – 0·10.
D indicates a probable error of 0·10 – 0·30 or less than 4 measures.

In the third column is given the estimated intensity of the line on a scale of ten; each time a plate was measured the line intensities were estimated and the values in this column are means of these estimates. The probable origin of the line is given in the fourth column. Arc lines are indicated by the symbol of the element, once enhanced lines by the symbol followed by a plus, thus C+ and doubly enhanced by the symbol followed by two plus signs, C++. Where the series relations of enhanced lines are not known the interpretation of these symbols is uncertain, and in the case of silicon and oxygen the

‡References at the end of Part III.

notation of Lockyer and Fowler has been followed. It is to be noted that also in carbon and nitrogen the series relations of the lines are still unknown so that the symbols C+, C++ simply indicate spark and "super-spark" lines. The fifth column contains the laboratory wave length of the line in I.A. Wave lengths with an asterisk are those used as velocity standards (see Table 5, sec. 2). The final column of the table contains the name of the authority for the laboratory wave length. Complete references will be found at the end of Part III.

TABLE 13—WAVE LENGTH (I.A.) IN 10 LACERTAE

Stellar Wave Length (I.A.)	p.e.	Int.	Probable Origin	Laboratory Wave Length (I.A.)	Remarks
3933·55	B	4	Ca^+ —K	3933·664	Crew and McCauley [22]
3961·62	B	4	O II	3961·60	Fowler[3]
3964·73	A	4	He	3964·727*	Merrill [4]
3968·32	B	4	Ca^+ —H	3968·465	Crew and McCauley [22]
3970·17	B	9	H —Hε	3970·075	Curtis [5]
4025·13	D	2	....	....	
4026·17	A	8	He	4026·189	Merrill.[4] Blend He^+ 4025.63
4067·92	D	2	....	....	
4070·06	C	3	O II, blend	4069·635	69·903. Clark [6]
4072·15	B	2	O II	4072·156*	Clark [6]
4075·88	B	2	O II	4075·869*	Clark [6]
4088·95	A	8	Si IV	4088·94	Lockyer [7] : 88·85 Lunt [8]
4097·32	A	6	N^+	4097·327*	Fowler [9]
4099·22	C	2	....	....	
4100·27	C	2	He^+	4100·05	Paschen [10]
4101·72	A	9	H —Hδ	4101·738*	Curtis [5]
4103·32	B	3	N^+	4103·393*	Fowler [9]
4116·12	A	5	Si IV	4116·36	Lockyer [7] : 16·20 Lunt [8]
4119·13	B	2	O II	4119·222*	Clark [6]
4120·81	A	3	He	4120·812*	Merrill [4]
4143·78	B	3	He	4143·759*	Merrill [4]
4152·84	B	2	....	....	
4156·51	B	1	....	....	4156·5. Stellar line, Lockyer [7]
4162·94	B	1	....	....	
4168·86	D	1	He?	4168·98	Runge and Paschen [11]
4186·93	A	2	....	....	
4189·79	D	1	O II	4189·793	Clark [6]
4200·06	C	2	Blend He^+, N^+	4199·86	He^+ Paschen [10] : N^+ 4200·06 Fowler [9]
4212·33	B	2	....	....	
4253·94	D	2	S^+?	4253·61	Lockyer [7]
4267·14	D	1	C^+	4267·14	Lockyer [7]
4275·86	D	1	....	....	4276·0 Stellar line, Lockyer [7]
4319·65	D	1	O II	4319·647*	Clark [6]
4337·76	B	2	Ti^+?	4337·92	Lockyer [12]
4338·87	C	2	He^+	4338·69	Paschen [10]
4340·42	B	9	H —Hγ	4340·467*	Curtis [5]
4349·37	B	2	O II	4349·435*	Clark [6]
4366·74	D	1	O II	4366·906*	Clark [6]
4379·21	B	2	N^+	4379·09	Fowler [9]
4387·96	A	6	He	4387·928*	Merrill [4]
4471·52	A	9	He	4471·477*	Merrill [4]

Stellar Wave Length (I.A.)	p.e.	Int.	Probable Origin	Laboratory Wave Length (I.A.)	Remarks
4481·20	B	2	Mg^+	4481·195*	Fowler [2]—Red comp. Int. 1, V. comp. Int. 2
4510·88	B	2	N^+	4510·91*	Fowler [9]
4514·87	B	2	N^+	4514·865*	Fowler [9]
4523·56	D	2	N^+	4523·59	Fowler [9]
4534·82	D	2	N^+ blend	4534·57	35·07. Fowler [9]
4541·62	B	3	He^+	4541·61	Paschen [10]
4552·48	B	1	Si III	4552·47	Lockyer [7]; 52·65 Lunt [8]
4567·76	C	1	Si III	4567·72	Lockyer [7]; 67·64 Lunt [8]
4631·16	B	2	....	....	
4634·20	D	2	N^+	4634·165	Fowler [9]
4640·54	D	1	N^+	4640·649	Fowler [9]
4641·68	B	2	Blend O II, N^+	4641·827	Clark [6]; 41·90 N^+ Fowler [9]
4647·38	B	5	C^{++}	4647·6	Merton [13]
4649·06	B	1	O II	4649·148	Clark [6]
4650·34	B	3	C^{++}	4650·4	Merton [13]
4651·59	A	3	C^{++}	4651·6	Merton [13]
4654·35	B	2	....	....	4654·4. Stellar line, Lockyer [7]
4685·70	A	9	He^+	4685·74	Paschen [10]. 85·72 in star Frost [14]
4713·18	B	5	He	4713·143*	Merrill [4]
4859·08	C	5	Blend He^+, N^+	4859·34	Paschen [10]; 58·82 N^+ Fowler [9]
4861·32	B	10	H —Hβ	4861·326*	Curtis [5]
4921·78	C	5	He	4921·929*	Merrill [4]
5015·70	B	4	He	5015·675*	Merrill [4]
5411·62	C	4	He^+	5411·55	Paschen [10]
5592·36	C	4	O III	5592·35*	Fowler [3]
5875·68	B	9	He	5875·618	Merrill [4]
5889·86	C	4	Na	5889·963	Wood and Fortrat [15]
5896·10	C	3	Na	5895·930	Wood and Fortrat [15]
5919·19	D	1	....	....	
6278·33	D	3	....	....	
6558·25	C	4	....	....	
6560·04	D	3	He^+	6560·13	Paschen [10]
6562·59	D	10	H —Hα	6562·793*	Curtis [5]
6678·39	B	4	He	6678·149*	Merrill [4]

TABLE 14—WAVE LENGTHS (I.A.) IN 9 SAGITTAE

Stellar Wave Length (I.A.)	p.e.	Int.	Probable Origin	Laboratory Wave Length (I.A.)	Remarks
3933·60	D	5	Ca^+ —K	3933·664	Crew and McCauley[22]
3968·38	D	5	Ca^+ —H	3968·465	Crew and McCauley[22]
4088·89	C	6	Si IV	4088·94	Lockyer[7] : 88·85 Lunt[8]
4097·24	B	7	N^+	4097·327*	Fowler[9]
4100·83	D	4	He^+	4100·05	Paschen[10]
4102·05	D	8	H —Hδ	4101·738	Curtis[5]
4103·53	D	5	N^+	4103·393	Fowler[9]
4116·04	D	2	Si IV	4116·36	Lockyer[7] : 16·20 Lunt[8]
4200·08	C	4	BlendHe^+,N^+	4199·86	Paschen[10] : 4200·06 N^+ Fowler[9]
4338·71	C	4	He^+	4338·69	Paschen[10]
4340·45	B	9	H —Hγ	4340·467*	Curtis[5]
4379·25	D	3	N^+	4379·09	Fowler[9]
4388·01	B	3	He	4387·928*	Merrill[4]
4471·44	B	10	He	4471·477*	Merrill[4]
4510·98	C	3	N^+	4510·91*	Fowler[9]
4514·98	C	3	N^+	4514·865*	Fowler[9]
4523·63	D	2	N^+	4523·59	Fowler[9]
4534·48	D	2	N^+ blend	4534·57	35·07. Fowler[9]
4541·76	C	6	He^+	4541·61	Paschen[10]
4634·25	D	4E	N^+	4634·165*	Fowler[9]
4640·60	D	5E	N^+	4640·649*	Fowler[9]
4647·60	D	3	C^{++}	4647·6	Merton[13]
4650·72	D	3	C^{++} blend	4650·4	4651·6, Merton[13]
4713·30	C	5	He	4713·143*	Merrill[4]
4858·49	D	4	N^+	4858·82	Fowler[9]
4860·74	D	9	BlendH,He^+	4861·326	Hβ Curtis[5]; 4859·34 He + Paschen[10]

TABLE 15—WAVE LENGTHS (I.A.) IN B.D. 35° 3930 N

Stellar Wave Length (I.A.)	p.e.	Int.	Probable Origin	Laboratory Wave Length (I.A.)	Remarks
3933·6	D	5	Ca^+ —K	3933·664	Crew and McCauley [22]
3965·3	D	3	....	....	
3968·4	D	4	Ca^+ —H	3968·465	Crew and McCauley [22]
3969·8	D	2	H —Hε	3970·075	Curtis [5]
4025·7	D	2	He^+	4025·63	Paschen [10]. Computed from his N_3
4085·2	D	2	....	....	
4091·6	D	1	....	....	Measured also as 4090·3, 92·9
4096·8	D	2	N^+?	4097·327	Fowler [9]
4100·2	D	6	He^+	4100·05	Paschen [10]
4101·8	D	9	H —Hδ	4101·738*	Curtis [5]
4199·8	D	6	He^+	4199·86	Paschen [10]
4212·1	D	3	....	....	
4338·5	D	6	He^+	4338·69	Paschen [10]
4340·5	D	9	H —Hγ	4340·467*	Curtis [5]
4391·6	D	2	....	....	
4470·9	D	0	He?	4471·477	Merrill [4]
4541·2	D	7	He^+	4541·61	Paschen [10]
4603·8	D	3	....	....	
4634·0	D	3E	N^+	4634·165*	Fowler [9]
4640·4	D	4E	N^+	4640·649*	Fowler [9]
4687·0	D	80E	He^+	4685·74	Paschen [10]. Line about 10A wide
4858·1	D	4	N^+?	4858·82	Fowler
4861·0	D	9	Blend H, He^+	4861·326	Hβ Curtis [5]: 4859·34 He^+ Paschen [10]
4390	....	3	....	....	Two measures differed by 4 A.

Comments on Wave Lengths. The following comments on the relative intensities of the lines and other interesting features in the wave lengths are arranged under different elements, the elements themselves being in order of increasing atomic number.

Hydrogen. In the three stars the Balmer lines are the most intense of any. Saha[16] has predicted their disappearance before the disappearance of Mg + 4481 and ordinary He lines. Their continued presence in 9 Sagittæ and B.D. 35° 3930 N shows this prediction is not verified.

Helium. In 10 Lacertæ the strongest lines are those of the δ series of doublets, and the next most conspicuous series is the D series of singlets. In 9 Sagittæ the only lines showing are, in order of intensity 4471 ($1\pi - 3\delta$), 4713 ($1\pi - 3\sigma$), 4388 (1P − 4D). In B.D. 35° 3930 N, 4471 is just on the point of disappearing and no other He lines appear. From the work of Merton and Nicholson[17] it appears that the enhancement of 4922, 4388, the later members of the D series of singlets, relative to 6678, indicates a comparatively low mode of excitation in 10 Lacertæ. The conditions in this star are apparently not unfavourable, judging from Merton and Nicholson's experiments for the appearance of the band spectrum of He[18], which is apparently due to the transitory existence of the He_2 molecule. None of the unknown lines, however, in the star spectrum coincide with conspicuous lines in the band spectrum.

The spectrum of He + in the three stars has been discussed at length in sec. 3. It is of interest here merely to draw attention to the way the Pickering lines increase in intensity relative to the Balmer lines from 10 Lacertæ to B.D. 35° 3930 N. The behaviour of 4686 appears to be anomalous. Its disappearance in 9 Sagittæ may be due to the fact that it is just on the point of appearing as an emission line.

Carbon. In 10 Lacertæ there is a trace of 4267 which Fowler[19] assigns to the once ionized atom. The triplet at 4647 found by Merton[13] consists of three very sharp lines in 10 Lacertae and the stellar wave lengths are probably accurate within ± 0·02 A. Fowler[19] has suggested that these lines are due possibly to C + +. In 9 Sagittæ this triplet just shows but in B.D. 35° 3930 N it has disappeared entirely.

Nitrogen. The enhanced lines of this element, recently discussed by Fowler[9], are conspicuous in 10 Lacertæ and are relatively stronger again in 9 Sagittæ. In B.D. 35° 3930 N they have practically disappeared save possibly for 4097 and traces of emission at 4634, 4640.

Oxygen. In 10 Lacertæ quite a number of the stronger OII lines appear. Also Fowler's[3] OIII lines 5592, 3962 are very strong in 10 Lacertæ but have completely disappeared in 9 Sagittæ and B.D. 35° 3930 N.

Sodium. The D lines appear in 10 Lacertæ. Their appearance in a star of this temperature is anomalous and they are probably related to the sharp Ca + lines[20].

