# STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

## SEVENTH PAPER: THE DISTANCES, DISTRIBUTION IN SPACE, AND DIMENSIONS OF 69 GLOBULAR CLUSTERS

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### I. PARALLAXES FROM VARIABLE STARS, APPARENT MAGNITUDES, AND ANGULAR DIAMETERS

Applying the methods discussed in the preceding *Contribution*,[2] the parallaxes of a few clusters are obtained directly from the periods and magnitudes of Cepheid variables; the parallaxes of a considerably larger number are derived from the mean magnitudes of the brightest cluster stars, and the survey is then made complete through measures of diameters of the photographic images of all globular systems. In Table I are listed the clusters for which the variable stars have received a sufficiently detailed discussion to permit a determination of the parallax directly from the luminosity-period curve of Cepheid variation. The apparent diameters and the adopted parallaxes are taken from tables appearing on following pages; the method of weighting the results is also subsequently described.

The parallaxes for Messier 3, 5, 15, and 22, depending almost entirely on the median magnitudes of numerous variable stars, are the most accurate. The computed probable error of the absolute magnitude is ±0.2; that of the apparent magnitude is estimated to vary from ±0.05 for Messier 3 to possibly ±0.2 for Messier 22. The corresponding limits of the probable error of the parallaxes are ±0″000007 and ±0″000015, that is, 10 and 13 per cent, respectively.

For most other clusters, of course, the errors are somewhat greater, particularly for those where the parallax depends solely upon either the magnitudes of the brightest stars or the measured

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 152.

[2] *Mt. Wilson Contr.*, No. 151.

diameter on photographic charts. For the former the estimated average probable error is 20 per cent, for the latter 25 per cent.[1] When two or three sources are available, as in Tables I and V, the errors are less; but in Table I the different parallaxes for each cluster are not all completely independent, as they were used in part to determine the reduction constants and curves. The residuals in the last column of Table I are expressed in millionths of a second of arc, and their minuteness indicates the validity of the methods involving diameters and the magnitudes of the bright stars.

TABLE I

COMPARISON OF CLUSTER PARALLAXES FROM VARIABLES, MAGNITUDES, AND DIAMETERS

| DESIGNATION | | APPARENT DIAMETER | PARALLAX (UNIT IS 0".000001) | | | | RESIDUALS |
|---|---|---|---|---|---|---|---|
| N.G.C. | Messier | | Adopted | From Variables | From Magnitudes | From Diameters | |
| 5272 | 3 | 7'.0 | 72 | 72 | 71 | 72 | 0, −1, 0 |
| 5904 | 5 | 8.6 | 80 | 80 | 80 | 81 | 0, 0, +1 |
| 6205 | 13 | 10.6 | 90 | 82: | 89 | 91 | −8:, −1, +1 |
| 6656 | 22 | 16.0 | 118 | 116 | 121 | 116 | −2, +3, −2 |
| 7078 | 15 | 5.0 | 68 | 67 | 69 | 59 | −1, +1, −9 |
| 7089 | 2 | 7.0 | 64 | 65 | 60 | 72 | +1, −4, +8 |
| 5139 | ........ | 30 | 153 | 150 | 170: | 155 | −3, +17:, +2 |
| Small Magellanic Cloud | | ........ | 52 | 52 | ........ | ........ | .............. |

The parallax of the Small Magellanic Cloud, which is given at the end of Table I, is relatively uncertain, for the value from variables can be checked by neither diameter measures nor maximum luminosities, and the zero-point error in the provisional magnitude scale used by Miss Leavitt is unknown.[2]

[1] These estimates appear to be safely conservative. After the smoothing operation, described on a later page, much smaller average errors are obtained for the final parallaxes in Tables V and VIII; thus, including the 10 per cent probable error in the parallaxes due to uncertainty of the zero-point of the absolute scale, the average probable error for all clusters is estimated to be less than 15 per cent. Cf. Table VII.

[2] See sec. III of the preceding *Contribution*. Previous values for the parallax of the Small Magellanic Cloud are: Hertzsprung, $\pi = 0''.0001$, *Astronomische Nachrichten*, **196**, 204, 1913; Kapteyn, $\pi = 0''.00004$, *Mt. Wilson Contr.*, No. 82, 71, 1914; Shapley, $\pi = 0''.00006$, *Mt. Wilson Contr.*, No. 116, 82, 1915. The new value in Table I is probably an improvement over the others because it allows for diversity in color of the variable stars and is based upon more definite knowledge of their absolute magnitudes.

In the preceding paper we found from the study of the bright stars and variables in several clusters that, after excluding the five brightest, the mean absolute photographic magnitude of the 25 most luminous objects is

$$M_{25} = -1.51 \pm 0.28.$$

Adopting this value, we have derived from measures of apparent magnitude the distances of practically all clusters north of declination $-30^{\circ}$, the southern limit reached with the 60-inch reflector.

Of 300 cluster photographs taken during the last three years as a part of the program, about 175 have been measured for the magnitudes of either the bright stars or the variables; but the material is too extensive to describe in detail. Nearly all of the photographs were made with full aperture and on Seed 27 plates of various emulsions. The exposures vary in length from 10 seconds to 2 hours, but are mostly between 1 and 12 minutes in duration. For the clusters south of $-20^{\circ}$ the altitude was frequently so low that plates with first-class images could not be secured. Mr. Hoge has assisted with all the observational work.

A summary of the work on each cluster is given in Table II. The designation in the first column is followed in the second with numbers indicating the total number of plates used in all phases of the work and those used in the derivation of the mean magnitude. For the latter purpose the plates, with few exceptions, involve direct polar comparisons on two or more nights; the images of between 50 and 100 cluster stars and of between 20 and 50 Polar Standards were measured at least twice on each plate; and the measures were corrected as usual for scale, distance from center, and differential atmospheric extinction. As the measures, reductions, and discussion cannot well be given for the individual clusters, the method of work is illustrated merely with a summary of the final magnitudes for the bright stars in Messier 2 (Table III) and Messier 53 (Table IV).

The radius of the concentric region in which all bright stars were measured, given in the third column of Table II, does not closely indicate the apparent size of the cluster either actually

## TABLE II

SUMMARY OF OBSERVATIONS

| N.G.C. | No. Plates | Photographic Magnitude 25 Stars | | | | Weight | Angular Diameter | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Radius | Mean | Av. Dev. | Extremes | | Melotte | Davis | Shapley | Mean |
| 288 | 2, 2 | 4′ | 14.81 | ±0.16 | 14.38–15.04 | *d* | 12′ | 4′.0 | 4′.4 | 4′.2 |
| 1904 | 3, 2 | 2 | 15.29 | 0.13 | 15.01–15.72 | *d* | 4.5 | 2.4 | 2.2 | 2.3 |
| 4147 | 4, 2 | 1.5 | 16.58 | 0.18 | 16.23–16.93 | *d* | 1 | 0.8 | 0.8 | 0.8 |
| 5024 | 2, 2 | 5 | 15.07 | 0.09 | 14.94–15.26 | *b* | 5 | 5.6 | 5.4 | 5.5 |
| 5139 | 3, 2 | ......... | (12.3:) | ......... | ............... | *d* | 45 | 32 | 28 | 30 |
| 5272 | 65, 4 | 9 | 14.23 | 0.13 | 13.92–14.45 | *a* | 18 | 5.8 | 8.3 | 7.0 |
| 5904 | 3, 3 | 4 | 13.97 | 0.15 | 13.74–14.27 | *a* | 15 | 9.1 | 8.2 | 8.6 |
| 6093 | 2, 2 | 3 | 14.88 | 0.08 | 14.72–15.09 | *d* | 5 | 4.0 | 3.0 | 3.5 |
| 6121 | 3, 2 | 6 | 13.84 | 0.35 | 13.18–14.40 | *c* | 20 | 10.0 | 11.0 | 10.5 |
| 6205 | 15, 4 | 6 | 13.75 | 0.11 | 13.45–13.92 | *b* | 12 | 10.2 | 11.0 | 10.6 |
| 6218 | 2, 2 | 6 | 13.97 | 0.22 | 13.56–14.31 | *b* | 9 | 8.8 | 8.8 | 8.8 |
| 6229 | 3, 2 | 2.5 | 16.18 | 0.13 | 15.90–16.37 | *d* | ......... | 1.0 | 1.2 | 1.1 |
| 6254 | 2, 2 | 5 | 14.06 | 0.24 | 13.35–14.38 | *c* | 10 | 10.9 | 10.0 | 10.4 |
| 6333 | 2, 2 | 4 | 15.61 | 0.16 | 15.08–15.88 | *b* | 3 | 3.0 | 3.5 | 3.2 |
| 6341 | 3, 3 | 7 | 13.86 | 0.13 | 13.60–14.16 | *c* | 8 | 8.4 | 7.6 | 8.0 |
| 6356 | 2, 2 | 1.5 | 17.16 | 0.14 | 16.86–17.44 | *d* | 1.5 | 2.0 | 1.8 | 1.9 |
| 6402 | 2, 2 | 5 | 15.44 | 0.24 | 14.85–15.86 | *b* | 6 | 3.1 | 3.8 | 3.4 |
| 6626 | 3, 3 | 3 | 14.87 | 0.16 | 14.49–15.11 | *b* | 5 | 5.2 | 4.2 | 4.7 |
| 6638 | 2, 2 | 2 | 16.22 | 0.19 | 15.90–16.60 | *b* | 1.5 | 1.6 | 1.8 | 1.7 |
| 6642* | 3, 2 | 1 | 16.07 | 0.26 | 15.51–16.46 | *c* | 1 | 1.2 | 1.2 | 1.2 |
| 6656 | 6, 3 | 5.5 | 13.08 | 0.19 | 12.58–13.55 | *a* | 20 | 14.5 | 17.5 | 16.0 |
| 6712 | 5, 2 | 1.5 | 16.10 | 0.19 | 15.65–16.36 | *d* | 2.5 | 2.2 | 2.0 | 2.1 |
| 6779 | 6, 2 | 2 | 15.31 | 0.20 | 14.98–15.70 | *c* | 1.5 | 2.2 | 2.6 | 2.4 |
| 6864 | 2, 2 | 1 | 17.06 | 0.13 | 16.76–17.35 | *c* | 2 | 1.6 | 1.5 | 1.6 |
| 6934 | 5, 3 | 3 | 15.78 | 0.19 | 15.33–16.11 | *c* | 1.5 | 1.4 | 1.5 | 1.4 |
| 6981 | 7, 2 | 3 | 15.92 | 0.17 | 15.53–16.20 | *b* | 2 | 2.2 | 2.4 | 2.3 |
| 7078 | 7, 3 | 6 | 14.31 | 0.11 | 14.13–14.55 | *a* | 6 | 4.8 | 5.2 | 5.0 |
| 7089 | 7, 3 | 4 | 14.61 | 0.09 | 14.25–14.76 | *b* | 8 | 7.3 | 6.8 | 7.0 |
| 7099 | 4, 3 | 2.5 | 14.63 | ±0.27 | 13.77–15.04 | *c* | 6 | 4.6 | 4.7 | 4.6 |

