RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS.*

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Investigations into the nature of the stars must necessarily be very largely based upon the average characteristics of groups of stars selected in various ways,—as by brightness, proper motion, and the like. The publication within the last few years of a great wealth of accumulated observational material makes the compilation of such data an easy process; but some methods of grouping appear to bring out much more definite and interesting relations than others, and, of all the principles of division, that which separates the stars according to their spectral types has revealed the most remarkable differences, and those which most stimulate attempts at a theoretical explanation.

In the present discussion, I shall attempt to review very rapidly the principal results reached by other investigators, and shall then ask your indulgence for an account of certain researches in which I have been engaged during the past few years.

Thanks to the possibility of obtaining with the objective prism photographs of the spectra of hundreds of stars on a single plate, the number of stars whose spectra have been observed and classified now exceeds one hundred thousand, and probably as many more are within the reach of existing instruments. The vast majority of these spectra show only dark lines, indicating that absorption in the outer and least dense layers of the stellar atmospheres is the main cause of their production. Even if we could not identify a single line as arising from some known constituent of these atmospheres, we could nevertheless draw from a study of the spectra, considered merely as line-patterns, a conclusion of fundamental importance.

The spectra of the stars show remarkably few radical differences in type. More than ninety-nine per cent of them fall into one or other of the six great groups which, during the classic work of the Harvard College Observatory, were recognized as of fundamental importance,

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* An address delivered before a joint meeting of the Astronomical and Astrophysical Society of America and Section A of the American Association for the Advancement of Science, at Atlanta, Georgia, December 30, 1913, with a few additions.

and received as designations, by the process of "survival of the fittest", the rather arbitrary series of letters B, A, F, G, K, and M. That there should be so few types is noteworthy; but much more remarkable is the fact that they form a continuous series. Every degree of gradation, for example, between the typical spectra denoted by B and A may be found in different stars, and the same is true to the end of the series, a fact recognized in the familiar decimal classification, in which B5, for example, denotes a spectrum half-way between the typical examples of B and A. This series is not merely continuous; it is *linear.* There exist indeed slight differences between the spectra of different stars of the same spectral class, such as A0; but these relate to minor details, which usually require a trained eye for their detection, while the difference between successive classes, such as A and F, are conspicuous to the novice. Almost all the stars of the small outstanding minority fall into three other classes, denoted by the letters O, N, and R. Of these O undoubtedly precedes B at the head of the series, while R and N, which grade into one another, come probably at its other end, though in this case the transition stages, if they exist, are not yet clearly worked out.

From these facts it may be concluded that the principal differences in stellar spectra, however they may originate, arise in the main from variations in a single physical condition in the stellar atmospheres. This follows at once from the linearity of the series. If the spectra depended, to a comparable degree, on two independently variable conditions, we should expect that we would be obliged to represent their relations, not by points on a line, but by points scattered over an area. The minor differences which are usually described as "peculiarities" may well represent the effects of other physical conditions than the controlling one.

The first great problem of stellar spectroscopy is the identification of this predominant cause of the spectral differences. The hypothesis which suggested itself immediately upon the first studies of stellar spectra was that the differences arose from variations in the chemical composition of the stars. Our knowledge of this composition is now very extensive. Almost every line in the spectra of all the principal classes can be produced in the laboratory, and the evidence so secured regarding the uniformity of nature is probably the most impressive in existence. The lines of certain elements are indeed characteristic of particular spectral classes; those of helium, for instance, appear only in Class B, and form its most distinctive characteristic. But negative conclusions are proverbially unsafe. The integrated spectrum of the Sun shows no evidence whatever of helium, but in that of the chromosphere it is exceedingly conspicuous. Were it not for the fact that we are near this one star of Class G, and can study it in detail, we might have erroneously concluded that helium was confined to the "helium stars". There are other cogent arguments against this hypothesis. For example, the members of a star-cluster, which are all moving together, and presumably have a common origin, and even the physically connected components of many double stars, may have spectra of very different types, and it is very hard to see how, in such a case, all the helium and most of the hydrogen could have collected in one star, and practically all the metals in the other. A further argument—and to the speaker a very convincing one—is that it is almost unbelievable that differences of chemical composition should reduce to a function of a single variable, and give rise to the observed linear series of spectral types.

I need not detain you with the recital of the steps by which astrophysicists have become generally convinced that the main cause of the differences of the spectral classes is difference of temperature of the stellar atmospheres. There is time only to review some of the most important evidence which, converging from several quarters, affords apparently a secure basis for this belief.

The first argument is based upon the behavior of the spectral lines themselves. To appreciate its full force, one must familiarize himself with a multitude of details. A typical instance is that of the heavy bands in the region of longer wave-length, which are the most characteristic feature of spectra of Class M, appear faintly in Class K5, and are absent in Class K and all those higher in the series. Fowler has shown[*] that these bands are perfectly reproduced in the spectrum of the outer flame of an electric arc charged with some compound of titanium, while the spectrum of the core of the arc, though showing conspicuously the bright lines of titanium, does not contain the bands. Here we are evidently dealing with some compound,—perhaps titanium oxide,—whose vapor is present in the relatively cool flame of the arc, and emits a spectrum of the banded type characteristic of compounds, while in the hotter core it is dissociated, and only the lines of the metal are seen. There seems then to be no escape from the conclusion that the atmospheres of stars of Class M are cool enough to permit the existence of this compound, and hence cooler than the core of the arc, and that the temperature of its dissociation is approached in Class K5, and surpassed in Class K. In general, those metallic lines which are relatively strong in the spectra produced in the oxyhydrogen flame or the electric furnace are also strong in spectra of Classes M and K; the lines most prominent in Class G are the typical arc lines; and the rela-

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[*] Proc. Roy. Soc. Vol. 72, pp. 219-225, 1904.

tively few metallic lines which persist into Classes A and B are those which appear exclusively, or with greatly enhanced intensity, in the spark spectra of the laboratory.

