THE ORBITS OF EIGHTY-SEVEN ECLIPSING BINARIES—A SUMMARY

By HARLOW SHAPLEY

The results which are catalogued and briefly summarized in the present communication have been obtained from an extensive study of all accessible published and unpublished observations of eclipsing variables. More than a hundred thousand light-measures have been discussed in detail, representing the photometric work of thirty observers on the light-curves of nearly a hundred stars. The observations have been made in many ways: with the sliding-prism polarizing photometer, with the meridian, selenium, Zöllner, and wedge photometers, by measures of extra-focal plates, by estimates on the Harvard photographs, and by Argelander's method of visual estimates. The new methods of obtaining orbits from light-curves have so greatly diminished the labor of computation that it has been possible to develop in a relatively short time this branch of double-star astronomy. The catalogue of orbits given below contains 87 systems—a number that compares favorably with the lists of orbits of spectroscopic binaries and of visual double stars. Two or more solutions were made for each system. Of the total of 199 orbits, two were computed by Dugan, two by Stebbins, three by Roberts, eight by Russell, and 184 by the writer. The reader is referred to papers published during the last year for the theory of the orbits of eclipsing stars, and for examples of the solution for well-observed stars with many different types of light-curves.[1] The detailed discussion of the observational and computational

[1] H. N. Russell, "On the Determination of the Orbital Elements of Eclipsing Variable Stars," *Astrophysical Journal*, **35**, 315, 1911, and **36**, 54, 1912; "Elements of the Variables *W Delphini, W Ursae Majoris*, and *W Crucis*," *ibid.*, **36**, 133, 1912; H. N. Russell and H. Shapley, "On Darkening at the Limb in Eclipsing Variables," *ibid.*, **36**, 239 and 385, 1912; H. Shapley, "Elements of the Eclipsing Variables *W Delphini, S Cancri, SW Cygni*, and *U Cephei*," *ibid.*, **36**, 269, 1912; "The Visual and Photographic Ranges and Provisional Orbits of *Y Piscium* and *RR Draconis*," *ibid.*, **37**, 155, 1913; "The Orbits of *RZ Ophiuchi* and ϵ *Aurigae* Treated as Eclipsing Binaries," *Astronomische Nachrichten*, **194**, 225 (1913).

work, together with the results of the statistical investigation of the orbits, is to appear as a publication of the Princeton University Observatory.

In a work of this kind the investigation of every star cannot, of course, be considered as exhaustive and definitive. I have not utilized all the existing observations of the variables considered, nor tried to harmonize, explain, and adjust non-homogeneous sets of measures. The computations for each star have been based on what appeared to be the most complete and reliable series of observations, generally the work of some one observer being used, but occasionally the combined results of two or more; and in many cases the work in whole or in part has been based on unpublished photometric observations of my own. Whenever a good series of photometric measures has been available, estimates made by the Argelander method have been rejected. I am under obligation to a number of astronomers for assisting me with this study in various ways, but particularly to Professor Russell, who has directed and encouraged the investigation throughout and has helped with the computations in many cases; to Professor Pickering and Miss Cannon, who generously put at our disposal extensive unpublished photometric data and made special investigations of the spectra of many stars; and to Professor Nijland, of Utrecht, who has sent in manuscript light-curves based on long series of observations of 35 eclipsing systems, nearly one-half of which were stars for which no other data would have been available.

The stars in the following table have been divided into three general classes. In each division I have attempted to arrange the individual systems in order of the completeness of the photometric data, rather than in order of the degree of determinateness of the orbit obtained. The classification and order can be only approximate; but in general, stars in the first group have been so well observed that further photometric work will not appreciably change the solutions; orbits in the second group are susceptible of more or less improvement, as they are based on observations that are not as complete or as accurate as might be desired; while in the third class are listed those stars for which the observational data are very meager and uncertain, but concerning whose light-variations enough is known to make it possible to derive approximate orbits. Further observations will probably alter greatly some of the orbits in the third group. But certain factors in these systems (for instance, the most important one of all—the density) are derived with a precision sufficient to aid materially in the generic studies of eclipsing variables. The manuscript sent by Professor Nijland contains only the co-ordinates of smooth curves drawn to represent his series of observations at primary eclipse. The precision of the resulting orbits cannot be estimated without a knowledge of the accuracy with which the normal points are represented and of the uniformity of the distribution of the observations. Consequently I have placed all the orbits that depend only on the Nijland curves in a group by themselves, arranging them in order of number of observations involved. Doubtless some of the curves are of high accuracy, while others must be considered only provisional.