Magnesium. The enhanced line 4481[2], forming the first line of the fundamental series with a constant 4N, is present only in 10 Lacertæ and there it has been measured in less than half of the possible plates.

Silicon. In 10 Lacertæ Lockyer's Si III lines 4552, 4568 are just on the point of disappearing. Fowler [19] provisionally assigns them to a twice ionized silicon atom Si ++. In both 10 Lacertae and 9 Sagittae the Si IV pair 4089, 4116, which Fowler [19] suggests is due to Si +++, is very conspicuous but has disappeared in B.D. 35° 3930 N.

Sulphur. This element is possibly represented in 10 Lacertae by Lockyer's[7] enhanced line 4253·61.

Calcium. The two sharp lines H and K appear in all three stars. In 10 Lacertæ and 9 Sagittæ they are definitely displaced to the violet, possibly also in B.D. 35°3930 N. On any theory of ionization it is improbable that these lines would appear even in 10 Lacertæ let alone the other two stars. Work on early type spectroscopic binaries[21] suggests that these lines and the D lines of sodium have their origin in a cloud either immediately surrounding the star or between the earth and the star.

Titanium. In 10 Lacertæ the line 4337·76 is possibly due to enhanced titanium.

Unknown Origin. All the lines in 9 Sagittæ have been identified but in 10 Lacertæ and B.D. 35° 3930 N there are a number of lines for which no identification has been found. Of the 75 lines in 10 Lacertæ there are 13 of unknown origin: three of these, viz., 4156·5, 4276·0, 4654·4 Lockyer[7] notes as occurring in B-type stars. Of the 24 lines in B.D. 35° 3930 N there are 7 to which no identification can be assigned. Though these lines are all faint they have been measured on two or more plates and there is no reason to doubt their presence.

An inspection of the preceding comments makes it evident that the three O-type stars form, as regards line intensities, a continuous sequence. This is shown by the gradual decrease in intensity and final disappearance of the ordinary helium lines, the increase in intensity of the Pickering lines, the gradual disappearance of the Si IV and C ++ lines and so on in passing from 10 Lacertæ to B.D. 35° 3930 N. In this sequence 10 Lacertæ

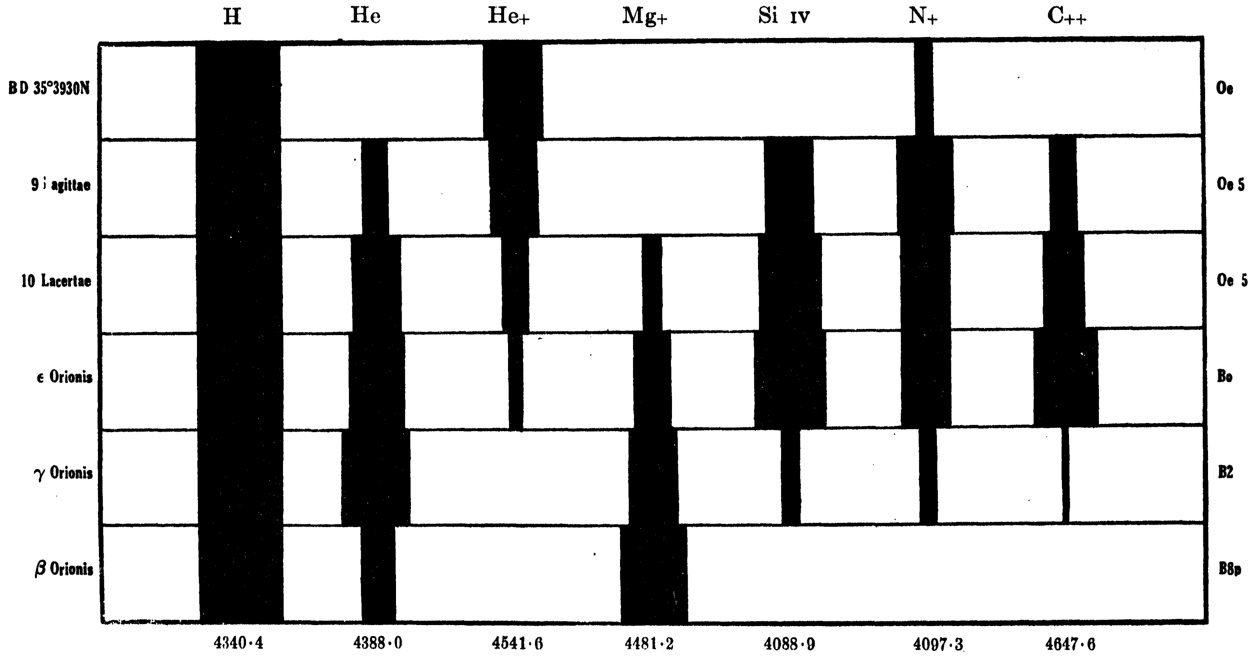

FIG. 7.—Intensities of Absorption Lines in B and O-Type Stars.

itself is closely related to the typical Bo star while 9 Sagittæ and B.D. 35° 3930 N are of "earlier" type. The general relations in line intensities are best shown by the type of diagram made familiar by Lockyer and his associates. In fig. 7 the intensities of a number of lines are shown graphically for the stars β Orionis (B8p), γ Orionis (B2), ε Orionis (Bo), the intensities of the lines in which are taken from Lockyer's tables[7], and for 10 Lacertæ, 9 Sagittæ and B.D. 35° 3930 N, the line intensities being taken from Tables 13, 14, 15. The breadth of the lines indicates the intensity. From this diagram, which summarizes the preceding information, it is clear that the three O-type stars form, as regards line intensities, a unique and continuous sequence with the Harvard class B.

SECTION 7—PHYSICAL CONDITIONS IN O-TYPE STARS

Of the various physical problems which are raised by O-type spectra it is proposed in this section to consider only two—the probable stellar temperatures and the relative abundance of hydrogen and helium. The more difficult problem of these two and the more important, for on its solution rests the attack on the other, is that of determining the stellar temperatures. The faintness of the O-type stars makes difficult at present a direct determination of temperatures from the intensity distribution in the continuous spectra. The only other mode of attack, that of an application of the methods developed in the Appendix to this paper on the "Physical Interpretation of Stellar Spectra," is contingent on a knowledge of the exact stage at which a line disappears, the series relations and ionization potentials of the element which gives rise to the line and the probable relative abundance of the element. While the paucity of lines and freedom from blends in these stars makes it possible to determine readily when a line has disappeared, the other two conditions are more difficult to fulfill in so far as, on the one hand, little or no laboratory work has yet been done on the characteristic O-type lines (enhanced silicon, carbon, oxygen and nitrogen) to ascertain their ionization potentials or series relations, and, on the other hand, the relative abundance of the element can only with more or less uncertainty be assumed identical with its abundance on the earth. As a result only the temperature of 9 Sagittæ has been determined directly from ionization theories; the temperature of the other two stars have been estimated from line intensity ratios. As for the other problem, the relative abundance of hydrogen and helium, its discussion arose from an attempt to obtain a physical interpretation of certain spectral peculiarities of O-type spectra noted in the previous section. These peculiarities were the persistence of the Balmer lines after the disappearance of enhanced magnesium and ordinary helium, contrary to Saha's prediction.[16] The physical interpretation is comparatively simple when the stellar temperatures have once been estimated.

Stellar Temperatures. The two methods of determining stellar temperatures developed in the Appendix depend upon the application of the fundamental equation $b = Ka\ (1 - x)$. In this equation b is the number of atoms required to absorb all the radiation emitted per sq. cm. per sec. by the photosphere within $\pm$ 25 km. of a given wave length, and is therefore a function of the temperature. K is a constant ($1 \cdot 5 \times 10^{20}$ for Saha's theory, $7 \cdot 7 \times 10^{20}$ for the electron collision hypothesis), a is the percentage abundance of the element and x is the fraction of once ionized atoms. The two methods developed in the Appendix, the one a slight modification of Saha's well known theory[16] and the other based on the assumption that ionization is due entirely to free electron collisions, differ in the mode of calculating x for an element of known ionization potential at a given temperature, and also, as mentioned, in the value of the constant K. It should be noted that as K was determined from stellar data based on the generally accepted temperatures of 6000° K for the sun and 10,400° K for B8 stars, the temperatures given by an application of these ionization theories should be consistent with the lower temperatures in the Wilsing, Scheiner and Münch [23] scale. As both sides of the fundamental equation are functions of the temperature, the method of determining the temperature of the star from the disappearance of an arc or spark line, when the value of a, the relative abundance of the element, is known, is one of trial and error. The values of b and $Ka\ (1 - x)$ must be computed for several temperatures until that temperature is found for which the equation becomes an identity.

In 9 Sagittæ, as Table 14, Sec. 6, shows, the line Mg_+ 4481·19 is not present. The value of b for this line for several temperatures may readily be computed and is the same for both theories. From the quantum relation $\Delta W = h\nu$ the amount of 4481 radiation absorbed by a single atom is $4 \cdot 384 \times 10^{-12}$ ergs. At 15,000° K each sq. cm. of the photosphere emits per sec. from Planck's law within ± 25 km of 4481·19, $2{\cdot}052 \times 10^8$ ergs. Therefore at 15,000° K, b = $(2{\cdot}052 \times 10^8)/(4{\cdot}384 \times 10^{-12}) = 4{\cdot}681 \times 10^{19}$. In a similar manner the value of b may be computed for other temperatures. Now the disappearance of 4481 means that there are no more or at least only a small fraction of once ionized magnesium atoms left. From Saha's theory, as revised by Russell[24], the fraction y of once ionized atoms for any given temperature may be readily computed as shown in the Appendix. Carrying through the necessary computations the following values result from Saha's theory on the assumption that a = 1·426, the same abundance in 9 Sagittæ that it has on the earth.

T = 15,000° K : b = $4{\cdot}681 \times 10^{19}$; Ka (1 - y) = $1{\cdot}5 \times 10^{20} \times 1{\cdot}426\ (1-0{\cdot}1410) = 1{\cdot}837 \times 10^{20}$.
T = 18,000° K : b = $7{\cdot}042 \times 10^{19}$; Ka (1 - y) = $1{\cdot}5 \times 10^{20} \times 1{\cdot}426\ (1-0{\cdot}6404) = 7{\cdot}693 \times 10^{19}$.
T = 20,000° K : b = $8{\cdot}782 \times 10^{19}$; Ka (1 - y) = $1{\cdot}5 \times 10^{20} \times 1{\cdot}426\ (1-0{\cdot}8587) = 3{\cdot}023 \times 10^{19}$.

Performing a graphical interpolation (see fig. 8) it is found that the fundamental equation becomes an identity at T = 18,150° K. It will be noted that this is a much lower temperature than Saha[16] himself assigns to the disappearance of Mg^+ 4481, viz., 23,000° K. The chief reasons for this difference in estimates are Saha's neglect of the relative abundance of elements, and even more of the fact that as the temperature rises a greater number of atoms are required to absorb the increasingly intense 4481 radiation from the photosphere.

For the electron collision hypothesis, while the values of b remain as before, the quantity Ka (1 - y) has to be recomputed in the manner shown in the Appendix. The results are, assuming as before that magnesium has the same abundance in 9 Sagittæ as on the earth

T = 15,000° K : b = $4{\cdot}681 \times 10^{19}$; Ka (1 − y) = $7{\cdot}7 \times 10^{20} \times 1{\cdot}426\ (1-0{\cdot}5576) = 4{\cdot}858 \times 10^{20}$.
T = 18,000° K : b = $7{\cdot}042 \times 10^{19}$; Ka (1 − y) = $7{\cdot}7 \times 10^{20} \times 1{\cdot}426\ (1-0{\cdot}8903) = 1{\cdot}205 \times 10^{20}$.
T = 20,000° K : b = $8{\cdot}782 \times 10^{19}$; Ka (1 − y) = $7{\cdot}7 \times 10^{20} \times 1{\cdot}426\ (1-0{\cdot}9531) = 5{\cdot}154 \times 10^{19}$.

From the graphical interpolation in fig. 8 it will be seen that on the electron collision hypothesis the temperature of 9 Sagittæ would be *18,925° K*. The temperatures given by the two methods, which are based on different sets of assumptions, are accordingly in good agreement, and the value *T = 18,500° K* will here be adopted. It is to be noted, however, that this temperature, quite apart from the uncertainty as to the relative abundance of magnesium in the star, is subject to two possible errors. There is no trace of 4481 in 9 Sagittæ and in 10 Lacertæ it has been measured in less than half of the possible plates. It is therefore possible that the line has disappeared before the 9 Sagittæ stage is reached, in which case the temperature of this star would be higher than 18,500° K. On the other hand the line 4481 is not absorbed by the normal once ionized magnesium atom, but only by one in which the atomic electron is in the 2δ orbit. Though a number of atoms owing to collisions with free electrons of appropriate velocity will be in this 2δ state, clearly not all of them will be, so that the temperature of 18,500° K. for the disappearance of Mg_+ 4481 will have to be revised downwards. As these two possible errors tend to balance each other, the temperature of 18,500° K for 9 Sagittæ is probably not greatly in error.