The second line of evidence is afforded by the distribution of intensity in the continuous background of the spectra, whose differences from type to type are obvious to the eye as differences in the color of the stars. This characteristic is fortunately capable of accurate measurement. For the brighter stars, spectro-photometric comparisons may be made with a terrestrial light-source whose energy curve is known, as has been done visually by Wilsing and Scheiner[*], and photographically by Rosenberg[†]. Much fainter stars may be reached by the comparison of their brightness as measured visually, (or on isochromatic plates with a suitable color screen) and photographically on ordinary plates. The "color-index" so obtained, which expresses, in stellar magnitudes, the relative photographic brightness of stars of equal visual brightness, is found to be very intimately related to the spectral type,—the differences within each spectral class being hardly greater than the errors of observation. The results of King,[‡] Parkhurst,[¶] and Schwarzschild,[||] working with different instruments and on stars of very different brightness, are in excellent agreement, as is shown in Table I. The near approach to equality among the differences in color-index from class to class is very remarkable, when it is considered that these types were picked out somewhat arbitrarily according to the general appearance of the photographic spectra. The judgment of the Harvard observers in selecting the really important points of difference was evidently very good.

TABLE I

Spectrum	Color-Index			Temperature
	King	Parkhurst	Schwarzschild	
B0	—0.32			20000 :
B5	—0.17	—0.21	—0.20	14000
A0	0.00	0.00	0.00	11000
A5	0.19	0.23	0.20	9000
F0	0.30	0.43	0.40	7500
F5	0.52	0.65	0.60	6000
G0	0.71	0.86	0.84	5000
G5	0.90	1.07	1.10	4500
K0	1.16	1.30	1.35	4200
K5	1.62	1.51	1.80	3200
M	1.62	1.68		3100
N		2.5		2300 :

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[*] Potsdam Publications, Vol. 19, part 1.

[†] *A. N.* 4628, 1913.

[‡] Harvard Annals, Vol. 59, p. 179.

[¶] Astrophys. Jour. Vol. 36, p. 218, 1912.

[||] Göttingen Aktinometrie, Teil B. p. 19.

If the spectral sensitiveness of the plates used in such investigations has been determined, (as Parkhurst has done) it is possible to calculate the temperature at which a black body would emit light of the same color as that observed; and similar calculations can be made, with greater accuracy, from the spectro-photometric data. The last column of Table I gives the effective temperatures thus derived, (based mainly on the work of Wilsing and Scheiner). The absolute values of the temperatures here given may be considerably in error, especially at the top of the scale, (in fact, Rosenberg's work indicates a much greater range), but there can be no doubt about the relative order.

Of a third independent confirmation of the temperature hypothesis, based on the determination of the surface brightness of the stars, I shall have occasion to speak later.

It should be expressly stated that the "temperatures" here spoken of are the effective "black body" temperatures corresponding to the spectral distribution of the radiation. Unless the surfaces of the stars possess decided selective emissivity for certain wave-lengths, these effective temperatures should also indicate with tolerable accuracy the energy density of the flux of radiation which escapes from them. This tells us little about the temperature of the deeper regions; but it must be the main, if not the only, factor in determining the temperature of those outer and nearly transparent layers of the atmospheres in which the characteristic line absorption takes place. If we further assume, in accordance with Abbot's studies of the solar atmosphere,* that the absorption is nearly complete in so small a thickness of the atmosphere that wide variations in its depth and density would modify its total absorption but little, it becomes easy to see how the influence of its temperature, (which presumably determines the relative strengths of absorption in different lines) may predominate so greatly over that of all other factors in determining the spectral type.

We may now rapidly review some of the relations which have been brought to light between other characteristics of the stars and their spectral types. First, as regards the relative numbers of stars of the different classes, we have in Table II some results of counts made at Harvard.†

TABLE II

Spectrum	O	B	A	F	G	K	M	N
Number above $3^m.25$	3	52	32	16	20	35	21	0
Number above $6^m.25$	20	696	1885	720	609	1719	457	8
Percentage in Galactic Region	100	82	66	57	58	56	54	87

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* Abbot, The Sun, p. 252, 1911.

† Harvard Annals, Vol. 64, p. 134.

Classes A and K make up more than half of all the stars brighter than $6^m.25$,—that is, of the stars visible to the naked eye. The remaining stars are divided fairly evenly among the other four principal classes, while only one star in 300 is of Class O, and only one in 800 of Class N. The relative proportions of the different classes are however different in different parts of the heavens, as is indicated by the last line of the table, which gives the percentage of stars of each class which lie in a belt covering one-half of the celestial sphere, and extending for 30° each side of the Milky Way. All the stars of Class O are close to the central line of the Galaxy, (except for a few in the Magellanic Clouds). The stars of Class B are very strongly concentrated in the galactic region; those of Class A are considerably so; those of the following classes very little, except in the case of Class N (for which the tabular percentage is derived, not from the eight brightest stars of this class alone, but from a much larger number of fainter ones).[*]

The relative proportions of the different classes vary also with the apparent brightness of the stars. Among the stars brighter than $3^m.25$, as the table shows, Class B has more representatives than any other, but the percentage of this type steadily diminishes as we pass to fainter stars. The percentage of stars of class A at first increases with diminishing visual brightness; but there is good reason to believe that, at least in regions remote from the Galaxy, the relative proportion of these too falls off rapidly in the neighborhood of the ninth magnitude;[†] and Fath's work on the integrated spectrum of the Milky Way[‡] shows that even there, the bulk of the very faint stars which form the galactic clouds must be of Secchi's second type (F, G, or K).