EXPLANATION OF THE TABLE OF ORBITS

Letters in column (3), indicating the observer, have the following significance (the complete bibliography will be given in the later publication):

B =Baker	L =Miss Leavitt	Ro =A. Roberts
C =Miss Cannon	Le =Lehnert	Se =Seares
D =Dugan	Lu =Ludendorff	Sh =Shapley
E =Enebo	Lz =Luizet	St =Stebbins
G =Graff	N =Nijland	Sw =Stratonow
H =Haynes	P =E. C. Pickering	W =Wendell
I =Ichinohe	Pa =J. A. Parkhurst	Wh =Miss Whiteside
In =Innes	Pr =Pračka	Wy =Wylie
J =Jordan	*r* =see remark	

Unpublished observations are indicated by "ms" in this column. In column (4) the period of revolution is rounded off to the third decimal place.[1] The magnitude at maximum is only approximate for most faint stars; its precise value is not important. The ranges, column (6), are "unrectified," that is, the variations due to

[1] For corrections to the light-elements of many stars, obtained during the course of the work, see *Popular Astronomy*, December 1912; March 1913; and *Astronomische Nachrichten*, **192**, 79, 1912. Also see note below on *RS Cephei*.

eclipse, ellipticity, and "reflection" are combined. The first number for each system pertains to primary minimum, and the second number to secondary minimum. When the secondary is computed, but has not been observed, the value is inclosed in brackets; when it is assumed for purposes of solution, it is in parentheses, and when observed, no brackets or parentheses are used. The computed secondary minima are always for "uniform" disks, the "darkened" values being about twice as great except where central transit restrictions exist. Spectra are taken from *H.A.*, **56**, VI, with many revisions and additions furnished by Miss Cannon. "*tf*" signifies "too faint to classify."

In the absence of definite knowledge concerning the degree of darkening toward the limb of the stellar disks, I have computed double sets of elements for all systems on the two extreme hypotheses of uniform disks and disks completely darkened at the edge. These solutions are designated by "U" and "D" in the eighth column. For some stars, for which the orbit is indeterminate between certain limits, I have given the limiting solutions, "uniform" or "darkened"; and for some (*RZ Cassiopeiae* and *U Coronae*, for instance), solutions depending on different assumptions concerning the secondary minimum. The units of light, length, and density are respectively the maximum light of the system, the radius of the relative orbit, and the solar density. L_b is the light of the brighter star; that of the fainter is $L_f = 1 - L_b$. Columns (10) and (11) contain r_b and r_f, the radii of the two components. When the stars are elliptical these columns contain their longest axes, a; the shorter equatorial axes, b, may be obtained from column (13), which contains b/a. Column (12) contains the cosine of the orbital inclination, that is, the projected distance of centers at the time of mid-eclipse. When cos i is given as (0) the assumption of a central eclipse was necessary—the elements that would naturally develop from the observations yielding an imaginary value of i. With the third-class stars, however, cos i = (0) often means that in the absence of a good light-curve the simpler solution of central transit was found to represent the observations satisfactorily. Columns (14) and (15) contain the densities of the brighter and fainter stars, computed in all cases on the assumption

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES

GRADE I. STARS WELL OBSERVED

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	*Z Draconis*..........	D	1^{d}357	10^{m}46	2^{m}55	*tf*	U_1	0.911	0.217	0.270	0.074	0.986	0.36	0.19	15.8
					0.06		U_2	.886	.214	.270	.070	.990	.39	.19	12.3
							D_2	.927	.257	.262	.054	.997	.22	.21	13.1
2	*RT Persei*...........	D	0.849	10.62	1.37	F?	U_1	.861	.274	.274	.076	.980	.45	.45	6.2
					0.17		U_2	.849	.272	.272	.078	.980	.48	.48	5.62
							D_2	.879	.320	.256	.026	.987	.29	.57	4.64
3	*β Aurigae*...........	St	3.960	2.07	0.09	Ap	U	.50	.146	.146	.220	.990	.14	.14	1.0
					0.09		D	.50	.159	.159	.229	.993	.11	.11	1.0
4	*β Persei*............	St	2.867	2.2	1.22	B8	U_1	.898	.210	.239	.134	....	.088	.060	11.4
					0.06		U_2	.907	.208	.228	.121	....	.092	.070	11.6
							D_2	.926	.241	.229	.129	....	.060	.069	11.3
5	*RZ Centauri*..........	L	1.876	8.48	0.46	A	U_1	.74	.490	.245	.221	.896	.020	.16	0.7
					0.34		D_1	.74	.481	.226	.239	.935	.020	.19	0.6
							D_2	.82	.491	.233	.000	.940	.018	.17	1.0
6	*U Pegasi*............	W	0.375	9.32	0.60	F?	U_1	.57	.50	.50	.264	.887	.....	.....	.....
					0.46		U_2	.603	.450	.450	.300	.770	.88	0.88	1.52
							U_3	.788	.544	.348	.223	.782	.49	1.86	1.52
							D_2	.603	.454	.454	.352	.858	.70	0.70	1.52
7	*WZ Cygni*...........	Sh	0.584	9.9	1.45	A	U	.794	.455	.387	.058	.842	.30	.49	2.8
		ms			0.44		D	.794	.473	.369	.094	0.904	.23	.48	2.4
8	*S Cancri*............	W	9.485	7.98	2.12	A	U	.858	.075	.203	.119	....	.178	.009	44.0
					0.04:		D	.858	.096	.184	.080	....	.084	.012	22.0
9	*SW Cygni*...........	W	4.573	9.06	2.66	A	U	.914	.131	.266	.110	....	.141	.017	43.0
					[0.02]		D	.914	.166	.247	.044	....	.070	.021	24.0
10	*U Cephei*............	W	2.493	6.78	2.39	A	U	.890	.205	.324	.000	....	.126	.032	20.0
					0.05:		D	.890	.219	.319	(0)	....	.104	.034	17.0
11	*W Delphini*..........	W	4.806	9.40	2.70	A	U	.917	.135	.256	.114	....	.118	.017	40.0
					[0.03]		D	0.917	0.170	0.241	0.068	....	0.060	0.021	22.0