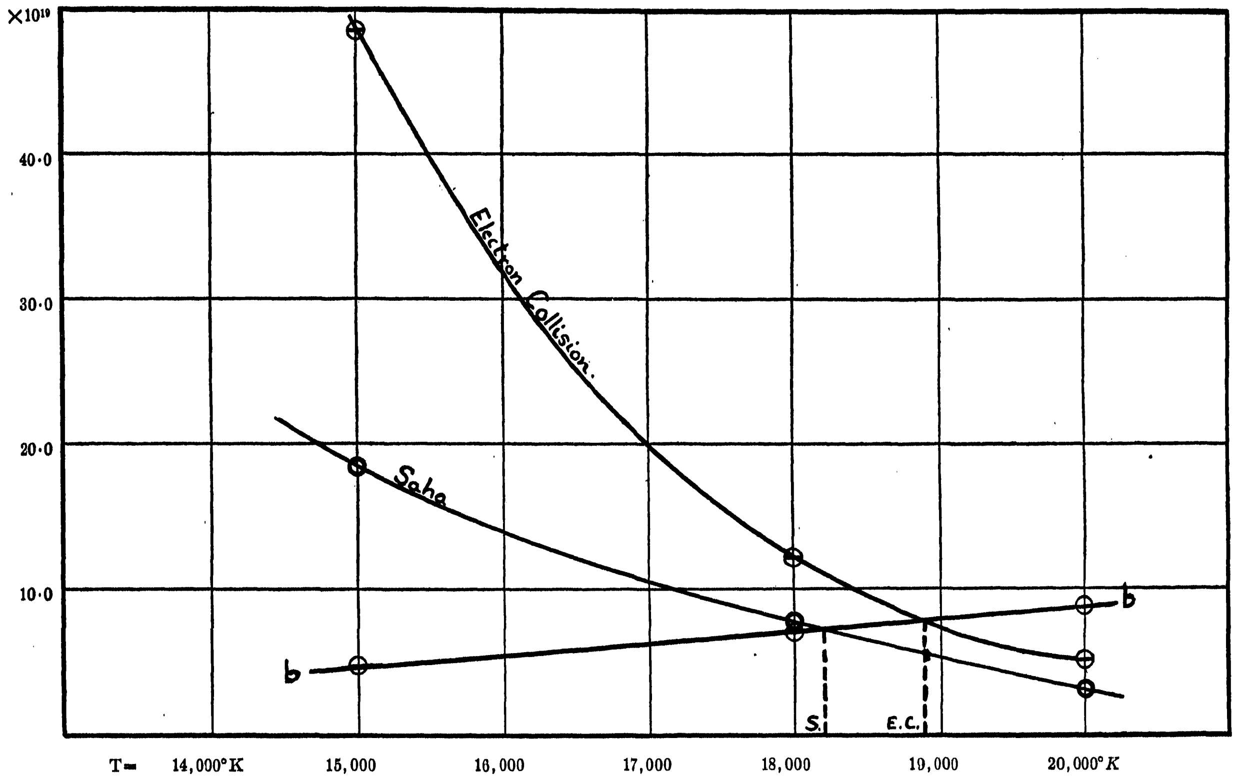

FIG 8—Temperature of 9 Sagittae from Disappearance of Mg_+ 4481

In *10 Lacertae*, since no lines disappear whose series relations or ionization potentials are known, the above ionization methods cannot be applied. However, since temperatues may be measured by any property of matter which varies continuously with temperature and since it has been shown that the intensity of lines in stellar spectra is a function of one main variable, the temperature[25], it appears probable that some estimate of the temperature of 10 Lacertæ may be arrived at from its line intensities. As noted in the previous section estimates of line intensities were made on all the plates and the means of these estimates are given in Tables 13, 14, 15, of Sec. 6 for the three O-type stars. Comparable with these estimates are those of Lockyer and his associates for the B-type stars.[7] In the accompanying schedule are given the ratios of intensities of 4388 (He)/Hγ; 4541 (He_+)/Hγ; 4471 (He)/4541 (He_+); 4088 (Si IV)/4097 (N_+); and for 4647 (C_{++})/Hβ for three B-type and three O-type stars. In the first column is given the star, in the second the Harvard type, in the third column the probable temperature, and the succeeding columns contain the line intensity ratios noted above. The temperatures of β Orionis (B8p) 10,000° K and of ϵ Orionis (Bo) 13,000° K are those given by Coblentz[26], as a result of his determination of intensity distribution in these stars in the spectral range $0\cdot3\mu$ to 10μ. The temperature of 12,000° K adopted for γ Orionis is the mean of the temperatures given by Wilsing, Scheiner and Münch for γ Orionis, γ Pegasi and β Lyræ. The temperature of the other B2 star, ζ Cassiopeiæ, given by them was not included as being probably too low by reason of selective absorption of the Milky Way material in its neighbourhood. The temperature of 18,500° K for 9 Sagittæ is of course that determined immediately above by means of the two theories of ionization. From the intensity ratios of the various pairs of lines given in this schedule it is now possible to interpolate graphically

Star	Harv. Type	Temperature	4388 / Hγ	4541 / Hγ	4471 / 4541	4088 / 4097	4647 / Hβ
β Orionis	B8p	10,000°K	0·40	....	....	....	....
γ Orionis	B2	12,000°K	0·80	....	....	1·0	....
ε Orionis	Bo	13,000°K	0·65	0·15	5·0	1·5	0·80
10 Lacertæ	Oe5	(15,000°K)	0·67	0·33	3·0	1·3	0·50
9 Sagittæ	Oe5	18,500°K	0·33	0·67	1·7	0·9	0·33
B.D. 35°3930 N	Oe	(22,000°K)	0·00	0·78	0·0	....	0·00

and arrive at an estimate of the temperature of 10 Lacertæ. As a result it is found that the various pairs of lines give the following temperatures:—4388/Hγ = 12,800° K 4541/Hγ = 14,400° K; 4471/4541 = 15,400° K; 4088/4097 = 15,200° K; 4647/Hβ = 16,600° K with a mean in the round numbers of *15,000° K*. This temperature, in brackets in column 3 of the schedule, is assumed in what follows to be that of 10 Lacertæ.

In *B.D. 35° 3930 N*, as the above schedule shows, helium arc lines are just on the point of disappearing. An estimate of the relative abundance of helium on the earth may be obtained in the following manner. The emission of helium by volcanoes, from mineral springs and natural gases, its escape into and ultimate retention [27] by the earth's atmosphere, even if the temperature of the outer layers of this atmosphere has been as high as 300° C in the early history of the world,—all these things make it probable that the greater part of the element is in the atmosphere. F. W. Clarke[28] gives the ratio of helium to nitrogen (also chemically inert) in the atmosphere to be $0\cdot0004/78\cdot122$. As all the nitrogen is effectively in the atmosphere and its total percentage in the first ten metres of the earth's crust is $0\cdot0383$, it follows that a lower limit to the percentage abundance of helium on the earth is $1\cdot96 \times 10^{-7}$. If this be the abundance of helium in the stars then from Saha's theory the highest possible value of Ka $(1-x)$ will be $1\cdot5 \times 10^{20} \times 1\cdot96 \times 10^{-7} = 2\cdot94 \times 10^{13}$ when $x = 0$. Since the individual atom absorbs from the quantum relation $4\cdot394 \times 10^{-12}$ ergs of 4471 radiation, it follows that the energy radiated per sq. cm. per sec. when the helium spectrum will appear will be $2\cdot94 \times 10^{13}$

$\times\ 4 \cdot 394 \times 10^{-12} = 1 \cdot 29 \times 10^{2}$ ergs, corresponding to a temperature of 250° K. At this temperature, however, there would be no helium atoms with their electrons in the 1 p orbit necessary for 4471 to appear as absorption [16]. Consequently if the abundance of helium be no greater than it probably is on the earth, the helium arc lines would never appear as absorption lines in stellar spectra. As, however, helium arc lines do appear it is evident that the relative abundance in the stars is different from the probable terrestrial abundance of helium. Consequently as no independent or probable value of a for helium can be obtained, it is impossible to use the two ionization theories to determine the temperature of B.D. 35° 3930 N from the disappearance of the helium arc lines in its spectrum.

For the present, until laboratory data on the series relations and ionization potentials of enhanced silicon, carbon and nitrogen are obtained, the temperature of B.D. 35° 3930 N can only be determined by extrapolation of the line intensity ratios. Using the values of these ratios given in the above schedule and including 10 Lacertæ at an estimated temperature of 15,000° K, the following graphically extrapolated temperatures result for B.D. 35° 3930 N; $-4388/H\gamma$ = 21,600° K; $4541/H\gamma$ = 19,700° K; 4471/4541 = 25,800° K; $4647/H\beta$ = 21,000° K with a mean in round numbers of *T = 22,000° K.* This somewhat uncertain temperature, given in brackets in the above schedule, will be assumed in what follows to be the temperature of B.D. 35° 3930 N.

Summarizing, the following are the adopted temperatures for the three O-type stars:—

10 Lacertæ	—15,000° K Interpolated by line intensity ratios from ε Orionis and 9 Sagittæ.
9 Sagittæ	—18,500° K From the disappearance of Mg + 4481 on the two theories of ionization.
B.D. 35° 3930 N	—22,000° K Extrapolated from line intensity ratios in the three O-type stars.

of these temperatures that assigned to 9 Sagittæ is entitled to the greatest weight. The other two temperatures are effectively based on its determination and on the temperature of 13,000° K given to ε Orionis by Coblentz[26].

Abundance of Hydrogen and Helium. The persistence of the Balmer lines in 9 Sagittæ and B.D. 35° 3930 N after the disappearance of enhanced magnesium and helium arc lines is, as previously noted, contrary to Saha's predictions on his original theory.[16] However, it is shown in the Appendix that the disappearance of an arc or spark line is a function not only of the stellar temperature but also of the relative abundance of the element. It is, therefore possible, and is a matter of some interest, from the temperatures of the O-type stars derived immediately above, to compute by the use of the two ionization theories, what the relative abundance of hydrogen and helium must be to account for the apparently anomalous behaviour of their spectra.

Taking the ionization potential of *Hydrogen* as 13·545 volts, it may be shown from Saha's modified theory (see Appendix) what its relative abundance must be for its spectrum to disappear in 9 Sagittæ or B.D. 35° 3930 N. If $H\gamma$ disappeared in 9 Sagittæ at a temperature of 18,500° K the relative abundance of hydrogen would be a = 7·773

$\times 10^{19} / [1 \cdot 5 \times 10^{20} (1-0 \cdot 8583)] = 3 \cdot 66$. As the relative abundance of hydrogen on the earth (see Table 17, Appendix) is 15·459, it follows that if the stellar abundance of the element is comparable with its terrestrial value, the Balmer lines will persist after the temperature 18,500° K is reached or after the disappearance of Mg_+ 4481. In other words, when Saha's theory is modified to take account of the relative abundance of elements, the observed behaviour of hydrogen and enhanced magnesium is precisely what would be anticipated if the stellar abundances of the two elements are comparable with their terrestrial values. Similarly on Saha's theory if $H\gamma$ is to appear in B.D. 35° 3930 N at a temperature of 22,000° K the relative abundance of hydrogen must be greater than $1 \cdot 101 \times 10^{20} / [1 \cdot 5 \times 10^{20} (1-0 \cdot 9810)] = 38 \cdot 63$; or the stellar abundance of hydrogen would have to be more than double its terrestrial value for the Balmer lines to appear in B.D. 35° 3930 N. In fact it may readily be shown that if a = 15·459 for hydrogen in the stars, then the Balmer lines will disappear on Saha's theory at 21,000° K. These numerical results obtained from Saha's theory may be checked by the electron collision hypothesis. On this theory if $H\gamma$ disappears in 9 Sagittæ $a = 7 \cdot 773 \times 10^{19} / [7 \cdot 7 \times 10^{20} (1-0 \cdot 9313)] = 1 \cdot 47$: if $H\gamma$ disappears in B.D. 35° 3930 N $a = 1 \cdot 101 \times 10^{20} / [7 \cdot 7 \times 10^{20} (1-0 \cdot 9800)] = 7 \cdot 15$. If then the stellar and terrestrial values of the percentage abundance of hydrogen are comparable, on the electron collision hypothesis the Balmer lines will persist even in B. D. 35° 3930 N, and it may be shown that $H\gamma$ will not disappear until a temperature of between 24,000° K and 25,000° K is reached. Summarizing, on Saha's theory the stellar abundance of hydrogen will have to be at least twice its abundance on the earth for the Balmer lines to be on the point of disappearing in B.D. 35° 3930 N; on the electron collision hypothesis if the stellar abundance is less than half the terrestrial value the Balmer lines will still appear at 22,000° K. It may, therefore, be concluded with a fair degree of probability that the stellar relative abundance of hydrogen is of the same order as its abundance in the first ten miles of the earth's crust.