Counts of the stars down to any given magnitude may however be very misleading unless we bear in mind the enormous preference which this method of observation gives to the stars of great actual luminosity, which can be seen afar off, and hence are being sought in a much greater volume of space than those of small luminosity. A difference of but five magnitudes in the real brightness of two groups of stars gives the brighter kind (if both are uniformly distributed in space) a thousand-fold better chance of getting into our catalogues;—and this example understates the actual conditions in some cases. Mere counts of stars need therefore to be supplemented by such knowledge as we can obtain concerning their distances.

Much information can be obtained from the average proper motions of the stars of the various classes, and still more by deriving their average parallaxes from the mean parallactic drift due to the motion

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[*] Harvard Annals, Vol. 66, p. 213.
[†] Astronomical Journal, Vol. 26, p. 153, 1910.
[‡] Astrophys. Jour. Vol. 36, pp. 362-367, 1912.

of the solar system in space. Studies of this character have been made by several investigators of the first rank. Their results, which are summarized in Table III, show certain apparent discrepancies, which however arise principally from differences in the methods according to which the various workers have selected the groups of stars for investigation.

TABLE III

Spectrum	Mean Centennial Proper Motion			Mean Parallax			
	Kapteyn	Boss	% Rej.	Kapteyn	Boss	Campbell	% Rej.
O	″	1″.6	0	″	0″.004	″	
B	2.6	2.4	0	0.007	0.007	0.006	0
A	5.8	4.6	3	0.010	0.010	0.016	3
F	14.5	7.7	28	0.022	0.012	0.035	3
G	27.0	5.2	20		0.008	0.022	8
K	13.0	5.7	6		0.010	0.015	9
M	5.9	5.0	6	0.011	0.008	0.011	3
N		3.2		0.000			

Kapteyn's data* represent the mean proper motions and parallaxes of all the stars of the fifth magnitude of each class, except for Class N, in which, to get enough stars, it was necessary to include faint objects, so that the average magnitude is here 8.3. His results show a conspicuous maximum of average proper motion and parallax for Class G, with a rapid fall on both sides of it. The stars of Class N would have to be brought about five times nearer to appear as bright as the others, but even then they would have the smallest mean parallax of all.

Boss,† in his investigation of the solar motion, had at his disposal very accurate proper motions of all the stars down to $5^{m}.7$, and about half as many more between this and the seventh magnitude. The average magnitude of his stars is therefore nearly the same as that of Kapteyn's. But, for very good reasons, he excluded from his main solution all stars with proper motions exceeding 20″ per century. The percentage of stars thus excluded (which differs greatly from class to class) is given in the fourth column of Table III. It is natural that this often drastic rejection of the large proper motions, and hence in general of the nearer stars, should greatly diminish his mean values. Among the classes in which the mean proper motion is small, the percentage of exclusion is also small, and the results are but little modified. But it is noteworthy that the exclusion of six per cent of the stars of

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* Astrophys. Jour. Vol. 30, p. 295; Vol. 32, p. 91. 1909-10.

† Astronomical Journal, Vol. 26, pp. 187-201, 1911. The mean proper-motions of the few stars of Classes O and N which appear in Boss's Catalogue have been added by the writer.

Class K has reduced the mean proper motion in a greater ratio than that of 28 per cent of those of Class F, and also that the removal of one fifth of the stars of Class G decreases the mean for the remainder to less than one fifth of its initial value. It appears from these results that a large majority of the stars of Classes F, G, and K have nearly if not quite as small parallaxes and proper motions as those of Classes A and M, though they are not quite so remote as the stars of Class B. The large mean values obtained for all the stars of these classes are due to the presence of a relatively small proportion of near and apparently rapidly moving stars, of which the percentage decreases, but the mean proper motion and parallax increase, from F to K.

Campbell's results* are derived from a comparison of the radial velocities and proper motions of nearly 1200 stars, mostly brighter than the fifth magnitude, and averaging about a magnitude brighter than Boss's stars, which would lead us to expect that their mean parallaxes should be forty or fifty per cent greater. In his work "a few stars having proper motions abnormally large for their classes were omitted in accordance with definitely set limits" (which unfortunately are not described more specifically). The approximate percentage of exclusion is given in the last column of the table. It appears on inspection that the differences between Campbell's and Boss's results for stars of Classes A, K, and M arise mainly from the greater brightness of Campbell's stars; those for Classes F and G are due mainly to the different percentages of exclusion; and that the only significant difference is that Campbell's B-stars, though averaging much brighter to the eye than Boss's, have a slightly smaller mean parallax, and therefore must be, on the average, of greater real brightness.

Closely allied with these investigations is the determination of the mean peculiar velocity of the stars of each spectral class. The results of Boss and Campbell, reached almost simultaneously, and from quite independent data,—proper motions in one case and radial velocities in the other,—are in extraordinary agreement. The values found for the average component of motion in any arbitrary direction are, (in kilometers per second):

Spectrum	B	A	F	G	K	M
Campbell	6.5	10.5	14.4	15.9	16.8	17.1
Boss	6.3	10.2	16.2	18.6	15.1	17.1

The rapid increase of the mean velocity from B to F is very remarkable. The slow further gain from F to M would attract little attention if it were not in the same direction.

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* Lick Observatory Bulletin, Vol. 6, p. 134, 1911.

It should here be added that the phenomenon known as preferential motion, or "star-streaming"—the excess of the average peculiar velocity of the stars in a certain direction above those in the perpendicular directions,—is almost absent in Class B, very conspicuous in Class A, and somewhat less so in the following classes, (being partially concealed by the greater average magnitude of the velocities).