12	*RZ Cassiopeiae*........	PaJ	1.195	6.43	1.22	A	U_1	1.00	0.273	0.257	0.118	....	0.24	0.28	∞
					(0.00)		U_2	0.913	.261	.269	.118	....	.27	.24	11.0
					or		D_1	1.00	.320	.256	.111	....	.14	.28	∞
					(0.07)		D_2	0.936	.313	.263	.106	....	.15	.26	10.0
13	*RX Herculis*..........	Sh	1.779	7.0	0.49	B9	U_1	.50	.190	.190	.084	....	.31	.31	1.0
		ms			0.49		U_2	.639	.213	.162	.051	....	.22	.50	1.0
							D_1	.50	.202	.202	.110	....	.26	.26	1.0
14	*V Serpentis*...........	L	3.454	9.52	0.94	A	U	.587	.185	.296	.131	.947	.099	.024	3.6
					0.24		D	.587	.224	.280	.098	.968	.053	.027	2.2
15	*U Sagittae*............	W	3.381	6.43	2.76	B8	U	.921	.220	.291	.000	....	.055	.024	20.0
					[0.05]		D	.917	.238	.288	(0)	....	.043	.025	17.0
16	*RX Draconis*..........	HSh	3.786	10.20	0.50	F	U_1	.50	.099	.099	.037	....	.49	.49	1.0
		ms			0.50		U_2	.63	.111	.086	.024	....	.35	.75	1.0
							D_1	.50	.104	.104	.170	....	.42	.42	1.0
17	*u Herculis*............	W	2.051	4.61	0.71	B3	U	.715	.312	.372	.249	.887	.067	.039	3.6
					0.24		D	.715	.308	.362	.278	.931	.064	.039	3.5
18	*U Ophiuchi*...........	PW	1.677	5.67	0.69	B8	U_1	.637	.224	.276	.048	.910	.14	.26	1.15
					0.59		U_2	.535	.252	.252	.079	.909	.18	.18	1.15
							D_2	.535	.252	.252	.109	.945	.17	.17	1.15
19	*ST Carinae*...........	L	0.902	9.31	0.87	A	U	.865	.273	.345	.245	....	.40	.20	10.3
					0.24		D	.868	.291	.363	.276	....	.33	.17	10.3
20	*RW Tauri*...........	Sh	2.769	8.05	3.42	B5	U_1	.957	.135	.254	.110	....	.36	.054	79.0
		ms			[0.02]		D_1	.957	.169	.241	.060	....	.18	.063	45.0
							U_2	.950	.136	.234	.084	....	.35	.069	56.0
21	*ZZ Cygni*............	Sh	0.629	10.59	1.09	A?	U	.893	.287	.354	.215	....	.71	.38	12.7
		ms			0.06:		D_1	.918	.332	.357	.226	....	.46	.38	13.0
							D_2	.933	.361	.340	.197	....	.36	.43	12.3
22	*UW Cygni*...........	W	3.451	10.55	2.57	A?	U	.907	.175	.208	.000	....	.099	.063	13.0
					[0.08]		D	.907	.196	.209	.000	....	.075	.062	11.0
23	*R Canis Majoris*......	W	1.136	5.38	0.60	F	U_1	.871	.346	.277	.220	....	.13	.25	4.3
					0.09		D_1	.782	.336	.336	.308	....	.14	.14	3.6
							U_2	.904	.266	.213	.140	....	.28	.55	6.0
							U_3	0.869	0.288	0.257	0.196	....	0.22	0.31	5.3

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES—*Continued*

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos *i*	*b/a*	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
24	*W Ursae Majoris*....	r	0.334	7.91	0.60	G	U_1	0.760	0.431	0.242	0.143	0.757	1.32	7.39	1.0
					0.60		U_2	.500	.366	.366	.234	.745	2.22	2.22	1.0
							D_2	.500	.368	.368	.260	.850	1.72	1.72	1.0
25	*W Crucis*..........	L	198.5	8.90	0.60	Gp	U	.676	.520	.197	.150	.855	2×10^{-6}	3×10^{-5}	0.3
					0.28		D	.904	.613	.343	.247	.912	10^{-6}	5×10^{-6}	3.0
26	*RR Centauri*........	Ro	0.606	7.38	0.44	F	U	.563	.506	.506	.535	.631	0.35	0.35	1.3
					0.42		D	.574	.500	.500	.668	0.685	.31	.31	1.4
27	*RS Sagittarii*	Ro	2.416	6.10	0.74	A	U	.824	.324	.253	.076	....	.034	.071	2.85
					0.21		D	.813	.347	.277	.148	....	.028	.055	2.8
28	*S Velorum*..........	Ro	5.934	7.85	1.39	A	U	.722	.104	.230	.000	....	.170	.016	13.0
					[0.06]		D	0.722	0.121	0.233	(0)	....	0.106	0.015	9.6
	Grade II. Stars Fairly Well Observed														
29	*RW Monocerotis*.....	HSh	1.906	8.75	1.72	A	U	0.795	0.207	0.287	(0)	....	0.21	0.078	7.5
		ms			0.10:		D	.795	.218	.282	(0)	....	.18	.082	6.5
30	*V Puppis*...........	Ro	1.454	4.14	0.64	B1	U	.60	.456	.420	0.236	0.813	.051	.065	1.27
					0.53	B3	D	.60	.461	.420	.289	.885	.042	.055	1.24
31	*Y Piscium*..........	C	3.766	9.00	3.40	A	U	.975	.144	.240	.107	....	.157	.034	84.0
		ms			(0.01)		D	0.975	.178	.231	.075	....	.084	.038	66.0
32	*U Coronae*..........	W	3.452	7.52	1.18	A	U_1	1.00	.188	.265	.193	....	.084	.030	∞
					(0.00)		U_2	0.95	.183	.269	.189	....	.093	.029	37.0
					(0.02)		U_3	.86	.176	.274	.180	....	.103	.028	15.0
					(0.05)		U_4	.74	.167	.273	.148	....	.120	.028	8.0
					(0.10)		D_3	.83	.202	.273	.168	....	.069	.028	9.0
33	*SZ Centauri*	L	4.108	8.18	0.65	A	U_1	.59	.302	.242	.046	.933	.017	.032	0.9
					0.58		U_2	.626	.302	.242	.046	.933	.017	.032	1.1
							D	0.59	0.296	0.219	0.074	0.959	0.017	0.041	0.8