It has been shown in the discussion on the temperatures of B.D. 35° 3930 N that the relative terrestrial abundance of *Helium* is probably of the order of $1 \cdot 96 \times 10^{-7}$. If the element is no more abundant than this in the stars, its spectrum will never appear. Once again it is of interest to compute (1) on Saha's theory and (2) on the electron collision hypothesis what the minimum values of the relative abundance of helium must be for 4471 to appear in 9 Sagittæ and B.D. 35° 3930 N. Taking the ionization potential as 25·4 volts[29], on Saha's theory for 4471 to appear in 9 Sagittæ at a temperature of 18,500° K, $a > 7 \cdot 579 \times 10^{19} / [1 \cdot 5 \times 10^{20} (1- \cdot 0037)] = 0 \cdot 507$; similarly for 4471 to be on the point of disappearing in B.D. 35° 3930 N at 22,000° K $a = 1 \cdot 064 \times 10^{20} / [1 \cdot 5 \times 10^{20} (1- \cdot 0663)] = 0 \cdot 760$. On Saha's theory then the stellar abundance of helium lies between 0·507 and 0·760. It may be noted here that at 22,000° K Saha obtains x = 0·53 in place of the value ·0663 given above. The principal cause of this difference is that Saha used the resonance potential of 20·5 volts for helium in place of the ionization potential of 25·4 volts. Carrying through the computations on the electron collision hypothesis, it is found that for the disappearance of 4471 in 9 Sagittæ or in B.D. 35° 3930 N. the stellar relative abundance of helium must lie between $7 \cdot 579 \times 10^{19} / [7 \cdot 7 \times 10^{20} (1-0 \cdot 0101)] = 0 \cdot 099$ and $1 \cdot 064 \times 10^{20} / [7 \cdot 7 \times 10^{20} (1-0 \cdot 1078)] = 0 \cdot 155$. Taking means from the two theories the stellar relative abundance of helium must lie between 0·30 and 0·46, or it is probable that there is about two million times more helium in stellar atmospheres than there is in the first ten miles of the earth's crust.

From the observed spectra and the probable temperatures of O-type stars it has been concluded,—(1) that the stellar and terrestrial abundance of hydrogen are comparable, and (2) that helium is probably much more abundant in the stars than on the earth. These conclusions are scarcely surprising when the high temperatures and pressures that must exist towards the centre of an O-type star are recalled. Under these conditions it is to be expected that a certain small fraction of atomic collisions will result in nuclear disintegration, and the products of this disintregration will be helium and hydrogen nuclei[30]. If then initially, say, there are 10^{14} hydrogen atoms and 10^{6} helium atoms per cu. cm. (the ratio of hydrogen to helium on the earth approximately 10^{8} : 1), then after nuclear disintregration while the addition of 10^{10} hydrogen atoms per cu. cm. will not alter appreciably the relative abundance of that element, the addition of 10^{10} helium atoms per cu. cm. will increase the relative abundance of helium by 10,000 times of its terrestrial value. Further Rutherford's[30] working hypothesis suggests that at most there are not more than three hydrogen nuclei to a complex nucleus while there are, for example, as many as ten helium nuclei in the calcium nucleus. That is nuclear disintegration in the lower layers of the star will probably supply a greater number of helium atoms than of hydrogen atoms, and the final result will be a great increase in the relative abundance of helium over its initial value while the abundance of hydrogen will remain practically unchanged. This is precisely the result which is suggested by the observed O-type spectra. Accordingly nuclear disintegration in the lower layers of the star with subsequent diffusion of the products to the chromosphere is probably a satisfactory working hypothesis to account for the unchanged abundance of hydrogen and the great increase of helium.

Section 8—Classification of Absorption Line O-Type Stars

One of the most important and informative modes of classifying stars is by their spectra. The Harvard system alone, of all the various classifications that have been proposed, is universally accepted and the reasons for its survival are not far to seek. For from the intensities of arc and spark lines more than 99 per cent of all the stars fall into one or another of the six great classes M, K, G, F, A, B: the remaining 1 per cent of the stars are divided among the minor classes N, R, O. Nor are these six main types unrelated, but are found to form, when expressed in the order M, K, G, F, A, B, a unique continuous sequence as regards the intensities of lines. From the investigations of Wilsing and Scheiner, Coblentz and others it has further become evident that this linear sequence is due to the fact that the spectral type is a function primarily of the temperature, a relation for which the theoretical investigations of Saha afford the soundest basis. The Harvard system is therefore, on account of its comprehensiveness, its facility and its firm, though unforeseen physical foundation, of fundamental importance in the classification of stars by their spectra.

Harvard subdivisions. In this final section of the paper, in the light of the data acquired in the previous sections, it is proposed to discuss the numerically insignificant class O in an endeavour to bring its subdivisions more into harmony with the rest of the Harvard system. Within the Class O are comprised effectively all those stars, with the exception of those classified as B1, B0, which show the Pickering lines. The stars so classified may be divided into two main groups,—(1) absorption line stars showing occasional emission lines, and (2) stars showing nothing but broad emission bands. It is with the first of these groups only that this section is concerned, a group in which the present Harvard system recognizes three subdivisions:—

Oe5 Pickering lines and ordinary helium lines intense. 4089 > 4097. *No* emission.

Oe Pickering lines and ordinary helium as in Oe5. 4097 max. intensity. Emission 4638, 4686.

Od Pickering lines stronger, helium weaker. Emission 4638, 4686 stronger than in Oe.

In these subdivisions the practical distinction between Oe5 on the one hand, and Oe and Od on the other, is afforded by the presence of emission at 4638, 4686. Now this procedure is inconsistent with the remainder of the Harvard classification, it is physically unsound, and it leads to the inclusion in the class Oe5 of stars differing widely in the type as estimated by the absorption line intensities. The procedure is inconsistent in so far as the spectral class of later type stars from B to M which show emission lines is determined by the intensity of the absorption lines, the presence of bright lines being regarded merely as peculiarity (symbol p)—thus γ Cassiopeiæ Bop. In the Report of the Committee on Spectral Classification, adopted at the Rome meeting of the International Astronomical Union[31], it was recommended that the presence of emission lines be denoted by the small letter e—thus γ Cassiopeiæ Boe; it is thus implicitly recognized in this Report that the presence or absence of emission lines does not determine type. Finally such a procedure leads to the inclusion of all O-type stars which show no bright lines in the class Oe5, irrespective of the strength of the absorption lines. The anticipated result is that the class Oe5 has no physical significance and includes for example 10 Lacertæ and 9 Sagittæ, two stars whose temperatures differ some 3500° K.

The anomalous results brought about by this definition of the sub-class Oe5 can best be seen when a greater number of stars are considered. In the course of his radial velocity programme of O-type stars, Dr. J. S. Plaskett has secured spectra of some thirty absorption line O's which he has kindly placed at the disposal of the writer for the purposes of this discussion. The B.D. numbers of these stars, their 1900 coordinates, their visual magnitudes and Harvard types (sent by Miss Cannon in advance of publication) are given in the first five columns of Table 16. In the succeeding three columns are given, from estimates with an eyepiece, the intensity ratios of the three pairs of lines 4471/4541, 4541/Hγ, 4088/4097—intensity ratios which, as was shown in the previous section, change rapidly with the temperature and therefore rapidly with type. The two final columns contain the subdivision of the star as tentatively suggested in the next paragraph and remarks. An inspection of this table immediately shows that of 21 stars classified by Harvard as Oe5, from the intensity ratios 2 are more nearly B, 5 are similar to 10 Lacertæ, 4 lie between 10 Lacertæ and 9 Sagittæ, 3 are similar to 9 Sagittæ and 7 are earlier than 9 Sagittæ and approximating to B.D. 35° 3930 N. In other words, two-thirds of the stars placed in the class Oe5 because they showed no trace of emission, are actually from the

TABLE 16—ABSORPTION LINE O-TYPE STARS

B.D.		R.A. 1900		Dec. 1900	Vis. Mag.	Harv. Type	4471 / 4541	4541 / Hγ	4088 / 4097	Prop. Type	Remarks
°		h	m	° ′							
40	501	2	16·7	41 02	7·5	Bo	2·8	0·25	0·53	O9	4686 absorption
58	467	2	19·5	58 25	8·04	Oc	0	0·6	—	O5e	4686 broad emission
56	693	2	35·5	56 28	8·4	Oe	0·7	0·6	—	O6e	4686 emission. One plate only.
52	726	3	48·0	52 22	6·70	Oe5	3·0	0·2	0·9	O9	4686 absorption
34	980	5	09·7	34 12	5·81	Oe5	4	0·2	1·5	O9	Almost identical 10 Lacertæ
37	1146	5	14·0	37 20	6·71	Oe5	0·8	0·6	—	O6	4686 faint absorption
9	879	5	29·6	9 52	3·66	Oe5	2	0·3	1·0	O8	λ_1 Orionis
......	......	....	.....	....	5·56	Oe5	∞	0	∞	B1	λ_2 Orionis
−5	1315	5	30·4	−5 27	5·36	Oe5	1·0	0·5	—	O6	θ Orionis C
......	......	....	.....	....	.....	....	∞	0	—	B1	θ Orionis D.
−6	1241	5	30·5	−5 59	2·87	Oe5	3	0·2	2	O9	ι Orionis.
20	1284	6	03·7	20 31	7·40	Oe5	0·7	0·6	—	O6	
6	1303	6	31·1	6 10	7·3	B2	2	0·3	1·0	O8	Discovered by J.S.P.
6	1309	6	32·0	6 13	6·06	Bop	2	0·3	1·1	O8	Very massive star J.S.P., D.A.O.
10	1220	6	35·5	9 59	4·68	Oe5	1·1	0·45	0·8	O7	S Monoc.
−10	1892	7	04·6	−10 11	6·20	Oe5	0·7	0·4	—	O7	
−24	5176	7	14·5	−24 47	4·40	Oe5	5	0·15	1·4	O9—Bo	τ Can.Maj. Typical Oe5 star.
−24	13814	17	57·7	−24 22	5·86	Oe5	0	0·5	—	O5	9 Sagitarii
−24	13864	17	59·0	−24 24	6·79	Oe5	1	0·5	—	O6	
−18	4886	18	11·6	−18 30	6·37	Oe5	2	0·4	—	O8	
−20	5344	18	52·3	−20 33	6·73	Oe5	1	0·3	1	O8	
18	4276	19	47·9	18 25	6·29	Oe5	1·4	0·4	0·8	O7	9 Sagittæ. Eye-piece estimates.
35	3930N	19	59·8	35 45	7·2	Oe	0	0·6	—	O5e	Eyepiece estimates.
	S	....	.....	....	7·7	....	4	0·2	2	O9—Bo	
35	3949	20	01·9	35 19	7·4	....	0·9	0·5	—	O6	
35	3953	20	02·2	35 31	7·01	Op	5	0·2	2	O9—Bo	Broad emission 4686.
43	3571	20	17·1	43 32	6·83	Oa	0	0·6	—	O5e	Broad emission band at 4686.
44	3639	20	53·1	44 33	6·01	Oe5	0·8	0·5	0·6	O6	
43	3877	21	14·8	43 31	6·06	Oe5	1·7	0·3	—	O8	A Cygni. Broad weak lines.
56	2617	21	35·9	57 02	5·64	Oe5	0·6	0·5	—	O6	
61	2246	22	02·1	61 48	5·17	Oe5	8	0·2	1·3	O9—Bo	
58	2402	22	08·1	58 56	5·19	Od	1	0·5	—	O6e	λ Cephei. Diffuse lines.
38	4826	22	34·8	38 32	4·91	Oe5	2·7	0·2	1·4	O9	10 Lacertæ. Eye-piece estimates.

intensities of the absorption lines much earlier in type and approach more nearly the subtypes Od, Oe. Finally attention may be drawn to two other anomalies which have resulted from the use of the presence or absence of emission lines as a criterion of type. The one is the star B.D. 35° 3953 (R.A. $20^h\ 02^m \cdot 2$.) classified by Harvard as Op because it shows a broad emission band at 4686. Actually from its absorption lines it is an Oe5 star but it cannot so be classified because it shows emission. The other case is the star B.D. 43° 3571 (R.A. $20^h\ 17^m \cdot 1$) classified by Harvard as Oa because it shows a broad emission near 4686, a characteristic feature of this class. Actually, however, the star has absorption lines and from their intensity ratios the star is similar in type to B.D. 35° 3930 N.

Proposed Subdivisions. The preceding discussion has made it clear that the present subdivisions for the absorption line O-type stars are unsatisfactory. In tentatively suggesting any new mode of classifying it must be recalled, as was clearly shown in sec.

6 and 7 of this paper and as has long been more or less distinctly recognized, that the absorption line O-type stars form from the intensities of their lines and from their probable temperatures a unique and continuous sequence with the early B-type stars. It is, therefore, advisable in any proposed alterations of the subdivisions to use the decimal scale as in the major types B, A, F, G, K, M, where continuous sequences of line intensities also exist, and abandon the letter subdivisions of the present Harvard system; the more so as the Committee on Spectral Classification state that old symbols should not be used with new meanings.[31] One other general condition that must be met by any satisfactory working classification of absorption line O's is, that a place be left for stars earlier in type than any in Table 16. Recalling that within the Harvard class O are comprised all those stars which show the Pickering lines, the class Oo might well be typified by a star in which these lines are on the point of disappearing. From Saha's theory, as modified in the Appendix for the relative abundance of elements, it is possible to compute at what temperature the Pickering series will disappear. It is found that taking the relative abundance of helium as 1·1 (probabaly a safe upper limit, see Sec. 7) the temperature at which the Pickering lines will be on the point of disappearing is about 29,000° K. If now B.D. 35° 3930 N (T = 22,000° K) be taken as the typical star of the class O5, 9 Sagittæ (T = 18,500° K) as the typical star of class O7 and 10 Lacertæ (T = 15,000° K) of O9, the main framework of the tentative scheme is thus clearly outlined. A brief description of each one of the proposed types follows:—

Class Oo. Typical star—none known (Prob T = 29,000° K). This class is characterized by the disappearance of the Pickering lines. The probable temperature is a very sensitive function of the relative abundance of the element helium.