* See n. 1, p. 160.

or relatively. The basis of its choice has been described in the fifth section of the preceding paper.

TABLE III

MAGNITUDES OF BRIGHT STARS IN MESSIER 2 (N.G.C. 7089)

| STAR* | PHOTOGRAPHIC MAGNITUDES | | | | RESIDUALS† | DEVIATION |
|---|---|---|---|---|---|---|
| | 3883P | 3902P | 3904P | Means | | |
| 1 | 14.09 | 14.16 | 14.04 | Bright | ........ | ........ |
| 2 | 14.68 | 14.59 | 14.53 | 14.60 | + 8, − 1, − 7 | 0 |
| 3 | 14.65 | 14.64 | 14.70 | 14.66 | − 1, − 2, + 4 | + 6 |
| 4 | 13.54 | 13.57 | 13.71 | Bright | ........ | ........ |
| 5 | 14.87 | 14.69 | 14.62 | 14.73 | +14, − 4, −11 | +13 |
| 6 | 14.77 | 14.69 | 14.65 | 14.70 | + 7, − 1, − 5 | +10 |
| 7 | 14.73 | 14.69 | 14.65 | 14.69 | + 4, 0, − 4 | + 9 |
| 10 | 14.60 | 14.49 | 14.51 | 14.53 | + 7, − 4, − 2 | − 7 |
| 11 | 14.80 | 14.72 | 14.70 | 14.74 | + 6, − 2, − 4 | +14 |
| 13 | 14.68 | 14.69 | 14.57 | 14.65 | + 3, + 4, − 8 | + 5 |
| 14 | 14.65 | 14.57 | 14.61 | 14.61 | + 4, − 4, 0 | + 1 |
| 16 | 14.68 | 14.66 | 14.57 | 14.64 | + 4, + 2, − 7 | + 4 |
| 17 | 13.61 | 14.19 | 13.92 | Bright | ........ | ........ |
| 18 | 14.34 | 14.56 | 14.31 | 14.40 | − 6, +16, − 9 | −20 |
| 19 | 14.29 | Contact‡ | 14.17 | 14.23 | + 6, ...., − 6 | −37 |
| 20 | 13.54 | 13.78 | 13.70 | Bright | ........ | ........ |
| 21 | 14.68 | 14.69 | 14.57 | 14.65 | + 3, + 4, − 8 | + 5 |
| 23 | 14.77 | 14.69 | 14.57 | 14.68 | + 9, + 1, −11 | + 8 |
| 25 | 14.57 | 14.54 | 14.57 | 14.56 | + 1, − 2, + 1 | − 4 |
| 26 | 14.65 | 14.72 | 14.57 | 14.65 | 0, + 7, − 8 | + 5 |
| 27 | 13.60 | 14.02 | 13.99 | Bright | ........ | ........ |
| 28 | 14.33 | 14.40 | 14.23 | 14.32 | + 1, + 8, − 9 | −28 |
| 29 | 14.73 | 14.85 | 14.70 | 14.76 | − 3, + 9, − 6 | +16 |
| 30 | 14.57 | 14.72 | 14.46 | 14.58 | − 1, +14, −12 | − 2 |
| 31 | 14.73 | 14.72 | 14.57 | 14.67 | + 6, + 5, −10 | + 7 |
| 33 | 14.52 | 14.69 | 14.40 | 14.54 | − 2, +15, −14 | − 6 |
| 36 | 14.65 | 14.69 | 14.46 | 14.60 | + 5, + 9, −14 | 0 |
| 39 | 14.73 | 14.66 | 14.72 | 14.70 | + 3, − 4, + 2 | +10 |
| 40 | 14.73 | 14.66 | 14.53 | 14.64 | + 9, + 2, −11 | + 4 |
| 43 | 14.65 | 14.59 | 14.49 | 14.58 | + 7, + 1, − 9 | − 2 |
| Mean | | | | 14.60 | Mean | ±0.09 |

* Stars fainter than the brightest thirty are omitted.
† Residuals and deviations from mean expressed in hundredths of a magnitude.
‡ Cluster star in contact with a Polar Standard.

For the average cluster the mean value of the photographic magnitude, in the fourth column, depends upon about 500 measures. Its estimated probable error varies from two- to four-tenths of a magnitude, the principal uncertainty coming from possible non-homogeneity of the clusters and from the error in choosing the area to be measured. The average deviation shows the dispersion of the magnitudes entering the mean, but gives little indication of the certainty of the result; the extremes also show the dispersion, and the upper limit records the highest luminosity of the individual stars.

TABLE IV

Magnitudes of Bright Stars in Messier 53 (N.G.C. 5024)

| Star* | Photographic Magnitudes | | | Residuals† | Deviation |
|---|---|---|---|---|---|
| | 2357P | 2568P | Means | | |
| 2 | 15.03 | 14.98 | 15.00 | + 3, − 2 | − 8 |
| 4 | 15.25 | 15.21 | 15.23 | + 2, − 2 | +15 |
| 6 | 14.99 | 15.01 | 15.00 | − 1, + 1 | − 8 |
| 7 | 14.97 | 14.93 | 14.95 | + 2, − 2 | −12 |
| 9 | 14.95 | 15.10 | 15.02 | − 7, + 8 | − 6 |
| 11 | 15.17 | 15.26 | 15.22 | − 5, + 4 | +14 |
| 12 | 15.10 | 15.26 | 15.18 | − 8, + 8 | +10 |
| 13 | 15.00 | 14.93 | 14.96 | + 4, − 3 | −12 |
| 15 | 15.03 | 14.91 | 14.97 | + 6, − 6 | −11 |
| 16 | 15.03 | 15.21 | 15.12 | − 9, + 9 | + 4 |
| 17 | 15.03 | 14.88 | Bright | ........... | ........... |
| 18 | 14.50 | 14.57 | Bright | ........... | ........... |
| 20 | 15.27 | 15.21 | 15.24 | + 3, − 3 | +16 |
| 22 | 15.15 | 14.93 | 15.04 | +11, −11 | − 4 |
| 24 | 14.87 | 14.57 | Bright | ........... | ........... |
| 26 | 15.05 | 15.01 | 15.03 | + 2, − 2 | − 5 |
| 27 | 15.05 | 14.83 | Bright | ........... | ........... |
| 28 | 12.90 | 13.18 | Bright | ........... | ........... |
| 29 | 15.28 | 15.24 | 15.26 | + 2, − 2 | +18 |
| 30 | 14.95 | 15.10 | 15.02 | − 7, + 8 | − 6 |
| 31 | 15.03 | 15.07 | 15.05 | − 2, + 2 | − 3 |
| 32 | 15.30 | 15.14 | 15.22 | + 8, − 8 | +14 |
| 33 | 15.05 | 14.93 | 14.99 | + 6, − 6 | − 9 |
| 34 | 15.15 | 15.33 | 15.24 | − 9, + 9 | +16 |
| 35 | 14.94 | 15.03 | 14.98 | − 4, + 5 | −10 |
| 36 | 15.05 | 15.10 | 15.08 | − 3, + 2 | + 0 |
| 37 | 15.05 | 15.13 | 15.09 | − 4, + 4 | + 1 |
| 38 | 15.05 | 15.03 | 15.04 | + 1, − 1 | − 4 |
| 42 | 14.94 | 15.01 | 14.98 | − 4, + 3 | −10 |
| 45 | 14.94 | 15.04 | 14.99 | − 5, + 5 | − 9 |
| Mean | | | 15.08 | Mean | ±0.09 |

* Stars fainter than the brightest thirty are omitted.

† Residuals and deviations from mean expressed in hundredths of a magnitude.

Adding the difference between the brighter extreme and the mean magnitude to the adopted absolute value of the latter, we get the maximum brightness in each cluster. Thus we find that the highest photographic luminosity never exceeds magnitude −2.5 (unless some of the excluded five are actually cluster stars), and the maximum usually falls slightly below −2. Contrary to expectation, the mean magnitude is nearer the brighter extreme in nearly one-third of the clusters, suggesting that among the giants the number of stars does not always increase regularly with decreasing luminosity.