Another notable difference between the various spectral classes may be found in the number of binary stars, both visual and spectroscopic, among them. We may distinguish two classes of visual double stars; binary stars for which orbits have been computed, (with periods rarely exceeding two centuries), and physical pairs, whose real connection is proved by common proper motion, but whose relative motions are slow, and periods long,—probably, often thousands of years. The counts of these two classes here given are from a list prepared in the course of my work, and include all stars for which the necessary data could be obtained, including many stars for which unpublished observations of spectra have been generously furnished me from Harvard. For the spectroscopic binaries, Campbell's counts have been taken from his Catalogue of 1910.* They include all the systems whose periods were then known, and are divided into two groups,—one including all whose periods are less than ten days, and also all those whose periods, though not exactly known, are described as short; the other all the known periods exceeding ten days, and those which, though not precisely determined, are known to be long.

TABLE IV

Spectrum	Visual Binaries	Physical Pairs	Spectroscopic Binaries	
			Short Period	Long Period
B	0	52	33	15
A	14	152	15	14
F	33	115	11	9
G	24	74	8	14
K	12	62	0	13
M	0	11	0	2

It appears that, in Campbell's picturesque phrase, visual double stars of relatively short period "abhor" Classes B and M, the greatest number being of Class F, with G a good second. Among the physical pairs, of long period, the most favored class is A. Class B is abundantly represented, and Class M very sparingly.

The percentage of stars which are found to be spectroscopic binaries is very probably greater among Classes B and A than lower down the list. As time goes on, indeed, more and more of the stars of these

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* Lick Observatory Bulletin, Vol. 6, p. 38, 1910.

"later" types are found to be spectroscopically double but of long period, but among these classes the detection of such systems, where the range of velocity is small, is much easier than among the stars of the first type, whose lines are diffuse. In any case it is certain that short periods are almost confined to Classes B, A, and F, and are especially abundant in the first of these. The few short period stars of Class G which appear in the table are all Cepheid variables, most of which were selected for observation on this account, and would not otherwise have got into the list.

Finally, we may note that, among variable stars, those of the eclipsing type, such as Algol or Beta Lyrae, are for the most part of Classes A and B, though there are a number of Classes F and G, and one at least of Class K; that the Cepheid variables are almost all of Classes F and G, with a few of A and K; and that almost all the irregular variables, and all the variables of long period, are of Classes M or N; Stars of Class M whose spectra show bright hydrogen lines are without exception variable, and almost all the stars of Class N are also subject to changes in brightness.

Having thus made a rapid survey of the general field, I will now ask your attention in greater detail to certain relations which have been the more special objects of my study.

Let us begin with the relations between the spectra and the real brightness of the stars. These have been discussed by many investigators,—notably by Kapteyn and Hertzsprung,—and many of the facts which will be brought before you are not new; but the observational material here presented is, I believe, much more extensive than has hitherto been assembled. We can only determine the real brightness of a star when we know its distance; but the recent accumulation of direct measures of parallax, and the discovery of several moving clusters of stars whose distances can be determined, put at our disposal far more extensive data than were available a few years ago.

Figure 1 shows graphically the results derived from all the direct measures of parallax available in the spring of 1913 (when the diagram was constructed). The spectral class appears as the horizontal coördinate, while the vertical one is the absolute magnitude, according to Kapteyn's definition,—that is, the visual magnitude which each star would appear to have if it should be brought up to a standard distance, corresponding to a parallax of $0''.1$ (no account being taken of any possible absorption of light in space.) The absolute magnitude -5, at the top of the diagram, corresponds to a luminosity 7500 times that of the Sun, whose absolute magnitude is 4.7. The absolute magnitude 14, at the bottom, corresponds to $\frac{1}{5000}$ of the Sun's luminosity. The larger dots denote the stars for which the computed probable error of the parallax is less than 42 per cent of the parallax itself, so that the probable error of the resulting absolute magnitude is less than ±1ᵐ.0.

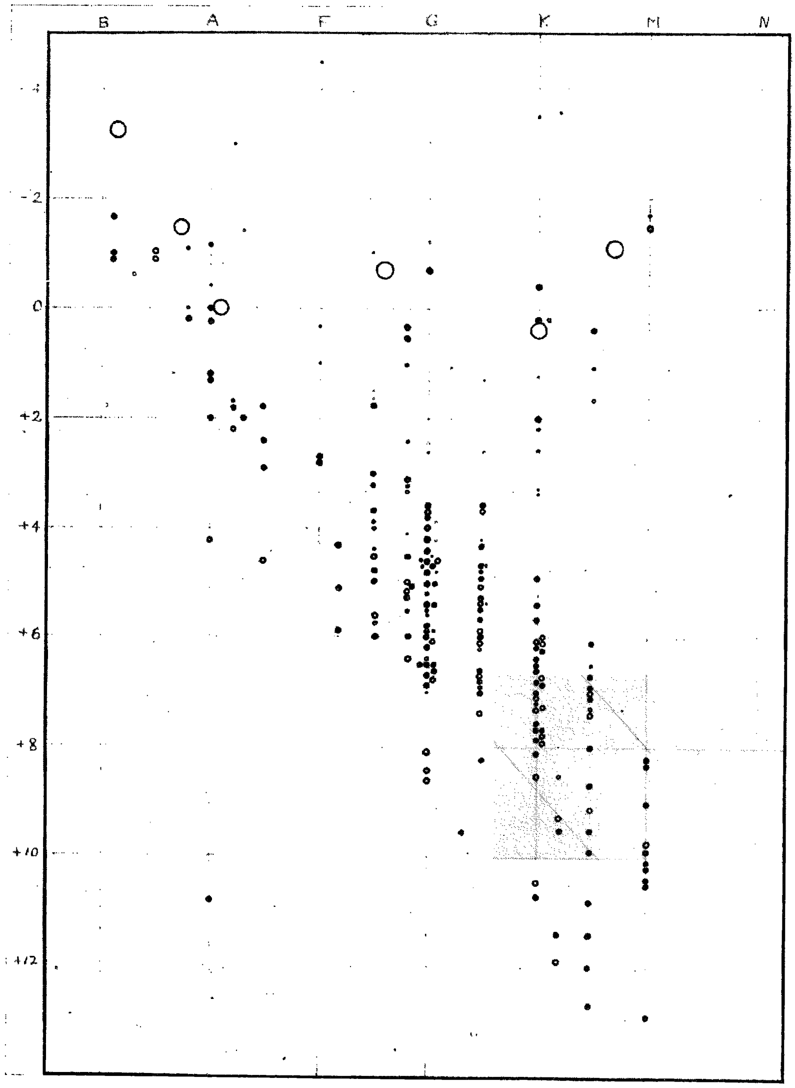

FIGURE 1.