34	*Z Herculis*..........	W	3.993	7.10	0.80	F	U	0.508	0.097	0.207	0.110	....	0.45	0.047	4.7
					0.12		D	.570	.117	.217	.139	....	.26	.040	4.5
35	*U Scuti*............	W	0.955	9.67	0.96	A	U	.900	.393	.295	.077	0.841	.17	.40	5.1
					0.30		D	.900	.418	.284	.121	.904	.12	.39	4.2
36	*Z Vulpeculae*........	B	2.455	7.80	1.65	A	U	.749	.251	.314	.055	.872	.094	.048	4.7
					0.34		D	.749	.281	.296	.014	.922	.060	.051	3.3
37	*δ Librae*............	W	2.327	4.83	1.10	A	U	.945	.324	.278	.104	....	.037	.058	13.0
					0.05:		D	.95	.386	.290	.098	....	.022	.051	11.0
38	*β Lyrae*............	Sw	12.916	3.36	0.97	B8 } p	D	.60	.271	.678	.499	.758	.0035	.0002	9.4
					0.45	B5 }									
39	*RT Lacertae*........	LzE	5.074	9.06	1.06	G5?	U	.602	.343	.343	.000	.949	.007	.007	1.5
					0.61		D	.602	.295	.295	.000	0.968	.011	.011	1.5
40	*SU Centauri*........	L	5.354	8.73	0.87	F2	U	.926	.225	.203	.115	....	.021	.028	10.2
					0.06:		D	.934	.253	.213	.136	....	.014	.024	10.0
41	*RR Draconis*........	Sh	2.831	9.98	2.96	A	U	.934	.099	.249	.136	....	.86	.054	90.0
		ms			[0.01]		D	.934	.131	.226	.076	....	.37	.072	42.0
42	*UZ Cygni*..........	W	31.304	10.29	1.88	A	U	.823	.070	.174	.049	....	.020	.0013	29.0
					[0.03]		D	.823	.084	.167	.000	....	.012	.0011	18.5
43	*VW Cygni*.........	G	8.431	10.32	1.94	A	U	.832	.106	.222	.036′	....	.078	.009	22.0
					[0.04]		D	.832	.117	.217	(0)	....	0.059	0.009	17.0
44	*ε Aurigae*..........	Lu	9905.	3.26	0.75	F8p	U_1	.50	.030	.298	.249	....	3×10^{-6}	3×10^{-9}	100.0
							U_2	.50	.059	.170	(0)	....	3×10^{-7}	10^{-8}	8.0
							D_1	.50	.031	.307	.225	....	2×10^{-6}	2×10^{-9}	100.0
							D_2	.50	.069	.173	(0)	....	2×10^{-7}	10^{-8}	6.0
45	*RW Capricorni*......	Sh	3.392	9.2	1.45	A	U	.737	.206	.237	.031	....	0.067	0.044	3.7
		ms			0.24		D	.788	.229	.229	.045	....	.049	.049	3.7
46	*RT Scuti*...........	Wh	0.512	9.65	0.71	F	U	.941	.435	.524	.442	0.800	.49	.28	23.0
					0.20:		D	.955	.498	.498	.452	0.880	.27	.27	21.0
47	*RS Cephei*..........	W	12.42	10.19	1.66	Ap	U	.783	.057	.229	.133	....	.23	.004	58.0
		ms			[0.02]		D	.783	.079	.197	.026	....	.089	.006	22.0
48	*SV Centauri*........	L	1.661	8.80	0.86	A	U	.72	.323	.323	.122	....	.072	.072	2.6
					0.26		D	.771	.369	.314	.142	....	.049	0.079	4.7
49	*RZ Ophiuchi*........	SeN	261.8	9.75	0.83	G8	U	.53	.037	.185	.112	....	.002	2×10^{-5}	28.0
		ms			[0.02]		D	0.53	0.046	0.154	0.047	....	0.001	3×10^{-5}	12.0