In the seventh column of Table II weights are assigned each cluster on the basis of the quality of the plates, their number, the character of the surrounding stellar field, the certainty of the result for the mean magnitude, and other factors. The remaining columns are described later. Except to remark that the plates for N.G.C. 5139 ($\omega$ Centauri) were made with the 10-inch refractor and (because of the very low altitude) are of little value except as a check on Bailey's magnitudes, further space will not be taken for the extensive notes compiled relative to the peculiarities of individual clusters, the observations, the measures, and the investigations of errors.

The equatorial and galactic co-ordinates, the parallaxes, and the co-ordinates in space of the 28 globular clusters[1] for which magnitudes have been measured appear in Table V. For a few clusters the values of the parallax in the sixth column are taken from Table I and for the remainder are computed directly from the mean magnitudes of Table II.

If we plot the parallaxes derived from magnitudes against the diameters of the clusters, as recorded by Melotte,[2] a very definite progression of size with parallax is apparent. Melotte's estimates were made directly from the original Franklin-Adams chart plates; but there is no record of what accuracy was sought or what homogeneity may be expected in the results. Accordingly the diameters have been redetermined from the photographic copies of the plates with the special purpose in view of obtaining results as nearly comparable as possible for all parts of the sky. Measures of the diameters of the images were made independently by two observers, using a finely divided scale under low magnification. By estimating the

[1] N.G.C. 6642 is retained in the table as a twenty-ninth entry. Melotte, with some doubt, classifies it as a globular cluster; Bailey thinks that its stars, few in number, are involved in nebulosity. Mount Wilson plates show a few stars closely crowded, but almost certainly not forming a typical globular cluster. The group is in a rich galactic field.

[2] *Memoirs of the Royal Astronomical Society*, **60**, Part 5, 1915.

## TABLE V

PARALLAXES OF 28 GLOBULAR CLUSTERS FROM MAGNITUDES AND DIAMETERS

| Designation | | Position in 1900 | | Galactic | | Parallax (Unit Is 0″.000001) | | | Distance (Unit Is 100 Parsecs) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N.G.C. | Messier | R.A. | Decl. | Long. | Lat. | From Mag. | From Diam. | Adopted | Radial | Projected on Galactic Plane | From Galactic Plane |
| 288 | ........ | $0^h 47^m.8$ | −27° 8′ | 214° | −88° | 55 | 52 | 53 | 189 | 7 | −189 |
| 1904 | 79 | 5 20.1 | −24 37 | 195 | −28 | 44 | 36 | 39 | 256 | 226 | −120 |
| 4147 | ........ | 12 5.0 | +19 6 | 227 | +78 | 24 | 17 | 19 | 526 | 109 | +514 |
| 5024 | 53 | 13 8.0 | +18 42 | 307 | +79 | 48 | 62 | 53 | 189 | 36 | +186 |
| 5139 | ........ | 13 20.8 | −46 47 | 277 | +16 | 150 | 155 | 153 | 65 | 62 | + 18 |
| 5272 | 3 | 13 37.6 | +28 53 | 8 | +77 | 72 | 72 | 72 | 139 | 31 | +135 |
| 5904 | 5 | 15 13.5 | + 2 27 | 333 | +45 | 80 | 81 | 80 | 125 | 88 | + 88 |
| 6093 | 80 | 16 11.1 | −22 44 | 320 | +18 | 53 | 48 | 50 | 200 | 190 | + 62 |
| 6121 | 4 | 16 17.5 | −26 17 | 319 | +15 | 85 | 90 | 88 | 114 | 110 | + 30 |
| 6205 | 13 | 16 38.1 | +36 39 | 26 | +40 | 89 | 91 | 90 | 111 | 85 | + 71 |
| 6218 | 12 | 16 42.0 | − 1 46 | 344 | +25 | 80 | 82 | 81 | 123 | 111 | + 52 |
| 6229 | ........ | 16 44.2 | +47 42 | 41 | +39 | 29 | 20 | 23 | 435 | 338 | +274 |
| 6254 | 10 | 16 51.9 | − 3 57 | 343 | +22 | 77 | 89 | 83 | 120 | 111 | + 45 |
| 6333 | 9 | 17 13.3 | −18 25 | 334 | + 9 | 38 | 44 | 40 | 250 | 247 | + 39 |
| 6341 | 92 | 17 14.1 | +43 15 | 35 | +34 | 84 | 78 | 81 | 123 | 102 | + 69 |
| 6356 | ........ | 17 17.8 | −17 43 | 335 | + 9 | 19 | 30 | 26 | 385 | 380 | + 60 |
| 6402 | 14 | 17 32.4 | − 3 11 | 349 | +14 | 41 | 46 | 43 | 233 | 226 | + 56 |
| 6626 | 28 | 18 18.4 | −24 55 | 336 | − 7 | 53 | 57 | 54 | 185 | 184 | − 23 |
| 6638 | ........ | 18 24.8 | −25 34 | 335 | − 8 | 29 | 28 | 29 | 345 | 342 | − 48 |
| 6642* | ........ | 18 25.8 | −23 32 | 337 | − 7 | 31 | 22 | 26 | 385 | 382 | − 47 |
| 6656 | 22 | 18 30.3 | −23 59 | 338 | − 9 | 118 | 116 | 118 | 85 | 84 | − 13 |
| 6712 | ........ | 18 47.6 | − 8 50 | 353 | − 6 | 30 | 33 | 32 | 312 | 310 | − 32 |
| 6779 | 56 | 19 12.7 | +30 0 | 30 | + 7 | 43 | 37 | 40 | 250 | 248 | + 30 |
| 6864 | 75 | 20 0.2 | −22 12 | 348 | −28 | 19 | 26 | 22 | 455 | 401 | −213 |
| 6934 | ........ | 20 29.3 | + 7 4 | 20 | −20 | 35 | 24 | 30 | 333 | 313 | −114 |
| 6981 | 72 | 20 48.0 | −12 55 | 3 | −34 | 33 | 35 | 34 | 294 | 244 | −164 |
| 7078 | 15 | 21 25.2 | +11 44 | 33 | −29 | 68 | 59 | 68 | 147 | 129 | − 71 |
| 7089 | 2 | 21 28.3 | − 1 16 | 22 | −37 | 60 | 72 | 64 | 156 | 125 | − 94 |
| 7099 | 30 | 21 34.7 | −23 38 | 356 | −48 | 59 | 56 | 58 | 172 | 115 | −128 |

*See n. 1 on p. 160.

elongation of similarly situated bright stars corrections have been made for distortion when a cluster is near the edge of the chart;[1] and to counteract the frequently observed actual ellipticity of figure the mean of the diameters in several directions has been determined for all clusters. The last three columns of Table II contain these closely agreeing measures and their mean, thus affording a fairly homogeneous record of relative apparent dimensions. The diameters estimated from the original plates are usually larger, as would be expected, but even they fall short of the actual dimensions for many clusters.

Plotting the adopted diameters of the last column of Table II against the parallaxes derived from magnitudes in Table V, we obtain the curve in Fig. 1—a curve that demands and apparently justifies the hypothesis that all globular clusters are of nearly the same linear dimensions. There is no reason for supposing that the more distant clusters are actually and systematically smaller than nearer ones; hence, as is reasonable a priori, the non-linear form of this curve may be attributed to the characteristic distribution of luminosity in a cluster and its resulting effect on photographic reproduction. Normal points for the data underlying the figure are given in Table VI and are plotted as black circles.[2] The lower part of the curve is somewhat uncertain, depending only on Messier 22 and $\omega$ Centauri, but the uncertainty is not important, for only two or three clusters are large enough to make their parallaxes depend on that part of the curve.

Undoubtedly an improvement can be made in at least some of the parallaxes through the process of smoothing with the aid of the

[1] Many of the clusters appear on two or more charts. The differences in quality from plate to plate, which may be greater for the charts, are not nearly so effective in measures of angular diameter as they would be in estimates of magnitudes. Fortunately about 90 per cent of the clusters are on the Johannesburg plates, which attain a fainter limit of magnitude and are more uniform than those made for the northern sky at Mervel Hill (*Memoirs of the Royal Astronomical Society*, **60**, 167, 1914).

[2] Half weight is given to N.G.C. 6642; also to N.G.C. 6712 because of a slight doubt as to its nature and because it is in such a rich region of the galactic clouds that the measured diameter is a little uncertain. Cf. *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Possibly a doubt should also be expressed as to the perfectly normal nature of N.G.C. 4147. Cf. the eleventh paper of this series.

parallax-diameter curve of Fig. 1. Accordingly, parallaxes corresponding to the measured diameters are entered in the seventh column of Table V and are combined with the parallaxes from magnitudes to obtain the smoothed values of the eighth column. The combination is made with due regard for the relative quality of the magnitude work, assigning for convenience weight unity

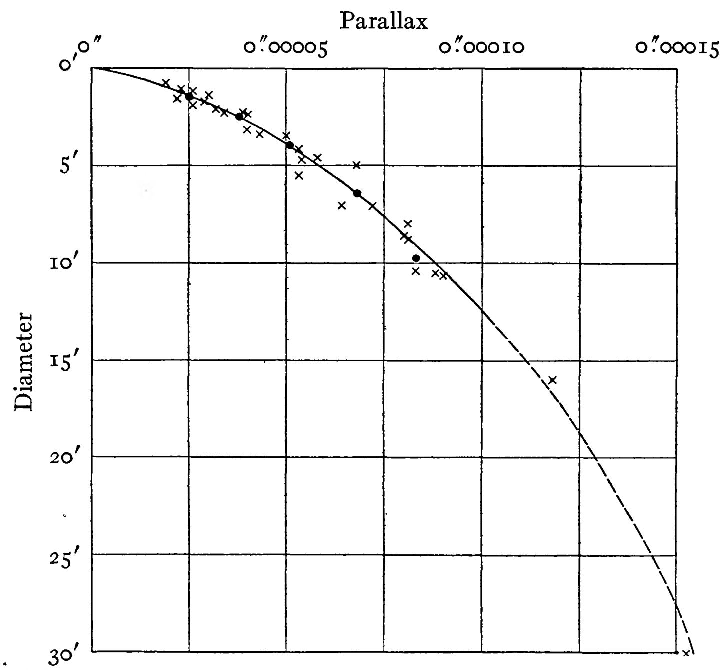


FIG. 1.—The parallax-diameter curve for globular clusters (diameters from Franklin-Adams charts). Dots are normals based upon parallaxes from magnitudes alone; crosses are the finally adopted values for the 29 clusters of Table V.

to all the parallaxes from magnitudes, and, to the corresponding values from angular measurements, the weight zero for group *a*, one-half for group *b*, one for group *c*, and two for group *d*. The adopted parallaxes are plotted in Fig. 1 as crosses.