Class O5. Typical star—B.D. 35° 3930 N (Prob. T = 22,000° K). In this class the ordinary helium arc lines and the enhanced nitrogen lines have disappeared. In the typical star there is a trace of enhanced nitrogen emission at 4634, 4640 and also a very characteristic line of unknown origin at 4603·8. The intensity ratios for the class are $\frac{4471}{4541} = 0$; $\frac{4541}{H\gamma} = 0{\cdot}6$.

Class O6. Typical star—B.D. 44° 3639 (Prob. T = 20,000° K). In the typical star Si IV 4088·9 is weak and 4116·2 is absent or very faint. There is no trace of the enhanced carbon triplet at 4647. The intensity ratios which define the class are $\frac{4471}{4541} = 0{\cdot}8$; $\frac{4541}{H\gamma} = 0{\cdot}5$; $\frac{4088}{4097} = 0{\cdot}6$.

Class O7. Typical star—9 Sagittæ (Prob. T = 18,500° K). In the typical star enhanced nitrogen reaches its maximum strength. Ordinary helium is well marked and the enhanced carbon triplet is just beginning to appear. The enhanced magnesium line 4481 has disappeared, which serves to define the temperature. The intensity ratios are $\frac{4471}{4541} = 1{\cdot}4$; $\frac{4541}{H\gamma} = 0{\cdot}4$; $\frac{4088}{4097} = 0{\cdot}8$.

Class O8. Typical star — λ_1 Orionis (Prob. T = 17,000° K). This spectrum is characterized by the decreased strength of the enhanced nitrogen lines, the strengthening of ordinary helium and of the enhanced carbon triplet at 4647. In the typical star the fainter helium lines at 4713, 4388, 4143, 4120 are quite strong, and enhanced magnesium 4481 is barely present. The intensity rates are $\frac{4471}{4541} = 2{\cdot}0$; $\frac{4541}{H\gamma} = 0{\cdot}3$; $\frac{4088}{4097} = 1{\cdot}0$.

Class O9. Typical star—10 Lacertæ (Prob. T = 15,000° K). The typical star is characterized by the strength and sharpness of its lines. Si IV 4088, 4116 reach their maximum; Si III 4552, 4567 are on the point of appearing and Mg+ 4481 is clearly present. The enhanced nitrogen lines are present and the carbon triplet at 4647 is conspicuous. Fowler's O III lines at 3961, 5592 are strong. The intensity ratios are $\frac{4471}{4541} = 2{\cdot}7$; $\frac{4541}{H\gamma} = 0{\cdot}2$; $\frac{4088}{4097} = 1{\cdot}4$.

Using these criteria it is now a simple matter, from eyepiece estimates of the intensity ratios of these three pairs of lines, to classify any absorption line O-type star. These tentative classes for some thirty-three O-type stars are given in the ninth column of Table 16. The suggestion embodied in the Stellar Classification Report [31] is here adopted, and the letter e following the type indicates the presence of an emission line or lines in the spectrum. That the proposed classification is on the whole more satisfactory than the present Harvard system is shown by the distribution of stars among the various subdivisions. Of the 33 stars in Table 16 Harvard classifies as B2–1; Bo–2; Oe5–21; Oe–3; Od–1; Op–1; Oa–1; and unclassified 3. On the face of the matter it seems unreasonable to suppose that the vast majority of the absorption line O's should fall into one subdivision of the classification. On the proposed classification of the 33 stars there is in the subclass B1–2; O9–9; O8–6; O7–3; O6–9; O5–4;—on the whole a much more uniform and probable distribution amongst the various temperatures.

In conclusion there is at least one criticism that may be urged against this tentative scheme—namely it contains no place for that group of O-type stars showing emission bands with no trace of absorption lines. Unfortunately knowledge of this group of stars is still so meagre as to make a discussion of their place in any scheme of classification difficult. Two things at least, however, are evident. First the pioneer investigation of W. W. Campbell [32] has made it clear that typical stars of this group B.D. 35° 4001 (Ob), B.D. 35° 4013 (Oa), and B.D. 37° 3821 (Ob), spectra of which secured here show no trace of absorption lines, have bright bands coincident with ordinary helium at λ5876, with Mg+ 4481 and with O III 5593. The presence of these lines at once makes it clear that these stars are not of higher temperature than the tentative type O7, and that as a group the emission band or Wolf Rayet stars are not of earlier type than the absorption line O's. In the second place Wright [33] has shown in a number of typical cases that these emission band stars are most closely related to the planetary nebulæ. Obviously the width of the emission lines and the weakness of the continuous spectrum suggest, that the radiation has had its origin in something quite different from a star with a well defined photosphere and a tenuous chromosphere. The observed spectra indicate if anything a veiled photosphere and a high pressure chromosphere. Notwithstanding these two facts it must, however, be remembered that the absorption line O's and the Wolf-Rayet stars quite frequently have in common more or less ill-defined emission bands in the neighbourhood of 4686, a fact which suggests some connection between the two groups. Until more is known of this interesting group of emission band stars, it may be assumed as a possible working hypothesis that the Wolf Rayet stars form a side chain with the O's thus A, B, O and A,B, Wolf Rayet, Planetary Nebulæ? just as the spectral series divides into two branches at the other end, viz., G, K, M and G, K, R, N. The physical interpretation of such an assumption would be that the exceptionally massive stars followed the Wolf Rayet branch until they were "blown out" by radiation pressure[34] into planetary nebulæ. In any case the assumption fits the facts about these stars as they are now known, and justifies to some extent the complete omission of the Wolf Rayet stars from the tentative scheme for the absorption line O's.

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APPENDIX—PHYSICAL INTERPRETATION OF STELLAR SPECTRA

The purpose of this Appendix is to explain the methods which are used in Sec. 7, Part III, to determine the temperatures and the relative abundance of elements in O-type stars. This problem of the physical interpretation of stellar spectra is made complex by the fact that the star spectrum is a function probably of at least four independent variables—the chromospheric temperature and pressure, the relative abundance of elements and the free electron concentration. That any progress in the solution of the problem has been made at all is largely due to the pioneer work of M. N. Saha, who in a series of papers has laid down the guiding principles for a successful attack. In this Appendix two independent methods of determining temperature and relative abundance of elements from an observed stellar spectrum are outlined, the more important one of which is a slight extension of Saha's theory. The other method was devised primarily to act as a check on the numerical results which applications of his theory gave.

The appendix is divided into three sections. In the first of these a review is given of current and well established theories on the origin of spectra on which basis any physical theory of stellar spectra must rest. In the second section Saha's theory is outlined and modified to allow for the relative abundance of elements. It is shown that with this modified theory the "anomalous" behaviour of barium and sodium in the sun can be explained without any additional hypothesis. Finally in the third section certain possible imperfections in Saha's postulates are considered and an alternative hypothesis of ionization by electron collisions is developed.

SECTION. A—THE ORIGIN OF SPECTRA

Bohr's[1]* theory of the atom with one electron shows that there is associated with the appearance of a given line a definite amount of energy. In the normal atom where the sum of potential and kinetic energies is a minimum, the electron revolves in the 1 quantum orbit. In order to produce an emission line the electron must first be lifted out of that 1 quantum orbit, either to some more distant orbit, or to infinity. Thus the line 1216, the first line of Lyman's ultra violet hydrogen series, is emitted by a hydrogen atom in which the electron falls from the 2 to the 1 quantum orbit. In order, therefore, to bring the hydrogen atom into the necessary preliminary condition to radiation, the electron will have to be lifted from the normal 1 quantum to the 2 quantum orbit. Directly from Bohr's theory it is seen that this involves the expenditure of $Nh\,(1 - \frac{1}{2^2}) = 1\cdot617 \times 10^{-11}$ ergs of energy per atom. In order for the hydrogen atom to emit any possible line of its spectrum it is necessary to lift the electron to infinity. Again from Bohr's theory this requires $Nh = 2\cdot156 \times 10^{-11}$ ergs. Thus it will be noted that, for an atom with stationary orbits (like the Bohr model), the amount of energy required for the production of a given line is obtained from the quantum relation $\Delta W = h\nu$,—the frequency times Planck's constant.

*References will be found at the close of the Appendix

Now the work of Franck and Hertz[2], McLennan[3] and others has shown that relations of precisely this form hold also for more complex atoms,—atoms with more than one electron. Their experimental procedure has been to bombard the monatomic atoms of some metallic vapour with free electrons emitted by an incandescent filament and accelerated by known potentials. Thus it has been shown that for sodium, when the free electrons have a potential drop of 2·09 volts, known as the resonance potential, a "single line spectrum" occurs,—the well known D lines. No further spectral lines appear until the electrons have a potential drop of 5·11 volts, known as the ionization potential, when suddenly the whole spectrum flashes out. Now the frequency of D_2, λ 5889·963 is $\nu = 5{\cdot}09 \times 10^{14}$. Hence from the quantum relation the energy to produce D_2 is $6{\cdot}554 \times 10^{-27} \times 5{\cdot}09 \times 10^{14} = 3{\cdot}34 \times 10^{-12}$ ergs $= 2{\cdot}09$ volts. Similarly the frequency of the limit of the π series of sodium is $1{\cdot}24 \times 10^{15}$, and the energy necessary to produce this limit is again given by the quantum relation to be $8{\cdot}15 \times 10^{-12}$ ergs $= 5{\cdot}11$ volts. The close similarity between the energy relations in the hydrogen and sodium spectrum is at once evident, and strongly suggests the existence in the sodium atom of stationary orbits. Relations similar to these have been found for the alkalis and alkaline earths[4] in the first two columns of the periodic table.

These experiments have led to the formulation of the following theory by Sommerfeld[5] for atoms with more than one electron. Fixing attention on one electron of an atom of atomic number n, the charge on the rest of the atom, or the "core," is $+ ne - (n-1) e = +e$. When the single electron is sufficiently far removed, the atomic core (1 nucleus, n–1 electrons) will act like a point charge + e. The single electron will then revolve in stationary orbits about this core, and various lines will be emitted as the electron drops from outer to inner orbits so that their frequencies will form series of the form $N \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ The innermost or normal orbit of the electron is that to which it falls in emitting the limit of the principal series. It is, therefore, designated by the symbol 1s; other orbits are indicated by the terms of the usual series notation[6]. To a second approximation, considering the residual repulsive actions of the n–1 electrons in the atomic core, the potential energy of the single electron will be $-\frac{e^2}{r}\left(1 - \frac{c_1}{r^2}\right)$ where c_1 is a constant and r is the radius of the orbit. This leads to frequencies connected by the well known Rydberg relation $\nu = N\left[\frac{1}{(n + \mu_1)^2} - \frac{1}{(m + \mu_2)^2}\right]$, where N is the universal Rydberg constant. In this way a satisfactory theory is obtained of the spectral emission of more complex atoms, which are shown to behave in a manner closely similar to the one electron atom.

Two consequences of this complete Bohr–Sommerfeld atomic theory must be noted, An *emission line spectrum* is given when the electron falls from outer orbits inwards, the loss of energy of the atomic system appearing as monochromatic radiation whose frequency is given by the quantum relation. On the other hand, an *absorptiom line spectrum* is given when the electron is lifted from within outwards by the absorption of radiation, whose frequency, from the quantum relation, corresponds to the necessary increase in energy of the atomic system. The principal series of an element will appear as absorption lines for undisturbed atoms since the electrons are in the 1 s orbit or normal orbit and the various lines are given by lifting the electron from this to outer orbits,—the 1p, 2p, 3p,

. . . . . . In order, however, for the subordinate series to appear as absorption, it is necesasry to disturb the atom in some manner so as to lift the electron to the 1p orbit when the sharp and diffuse series can appear, or to the 2d orbit, when the fundamental series can appear by the absorption of radiation. The classical example is sodium whose "cold" vapour shows 33 members of the principal series as absorption, but whose atoms must be electrically excited to show the subordinate series in absorption.