This is a fairly tolerant criterion for a "good parallax", and the small dots, representing the results derived from the poor parallaxes, should hardly be used as a basis for any argument. The solid black dots represent stars whose parallaxes depend on the mean of two or more determinations; the open circles, those observed but once. In the latter case, only the results of those observers whose work appears to be nearly free from systematic error have been included, and in all cases the observed parallaxes have been corrected for the probable mean parallax of the comparison stars to which they were referred. The large open circles in the upper part of the diagram represent mean results for numerous bright stars of small proper motion (about 120 altogether) whose observed parallaxes hardly exceed their probable errors. In this case the best thing to do is to take means of the observed parallaxes and magnitudes for suitable groups of stars, and then calculate the absolute magnitudes of the typical stars thus defined. These will not exactly correspond to the mean of the individual absolute magnitudes, which we could obtain if we knew all the parallaxes exactly, but they are pretty certainly good enough for our purpose.

Upon studying Figure 1, several things can be observed.

1. All the white stars, of Classes B and A, are bright, far exceeding the Sun; and all the very faint stars,—for example, those less than $\frac{1}{50}$ as bright as the Sun,—are red, and of Classes K and M. We may make this statement more specific by saying, as Hertzsprung does,[*] that there is a certain limit of brightness for each spectral class, below which stars of this class are very rare, if they occur at all. Our diagram shows that this limit varies by rather more than two magnitudes from class to class. The single apparent exception is the faint double companion to o^2 Eridani, concerning whose parallax and brightness there can be no doubt, but whose spectrum, though apparently of Class A, is rendered very difficult of observation by the proximity of its far brighter primary.

2. On the other hand, there are many red stars of great brightness, such as Arcturus, Aldebaran and Antares, and these are as bright, on the average, as the stars of Class A, though probably fainter than those of Class B. Direct measures of parallax are unsuited to furnish even an estimate of the upper limit of brightness to which these stars attain, but it is clear that some stars of all the principal classes must be very bright. The range of actual brightness among the stars of each spectral class therefore increases steadily with increasing redness.

3. But it is further noteworthy that all the stars of Classes K5 and M which appear on our diagram are either very bright or very faint. There are none comparable with the Sun in brightness. We must be very careful here not to be misled by the results of the methods of selection employed by observers of stellar parallax. They have for the most part observed either the stars which appear brightest to the naked eye or stars of large proper motion. In the first case, the method of selection gives an enormous preference to stars of great luminosity, and, in

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[*] *A. N.* 4422, 1910.

the second, to the nearest and most rapidly moving stars, without much regard to their actual brightness. It is not surprising, therefore, that the stars picked out in the first way (and represented by the large circles in Figure 1) should be much brighter than those picked out by the second method, (and represented by the smaller dots). But if we consider the lower half of the diagram alone, in which all the stars have been picked out for proper-motion, we find that there are no very faint stars of Class G, and no relatively bright ones of Class M. As these stars were selected for observation entirely without consideration of their spectra, (most of which were then unknown) it seems clear that this difference, at least, is real, and that there is a real lack of red stars comparable in brightness to the Sun, relatively to the number of those 100 times fainter.

The appearance of Figure 1 therefore suggests the hypothesis that, if we could put on it some thousands of stars, instead of the 300 now available, and plot their absolute magnitudes without uncertainty arising from observational error, we would find the points representing them clustered principally close to two lines, one descending sharply along the diagonal, from B to M, the other starting also at B, but running almost horizontally. The individual points, though thickest near the diagonal line, would scatter above and below it to a vertical distance corresponding to at least two magnitudes, and similarly would be thickest near the horizontal line, but scatter above and below it to a distance which cannot so far be definitely specified, so that there would be two fairly broad bands in which most of the points lay. For Classes A and F, these two zones would overlap, while their outliers would still intermingle in Class G, and probably even in Class K. There would however be left a triangular space between the two zones, at the right-hand edge of the diagram, where very few, if any, points appeared; and the lower left-hand corner would be still more nearly vacant.

We may express this hypothesis in another form by saying that there are two great classes of stars,—the one of great brightness, (averaging perhaps a hundred times as bright as the Sun) and varying very little in brightness from one class of spectrum to another; the other of smaller brightness, which falls off very rapidly with increasing redness. These two classes of stars were first noticed by Hertzsprung,[*] who has applied to them the excellent names of *giant* and *dwarf* stars. The two groups, on account of the considerable internal differences in each, are only distinctly separated among the stars of Class K or redder. In Class F they are partially, and in Class A thoroughly intermingled, while the stars of Class B may be regarded equally well as belonging to either series.

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[*] Zeitschrift fur Wissenschaftliche Photographie, Vol. 3, p. 442, 1905.