Before obtaining the parallaxes of other clusters from their diameters alone, we shall note what accuracy may be expected in the results. In Table VII are given the percentage deviations from the parallax-diameter curve, both for the original parallaxes from magnitudes and for the adopted values. Without assigning weights, the arithmetical[1] mean for the first is 14 per cent, and for the second, 7 per cent. The latter shows the average amount of the discrepancy that will affect the parallaxes of the clusters for which no magnitudes are available. The deviations are partly

TABLE VI

DIAMETERS AND PARALLAXES

| Number of Clusters | Mean Parallax | Mean Diameter |
|---|---|---|
| 7.............. | 0″.000025 | 1′.5 |
| 5.............. | 0.000038 | 2.5 |
| 5.............. | 0.000051 | 4.0 |
| 5.............. | 0.000068 | 6.4 |
| 5.............. | 0.000083 | 9.7 |
| 1.............. | 0.000118 | 16 |
| 1.............. | 0.000150 | 30 |

TABLE VII

DEVIATIONS FROM THE PARALLAX-DIAMETER CURVE

| N.G.C. | $\frac{\pi \text{ Mag.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$ | $\frac{\pi \text{ Adopt.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$ | N.G.C. | $\frac{\pi \text{ Mag.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$ | $\frac{\pi \text{ Adopt.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$ |
|---|---|---|---|---|---|
| 288.... | +0.06 | +0.02 | 6356... | −0.42 | −0.15 |
| 1904.... | + 20 | + 8 | 6402... | − 12 | − 7 |
| 4147.... | + 37 | + 11 | 6626... | − 7 | − 6 |
| 5024.... | − 26 | − 17 | 6638... | + 3 | + 3 |
| 5139.... | − 3 | − 1 | 6656... | + 2 | + 2 |
| 5272.... | 0 | 0 | 6712... | − 9 | − 3 |
| 5904.... | − 1 | − 1 | 6779... | + 15 | + 8 |
| 6093.... | + 10 | + 4 | 6864... | − 32 | − 18 |
| 6121.... | − 6 | − 2 | 6934... | + 37 | + 20 |
| 6205.... | − 2 | − 1 | 6981... | − 6 | − 3 |
| 6218.... | − 2 | − 1 | 7078... | + 13 | + 13 |
| 6229.... | + 39 | + 13 | 7089... | + 19 | − 12 |
| 6254.... | − 14 | − 7 | 7099... | +0.05 | +0.03 |
| 6333.... | −0.15 | −0.10 | | | |

due to real differences in the clusters, but most of the error is within the uncertainty of angular measurement, for the average difference between the estimates of diameter by the two observers is but slightly less than 10 per cent.

Nearly all of the 41 clusters included in Table VIII are south of declination −30°. The few exceptions will be photographed with the 60-inch reflector, when opportunity permits, and the

[1] For brevity the terms "algebraic mean" and "arithmetical mean" are used in the sense of *with* and *without* regard to sign.

TABLE VIII

PARALLAXES OF 41 GLOBULAR CLUSTERS FROM DIAMETERS

| N.G.C. | R.A. 1900 | Decl. 1900 | Galactic | | Angular Diameter D, S | Parallax (Unit Is 0".000001) | Distance (Unit Is 100 Parsecs) | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Long. | Lat. | | | Radial | Projected on Galactic Plane | From Galactic Plane |
| 104 | 0h 19m.6 | −72° 38′ | 272° | −44° | 26′.0, 27′.6 | 148 | 68 | 49 | − 47 |
| 362 | 0 58.9 | −71 23 | 268 | −46 | 5.4, 6.6 | 66 | 152 | 106 | −109 |
| 1261 | 3 9.5 | −55 36 | 238 | −51 | 2.6, 2.5 | 39 | 256 | 161 | −199 |
| 1851 | 5 10.8 | −40 9 | 211 | −34 | 4.8, 5.0 | 58 | 172 | 143 | − 96 |
| 2298 | 6 45.4 | −35 54 | 213 | −15 | 3.0, 2.7 | 41 | 244 | 236 | − 63 |
| 2808 | 9 10.0 | −64 27 | 249 | −11 | 5.2, 4.9 | 59 | 170 | 167 | − 32 |
| 3201 | 10 13.5 | −45 54 | 244 | +10 | 6.6, 6.3 | 68 | 147 | 145 | + 26 |
| 4372 | 12 20.1 | −72 7 | 269 | − 9 | 10.8, 9.3 | 88 | 114 | 113 | − 18 |
| 4590 | 12 34.2 | −26 12 | 268 | +37 | 4.8, 5.8 | 62 | 161 | 129 | + 97 |
| 4833 | 12 52.7 | −70 20 | 271 | − 8 | 5.6, 4.9 | 61 | 164 | 163 | − 23 |
| 5286 | 13 40.1 | −50 52 | 279 | +11 | 3.6, 4.5 | 51 | 196 | 193 | + 37 |
| 5634 | 14 24.4 | − 5 32 | 309 | +49 | 2.2, 2.0 | 33 | 303 | 199 | +229 |
| 5897 | 15 11.7 | −20 39 | 311 | +30 | 6.4, 6.2 | 67 | 149 | 129 | + 75 |
| 5986 | 15 39.5 | −37 27 | 305 | +13 | 3.3, 4.1 | 48 | 208 | 203 | + 47 |
| 6101 | 16 14.4 | −71 58 | 284 | −15 | 3.0, 3.7 | 47 | 213 | 206 | − 55 |
| 6144 | 16 21.2 | −25 49 | 319 | +15 | 2.8, 2.8 | 41 | 244 | 236 | + 63 |
| 6171 | 16 26.9 | −12 50 | 331 | +22 | 5.8, 5.0 | 62 | 161 | 149 | + 60 |
| 6235 | 16 47.4 | −22 1 | 326 | +13 | 1.0, 1.2 | 20 | 500 | 487 | +112 |
| 6266 | 16 54.8 | −29 58 | 320 | + 7 | 6.4, 5.9 | 66 | 152 | 151 | + 19 |
| 6273 | 16 56.4 | −26 7 | 324 | + 9 | 5.6, 5.4 | 63 | 159 | 157 | + 25 |
| 6284 | 16 58.4 | −24 37 | 325 | +10 | 1.4, 1.9 | 27 | 370 | 364 | + 64 |
| 6287 | 16 59.1 | −22 34 | 327 | +11 | 1.3, 1.4 | 23 | 435 | 428 | + 83 |
| 6293 | 17 4.0 | −26 26 | 325 | + 8 | 2.9, 2.2 | 38 | 263 | 261 | + 37 |
| 6304 | 17 8.2 | −29 20 | 323 | + 5 | 2.0, 1.9 | 31 | 322 | 320 | + 28 |
| 6316 | 17 10.3 | −28 1 | 325 | + 5 | 1.0, 1.0 | 19 | 526 | 524 | + 46 |
| 6352 | 17 17.8 | −48 19 | 308 | − 7 | 3.4, 3.0 | 44 | 227 | 226 | − 28 |
| 6362 | 17 21.5 | −66 58 | 293 | −17 | 7.3, 8.4 | 77 | 130 | 124 | − 38 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6388........... | 17 29.0 | −44 40 | 312 | − 7 | 2.5, 2.2 | 36 | 278 | 276 | − 34 |
| 6397........... | 17 32.5 | −53 37 | 304 | −12 | 18.0, 16.2 | 120 | 83 | 81 | − 17 |
| 6441........... | 17 43.4 | −37 1 | 321 | − 5 | 1.3, 1.2 | 22 | 455 | 453 | − 40 |
| 6541........... | 18 0.8 | −43 44 | 316 | −11 | 6.8, 6.2 | 68 | 147 | 144 | − 28 |
| 6584........... | 18 10.6 | −52 15 | 309 | −16 | 2.4, 2.6 | 38 | 263 | 253 | − 73 |
| 6624........... | 18 17.3 | −30 24 | 330 | − 8 | 2.4, 2.2 | 35 | 286 | 283 | − 40 |
| 6637........... | 18 24.8 | −32 25 | 329 | −11 | 3.3, 3.6 | 47 | 213 | 209 | − 41 |
| 6652........... | 18 29.2 | −33 4 | 328 | −12 | 2.0, 2.0 | 32 | 312 | 305 | − 65 |
| 6681........... | 18 36.7 | −32 23 | 329 | −13 | 4.4, 4.6 | 55 | 182 | 177 | − 41 |
| 6715........... | 18 48.7 | −30 36 | 333 | −16 | 5.7, 5.2 | 62 | 161 | 155 | − 44 |
| 6723........... | 18 52.8 | −36 46 | 327 | −18 | 7.3, 9.2 | 79 | 127 | 121 | − 39 |
| 6752........... | 19 2.0 | −60 8 | 303 | −26 | 16.8, 14.4 | 114 | 88 | 79 | − 39 |
| 6809........... | 19 33.7 | −31 10 | 335 | −24 | 11.7, 13.2 | 100 | 100 | 91 | − 41 |
| 7006........... | 20 56.8 | +15 48 | 32 | −20 | 0.8, 0.7 | 15 | 667 | 626 | −228 |

parallax, as determined from diameters, will receive an independent check from the study of magnitudes.[1] The right ascension, declination, and galactic co-ordinates in Table VIII, as in previous tables, have been taken, whenever possible, from Melotte's catalogue, the results being checked with Bailey's lists and other catalogues. The angular diameters in the fifth column were obtained at the same time as those measured for Table V; their mean value is the basis of the parallaxes in the sixth column. The first cluster of this list is 47 Tucanae.