The second consequence of the Bohr-Sommerfeld atomic theory has to do with the ionization of atoms. When the atom permanently loses through ionization 1 electron, the charge on the atomic core becomes ne − (n − 2) e = + 2e. As a result the series lines will have their Rydberg constant multiplied by 4 in precisely the same way as the series constant for the ionized helium atom is 4N. If the atom permanently loses two electrons, the charge on the core will be + 3e and the series constant will be 9 N. Thus from the series relations in the line spectrum it will be possible to tell the state of ionization of the atom; once ionized atoms will give series with a constant 4N like the Pickering lines, twice ionized atoms will give series with a constant 9N, possibly like 4647 C + + and so on. Now Kossell and Sommerfeld[7] have shown that it is possible to predict one other thing about the spectrum of an ionized atom. Clearly when an atom has lost one electron, in the number of remaining electrons it will resemble the atoms of the element immediately preceding it in the periodic table. Kossel and Sommerfeld, therefore, predict that the spectrum of the once ionized atom or the once enhanced spectrum of an element will resemble the arc spectrum of the element immediately preceding it in the periodic table. This relation, called the Displacement Law, they were able to verify substantially at the time of announcement, and it has since received important additional support.[8]

Section B—Saha's Theory and the Abundance of Elements

In a series of four papers Dr. M. N. Saha[9] has recently brought forward a very remarkable and highly satisfactory theory of the solar and stellar spectra, which is based on the well established theories of atomic structure and spectral emission outlined above. The fundamental postulate of his theory is that the disappearance of the arc spectrum of an element in a star means that all the atoms are once ionized and that no neutral atoms are left, the disappearance of the first spark spectrum with constant 4N means that there are neither neutral nor once ionized atoms left and so on. In short the presence or absence of arc, spark and "super spark" spectra is an indicator of the degree of ionization of the atoms of that element. Saha treats this ionization or removal of one or more electrons as a reversible chemical reaction of the form, say for sodium, $Na = Na_{+} + e - U$, where a sodium atom is reversibly dissociated into an ionized sodium atom and a free electron by a certain amount of energy U. From the law of mass action, if x be the fraction of neutral atoms so dissociated, it follows that $x^2/(1-x^2) = K/P$ where K is a function of the temperature alone and P is the total pressure. The exact form of K Saha deduces from thermodynamical considerations and from the Nernst heat theorem, so that the following expression results, where the numerical values of the various entropy constants and specific heats have been inserted,

$$\log \frac{x^2}{1-x^2} \cdot P = -\frac{U}{4{\cdot}571T} + 2{\cdot}5 \log T - 6{\cdot}5 \quad \ldots\ldots\ldots \quad (1)$$

U, the heat of dissociation in calories per gram molecule, may be readily deduced from the ionization potential for a single atom. Thus the work required to dissociate 1 gram atom of sodium, whose ionization potential is 5·11 volts, is $1\cdot176 \times 10^5$ calories. Accordingly from equation (1) knowing the temperature and total pressure in the medium, the fraction x of once ionized atoms of elements of known ionization potential may be readily calculated. Saha has performed such computations, and in his papers he gives tables for x for various elements at different temperatures and pressures. Assuming the pressure in stellar chromospheres to lie between an atmosphere and one-tenth of an atmosphere, from the disappearance of the Ca line 4227, the Balmer lines, the Mg_+ line 4481 and so on, he is able to calculate stellar temperatures which are in good agreement with the values determined from spectrophotometric data.

In his physical theory of stellar spectra, as outlined above, Saha has not considered the question of the relative abundance of elements. This, it may, however, be readily shown is a factor of vital importance. Consider two elements of the same ionization potential, the one with n_1 atoms in the chromosphere, the other with n_2, where $n_2 > n_1$. Let n be the number of neutral atoms required for the arc spectra to be just on the point of appearing; to a first approximation n will be the same for both elements. Then the arc spectrum of the less abundant element will disappear when $n_1 (1 - x_1) = n$ and of the more abundant when $n_2 (1 - x_2) = n$. The more abundant will, therefore, disappear at the higher temperature where the difference between the fractions of once ionized atoms for the two elements is $x_2 - x_1 = \frac{n}{n_1}\left(1 - \frac{n_1}{n_2}\right)$. If n, the minimum number of atoms for an arc spectrum to be on the point of appearing, is comparable with n_1, then the difference between x_2 and x_1 will be directly dependent upon the relative abundance of the two elements and the more abundant element will act precisely as if it had a much higher ionization potential. If n, however, is small compared with n_1, then the difference $x_2 - x_1$ is numerically small, but the more abundant element will still act as if it had the higher ionization potential, and the temperatures at which the two arc spectra disappear will differ by probably as much as before. For note that if n/n_1 is a very small fraction then x_1 differs but little from unity and x_2 even less. But as the form of equation (1) shows, x increases to the value unity with increase of temperature asymptotically, so that, provided the values of x are sufficiently near unity, no matter how small numerically the difference $x_2 - x_1$ may be, it will still correspond to large differences in temperature. It is, therefore, evident that whether the minimum number of atoms for the bare appearance of a spectrum is large or small compared with the total number available, neglect of the relative abundance of elements is likely to lead upon occasion to grave incongruities.

The complete treatment of relative abundance is as follows. A given line in a spectrum will be on the point of disappearing when the number of atoms which absorb it is a certain minimum. This minimum number is determined by the wave length of the line and the temperature of the photosphere. The amount of energy of this wave length absorbed by a single atom may be determined from the quantum relation $\Delta W = h\nu$, using the frequency of the line under consideration. Thus when sodium vapour is absorbing D_2 radiation, each atom absorbs $3\cdot34 \times 10^{-12}$ ergs. If now the photosphere is radiating per sq. cm. per sec. 10^6 ergs of D_2 radiation (temperature about 5000° K.), then to completely absorb this radiant energy will require 3×10^{17} sodium atoms per sq. cm. column per sec. in the stellar chromosphere. If the photosphere is radiating 10^{9} ergs per sq. cm. per sec. (corresponding to 15,000° K), then 3×10^{19} sodium atoms per sq. cm. column per sec. are required for complete absorption. The number of atoms required for complete absorption is, therefore, a function of the temperature of the photosphere. For the line to be just on the point of appearing there must be a certain fraction of this number available. Wedge spectra secured here indicate that the ratio of intensity absorption line: continuous spectrum for the faintest lines is about 1 : 10. Then if 3×10^{17} sodium atoms give complete absorption, 9/10ths of that number will give a line which just barely appears.* Irrespective of the number required for complete absorption, some constant fraction k_1 of this number will give the number of atoms for which the line will barely appear. If b is the number of atoms for complete absorption (a number readily calculable from the wave length of the line and the temperature of the photosphere), then k_1 b is the number of atoms for the line to be just appearing where k_1 is some constant fraction.

Now this number k_1 b will be attained at different temperatures by elements of the same ionization potential depending upon their relative abundance. If a is the percentage abundance of the atoms of a given element and k_2 is the number of atoms per sq. cm. column for an element with percentage abundance unity, then k_2 a is the number of atoms of the element per sq. cm. column. At the temperature where $k_1 b = k_2 a (1 - x)$ or where K is a constant when

$$b = Ka (1 - x) \quad \ldots\ldots \quad (2)$$

then, and only then, will the given line of the given element disappear. In this equation b can be computed from the temperature of the photosphere by Wien's or Planck's law in the form $E\nu d\nu = \frac{2\pi h\nu^3}{c^2} e^{-h\nu/kT} d\nu$, where $d\nu$ has been taken uniformly throughout this paper to correspond to an increment of frequency of 50 kms. and k is the Boltzman gas constant. This gives the radiation in ergs per sq. cm. per sec. at a temperature T. The amount of energy of given wave length absorbed by a single atom is given by the quantum relation. Then the value of b is,--radiation of given wave length emitted per sq. cm. per sec. divided by energy of given wave length absorbed by a single atom. Of the value of a, the percentage abundance of different elements in the stellar chromospheres, nothing definite is known. In order, however, to arrive at some estimate of the relative

*An alternative and possibly a clearer mode of arriving at these results is as follows. The basic assumption is that when a line is just on the point of appearing or disappearing the ratio of intensity in the absortion line (I) to the intensity in the continuous spectrum (I_o) is a certain constant, viz $I/I_o = j$. Now if the ordinary laws of absorption held (i.e. Lambert's law where the fraction of light absorbed is independent of the original intensity), then for a certain fixed number of atoms $I/I_o = j$ would be true irrespective of the value of I_o, that is, irrespective of the photospheric temperature. But this is not the case. Each atom can only absorb a finite quantum of radiation $\triangle W = h\nu$ ergs. Thus if n atoms are enough to make an absorption line just appear for photospheric intensity I_o, then $I = I_o - n. \triangle W$ and $j = (I_o - n. \triangle W) / I_o$. If now I_o is increased by some factor α owing to a rise in temperature of the photosphere, then n will have to be increased to n' where $(\alpha I_o - n'. \triangle W) / \alpha I_o = j$ or $n'/n = \alpha$. In short the number of atoms necessary for a line to appear must increase in the same ratio as the photospheric radiation. Further if b is the number of atoms required for total absorption, then b. $\triangle W = I_o$ and $I/I_o = j$ becomes $(b - n)/b = j$ or $n = b (1 - j)$. That is, the number of atoms necessary for a line to be on the point of appearing is a function of the photospheric temperature and is also a constant fraction of the number required for complete absorption.

abundance of elements, it has been assumed that the distribution of elements in stellar atmospheres is roughly the same as it is in the first ten miles of the earth's crust. F. W. Clarke[10] from the means of numerous analyses has given and recently revised[11] the percentage abundance *by weight* of the thirty-one most abundant elements in the first ten miles of the earth's crust. What is here desired is the percentage abundance *by atoms*, which may be obtained from Clarke and Washington's[11] figures by dividing by the respective atomic weights and reducing to percentages. In Table 17 the thirty-one elements are arranged in order of abundance by atoms with the percentages by weight and by number of atoms following. It is assumed then as a first approximation to the truth that the value of a in equation (2), is the percentage abundance of atoms in the third column of Table 17. The value of x, the fraction of once ionized atoms is computed for a given temperature T from a modified form of Saha's equation (1). The modification is due to Russell[12], and is to take account of the electron products from atoms other than those of the element under consideration. The effective pressure P_1 is taken as 1 atmosphere and

TABLE 17—RELATIVE ABUNDANCE OF ELEMENTS IN EARTH'S CRUST

Element	% Weight	% Atoms	Element	% Weight	% Atoms	Element	% Weight	% Atoms
Oxygen	49·19	54·940	Chlorine	0·228	0·1149	Nickel	0·030	0·0091
Silicon	25·71	16·235	Phospherous	0·142	0·0818	Strontium	0·032	0·0065
Hydrogen	0·872	15·459	Sulphur	0·093	0·0518	Cerium, Yttrium	0·019	0·0030
Aluminium	7·50	4·946	Nitrogen	0·030	0·0383	Copper	0·010	0·0028
Sodium	2·61	2·028	Manganese	0·108	0·0351	Beryllium	0·001	0·0020
Calcium	3·37	1·503	Fluorine	0·030	0·0282	Boron	0·001	0·0016
Iron	4·68	1·485	Chromium	0·062	0·0213	Zinc	0·004	0·0011
Magnesium	1·94	1·426	Vanadium	0·038	0·0133	Cobalt	0·003	0·0009
Potassium	2·38	1·088	Lithium	0·005	0·0129	Lead	0·002	0·0002
Titanium	0·648	0·2407	Barium	0·075	0·0098			
Carbon	0·139	0·2069	Zirconium	0·048	0·0095			

the value of x computed for lithium for the various temperatures. The value of x for other elements with different ionization potentials is then calculated from Russell's[12] equation (6) viz., $\frac{x_1}{1 - x_1} = \frac{K_1}{K_2} \cdot \frac{x_2}{1 - x_2}$.

In equation (2) everything can now be computed except the value of K. In order to determine this constant, which physically means about 10/9ths of the number of atoms per sq. cm. column of an element of percentage abundance unity, it is sufficient to use the disappearance of an arc line of an element of known ionization potential in a star whose temperature has been spectrophotometrically determined. Russell[12] has shown that in the sun, temperature 6000° K, the line 6707·85 ($1\sigma - 1\pi$ of lithium whose I.P. is 5·362 volts[4]) has disappeared. Saha[9] is the authority for the statement that in B8 stars, average temperature Wilsing, Scheiner and Münch[13] 10,400° K, the line 4226·73 (1S — 1P of calcium whose I.P. is 6·087 volts[4]) has disappeared. Performing the necessary computations and noting that a for lithium from Table 17 is 0·0129 and x at 6000° K is 0·1646 there results from (2)

$$\frac{8 \cdot 913 \times 10^{6}}{2 \cdot 925 \times 10^{-12}} = K\ (0 \cdot 0129)\ (1 - 0 \cdot 1646) \quad \text{or } K = 2 \cdot 83 \times 10^{20}$$

Similarly for calcium whose a = 1·503 and whose x at 10,400° K is 0·8900

$$\frac{7 \cdot 697 \times 10^{7}}{4 \cdot 647 \times 10^{-12}} = K\ (1 \cdot 503)\ (1 - 0 \cdot 8900) \quad \text{or } K = 1 \cdot 00 \times 10^{20}$$

Of course theoretically K is a constant and the uncertainty in values here is due to the uncertainty in the actual abundance of elements in the stars. In what follows the value $K = 1 \cdot 5 \times 10^{20}$ has been adopted which gives more weight to calcium than to lithium. Equation (2), therefore, becomes

$$b = 1 \cdot 5 \times 10^{20}\, a\, (1 - x) \qquad \ldots\ldots (3)$$

An interesting and an important test of the value of these considerations on the relative abundance of elements is afforded by the behaviour of barium and sodium in the sun, a matter which has recently been discussed by Russell[14]. Barium with an I.P. of 5·188 volts[4] has the line 5535·53 (1S − 1P) absent or excessively faint in the sun and the line 3071·59 (1S − 2P) just present. Sodium, on the other hand, with the smaller I.P. of 5·111 volts[4] has the well known D lines ($1\sigma - 1\pi$) excessively strong in the sun. To account for this Russell [14] has advanced a very ingenious *ad hoc* hypothesis which requires the photospheric radiation to play the rôle of a partial ionizing agent, and assumes that recombination of a free electron and a once ionized atom is less likely to take place when the atom has its electron in an orbit other than 1S. Now if equation (3) be applied to the question and the value of a, the relative abundance of barium in the sun, be computed from the disappearance of 5535 and 3071, the following values result.