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES—*Continued*

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
50	*SS Centauri*	L	2.479	8.73	1.55	B9	U	0.939	0.174	0.272	0.167	....	0.21	0.055	30.0
					(<0.05)		D	1.00	.213	.263	.146	....	.11	.060	∞
51	*SS Carinae*	L	6.602	12.29	0.66	*tf*	U_1	0.544	.113	.123	.010	....	.106	.083	1.0
					0.66		U_2	.500	.118	.118	.016	....	.094	.094	1.0
							D_2	.500	.115	.115	.016	....	.10	.10	1.0
52	*WW Cygni*	G	3.318	10.00	2.91	Ap	U	.931	.188	.265	.073	....	.091	.032	27.0
					[0.04]		D	.931	.226	.258	.016	....	.053	.035	14.0
53	*RR Velorum*	Ro	1.854	10.00	0.91	A	U	.691	.140	.197	.108	....	.71	.25	4.4
					0.15		D	0.767	0.166	0.195	0.120	....	0.42	0.26	4.5
	STARS WITH LIGHT-CURVES DETERMINED BY NIJLAND														
54*	*RY Persei*	N	6.864	8.20	2.37	A2	U	0.887	0.144	0.252	0.081	....	0.048	0.009	24.0
		ms			[0.04]		D	.887	.179	.242	.000	....	.025	.010	14.0
55	*RW Geminorum*	N	2.865	9.80	2.14	A	U	.861	.217	.319	.004	....	.080	.025	14.0
		ms			[0.07]		D	.861	.248	.310	(0)	....	.054	.027	10.0
56	*Z Persei*	N	3.056	9.60	2.70	A	U	.917	.116	.232	(0)	....	.46	.057	44.0
		ms			[0.02]		D	0.917	.132	.231	(0)	....	.31	.058	34.0
57	*Y Camelopardalis*	N	3.306	9.75	2.02		U	1.00	.180	.243	.121	....	.106	.043	....
		ms			(0.00)		U	0.844	.171	.244	.073	....	.123	.043	11.0
					(0.09)		D	0.844	.207	.238	.030	....	.069	.045	7.0
58*	*RR Delphini*	N	4.599	10.50	1.20	A	U_1	1.00	.092	.341	.313	....	.40	.008	∞
		ms			(0.00)		U_2	0.669	.106	.234	.128	....	.27	.025	9.6
					(0.08)		D_2	.673	.136	.216	.087	....	.13	.032	5.2
59	*ST Persei*	N	2.648	9.80	2.08	A	U	.853	.169	.264	.083	....	.20	.052	14.0
		ms			[0.07]		D	.853	.206	.253	.000	....	.11	.058	8.8
60	*RV Persei*	N	1.974	10.75	2.20	A	U_1	.954	.291	.434	.208	....	.070	.021	45.0
		ms			(0.02)		U_2	.868	.290	.427	.137	....	.070	.022	14.0
					(0.07)		D_2	0.870	0.355	0.418	0.065	....	0.039	0.024	9.0

61	*SY Andromedae*.....	N	34.912	10.65	1.50	A?	U	0.749	0.036	0.119	(o)	....	0.120	0.0033	33.0
		ms			[0.02]		D	.749	.042	.120	(o)	....	0.075	.0032	24.0
62	*RW Ursae Majoris* ..	N	7.328	10.35	1.05	G?	U	.62	.025	.249	0.186	....	1.00	.008	41.0
		ms			[0.01]		D	.62	.071	.202	.112	....	0.35	.015	13.0
63	*TW Draconis*.......	N	2.806	7.30	1.60	B9	U	.771	.130	.371	.244	....	.39	.017	26.0
		ms			(0.03)		D	0.780	.180	.334	.177	....	.14	.023	12.0
64	*TT Lyrae*..........	N	5.244	9.45	2.20	A	U_1	1.00	.128	.284	.198	....	.118	.011	∞
		ms			(0.00)		U_2	0.868	.132	.254	.122	....	.106	.015	25.0
					(0.04)		D_2	.877	.152	.253	.116	....	.070	.015	20.0
65	*TT Andromedae*.....	N	2.765	11.30	1.30	A	U	.698	.158	.244	.084	....	.22	.060	5.5
		ms			[0.15]		D	.803	.177	.253	.141	....	.16	.055	8.4
66	*SY Cygni*..........	N	6.006	10.90	2.30	G5?	U	.88	.090	.237	(o)	....	.25	.014	51.0
		ms			[0.02]		D	.88	.100	.237	(o)	....	.19	.014	42.0
67	*RS Vulpeculae*	N	4.477	7.35	0.68	A	U	.535	.272	.082	.030	....	.017	.61	0.10
		ms			[0.05]		D	.535	.252	.088	(o)	....	.021	.50	0.14
68	*TV Cassiopeiae*......	N	1.813	7.35	1.00	B9	U	.925	.248	.261	.155	....	.14	.12	14.0
		ms			(0.05)		D	.923	.272	.272	.176	....	.10	.10	12.0
69	3.1911 (*Cancri*).....	N	10.174	10.10	1.55	*tf*	U	.76	.048	.194	.105	....	.57	.009	51.0
		ms			[0.02]		D	.76	.057	.190	.096	....	.35	.010	35.0
70	*VV Cygni*..........	N	1.477	12.85	0.75	*tf*	U	.832	.211	.222	.149	....	.33	.28	5.5
		ms			(0.10)		D	.874	.251	.224	.161	....	.20	.27	5.5
71	*RV Lyrae*..........	N	3.599	11.60	1.90	A	U	.826	.089	.296	.191	....	.73	.020	53.0
		ms			[0.02]		D	0.826	0.126	0.258	0.107	....	0.26	0.031	20.0