Tables V and VIII contain all clusters now thought to be definitely globular—a total of 69. The 15 or 20 others[2] frequently

[1] Since writing the above, one highly satisfactory confirmation has been secured. The object N.G.C. 7006 was noted as a small faint cluster by Curtis (*Lick Observatory Bulletins*, **7**, 84, 1912). On the Franklin-Adams plates and charts it appears as a nebulous star and was excluded from Melotte's catalogue of globular clusters. It is not mentioned by Bailey. The apparent diameter as given in Table VIII is only 0′75—the smallest cluster of the two lists. If our hypotheses are right, therefore, it should be the most distant cluster, and the mean magnitude should be fainter than any hitherto measured. Two polar-comparison photographs were secured in December 1917. A seven-minute exposure on a fast plate shows about two hundred stars brighter than magnitude 18.5, and one star near the center is nearly as bright as magnitude 15; but the mean of the 25 brightest, according to the preliminary measures, is 17.7. The corresponding parallax is a little more than 0″000014, differing by less than a millionth of a second from the value in Table VIII.

[2] Among the clusters thought by some to be globular are the following:

| N.G.C. | Angular Diameter | Galactic Long. | Galactic Lat. | Remarks |
|---|---|---|---|---|
| 371 | 2′1 | 268° | −44° | In Small Mag. Cloud. Not typical, Bailey. If globular, it gives for the distance of the cloud $\pi = 0''000033$. Cf. Table I. |
| 2660 | 1.5 | 234 | − 2 | Appears to be globular cluster, Melotte. If so, $R \sin\beta = -1400$ parsecs; $R \cos\beta = 39{,}700$ parsecs. |
| 5466 | 5.4 | 10 | +72 | Apparently an open cluster, Shapley. If globular, $\pi = 0''00006$. |
| 6496 | 1.5 | 315 | −11 | Probably a nebula. Bailey does not mention it. |
| 6535 | 0.8 | 354 | +10 | A group of a few faint stars, Shapley. |
| 6569 | 1.3 | 328 | − 7 | A nebula according to Bailey. If a globular cluster, $\pi = 0''00002$. |
| 6760 | 1.0 | 3 | − 5 | Small cluster of very faint stars, Curtis, Pease. An open cluster, Shapley. |
| 7492 | 3.5 | 21 | −64 | An open cluster, Curtis, Shapley. Very loose globular cluster, Melotte. If globular, $\pi = 0''00005$. |

Bibliography: Bailey, *Harvard Annals*, **60**, No. 8, 1908; **76**, No. 4, 1915. Curtis, *Lick Observatory Bulletins*, **7**, 81, 1912; **8**, 43, 1913. Melotte, *Memoirs of the Royal Astronomical Society*, **60**, Part V, 1915. Pease, *Publications of the Astronomical Society of the Pacific*, **26**, 204, 1914. Shapley, *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Shaw, *Helwan Observatory Bulletins*, No. 9, 1912; No. 15, 1915; *Monthly Notices*, **76**, 105, 1915.

admitted to the catalogues are rejected temporarily on the basis of either the Mount Wilson plates or the published opinions of Bailey, Curtis, Shaw, or Melotte. Some questioned groups may be admitted later, and other faint objects now considered nebulous stars or open clusters or faint nebulae may be proved by the large reflectors to be globular clusters. But as far as systems containing stars brighter than the sixteenth photographic magnitude are concerned, the present work may be considered exhaustive.

## II. DISTRIBUTION IN SPACE

Some striking features of the arrangement of clusters in space are brought to light by a study of the data in Tables V and VIII. Fig. 2 illustrates the distribution in three dimensions, showing on the plane of the Milky Way the galactic longitudes and projected distances, while the distances from the Galaxy are shown by vectors drawn to scale in the plane of the figure. Distances above (north of) the galactic plane are represented by full heavy lines drawn upward from black circular bases, those below by broken lines downward from open circular bases. To visualize the actual positions in space one needs only to imagine the full-line vectors standing erect on their bases and the broken lines hanging vertically from theirs; the arrow points are then at the positions of the clusters.

The most remote of all the clusters[1] is N.G.C. 7006 with a distance of 67,000 parsecs, the equivalent of 220,000 light-years; the clusters N.G.C. 4147, 6229, 6235, 6287, 6441, and 6864 (M 75) are nearly as far away. Fortunately, of these seven most distant systems six are within reach of the Mount Wilson reflectors. One-fourth of all globular clusters appear to be more distant than 30,000 parsecs (100,000 light-years); and one (N.G.C. 4147) is more than 50,000 parsecs from the plane of the Milky Way. ω Centauri and 47 Tucanae, with distances somewhat less than 7000 parsecs, are the clusters nearest to the sun.

The concentration into a limited interval of galactic longitude is conspicuous; the region from 41° to 195° is completely void of globular clusters. The mean value of all longitudes is 316°, or,

[1] See n. 1, p. 167, of this paper and the later discussion in the eleventh paper.

excluding the five largest and five smallest values, is 318°; but the mean is too much affected by widely divergent values, and the

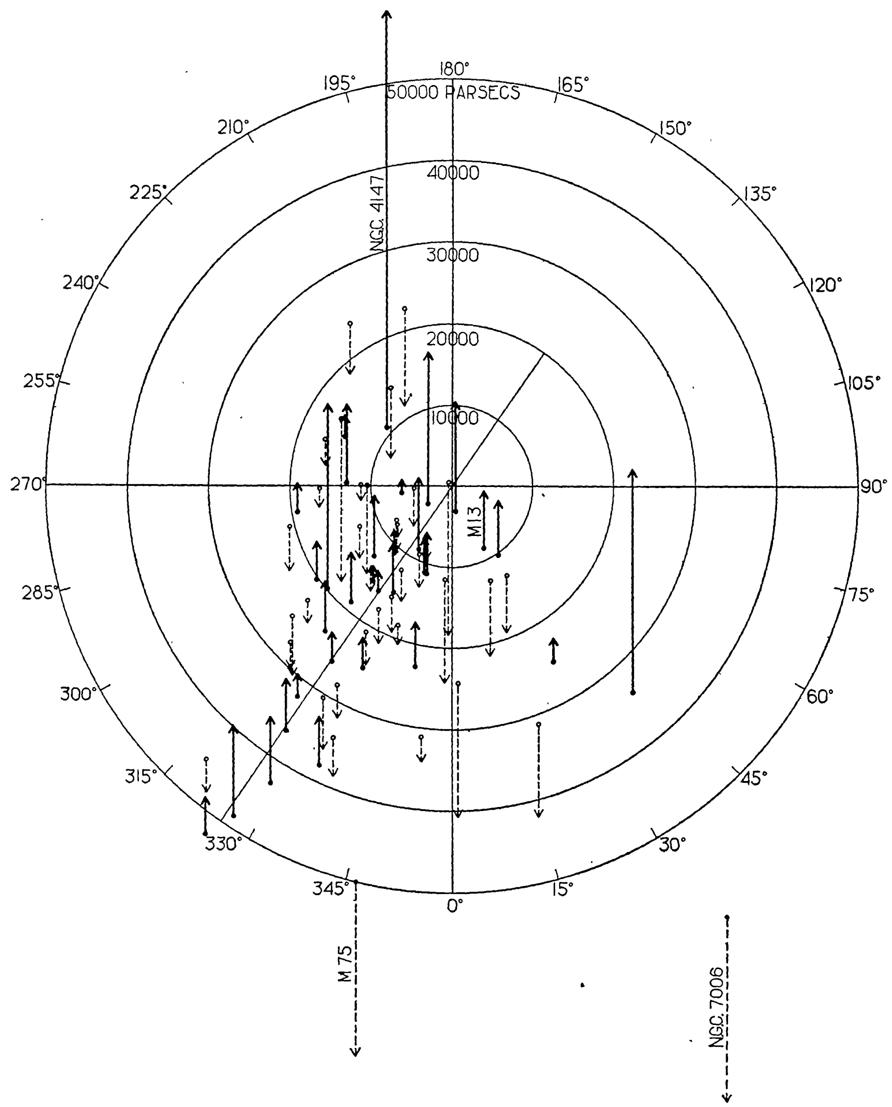


FIG. 2.—Distribution in space of globular clusters. The galactic plane is the plane of the diagram; distances above and below are shown to scale by full-line and broken-line vectors, respectively. Galactic longitudes are indicated in the margin and the scale of distances along the vertical radius. The sun is at the origin of co-ordinates. The diagram illustrates the remarkable distribution in longitude, with a maximum frequency at 325°, and by the absence of very small or zero vectors shows that globular clusters are not found within 1000 parsecs of the plane of the Milky Way. Cf. Fig. 1 of the twelfth paper.

median value,[1] 325°, is preferable in determining the central line of the system of globular clusters. The frequency of longitudes shown by Fig. 3 agrees in placing the center in longitude 325°, the points for the curve depending on the data of Table IX.