$$\frac{1 \cdot 034 \times 10^{7}}{3 \cdot 547 \times 10^{-12}} = 1 \cdot 5 \times 10^{20}\, a\, (1 - 0 \cdot 2159) \text{ or } a = 0 \cdot 0148 \text{ for disappearance of } 5535 \cdot 53.$$

$$\frac{2 \cdot 827 \times 10^{6}}{6 \cdot 397 \times 10^{-12}} = 1 \cdot 5 \times 10^{20}\, a\, (1 - 0 \cdot 2159) \text{ or } a = 0 \cdot 0038 \text{ for disappearance of } 3071 \cdot 59.$$

From Table 17 the relative abundance of barium on the earth is 0·0098. In short the disappearance of barium in the sun is precisely what would be expected when its probable relative abundance in the sun is considered. Similarly if the relative abundance of sodium, viz., a = 2·028 be taken from Table 17, and the temperature at which D_2 ought to disappear be computed, the following results are obtained:—

$$\text{At } 10{,}000^\circ \text{ K} : b = \frac{4 \cdot 944 \times 10^{7}}{3 \cdot 336 \times 10^{-12}} = 1 \cdot 482 \times 10^{19} : \text{R't side of (3)} = 1 \cdot 5 \times 10^{20} \times 2 \cdot 028\ (1 - 0 \cdot 9460) = 1 \cdot 643 \times 10^{19}$$

$$\text{At } 10{,}400^\circ \text{ K} : b = \frac{5 \cdot 459 \times 10^{7}}{3 \cdot 336 \times 10^{-12}} = 1 \cdot 636 \times 10^{19} : \text{R't side of (3)} = 1 \cdot 5 \times 10^{20} \times 2 \cdot 028\ (1 - 0 \cdot 9600) = 1 \cdot 217 \times 10^{19}$$

Accordingly if the relative abundance of sodium in the earth and in the sun is approximately the same, then the D lines should not disappear until the stellar type B8 is reached, and should, therefore, appear of great strength in the sun. In short the relative abundance of elements, which is a necessary extension of Saha's theory, accounts automatically for the behaviour of barium and sodium in the sun.* Furthermore it will account (see sec. 7) for other "anomalous" cases, such as the persistence in O-type stars of the Balmer lines after Mg+4481 and the He arc lines have disappeared, cases which Russell's hypothesis might have difficulty in meeting.

SECTION C—CAUSES OF IONIZATION AND THE ELECTRON COLLISION HYPOTHESIS

The validity of the conclusions drawn from Saha's theory as to the physical conditions in stars must rest in turn on the validity of the assumptions on which the theory is based. Three of these assumptions may be noted. (1) In his original presentation Saha effectively assumed that atoms of a given element could only recombine with free electrons dissociated from themselves. Milne[15] and Russell[12] pointed out this error and Russell[12] has derived modified formulæ (used in this paper) to take account of the free electrons from all the chromospheric atoms. (2) Saha has assumed that the rate of recombination of free electrons and ionized atoms upon encounter is independent of the temperature. However, it is probable that as the velocity of thermal agitation increases with the temperature so will the chances of recombination on encounter diminish. In fact it has been shown by Phillips[16] that the coefficient of recombination diminishes very rapidly with temperature. (3) Finally Saha has not indicated the physical processes by which ionization takes place, and has assumed that in some undefined manner the energy for the ionization is drawn from the medium. This question of the causes of ionization is of such importance as to merit further consideration.

Radiation from the photosphere and collisions with high velocity free electrons are two causes of ionization that must be at work in stellar atmospheres. The strong absorption in A-type stars at a probable temperature of 10,000° K of the Balmer lines, the mechanism of whose production is clearly understood, affords a not unfair test of the relative importance of these two causes. While, it is true that the resonance wave length of hydrogen lies in the far ultra violet (1216 A), yet in that respect the hydrogen atom does not differ from any of the once ionized atoms or from the neutral helium atom. To account for the strong Balmer lines in A-type stars is, therefore, a particularly appropriate test of the relative importance of these two causes of ionization in high temperature stars such as those comprised within the Harvard types A, B, O.

In order that a hydrogen atom may absorb lines of the Balmer series the electron must be lifted from its normal 1 quantum to the 2 quantum orbit. Absorption of photo-

*Prof. Russell has drawn my attention to the fact that there is in the case of barium a further unexplained anomaly. If the arc spectrum of barium disappears when the fraction of neutral atoms, 1–x, is 0.78 approximately, then the spark spectrum of barium will not appear since the fraction of once ionized atoms is only 0.22. But Russell has shown (Ap. J. *55*, 354) that the spark spectrum of barium is present in the sun. He has pointed out to me that if a pressure of 10^{-3} or 10^{-4} atmospheres (which recent investigations of his indicate) be assumed, then this anomaly will disappear. For with this pressure the fraction of neutral atoms will only be 0.02 while of once ionized atoms there will be 0.98. So that while the arc spectrum of barium will disappear at 6000°K if the relative abundance of the element is of the order of 0.007 the spark spectrum will be very conspicuous. It should be noted that this difficulty does not occur in the electron collision hypothesis developed in Sec. C. For this theory the fraction of neutral barium atoms is only 0.24 at 6000° K and of once ionized atoms 0.76. So that while the arc lines will disappear on this theory if the relative abundance is of the same order as on the earth, the spark lines will persist.

spheric radiation of the appropriate wave length (1216 A), or collision with a free electron with a velocity greater than 10·16 volts will lift the electron to the 2 quantum orbit. An upper limit to the number of H atoms brought into the 2 quantum condition by *radiation from the photosphere* can be readily calculated. The amount of energy radiated per sq. cm. per sec. by the photosphere at any given temperature and wave length can be computed from Wien's law $E\nu d\nu = \frac{2\pi h\nu^3}{c^2} e^{-h\nu/kT} d\nu$. As the hydrogen atoms are moving with their velocities of thermal agitation, they can absorb radiation in a wide range of frequencies; $d\nu$ has, therefore, been taken as varying from $7\cdot61 \times 10^{10}$ for Hα to $5\cdot33 \times 10^{11}$ for 938 A, corresponding to a range in atomic velocities of ± 25 km. In the accompanying schedule there are computed the total number of atoms which will have their electrons lifted to the 2, 3, 4, 5, 6, and from 7 to ∞ quantum orbits by the absorption of photospheric radiation. Of those atoms which have their electrons lifted to orbits higher than the 2 quantum only a small fraction will resume the 2 quantum state. Assuming, however, that all these atoms reach this state, then the sum of the numbers in the last column of the schedule gives an upper limit to the total number of H atoms with their electrons brought to the 2 quantum orbit by photospheric radiation. This upper limit is $1\cdot079 \times 10^{18}$ per sq. cm. column in the stellar atmosphere per sec. Assuming further that these atoms only absorb Hα, each atom of course absorbing from the quantum relation $2\cdot994 \times 10^{-12}$ ergs, then the total amount of Hα energy absorbed will be $3\cdot231 \times 10^6$ ergs per sq. cm. column per sec. But at 10,000° K each sq. cm. of the photosphere radiates per sec. of Hα radiation (± 25 km $= \pm 0\cdot55$ A) $4\cdot225 \times 10^7$ ergs. Accordingly under the most favourable conditions to photospheric radiation as the cause of ionization, the ratio of the intensities Hα absorption line: continuous spectrum will be 1 : 1·085. Wedge spectra of α Cygni taken here indicate that actually the Balmer absorption lines are less than one-thousandth of the intensity of the neighbouring continuous spectrum. It is evident, therefore, that radiation from the photosphere plays, in the production of the Balmer lines and probably of all enhanced lines in early type stars, an insignificant role.

Electron lifted from	Energy required per atom ergs	Radiation required to lift electron	Energy of this radiation in ergs per sq. cm. per sec.	No. of atoms partially ionized
1 – 2	$1\cdot617 \times 10^{-11}$	N $(1 - \frac{1}{4})$: 1216 A	$2\cdot156 \times 10^6$	$1\cdot333 \times 10^{17}$
1 – 3	1·916	N $(1 - \frac{1}{9})$: 1026	$4\cdot799 \times 10^5$	$2\cdot504 \times 10^{16}$
1 – 4	2·021	N $(1 - \frac{1}{16})$: 972	$2\cdot769 \times 10^5$	$1\cdot370 \times 10^{16}$
1 – 5	2·069	N $(1 - \frac{1}{25})$: 950	$2\cdot137 \times 10^5$	$1\cdot033 \times 10^{16}$
1 – 6	2·096	N $(1 - \frac{1}{36})$: 938	$1\cdot856 \times 10^5$	$8\cdot855 \times 10^{15}$
1 – 7 to 1 – ∞	$2\cdot111 \times 10^{-11}$	N $(1 - \frac{1}{49})$—N 931 to 911 A	$1\cdot875 \times 10^7$	$8\cdot882 \times 10^{17}$

On the other hand it may be shown that collisions with free electrons, moving with their velocities of thermal agitations, will produce the observed Balmer lines in A-type spectra. In order to arrive at any estimate of the amount of ionization produced by electron collisions, it is necessary to make some assumptions as to the probable concentrations of hydrogen atoms and electrons. From Richardson's [17] formula for thermionic emission $N = AT^{1/2}e^{-b/T}$ where N is number of electrons emitted per unit area per unit time at temperature T and the average values of $A = 5 \times 10^{25}$, $b = 5 \times 10^{4}$ are taken, the number of free electrons per cu. cm. at 10,000° K in the chromosphere should be 2×10^{18}. As at 6000° K the Zeeman effect[18] in sun spots requires between 10^{17} and 10^{18} free electrons per cu. cm., it is not unreasonable to suppose there are 10^{18} free electrons per cu. cm. at 10,000° K. Further assuming there are only 10^{13} hydrogen atoms per cu. cm. in the stellar atmosphere (less than there are in the earth's atmosphere at a height of 200 km), then it is possible to compute the number of collisions per cu. cm. per sec. between the free electrons and the hydrogen atoms. From the kinetic theory of gases[19] the number of collisions will be $n_1 \; n_2 \; S^2_{12} \sqrt{2\pi\,kT\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$ where n_1, n_2 are the number of free electrons and hydrogen atoms respectively, S_{12} is the distance between the centres of an electron and a hydrogen atom on collision ($0 \cdot 53 \times 10^{8}$ cms), k is the Boltzman gas constant, T the temperature and m_1, m_2 the masses of the free electron and hydrogen atom respectively. Accordingly at 10,000° K there will be $5 \cdot 56 \times 10^{22}$ collisions per cu. cm. per sec. between the 10^{18} electrons and 10^{13} hydrogen atoms. This is the minimum number because actually the charge on the electron will bring about impacts that otherwise would not take place. Now when a free electron is travelling with a velocity between $1 \cdot 895 \times 10^{8}$ cms. per sec. (10·16 volts) and $2 \cdot 063 \times 10^{8}$ cms. per sec. (12·04 volts) it will have sufficient kinetic energy to lift the hydrogen electron from the normal 1 quantum to the 2 quantum orbit. It is possible to compute from the kinetic theory of gases[20] the fraction of electrons at 10,000° K whose velocities of thermal agitation lie between these two limits. Its value is $2 \cdot 61 \times 10^{-5}$. Accordingly of the $5 \cdot 562 \times 10^{22}$ collisions per cu. cm. per sec. between electrons and hydrogen atoms, the fraction $2 \cdot 61 \times 10^{-5}$, or in all $1 \cdot 45 \times 10^{18}$, will take place between hydrogen atoms and these high velocity electrons. Even if only 1 in 10^{6} of these collisions is effective in lifting the hydrogen electron to the 2 quantum orbit, there will still be $1 \cdot 45 \times 10^{12}$ hydrogen atoms per cu. cm. per sec. in a condition to absorb the Balmer series. This will require of course that a number of these $1 \cdot 45 \times 10^{12}$ atoms have been partially or completely ionized several a second. As, however, recent work of Stark[21] and Wood[22] indicates that the time interval between partial ionization and return to the normal lies between 10^{-4} and 10^{-7} secs., this requirement offers no difficulty. In a sq. cm. column 1,000 km. high, in which the maximum chromospheric absorption may be supposed to take place, there will then be $1 \cdot 45 \times 10^{12} \times 10^{8} = 1 \cdot 45 \times 10^{20}$ hydrogen atoms in a condition to absorb the Balmer lines. It is to be noted here that the pressure of the chromosphere has been effectively assumed in computing the number of collisions to be 1/10th of an atmosphere. If the pressure is less than this while the number of effective collisions per cu. cm. will be smaller, yet the chromosphere will be correspondingly thicker so that an estimate of 10^{20} partially ionized hydrogen atoms per sq. cm. column per sec. is probably not far out. The number of atoms necessary for the complete absorption of a given line can be found by dividing the photospheric radiant energy of that wave length by the amount of energy absorbed by a single atom of that wave length. In this way at 10,000° K the numbers of H atoms per sq. cm. column per sec. with their electrons in the 2 quantum orbit required for complete absorption of the following lines are

$H\alpha$ $4{\cdot}225\times10^{7}/2{\cdot}994\times10^{-12}=1{\cdot}411\times10^{19}$: $H\beta$ $6{\cdot}137\times10^{7}/4{\cdot}042\times10^{-12}=1{\cdot}518\times10^{19}$
$H\gamma$ $6{\cdot}650\times10^{7}/4{\cdot}527\times10^{-12}=1{\cdot}469\times10^{19}$: $H\delta$ $6{\cdot}853\times10^{7}/4{\cdot}789\times10^{-12}=1{\cdot}431\times10^{19}$
$H\epsilon$ $6{\cdot}929\times10^{7}/4{\cdot}949\times10^{-12}=1{\cdot}400\times10^{19}$

The total number of atoms for *complete* absorption of the first five lines of the Balmer series, viz., $7{\cdot}23\times10^{19}$, is less than the number 10^{20} probably furnished by electron collisions, even when that cause of ionization is given a numerically unfavourable treatment. The contrast between the two causes of ionization, radiation from the photosphere and electron collisions, is thus most marked, and it is probably safe to conclude that electron collisions play an important role in the ionization of stellar atmospheres.