GRADE III. STARS INSUFFICIENTLY OBSERVED

72	*RZ Draconis*........	W	0.551	9.97	0.80	Ap	U	0.90	0.34	0.44	0.330	0.815	0.82	0.39	15.0
					0.22		D	.96	.40	.40	.332	.886	.43	.43	22.0
73	*SZ Herculis*.........	Sh	0.818	9.5	1.48	*tf*	U	.74	.25	.33	.019	....	.62	.28	4.9
		ms			[0.18]		D	.74	.27	.32	.000	....	.54	.31	4.2
74	*SV Tauri*..........	E	2.167	9.37	0.72	A	U	.96	.27	.19	.000	....	.075	.21	12.0
					0.05:		D	0.95	0.30	0.19	(o)	....	0.051	0.20	7.5

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES—*Continued*

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
75	*RX Cassiopeiae*	W	32.315	8.66	0.69	KO	U	0.57	0.27	0.27	0.136	0.809	0.0005	0.0005	1.3
					0.55		D	.57	.28	.28	.170	.880	.0004	.0004	1.3
76	*RZ Scuti*	BWy	15.194	7.47	1.38	B3	U	.88	.143	.298	.216	....	.010	.0011	29.0
					0.03:		D	.90	.157	.314	.238	....	.008	.0010	36.0
77	*Y Cygni*	W	2.996	6.95	0.60	A	U	.58	.167	.167	.070	....	.16	.16	1.4
		ms			0.40		D	.58	.166	.166	.088	....	.16	.16	1.4
78	*SX Cassiopeiae*	Lz	36.572	8.68	1.00	G3	U	.53	.21	.32	.115	.846	.0008	.0002	2.7
					0.41		D	.66	.26	.32	.180	.905	.0003	.0002	2.9
79	*SY Centauri*	L	6.631	9.88	0.87	A	U	.76	.15	.45	.39	....	.042	.002	28.0
					(0.02)		D	.79	.18	.45	.39	....	.026	.002	33.0
80	*SW Centauri*	L	5.219	9.12	2.33	A	U	.88	.09	.23	(0)	....	.32	.021	47.0
					0.02		D	.88	.10	.23	(0)	....	.22	.021	36.0
81	*RS Scuti*	I	1.329	8.86	0.93	F	U	.50	.26	.26	.00	.848	.30	.30	1.0
					0.93		D	.50	.27	.27	.00	.905	.24	.24	1.0
82	*RZ Aurigae*	Pr	3.011	10.5	1.74	*tf*	U	.80	.21	.27	.05	....	.080	.036	6.7
					[0.13]		D	.80	.26	.27	.00	....	.044	.036	4.2
83	*RW Persei*	P	13.199	9.5	2.2	A	U	.87	.06	.16	(0)	....	.14	.009	41.0
					[0.02]		D	.87	.08	.16	(0)	....	.09	.009	30.0
84	*RR Puppis*	In	6.430	9.45	1.11	A	U	.64	.10	.24	(0)	....	.17	.012	10.0
					[0.07]		D	.64	.11	.24	(0)	....	.12	.013	8.0
85	*Y Leonis*	Le	1.686	9.4	2.75	A	U	.92	.16	.26	.10	....	.60	.13	31.0
					[0.03]		D	.92	.20	.25	.07	....	.31	.14	19.0
86	*SX Sagittarii*	P	2.077	8.58	0.82	A5	U	.53	.19	.44	(0)	....	.22	.018	6.1
					[0.10]		D	.53	.25	.40	(0)	....	.11	.23	2.3
87	*X Carinae*	Ro	1.083	7.9	0.8	A	U	0.52	0.50	0.50	0.10	0.878	0.06	0.06	1.1

that the mass of each system is equally divided between the two components. This assumption will in general give the density for the bright star too low, and for the faint companion too high. The densities are usually given to the second significant figure, though they are often entirely uncertain in the last place. In the final column is given the ratio of the surface intensity of the star that has a majority of the light, to the surface intensity of the other. This ratio is greater than unity except in rare instances where the star that has the most light has a lower intensity per unit area.

AUXILIARY TABLES

1. Eccentricity of orbit has been determined for the stars listed below. In some other cases the orbits are known to be practically circular, but in most systems the evidence is insufficient. When only $e \cos \omega$ has been found from the displacement of the secondary minimum, I have assumed for this table $\omega = (0°)$ or $(180°)$, which gives minimum eccentricity; the uncertainty of such a determination justifies giving only circular elements in the catalogue. Spectrographic data were available for Nos. 17, 23, and 37.