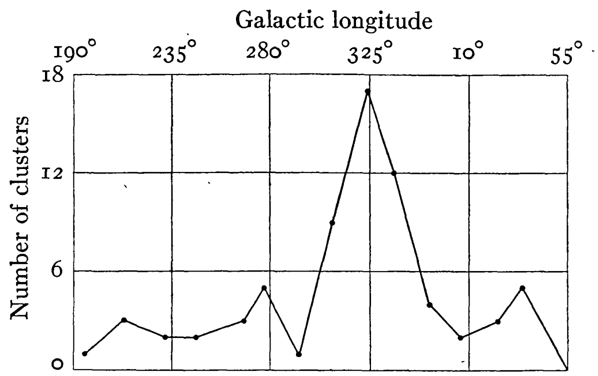


FIG. 3—Distribution of globular clusters in galactic longitude

TABLE IX

DISTRIBUTION OF GLOBULAR CLUSTERS IN GALACTIC LONGITUDE

| Interval of Longitude | Number of Clusters | Mean Longitude |
|---|---|---|
| 195° to 210°..... | 1 | 195° |
| 210 to 225 ..... | 3 | 213 |
| 225 to 240 ..... | 2 | 232 |
| 240 to 255 ..... | 2 | 246 |
| 255 to 270 ..... | 3 | 268 |
| 270 to 285 ..... | 5 | 277 |
| 285 to 300 ..... | 1 | 293 |
| 300 to 315 ..... | 9 | 308 |
| 315 to 330 ..... | 17 | 324 |
| 330 to 345 ..... | 12 | 336 |
| 345 to 360 ..... | 4 | 352 |
| 0 to 15 ..... | 2 | 6 |
| 15 to 30 ..... | 3 | 23 |
| 30 to 45 ..... | 5 | 34 |

Projecting all positions on to a plane through the sun perpendicular to the Galaxy and including the circle defined by galactic

[1] That is, the longitude of the thirty-fifth cluster when they are taken in order of increasing longitude, beginning with N.G.C. 1904.

longitude 325°, we get the diagram in Fig. 4, which represents the distribution as seen from a great distance in the direction of galactic latitude 0° and galactic longitude 235°. Black dots above the central line represent clusters north of the galactic plane; open circles below represent those south. Thus the ordinates are $R \sin \beta$ and the abscissae $R \cos \beta \cos (\lambda - 325°)$, where $R$, $\beta$, and $\lambda$ are respectively the distance, galactic latitude, and galactic longitude of a cluster.

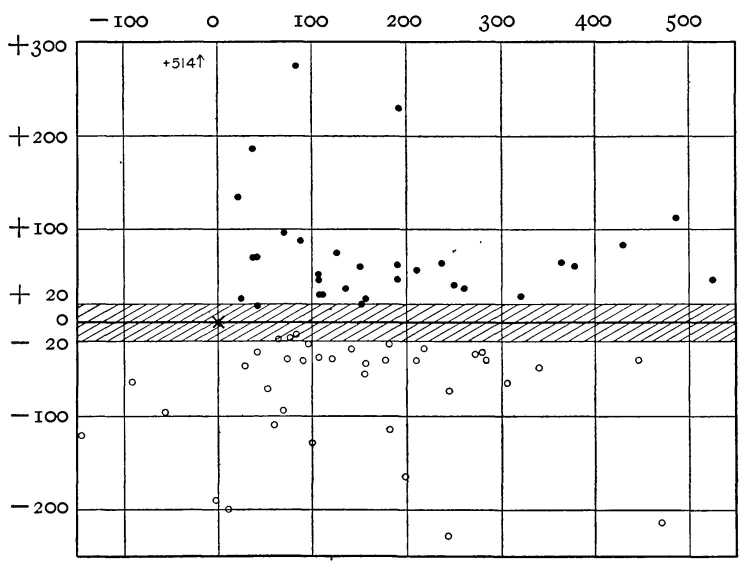


FIG. 4.—Projection of the positions of globular clusters on a plane perpendicular to the Galaxy, illustrating (1) the absence of clusters from the mid-galactic region, (2) their symmetrical arrangement with respect to the Galaxy, (3) the eccentric position of the sun (the cross) with respect to the center of the system of clusters. The ordinates are distances from the galactic plane, $R \sin \beta$; the abscissae are projected distances in the direction of the center, $R \cos \beta \cos (\lambda - 325°)$. The unit of distance is 100 parsecs; the side of a square is accordingly 10,000 parsecs. On this scale the actual diameter of the clusters is about one-fifth the diameter of the circles and dots. The cluster N.G.C. 4147 is outside the boundary of the diagram, as indicated by the arrow.

Fig. 5 differs from the preceding diagram only in having $R \cos \beta$ for abscissae. Hence the sun, as origin, is at the extreme left edge of the figure, and the actual distance of each cluster is represented by the radial distance from the origin.

Figs. 4 and 5 show more clearly than Fig. 2 the remarkable distribution of clusters with respect to the Galaxy. In the first place clusters are impartially distributed above and below, there being 32 north and 37 south of the plane. Although the average value without regard to sign is $\pm 79$ (in units of 100 parsecs), the algebraic mean of all distances from the plane is $-1$; rejection of

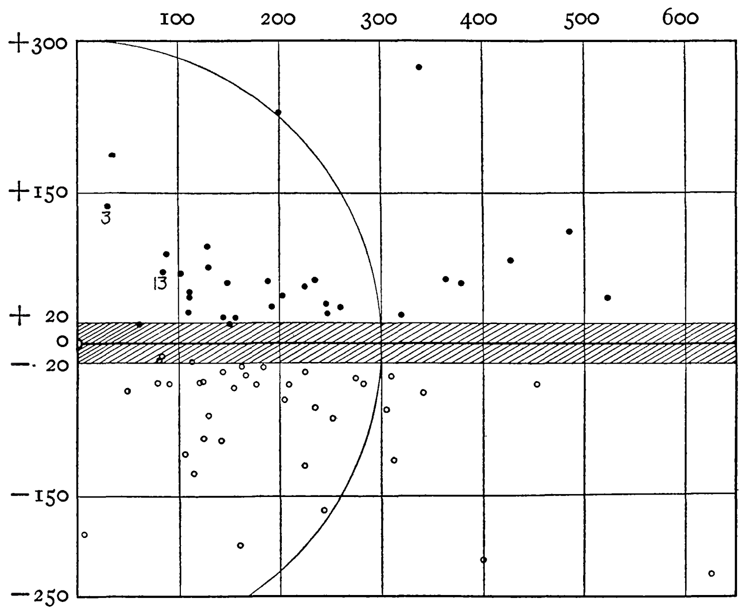


FIG. 5.—Distribution of globular clusters. The ordinates are $R \sin \beta$, as in Fig. 4; the abscissae are distances projected on the galactic plane, $R \cos \beta$. The unit of distance is 100 parsecs. The very small semicircle, with radius corresponding to a parallax of 0.″002, illustrates the region around the sun which contains all but a few of the stars in Charlier's B-type cluster. The large semicircle indicates the distance to which the present results are thought to be complete. Messier 3 and Messier 13 are indicated by numbers; the most distant cluster now known, N.G.C. 7006, is near the lower right-hand corner of the diagram. N.G.C. 4147, with co-ordinates 109 and 514, is not shown.

the very distant cluster, N.G.C. 7006, would change this to $+2$. Considering the accidental variation and the size of the distances involved, the algebraic mean is vanishingly small, and we may say confidently that the plane of the Milky Way is also a symmetrical plane in the great system of globular clusters. This relation to the Galaxy holds with good approximation at all distances from the sun (graphically shown best by Fig. 5), as may be seen from the following tabulation:

| | Interval of $R \cos \beta$ | | | | |
|---|---|---|---|---|---|
| | 0–100 | 100–200 | 200–300 | >300 | All |
| Number of Clusters | 11 | 29 | 15 | 14 | 69 |
| Mean* $R \sin \beta$ Algebraic | + 7 | + 9 | −23 | − 5 | − 1 |
| Mean* $R \sin \beta$ Arithmetical | ±70 | ±84 | ±59 | ±100 | ±79 |

*See n. 1, p. 164.

A second phenomenon clearly illustrated by the diagrams is the avoidance of the Milky Way—a result that may be of very exceptional significance. There is no cluster within 1300 parsecs of the plane of the Galaxy, and within 2000 parsecs of that plane there are only five, four of which are among the clusters nearest

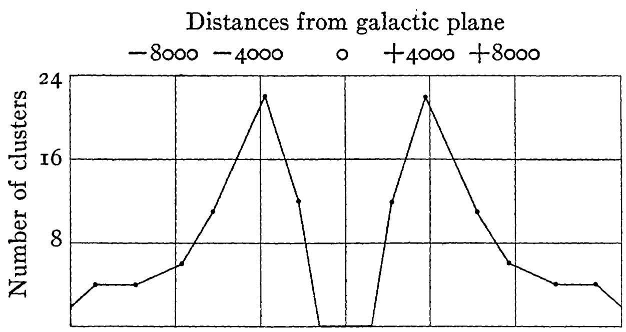


FIG. 6.—Reflected frequency-curve of the distances of globular clusters from the galactic plane, illustrating the equatorial region devoid of globular clusters. The unit of distance is one parsec.

the sun. In Figs. 4 and 5 a shaded region, 13,000 light-years in width, indicates the zone from which globular clusters are practically excluded.