Milne[15] and Anderson[23] have drawn attention to the fact that the use of the law of mass action by Saha in formulating his ionization theory implicitly assumes that collisions with free electrons do not play any part in ionization. From the preceding discussion this implicit assumption does not appear to be justified. In view of the uncertainty of this assumption, and also of the assumption that the co-efficient of recombination does not vary with the temperature, it is evident that conclusions as to stellar temperatures and the relative abundance of elements drawn from Saha's theory must be treated with caution. Accordingly in order to act as a check on the results given by Saha's theory, an independent hypothesis of ionization by electron collisions based on three more or less plausible assumptions, has been formed.

The three assumptions, upon which this hypothesis of ionization by electron collisions rests, are as follows:—(1) Collisions of atoms with free electrons is the preponderating cause of ionization. (2) Ionization takes place on single impact; cumulative effects, as discussed by K. T. Compton[24], are of secondary importance. (3) Recombination of a once ionized atom and a free electron only takes place when the latter has a velociyt corresponding to less than twice the ionization potential. The only justification for the first assumption lies in the preceding discussion on the causes of ionization. The question of the relative importance of single impacts and cumulative effects, the subject of the second assumption, has been discussed with great care by Compton[24]. Ionization by single impact takes place when an electron travelling with the ionization potential velocity completely removes the atomic electron. Ionization by multiple impact takes place when the atomic electron is removed to the 1P orbit by a free electron with the resonance velocity and before the atom can resume its normal condition, a second impact with a free electron (velocity = ionization minus resonance velocity) completely removes the atomic electron. This form of cumulative effect Compton[25] has shown to be of negligeable importance. If, however, the atom resumes its 1 S state before a second impact can take place, then there will be emitted 1 quantum of monochromatic radiation. The absorption of this quantum of radiation by a neighbouring normal atom will partially ionize it. In this way by a process of radiation transfer Compton points out that there should be a large number of atoms in a partially ionized condition, so that impact with an electron with less than the ionization velocity will result in complete ionization of the atom. Under laboratory conditions, monochromatic gases at relatively high pressures and subjected to dense electron bombardments show very pronounced cumulative effects due to this process of radiation transfer. Thus an arc can be struck in helium and maintained at the resonance voltage[24]. It is doubtful, however, whether under stellar conditions such cumulative effects are of importance. On the one hand the number of high velocity electrons is so small as to be far removed from the conditions that laboratory experience has shown to be essential for cumulative effects. On the other hand the process of radiation transfer will be greatly hindered by the changes in frequency of the monochromatic radiation produced by the Doppler effects due to the high velocities of the thermal agitation of the atoms. Accordingly the assumption of ionization by single impacts is probably a close approximation to the truth. The third and final assumption has to do with the conditions under which recombination can take place,—a problem of considerable difficulty. Fixing attention for simplicity on the hydrogen atom, in order to remove the atomic electron from the 1 quantum orbit to infinity, the additional kinetic energy of $2\pi^2 m_0 e^4/h^2$ ergs (corresponding to the ionization potential) must be communicated to it, making the kinetic energy of the electron in all $4\pi^2 m_0 e^4/h^2$ (=potential energy=twice the ionization potential). If an electron bound to the atom can be removed to infinity when it has this kinetic energy (corresponding to twice the ionization potential), it is probable that a free electron which enters the atomic system with this kinetic energy or greater cannot be retained by the nucleus. Similar considerations apply to more complex atoms and there thus appears to be justification for assuming that recombination can only take place when the electron velocity corresponds to less than twice the ionization potential.

Let n_1 be the number of neutral atoms of the element under consideration per cu. cm., and n_2 the number of free electrons per cu. cm. in the chromosphere at the temperature T. The number of collisions per sec. per cu. cm. between these free electrons and atoms will be proportional to[19] $n_1 n_2 \sqrt{T/m_1}$, where m_1 is the atomic weight of the element. Of these collisions only those which take place between atoms and free electrons travelling with a velocity equal to or greater than the ionization potential, according to the second assumption, will be effective in producing once ionized atoms. At any temperature the fraction of the total number of electrons travelling with any velocity greater than that produced by a potential drop of I volts can be computed. Denoting this fraction by the symbol $E\left(\frac{I}{T}\right)$, its value is given by[20] $E\left(\frac{I}{T}\right) = \text{erf } z + 2z/\sqrt{\pi}\, e^{z^2}$ where $z = 108{\cdot}03\sqrt{\frac{I}{T}}$, and $\text{erf } z = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-z^2}\, dz$. Then where A_1 is some unknown constant, the number of ionized atoms produced by electron collisions per cu. cm. per sec. is

$$q_1 = A_1 n_1 n_2 E\left(\frac{I}{T}\right)\sqrt{T/m_1} \quad \ldots\ldots (4).$$

If n'_1 is the actual number of once ionized atoms present per cu. cm., A_2 is an unknown constant, then from the third assumption the number of recombinations taking place per cu. cm. per sec. will be

$$A_2 n'_1 n_2 \left[1 - E\left(\frac{2I}{T}\right)\right]\sqrt{T/m_1} \quad \ldots\ldots (5).$$

When a steady state is reached there will be as many atoms recombining as there are new once ionized atoms being formed so that q_1 will be equal to the expression (5). Accordingly n'_1/n, the ratio of once ionized to neutral atoms will be given by

$$\frac{n'_1}{n_1} = \frac{x}{1-x} = A\,E\left(\frac{I}{T}\right)\Big/\left[1 - E\left(\frac{2I}{T}\right)\right] \quad \ldots\ldots (6).$$

where x is the fraction of once ionized atoms and A is some unknown constant.

Equation (6), when A is known, thus will give the fraction of once ionized atoms for an element of known I.P. at a given temperature in precisely the same way as equation (1) gives the fraction of ionized atoms on Saha's theory. Again consideration of the relative abundance of elements is necessary and equation (2) $b = K a (1 - x)$ will apply where x in this case is given by (6) and K is an unknown constant. The quantities b and a are of course determined in precisely the same way as explained for Saha's theory. In order to ascertain the values of the constants A and K, the disappearance of lithium 6707·85 in the sun at 6000°K[12] and of calcium 4226·73 in B8 stars at 10,400° K[9] have been used.

Lithium $E\left(\frac{I}{T}\right) = \text{erf}\ 3{\cdot}230 + 1{\cdot}1284 \times 3{\cdot}230/e^{10{\cdot}433} = 1{\cdot}1230 \times 10^{-4}$; $E\left(\frac{2I}{T}\right) = 4{\cdot}582 \times 10^{-9}$

Substituting in (6) and (2) there results $\frac{8{\cdot}913 \times 10^{6}}{2{\cdot}925 \times 10^{-12}} = \frac{0{\cdot}0129\,K}{1 + 1{\cdot}1230 \times 10^{-4}\,A}$

Calcium $E\left(\frac{I}{T}\right) = \text{erf}\ 2{\cdot}613 + 1{\cdot}1284 \times 2{\cdot}613/e^{6{\cdot}828} = 3{\cdot}417 \times 10^{-3}$; $E\left(\frac{2I}{T}\right) = 5{\cdot}047 \times 10^{-6}$

Substituting in (6) and (2) there results $\frac{7{\cdot}697 \times 10^{7}}{4{\cdot}647 \times 10^{-12}} = \frac{1{\cdot}503K}{1 \times 3{\cdot}417 \times 10^{-3}A}$

From these final equations for lithium and calcium there results $A = 2{\cdot}0 \times 10^{4}$ and $K = 7{\cdot}7 \times 10^{20}$. Accordingly the equations for the fraction of ionization and for the disappearance of an arc spectrum on the electron collision hypothesis become

$$\frac{x}{1-x} = 2{\cdot}0 \times 10^{4}\,E\left(\frac{I}{T}\right)\Big/\left[1 - E\left(\frac{2I}{T}\right)\right] \ \ldots (7);\quad b = 7{\cdot}7 \times 10^{20}\,a\,(1 - x) \ \ldots (8)$$

The theory may be applied as was Saha's modified theory, to investigate the behaviour ot barium and sodium in the sun[14]. Inserting the necessary ionization potentials and temperatures in (7) and (8), it is found that the barium lines 5535·53 and 3071·59 will disappear at 6000° K if the percentage abundance of the element in the sun lies between 0·0155 and 0·0024. The limits found from Saha's theory were 0·0148 and 0·0038 (Sec. B, Appendix) and the relative abundance on the earth is from Table 17 a = 0·0098. Further it is found from the electron collision hypothesis that if the abundance of sodium is the same in the stars as it is on the earth, the D lines will not disappear until a temperature of 9600° K is reached. From Saha's theory the D lines under the same conditions would disappear at about 10,200° K. (Sec. B, Appendix). It will be noted that the electron collision hypothesis, equally with Saha's theory, is thus able to explain the apparently anomalous behaviour of these elements.

Summarizing, the principal results of this appendix have been the development of two methods, Saha's modified theory and the electron collision hypothesis, by the use of which some of the physical conditions in the star can be determined from the disappearance of an *arc* spectrum of known ionization potential. It should be noted in conclusion hat the methods are equally applicable and can readily be extended to *spark spectra.*

For Saha's theory Russell[12] has given the correct formula for computing the ratio of twice ionized to once ionized atoms, namely $\frac{y}{x} = \frac{K'}{K_2} \cdot \frac{x_2}{1 - x_2}$ where $\log K = - 5036\ I/T + 2 \cdot 5 \log T - 6 \cdot 5$ and the subscript 2 refers in this paper to lithium with "effective pressure" P_1 taken as one atmosphere. For the electron collision hypothesis where n_1' is the number of once ionized, n_1'' the number of twice ionized atoms present per cu. cm. then in the same manner as was shown before $\frac{n_1''}{n_1'} = \frac{y}{x} = A' E\left(\frac{I'}{T}\right) / \left[1 - E\left(\frac{2I'}{T}\right)\right]$. The value of A' may be readily deduced from the value of A for arc spectra when the effect of the charges born by the free electrons and the once or twice ionized atoms in increasing the number of collisions is considered. A simple manner of treating the matter is to suppose that the charge acts as if it increased the radius of the sphere of influence of the charged particle. Then the effective radius for an electron and a once ionized atom will be βe, for a twice ionized atom $2\beta e$ and the radius of an uncharged atom is assumed to be negligeably small. Recalling that the distance between the centres of particles on collision enters as a square[19], it may then be immediately shown that $A' = (16/9)\ A = 3 \cdot 6 \times 10^4$. The resulting expression is then

$$\frac{y}{x} = 3 \cdot 6 \times 10^4\ E\left(\frac{I'}{T}\right) / \left[1 - E\left(\frac{2I'}{T}\right)\right] \quad \ldots . (9).$$

As the temperature becomes higher, the fraction of neutral atoms becomes vanishingly small and for either Saha's theory or the electron collision hypothesis $x + y = 1 - \epsilon$ where ϵ is a second order quantity. Accordingly for the disappearance of a once enhanced spectrum y/x may be written $y/(1 - y)$. From the values of y thus computed, for the Saha theory the abundance formula $b = 1 \cdot 5 \times 10^{20}\ a\ (1 - y)$ and for the electron collision hypothesis $b = 7 \cdot 7 \times 10^{20}\ a(1 - y)$ may be used to determine the temperature at which a spark spectrum will disappear.

In short the two methods are applicable equally to arc or spark spectra. They give from the disappearance of a spectrum of known series relations and ionization potential the *temperature* of the star if the abundance of the element is known, or the *abundance* of the element if the stellar temperature is known. As the two methods are based on completely different assumptions, it is probable that the true temperature or abundance will be somewhere near their numerical estimates. Interesting examples of their application and of the accordance of the results which they give will be found in Sec. 7,—"Physical Conditions in O-type Stars."

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END OF VOLUME I.