	Star	Eccent.	Long. of Periastron		Star	Eccent.	Long. of Periastron
1	*Z Drac*......	0.010	(0°)	25	*W Cruc*......	0.06	(180°)
2	*RT Pers*.....	.012	(0)	27	*RS Sag*......	.092	261.1
17	*u Herc*......	.053	66.9	37	*δ Libr*.......	.054	29.2
19	*ST Car*.....	.052	(0)	78	*SX Cass*.....	.043	(180)
23	*R Can. Maj*..	0.138	90	87	*X Carin*......	0.02	165

2. With the aid of spectrographic data it is possible to find for four systems the radius of the orbit, dimensions of the stars, and

	Star		Masses		Maxima Radii		Densities		Distance of Centers
			m_b	m_f	r_b	r_f	ρ_b	ρ_f	
3	*β Aurig.*	Unif........	2.38	2.34	2.58	2.58	0.14	0.14	17.7
		Dark.......	2.40	2.36	2.81	2.81	0.11	0.11	17.7
17	*u Herc.*	Unif........	7.50	2.87	4.60	5.48	0.097	0.022	14.7
		Dark.......	7.66	2.93	4.56	5.35	0.095	0.022	14.8
30	*V Pupp.*	Unif........	18.7	18.7	8.23	7.57	0.051	0.065	12.5
		Dark.......	19.4	19.4	8.45	7.70	0.042	0.055	12.7
38	*β Lyrae*	Dark.......	1.42	14.2	16.2	40.6	0.0006	0.0004	59.9

actual masses and densities—all in terms of the sun. The masses in the system of *V Puppis* are assumed equal. The mass ratio for *β Lyrae* was taken as 10/1 (see note).

3. A "reflection" effect has been detected in a few accurately observed stars, and no doubt its occurrence would be found quite general if the precision of the observations was increased. The bright side of the companion (toward the primary) gives out the light $1-L_b$ (see ninth column of the catalogue); the light of the opposite side is less by 0.040 for *Z Draconis*, 0.022 for *RT Persei*, 0.044 for *β Persei*, 0.025 for *RZ Centauri*, and 0.03 for *Z Herculis*.

NOTES TO THE CATALOGUE AND TABLES

1. *Z Draconis*.—The accuracy of the light-curves of the first four stars greatly exceeds that of all other eclipsing binaries. Their orbits will be discussed *in extenso* in a paper soon to be published. The solutions U_1 for *RT Persei* and for *Z Draconis* are by Dugan; other solutions for *Z Draconis* and D for *β Aurigae* are by Russell; U for *β Aurigae* and U_1 for *Algol* are by Stebbins.

6. *U Pegasi*.—First solution by Roberts, assuming stars in contact, *M.N.*, **66**, 135, 1906. Solutions indeterminate over a small range.

8. *S Cancri, SW Cygni, U Cephei, W Delphini.* See *Astrophysical Journal*, **36**, 269, 1912.

13. *RX Herculis*, 15. *U Sagittae*.—Harvard classifies spectra as A; Frost mentions helium lines; see *Astrophysical Journal*, **22**, 214, 215, 1905.

23. *R Canis Majoris*.—These orbits are based on Wendell's observations of 1898–1899 which show the secondary minimum exactly halfway between successive primaries. Jordan's elements from Allegheny spectrograms give $e=0.138$, $\omega=196°$ in 1908. Hence the line of apsides must be in motion, but there are no data to estimate its rate of revolution. With this value of the eccentricity, and considering that primary eclipse occurred at periastron, solutions U_1 and D_1 are obtained. They represent the observations satisfactorily. For the purpose of illustration U_2, which assumes the primary at apastron and fails to fit the secondary minimum well, and U_3, which is the set of circular elements, are given.

24. *W Ursae Majoris*.—Observations by Müller, Kempf, and Baldwin; solutions by Russell; see *Astrophysical Journal*, **36**, 139, 1912.

25. *W Crucis*.—Uniform solution by Russell; see *Astrophysical Journal*, **36**, 146, 1912.

26. *RR Centauri*.—Uniform solution by Roberts, *M.N.*, **63**, 545, 1904.

31. *Y Piscium*, 41. *RR Draconis*.—See *Astrophysical Journal*, **37**, 155, 1913.

38. *β Lyrae.*—The orbits previously obtained by Stein, Myers, Roberts, and von Hepperger are not consistent with the statement of Curtiss in *Alleg. Bull.*, **2**, 115, 1911, that the star eclipsed at primary minimum has the stronger continuous spectrum. In a recent letter Curtiss estimates that the primary star of type B8 has 60 per cent of the light of the system. The primary eclipse must then necessarily be partial, with large faint star in front. I am able to find no possible "uniform" orbit, but the "darkened" solution represents the light-variations quite satisfactorily, and at the same time conforms to the first hypothetical system deduced by Curtiss from his extensive spectroscopic investigation. He finds that the brighter star has at most one-tenth the mass of the other.

39. *RT Lacertae.*—Uniform solution very unsatisfactory; probably definite evidence of darkening toward the limb. *Astrophysical Journal*, **36**, 401, 1912.

42. *UZ Cygni.*—I find that Wendell's photometric observations contradict Hartwig's visual estimates relative to the secondary minimum. *A.N.*, **165**, 121, 1904, *V.J.S.*, **39**, 254, 1904; **40**, 329, 1904.