The increasing concentration toward the Galaxy from both sides stops almost abruptly at the boundary of the shaded zone. The frequency of distances from the plane is treated in more detail in Table X, and the undoubted dependence of the clusters on the galactic plane, noted numerically above, is further emphasized by the curve in Fig. 6. The completion of that curve, in a form naturally to be expected for the frequency of objects concentrated toward the

TABLE X

THE FREQUENCY OF DISTANCES FROM THE GALACTIC PLANE ($R \sin \beta$)

| LIMITS OF $R \sin \beta$ (UNIT IS 100 PARSECS) | $R \cos \beta < 200$ | | | | | $R \cos \beta > 200$ | | | | | ALL CLUSTERS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. of Clusters | | Mean $R \sin \beta$ | | | No. of Clusters | | Mean $R \sin \beta$ | | | | |
| | North | South | North | South | All | North | South | North | South | All | Number | Mean $R \sin \beta$ |
| <10 | 0 | 0 | ........ | ........ | ........ | 0 | 0 | ........ | ........ | ........ | 0 | ............ |
| 10– 30 | 4 | 6 | 22 | 20 | 21 | 1 | 1 | 28 | 28 | 28 | 12 | 22 |
| 30– 50 | 3 | 8 | 37 | 40 | 39 | 5 | 6 | 40 | 39 | 39 | 22 | 39 |
| 50– 70 | 4 | 0 | 61 | ........ | 61 | 4 | 3 | 61 | 61 | 61 | 11 | 61 |
| 70– 90 | 3 | 1 | 78 | 71 | 76 | 1 | 1 | 83 | 73 | 78 | 6 | 77 |
| 90–110 | 1 | 3 | 97 | 100 | 99 | 0 | 0 | ........ | ........ | ........ | 4 | 99 |
| 110–130 | 0 | 1 | ........ | 128 | 128 | 1 | 2 | 112 | 117 | 119 | 4 | 118 |
| 130–150 | 1 | 0 | 135 | ........ | 135 | 0 | 0 | ........ | ........ | ........ | 1 | 135 |
| 150–170 | 0 | 0 | ........ | ........ | ........ | 0 | 1 | ........ | 164 | 164 | 1 | 164 |
| 170–190 | 1 | 1 | 186 | 189 | 188 | 0 | 0 | ........ | ........ | ........ | 2 | 188 |
| 190–210 | 0 | 1 | ........ | 199 | 199 | 0 | 0 | ........ | ........ | ........ | 1 | 199 |
| 210–230 | 1 | 0 | 229 | ........ | 229 | 0 | 2 | ........ | 220 | 220 | 3 | 223 |
| >230 | 1 | 0 | 514 | ........ | 514 | 1 | 0 | 274 | ........ | 274 | 2 | 394 |
| Totals | 19 | 21 | ........ | ........ | ........ | 13 | 16 | ........ | ........ | ........ | 69 | ............ |

Galaxy, would require at least 50 globular clusters within 1500 parsecs of the plane; there is, however, only one, Messier 22, and its distance below the plane corresponds to a parallax of $0''.0008$. It should be observed, moreover, that not only at great distances, where insufficiency of observations may be urged, do we note this absence of clusters, but also within a distance from the sun of 20,000 parsecs, where the data are quite sufficient (37 clusters) and undoubtedly are complete. Hence we conclude that this great mid-galactic region, which is peculiarly rich in all types of stars, planetary nebulae, and open clusters, is unquestionably a region unoccupied by globular clusters.

To explain this remarkable condition several hypotheses have been considered, such as error in choosing the origin of galactic latitudes,[1] incompleteness of data, general absorption of light in space,[2] clouds or a ring of absorbing matter along the spine of the Milky Way analogous to the dark peripheral rings of spiral nebulae,[3]

[1] The adopted position of the north galactic pole is that given by Gould (*Uranometria Argentina*): $\alpha = 12^h41^m$, $\delta = +27°21'$. This position differs by less than a degree from those obtained through other reliable and definitive investigations. The most recent, and probably the best, is based on the Harvard Map of the Sky, by Nort, who finds $\alpha = 12^h44^m$, $\delta = +27°$ (*Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 84, 112, 1917). An error of $\pm 1°$ in the cluster latitudes might slightly displace, widen, or narrow the zone of avoidance, but nothing short of selectively operative errors of several degrees could seriously obscure it. The frequency-curve of galactic latitudes also shows the zone, but naturally to a less degree. Cf. Fig. 2 of the twelfth paper. The values in Tables V and VIII show no latitude less than 5°. See the remarks relative to N.G.C. 2660 in n. 2, p. 167.

[2] See the notes in a following paper (the eleventh of the series) on the color of stars in the two most distant clusters. The dark obstructing nebulae which are frequently found in and near the Milky Way are undoubtedly capable of obliterating or greatly diminishing the light of any cluster involved in the nebulosity or beyond it. N.G.C. 4372, a large southern cluster (almost certainly globular), which is very faint for its angular diameter, falls alongside a vacant space in the sky. N.G.C. 6144 is near the edge of the $\rho$ Ophiuchi dark nebulosity and appears large for the magnitude of its bright stars. This latter nebulosity may also affect the brightness of Messier 4 (N.G.C. 6121) to some extent. But it is interesting to note that in developing a parallax method that is independent of the magnitudes we have escaped from the errors in parallax that such obstructing material might occasionally have imposed.

[3] Possibly a hypothetical wedge-shaped ring might explain some of the divergence from the galactic plane with increasing distance (Figs. 4 and 5); but insufficiency of material for faint clusters would better account for most of it. The phenomenon, of course, may be real—a widening of the zone of avoidance in the direction of the

and finally the actual absence of globular clusters from the regions rich in stars because of the dynamical impossibility of existence. The first three seem clearly impossible, the fourth improbable or at least unquestionably insufficient, and, therefore, without going at present into the meaning and consequences of such a theory, the last hypothesis is tentatively adopted.

We have found that the center of the elongated and somewhat irregular system of globular clusters lies in the plane of the Milky Way on a line directed toward galactic longitude 325°. The distance along that line may be estimated from an inspection of Fig. 4, and perhaps obtained more accurately from the following consideration of the frequencies of $R \cos \beta \cos (\lambda - 325°)$, in which clusters more than 15,000 parsecs distant from the plane are excluded:

| | Interval of Distance in Direction of Center | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $<100$ | $-100$ to 0 | 0 to $+100$ | $+100$ to $+200$ | $+200$ to $+300$ | $+300$ to $+400$ | $+400$ to $+500$ | $>500$ | All |
| Number of clusters | 1 | 2 | 18 | 20 | 10 | 5 | 3 | 1 | 60 |
| Mean of $R \cos \beta \cos (\lambda - 325°)$... | $-145$ | $-74$ | $+59$ | $+138$ | $+246$ | $+342$ | $+454$ | $+525$ | $+158$ |

From a plot of these numbers we estimate provisionally the distance of the center to be 13,000 parsecs. Incompleteness of data because of faintness will not materially affect the galactic longitude and latitude of this point, but is likely to make the distance too small. The mean value of $R \cos \beta \cos (\lambda - 325°)$ for the 60 clusters, $+158$, is probably nearer the true value, but it is also liable to understate the distance. A definitive value is hardly possible, and, at least until the number of very faint clusters can be considerably increased, we may adopt as the center of the general system of globular clusters a point for which the parallax is between 0″.00006 and 0″.00004, with equatorial co-ordinates $\alpha = 17^h 30^m$, $\delta = -30°$, and galactic co-ordinates $\lambda = 325°$, $\beta = 0°$. The position

center of the globular cluster system, combined with a lack of observations for clusters far beyond the center. See n. 2, p. 167; if N.G.C. 2660 is globular, the apparent tendency to widen with distance is somewhat counteracted.

lies in the constellation Sagittarius, a few degrees east of its boundary with Scorpio and Ophiuchus.

## III. LINEAR DIMENSIONS OF CLUSTERS

A literal interpretation of the curve in Fig. 1 gives for the dimensions of globular clusters:

| | Parallax (Unit Is 0".000001) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 40 | 60 | 80 | 100 | 120 | 150 |
| Angular diameter | 1'.1 | 2'.7 | 5'.2 | 8'.3 | 12'.4 | 17'.2 | 27'.5 |
| Linear diameter in parsecs..... | 16 | 20 | 25 | 30 | 36 | 42 | 53 |

As remarked before, however, the diameters from the charts do not give a true measure of the clusters, and the hypothesis that size depends on distance from the sun, irrespective of distance from the Galaxy, is quite untenable.

There are, without doubt, real though relatively inconspicuous differences among globular clusters; and some of the differences, such as frequency of certain spectral types, degree of condensation and ellipticity, and possibly total numbers of stars, are being recognized and evaluated through the Mount Wilson studies. Two properties, however, so far seem to show little variation from cluster to cluster—the actual linear diameter and the mean magnitude of the brightest stars. We shall assume, therefore, on the basis of what observational evidence we now have at hand, that all globular clusters are of practically the same dimensions, explaining the apparent decrease in size with increasing distance (tabulated above) as a natural consequence of a central concentration of luminosity and of an intermingling near the edges with non-cluster stars. On that basis an investigation of the dimensions of one cluster will suffice for all.