In addition to the stars of directly measured parallax, represented in Figure 1, we know with high accuracy the distances and real brightness of about 150 stars which are members of the four moving clusters whose convergent points are known,—namely, the Hyades, the Ursa Major group, the 61 Cygni group, and the large group in Scorpius, discovered independently by Kapteyn, Eddington, and Benjamin Boss, whose motion appears to be almost entirely parallactic. The data for the stars of these four groups are plotted in Figure 2, on the same system as in Figure 1. The solid black dots denote the members of

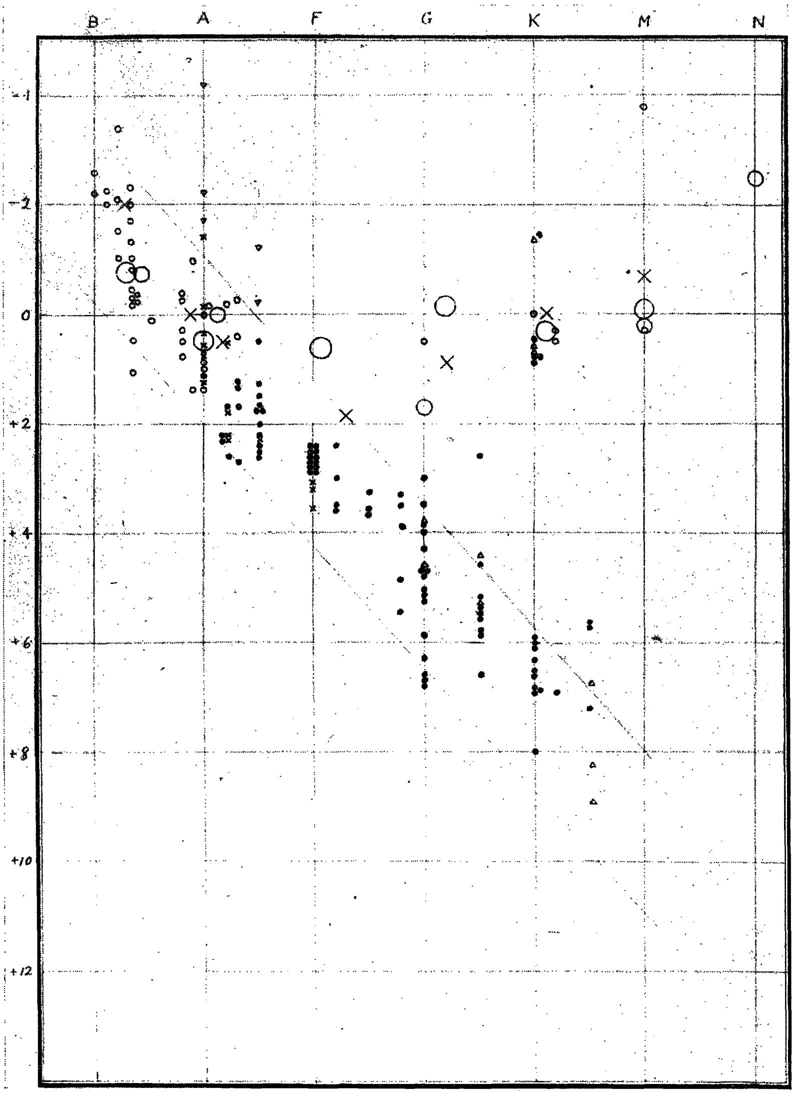

FIGURE 2.

the Hyades; the open circles, those of the group in Scorpius; the crosses the Ursa Major group, and the triangles the 61 Cygni group. Our lists of the members of each group are probably very nearly complete down to a certain limiting (visual) magnitude, but fail at this point, owing to lack of knowledge regarding the proper motions of the fainter stars. The apparently abrupt termination of the Hyades near the absolute magnitude 7.0, and of the Scorpius group at 1.5 arises from this observational limitation.

The large circles and crosses in the upper part of Figure 2 represent the absolute magnitudes calculated from the mean parallaxes and magnitudes of the groups of stars investigated by Kayteyn, Campbell, and Boss, concerning which data were given in Table III. The larger circles represent Boss's results, the smaller circles Kapteyn's, and the large crosses Campbell's.

It is evident that the conclusions previously drawn from Figure 1 are completely corroborated by these new and independent data. Most of the members of these clusters are dwarf stars, and it deserves particular notice that the stars of different clusters, which are presumably of different origin, are similar in absolute magnitude. But there are also a few giant stars, especially of Class K, (among which are the well known bright stars of this type in the Hyades); and most remarkable of all is Antares, which, though of Class M, shares the proper motion and radial velocity of the adjacent stars of Class B, and is the brightest star in the group, giving out about 2000 times the light of the Sun. It is also clear that the naked eye stars, studied by Boss, Campbell and Kapteyn, are for the most part giants. With this in mind, we are now in a position to explain more fully the differences between the results of these investigators.

All the stars of Class B are giants; and, so far as we may judge from the Scorpius cluster, they do not differ from one another very greatly in absolute brightness. It is therefore natural that the results of all three investigators are in this case fairly similar,—though Campbell, in employing stars that averaged brighter to the eye than did the others, has evidently been working with stars that are really brighter. In Class A, the giants and dwarfs differ so little, and are so thoroughly intermingled, that the situation is about the same. In Class M, even the nearest and brightest of the dwarf stars are invisible to the naked eye; hence the stars of this class studied by the three investigators are all giants, and once more their results agree.