44. *ε Aurigae*, 49. *RZ Ophiuchi.*—See recent discussion of orbits in *A.N.*, **194**, 225 (1913). Spectrum of *RZ Ophiuchi* is estimated G5 to K0.

47. *RS Cephei.*—I have determined new light-elements from Wendell's manuscript observations: Min. = J.D. 2417140.469, G.M.T. + $12^{d}4204$.E.

54–71. *Nijland's stars.*—Variables for which the series of observations have been completed by Professor Nijland are marked with asterisks.

72. *RZ Draconis.*—A complete study of this star is being made at Princeton.

73. *SZ Herculis.*—Comparison star probably variable.

77. *Y Cygni.*—Only circular elements are possible from existing data; according to Dunér the orbit is highly eccentric.

78. *SX Cassiopeiae.*—Spectrum G0 to G5.

85. *Y Leonis.*—Elements from rough light-curve; observed magnitudes not available.

CONCLUSIONS

Among the general results obtained from the present investigation the following points may be briefly mentioned. The complete statistical discussion will be published later.

1. The better the observations of an eclipsing binary are, the more satisfactory is the theoretical representation of the light-variations. Irregularities in the shape of light-curves disappear with increasing photometric accuracy. Halts and inflections in them have no objective existence, and the only apparently real peculiarities are occasional slight asymmetries and brightening

toward periastron (*Astrophysical Journal*, **36**, 278, 1912; **36**, 146, 1912).

2. The existence of darkening toward the limb of stellar disks is indicated in a large number of systems by the slightly better agreement of the "darkened" solution with the observed data, and its existence is actually demonstrated in a few cases. The degree of darkening, however, is as yet quite indeterminate.

3. In all but one of 28 first-grade stars, three of 25 second grade, and one of 16 of the third grade, there is a positive indication that the fainter star is self-luminous, and in no case is it necessary to assume one component completely black. In about two-thirds of the systems the difference in brightness of the components does not exceed two magnitudes, and no observed difference is greater than four magnitudes.

4. Regarding the relative sizes of the two components of an eclipsing system the following table shows that the conspicuous preponderance of systems in which the fainter star is the larger is entirely a matter of selection; and suggests, further, that there exist great numbers of eclipsing stars of small range in which the faint companion is smaller.

Range		Faint Star Large	Bright Star Large	Stars Nearly Equal
Greater than one magnitude	Unif.	47	2	2
	Dark.	44	4	3
Less than one magnitude	Unif.	12	10	13
	Dark.	9	12	14
Total	Unif.	59	12	15
	Dark.	53	16	17

5. Whenever the relative color-index of the components has been determined from the difference between the photographic and visual ranges at total eclipse, the large faint star has been found to be the redder. These stars are therefore presumably of "later" spectral type than their primaries, but columns (14) and (15) of the catalogue show that they are almost certainly less dense. See *Astrophysical Journal*, **37**, 155 ff., 1913, for more complete discussion.

6. The relation between the separation of the components of a close system and the gravitational elongation is shown in the following table where the ratio of the equatorial axes, b/a, for "uniform" and "darkened" solutions is compared with Darwin's theoretical value for homogeneous, incompressible fluid.[1] In forming the groups in order of separation of the stellar disks, I have excluded *X Carinae*, for which the observational data are not available.

Number Stars	Mean Separation $1-a_b-a_f$	Uniform b/a	Darkened b/a	Darwin b/a
5	0.501	0.971	0.983	0.944
5	.399	.900	.939	.902
5	.315	.847	.906	.857
4	.196	.809	.883	.772
4	0.106	0.700	0.788	0.692

7. In forming a table showing the distribution of densities relative to spectra, the "darkened" values have been used; the relative distribution would be altered but little if the "uniform" densities had been taken.

Density	B	A	F	G	K
>1.00	...	...	...	1	...
1.00 to 0.50	...	...	1	...	...
0.50 to 0.20	1	10	6	1	...
0.20 to 0.10	4	12	1	1	...
0.10 to 0.05	3	17	...	...	...
0.05 to 0.02	2	8	...	...	...
0.02 to 0.01	...	3	1	1	...
0.01 to 0.001	2	...	...	...	...
0.001 to 0.0001	...	...	...	2	1
<0.0001	...	...	1	1	...
Total	12	50	10	7	1

The first-type stars (spectra B and A) show a marked preference for an intermediate density, 75 per cent of them coming between the values 0.02 and 0.20, while out of the 18 second-type stars only two fall into that interval, and for one of them a small and permissible change of the elements would take it out of these limits.

[1] See *Astrophysical Journal*, **36**, 62, 1912.

The second-type stars fall apparently into two groups, of which one precedes and one follows the first-type stars in order of density. These two groups are obviously identical with the two classes of second-type stars of very greatly different luminosity discussed by Hertzsprung[1] and Russell,[2] and the facts collected here afford direct support of Russell's theory that the differences in brightness of the two groups are to be ascribed in the main to great differences in the mean density.[3]

PRINCETON UNIVERSITY OBSERVATORY
March 1913

[1] *Zeit. für wiss. Phot.*, **3**, 429; **5**, 86, 1907.

[2] *Astrophysical Journal*, **36**, 153, 1912.

[3] *Science*, N.S., **34**, 523, 1911; *Proc. Am. Phil. Soc.*, **51**, 569, 1912.