Long exposures with the 60-inch reflector upon the outer parts of some of the brightest clusters have confirmed the conclusion, obtained from a study of the distribution of variable stars, that the clusters are much greater in extent than would be inferred from ordinary visual or photographic observation. For instance, on the

Franklin-Adams charts the apparent diameter[1] of Messier 3, the cluster chosen for the present illustration of dimensions, is 7′; on the original plates it is 18′, according to Melotte; but the actual diameter is in excess of half a degree.

The distance of Messier 3 is 13,900 parsecs,[2] corresponding to nearly three thousand million times the distance of the sun from the earth. The distance north of the galactic plane is 13,500 parsecs. The accompanying plate is reproduced from a photograph of several hours' exposure made by Mr. Ritchey with the 60-inch reflector. The original negative shows more than twenty thousand stars outside the central burned-out area, the smallest images being fainter than the twentieth magnitude.

The cluster extends beyond the limits of the photograph in all directions. The most distant variable star (undoubtedly a member of the system) is 17′ from the center, corresponding to a projected distance of fourteen million astronomical units. As ordinarily seen and photographed, the cluster covers an area but little larger than one of the squares, but we may be sure that its actual projected area is at least twenty-five times as great; that is, the diameter is about thirty million astronomical units. To cross the cluster, light must travel 470 years.

If we suppose the sun situated at the center of the cluster, all stars with parallaxes greater than $0\overset{''}{.}1$ would be included within the concentric circle. Sirius would be at the distance indicated by the cross, and the projected distance of the bright triplet near the bottom of the picture equals the distance of the Hyades from the sun.

Inclosed in small circles are a few of the variable stars, chosen at random, the close equality of whose magnitudes is to be noted. In some cases they appear as doubles, but the actual separation

[1] The estimates from the charts, listed in Tables V and VIII, refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlying members of a cluster from the stars of its surrounding field.

[2] The diagram was made on the basis of a parallax of $0\overset{''}{.}000074$; the final value of Tables I and V indicates that the linear dimensions of the cluster on the plate are too small by 3 per cent, an amount, however, that is far within the probable error.

in the closest of the indicated pairs is more than half the distance separating the sun from $\alpha$ Centauri. In the small square is the image of an exceptional variable, No. 37, for which the period is less than eight hours.

The cluster variables, in the mean, have the absolute magnitude $-0.2$, photographically nearly six magnitudes brighter than the sun. A star of the brightness and color of the sun would not appear on the photograph, being nearly two magnitudes too faint (21.5). Sirius, located in this cluster, would be of the seventeenth apparent magnitude, corresponding to the star indicated on the photograph by an arrow-point.

The condensation of stars at the center of the cluster may be readily contrasted with that of stars around the sun. Within the circle, which marks a distance from the center corresponding to a parallax of $0\rlap{.}''1$ (approximately two million astronomical units), there are at least 15,000 stars brighter than magnitude 20. (This estimate deducts those stars not within the concentric sphere but appearing by projection within the circular area.) In a sphere of the same radius, with the sun as center, less than twenty stars brighter than the sun are known. But only those which are two magnitudes brighter appear on this photograph of the cluster; there are, accordingly, in the sphere around the sun only four or five stars to compare with the 15,000 in Messier 3.

Finally, we shall make some estimates relative to the probable mass of a globular cluster. We may go astray, to be sure, in assuming similar masses and analogous relations of mass to luminosity for stars in clusters and in the general galactic system. The dynamical conditions in these highly condensed globular systems may conceivably have some important effect upon the amount of matter that goes into a single star as well as upon the speed and nature of subsequent development. Eddington's recent theoretical work on the masses of stars, however, indicates that as long as we deal with typical giants the masses are definitely limited; and, further, the identity of Cepheid phenomena wherever studied—in the galactic system, in the more condensed Magellanic clouds, and in the extremely condensed globular clusters—tends to support the view of the universal comparability of stellar masses.

All that we know of the masses of stars from observation has been summarized recently by Russell.[1] The data come altogether from double stars, but we shall probably not commit serious error in applying the results without alteration to the single stars in the condensed globular clusters, although possibly slightly greater average values for each type would be appropriate for the isolated stars in our general galactic system. For the present approximation we shall simply take a mean value from Russell's data, say an average of four times the sun's mass for every star brighter than the sun. The number of such stars in Messier 3 may be fairly estimated at 40,000, only three-fourths of which have ever been photographed. Hence the entire mass, distributed among the stars which are brighter absolutely than photographic magnitude +5.6, is 160,000 times the solar mass; and something like three-fourths of this amount is within 10 parsecs of the center.

It seems quite futile at present to estimate the total mass of the cluster. Perhaps it is two or three million times the solar mass; probably it is several times the amount estimated for the stars brighter than 5.6—that we must admit on the basis of what little we now know of the relative frequency of dwarfs and giants in the vicinity of the sun; but as a matter of absolute certainty we do not know that there is a single star fainter than the sun, and even our estimate of 160,000 may be 50 per cent too great.

## SUMMARY

1. Following the methods outlined in the preceding paper the parallaxes and positions in space are obtained for 69 globular clusters—all that can now be definitely assigned to the globular class. The distances range from 6500 to 67,000 parsecs, the brightest stars in the most distant clusters being fainter than the seventeenth photographic magnitude. The average probable error of a parallax is of the order of 20 per cent, varying considerably with the method used and with the quality of the observational work. Something more than 15,000 measures of magnitudes were made for one phase of the work. Section I contains various

[1] *Popular Astronomy*, **25**, 666, 1917.

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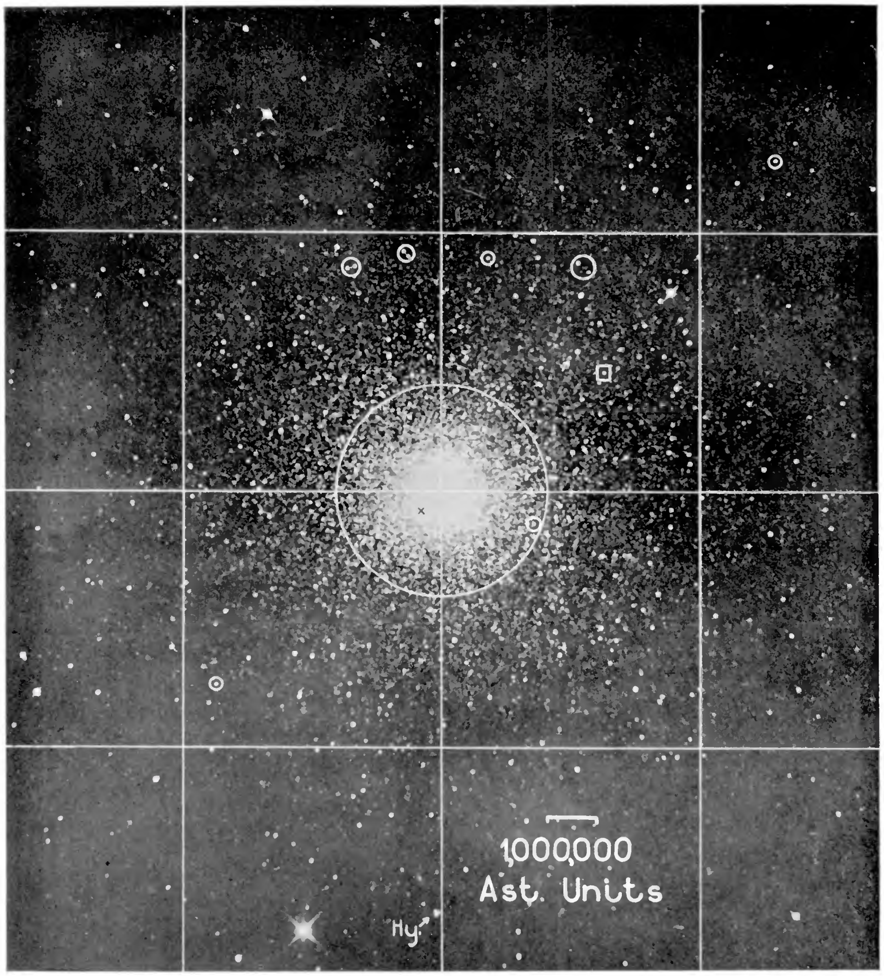
items relative to the method of investigation, comparative accuracy, maximum luminosities, and the frequency of giants.

2. The study of the distribution of clusters in space brings out a number of remarkable features (cf. sec. II), the most significant of which appear to be the absence from the denser stellar regions of globular clusters and the final proof that they are subordinate to the general galactic system. The center of the system of globular clusters is found, with some uncertainty in one co-ordinate. A discussion of the part played in a general theory by the distribution of clusters is postponed to a following paper.

3. The derivation of parallaxes has permitted the discussion of the actual dimensions of clusters and a comparison with familiar distances near the sun. Plate IV sufficiently summarizes the result for a typical system, Messier 3, whose distance from the earth is of the order of 250,000 million million miles. The total mass of a typical globular cluster is estimated to be from a quarter to a half of a million times the solar mass, with much uncertainty as to the upper limit.

Mount Wilson Solar Observatory
December 1917

PLATE IV




THE GLOBULAR CLUSTER MESSIER 3 (N.G.C. 5272)

The side of a large square is 5,000,000 times the distance of the earth from the sun. The radius of the concentric circle corresponds to a parallax of 0.″1 (2,062,650 astronomical units). To cross the circle light must travel for sixty-five years. Small circles contain typical variables; the small square, variable No. 37. If the sun were situated at the center, the Hyades would be at the distance of the triplet at the bottom of the picture; Sirius would be at the distance of the black cross near the center. A star of the luminosity of Sirius is indicated by the arrow-point. Stars of our sun's brightness are nearly two magnitudes too faint to appear on the photograph.