A number of the dwarf stars of Class K are visible to the naked eye, but these all lie very near us, and have such large proper motions that they are excluded as "abnormal" by both Campbell and Boss. The results of the two agree in indicating that the stars studied by them are typical giants. The few dwarfs, however, have such large parallaxes and proper motions that their inclusion more than doubles the mean proper motion, and presumably also the mean parallax of the whole, as shown by Kapteyn's figures in Table III. For Class G, the dwarf stars average much brighter, and a much greater number of them are visible to the naked eye. These have large parallaxes and proper motions, and raise the average for all the stars of this class to greater values than for any other. But Boss's rigorous limitation to small proper motions practically weeds them all out, leaving giant stars once more. Campbell's less drastic procedure admits only the nearer of the dwarfs, (to be precise, those with the larger proper motions) and his result lies about half way between the others, In the case of Class F, the dwarf stars are still brighter,—intermingling, in fact, with the giants. We can therefore see them farther off, and we get more of them in our catalogues, in proportion to the giants, than in any other class. Their mean parallax is however smaller than for the dwarfs of Classes G and K, and hence the mean proper motion and parallax of all the stars of this class is less than for Class G. Campbell's criterion here excludes very few stars, and even Boss's admits a good many of the remoter and slower moving dwarfs, causing his mean parallax and proper motion to be considerably greater for this class than for any other.

It should finally be added that Kapteyn's discussion shows that the stars of class N are exceedingly bright, possibly surpassing any of the other giant stars.

We are now in a position to define more precisely the brightness of a typical giant or dwarf star of a given class of spectrum, and also to obtain a measure of the degree of divergence of the individual stars from this typical brightness. Taking first the stars of class B and the dwarf stars of the other classes, we find for the mean absolute magnitudes of all the stars of each class, the following values.

TABLE V

MEAN ABSOLUTE MAGNITUDES

Spectrum	Stars of Measured Parallax				Stars in Clusters			
	No.	Abs. Mag.	Formula	O—C	No.	Abs. Mag.	Formula	O—C
B2	...	...	...	...	21	—1.2	—1.1	—0.1
B8	...	...	...	...	8	+0.3	+0.2	+0.1
A0	6	+1.4	+1.4	0.0	13	0.5	0.6	—0.1
A4	7	2.5	2.3	+0.2	26	1.7	1.5	+0.2
F0	...	...	...	...	15	2.4	2.7	—0.3
F1	5	4.2	3.7	+0.5	...	...	...	...
F3	...	...	...	...	7	3.3	3.3	0.0
F5	9	4.3	4.5	—0.2	...	...	...	...
F8	8	5.1	5.2	—0.1	5	4.2	4.4	—0.2
G0	29	5.7	5.6	+0.1	18	5.0	4.8	+0.2
G5	19	5.7	6.6	—0.9	9	5.1	5.8	—0.7
K0	28	7.1	7.7	—0.6	9	6.4	6.9	—0.5
K4	19	9.2	8.6	+0.6	7	+7.0	+7.7	(—0.7)
Ma	10	+9.9	+9.8	+0.1	...	...	...	...

The rate of decrease of brightness with increasing redness is very nearly the same for the stars with directly measured parallaxes and the stars in clusters, but the latter appear, with remarkable consistency, to be about $0^{m}.8$ brighter than the former. This seems at first sight very puzzling, but it is undoubtedly due to the way in which the stars observed for parallax were selected. Most observers, in preparing their working lists, have included mainly those stars which were brighter than a given magnitude, and had proper motions exceeding some definite limit. Of the stars above this limiting magnitude, those of greater actual luminosity will be, on the average, farther away, and have smaller proper motions, than those of small luminosity, and selection by proper motion favors the latter. The limitation of our present lists to stars whose parallaxes have been determined with a probable error not exceeding 42 per cent of their own amounts, though necessary to diminish the effects of casual errors of observation, works in the same direction, for, among the stars of any given visual magnitude, those of greatest luminosity have the smallest parallaxes, and are least likely to pass the test. The difference shown in our table need not therefore alarm us, but it is clear that the stars in clusters, rather than the others, should be taken as typical of the dwarf stars as a whole. For both sets of stars the absolute magnitude appears to be very nearly a linear function of the spectral class, (if B is regarded as 1, A as 2, &c). The columns headed "Formula" in Table V give the values calculated from the expressions $M = 1^{m}.4 + 2^{m}.1\,(\mathrm{Sp.}-\mathrm{A})$ for the stars of directly measured parallax, and $M = 0^{m}.6 + 2^{m}.1\,(\mathrm{Sp.}-\mathrm{A})$ for the stars in clusters. The residuals from these empirical formulae, for the mean absolute magnitudes of the observed stars of different classes, average $\pm 0^{m}.33$ in the first case, and $\pm 0^{m}.29$ in the second. They appear to be accidental in character, though in some cases, (notably in class G5), the residuals for the stars of the two sets are similar in sign and magnitude. The large negative residuals for Classes K and K5 in the clusters arise from the fact that in the Hyades, which contribute most of these stars, only the brighter ones have had their proper motions determined, and get into our lists,—as is clear from examination of Figure 2.

Among the dwarf stars, therefore, a typical star of any spectral class is about seven times fainter than one of the preceding class, and seven times brighter than one of the following class.

The giant stars of all the spectral classes appear to be of about the same mean brightness,—averaging a little above absolute magnitude zero, that is, about a hundred times as bright as the Sun. Since the stars of this series which appear in Figure 2 have been selected by apparent brightness, which gives a strong preference to those of the greatest luminosity, the average brightness, of all the giant stars in a given region of space must be less than this,—perhaps considerably so.

By tabulating the residual differences between the absolute magnitudes of the individual dwarf stars and the values given by the formulae just described, we find that the average difference, regardless of sign, for the stars of measured parallax is $\pm 0^{m}.88$ for spectra A to F8, $\pm 1^{m}.02$ for spectra G and G5, and $\pm 1^{m}.15$ for K and M. For the stars in clusters the average differences are $\pm 0^{m}.70$ for spectra B0 to B9 $\pm 0^{m}.66$ for A and A5, $\pm 0^{m}.56$ for spectra F to F8, and $\pm 0^{m}.80$ for G and G5.

These differences are larger for the stars of measured parallax than for the others, (probably on account of the greater average uncertainty of the individual parallaxes and spectra in this case) but show no marked systematic variation with the class of spectrum. Their distribution follows very approximately the law of accidental errors, as is shown by Table VI, in which the observed numbers lying between certain limits are compared with those given by this law.

TABLE VI

DISTRIBUTION OF DIFFERENCES FROM THE TYPICAL ABSOLUTE MAGNITUDES

Stars with Measured Parallax			Stars in Clusters		
Limits	Observed	Theory	Limits	Observed	Theory
$\pm 0^{m}.0$ to $\pm 0^{m}.8$	65	61	$\pm 0^{m}.0$ to $\pm 0^{m}.5$	59	58
±0.8 to ±1.6	41	44	±0.5 to ±1.0	42	42
±1.6 to ±2.4	21	23	±1.0 to ±1.5	21	24
±2.4 to ±3.2	10	9	±1.5 to ±2.0	10	8
±3.2 to ±4.0	3	3	±2.0 to ±2.5	4	4

The theoretical distribution for the stars in clusters corresponds to a probable error of $\pm 0^{m}.61$, and that for the others to one of $\pm 0^{m}.94$. Correction for the known influence of uncertainties of the parallaxes and spectra would reduce the latter to about $\pm 0^{m}.75$. It appears therefore that the absolute magnitude of a dwarf star can be predicted with surprising accuracy from a mere knowledge of its spectrum. Half of all the dwarf stars are not more than twice as bright or as faint as the typical stars of their spectral classes. The corresponding uncertainty in the estimated parallax would be about one-third of its amount.

The parallaxes of the giant stars are so small, in comparison with the errors of even the best present methods of observation, that direct observations are not well adapted to determine to what degree they differ in brightness among themselves. An indirect method of determining this is however practicable, among those classes in which all the naked-eye stars are giants, by comparing the parallactic motions of those stars whose proper motions at right angles to the direction of the parallactic drift are large and small. A discussion by this method of the typical case of Class M, (the details of which will be given elsewhere), shows that, if the distribution of the absolute magnitudes of these stars also follows the "law of errors," the probable error corresponding to it is approximately $\pm 0^{m}.6$ —almost exactly the same as has already been found for the dwarf stars. The mean absolute magnitude of all the stars of this class which are visible to the naked eye is −0.5 and that of all the stars in a given region of space is +0.6. This method can hardly be applied to the naked-eye stars of the other spectral classes, (unless some way can be devised for weeding out the dwarf stars from among the giants); but it seems probable that they do not differ greatly from the stars of Classes B and M as regards the degree of their similarity to one another in brightness. With such a probable error of distribution of the absolute magnitudes as has here been derived, the giant and dwarf stars would overlap perceptibly in Class G, be just separated in Class K, and widely so in Class M, as the observational data indicate.

The questions now arise:—

What differences in their nature or constitution give rise to the differences in brightness between the giant and dwarf stars, and why should these differences show such a systematic increase with increasing redness, or "advancing" spectral type?

We must evidently attack the first of these questions before the second. The absolute magnitude (or the actual luminosity) of a star may be expressed as a function of three physically independent quantities,—its mass, its density, and its surface brightness. Great mass small density, and high surface brightness make for high luminosity, and the giant stars must possess at least one of these characteristics in a marked degree, while the dwarf stars must show one or more of the opposite attributes.

A good deal of information is available concerning all these characteristics of the stars. The masses of a considerable number of visual and spectroscopic binaries are known with tolerable accuracy, the densities of a larger number of eclipsing variable stars have recently been worked out; and the recent investigations on stellar temperatures lead directly to estimates of the relative surface brightness of the different spectral classes, (subject, of course, to the uncertainty whether the stars really radiate like black bodies, as they are assumed to do). We will take these matters up in order.

First, as regards the masses of the stars, we are confined to the study of binary systems, which may or may not be similar in mass to the other stars. There appears however to be no present evidence at all that they are different from the other stars, and in what follows we will assume them to be typical of the stars as a whole.

The most conspicuous thing about those stellar masses which have been determined with any approach to accuracy is their remarkable similarity. While the range in the known luminosities of the stars exceeds a million-fold, and that in the well-determined densities is nearly as great, the range in the masses so far investigated is only about fifty-fold. The greatest known masses are those of the components of the spectroscopic binary and eclipsing variable V Puppis, which equal nineteen times that of the Sun; the smallest masses concerning which we have any reliable knowledge belong to the faint components of Zeta Herculis and Procyon and are from one-third to one-fourth of the Sun's mass. These are exceptional values, and the components of most binary systems are more nearly similar to the Sun in mass.

There appears, from the rather scanty evidence at present available, to be some correlation between mass and luminosity. Those stars which are known to be of small mass (say less than half the Sun's) are all considerably fainter than the sun. On the other hand, Ludendorff* has shown conclusively that the average mass of the spectroscopic binaries of spectrum B (which are all of very great luminosity) is three times as great as that of the spectroscopic binaries of other spectral types, and may exceed ten times that of the Sun. Further evidence in favor of this view is found in the fact that the components of a binary, when equal in brightness, are nearly equal in mass, while in unequal pairs the brighter star is almost (if not quite) always the more massive, but the ratio of the masses very rarely exceeds 3:1, even when one component is hundreds of times as bright as the other. Very large masses (such as 100 times the Sun's mass) do not appear, though they would certainly be detected among the spectroscopic binaries, if they existed. It is equally remarkable that there is no reliable evidence that any visible star has a mass as small as one-tenth that of the Sun. The apparent exceptions which may be found in the literature of the subject may be shown to arise from faulty determinations of parallax, arbitrary estimates of quantities unobtainable by observation, (such as the ratio of the densities of the two components of Algol), and even numerical mistakes.

[*To be Continued.*]

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* *A. N.* 4520, 1911.