although brighter, being only 240 miles in diameter. Dr. Berberich's elements follow; it will not be difficult with their aid to insert the orbit in a diagram of the Solar System.

Epoch 1906, Feb. 22. Berlin	Midnight.	Period 12.02 years.	
Mean Longitude	48° 57′	Mean distance from Sun	5.248
Longitude of Perihelion	76° 0′	Least " "	4.369
" " Asc. Node	315° 34′	Greatest " "	6.127
Inclination to Ecliptic	10° 21′	—	
Eccentricity	0.1676	—	

KNOWLEDGE AND SCIENTIFIC NEWS. September, 1906.

THE MILKY WAY AND THE THEORY OF GASES.*

H. POINCARÉ.†

The matter of which I wish to speak to you today has up to this time attracted but little attention from astronomers; I could scarcely make note of anything except an ingenious idea of Lord Kelvin, which has opened a new field of research but which is still waiting for someone to enter upon it. Neither have I any original results of which to tell you, and all that I can do is to give you an idea of the problems that present themselves but which no one has as yet undertaken to solve.

You all know how a large number of modern physicists imagine the composition of gas; gases are formed by an innumerable multitude of molecules which, animated with great velocity, cross and intercross in all directions. These molecules probably act upon each other at a distance, but this action decreases very rapidly with the distance so that their trajectories remain apparently rectilinear; they do not cease to be so except when two molecules happen to pass quite near to each other; in this case their mutual attraction or repulsion makes them deviate to the right or to the left. This is what is sometimes called a shock; but there is no occasion to understand the word *shock* in its usual significance; it is not necessary that the two molecules come in contact: it suffices that they approach each other near enough to become sensible of their mutual attractions. The laws of deviation which they follow are the same as if there had been an actual shock.

It seems at first that the disordered shocks of this countless dust can only create an inextricable chaos before which the ana-

* Bulletin Astronomique de la Société Astronomique de France, April, 1906.

† M. Poincaré former President of the Astronomical Society of France, is now of the Institute.

lyst must draw back. But the law of large numbers, that supreme law of chance, comes to our aid; face to face with a semi-disorder we should have to despair, but in extreme disorder this static law re-establishes a sort of mean order to which the mind can adjust itself. It is the study of this mean order which constitutes the kinetic theory of gases; it shows us that the velocity of the molecules is equally distributed in all

FIGURE 1. THE MILKY WAY IN THE NORTHERN HEMISPHERE.

directions, that the magnitude of these velocities varies from one molecule to another, but that this variation even is subject to a law called Maxwell's law. This law tells us how many molecules there are animated with such and such velocity. As soon as the gas departs from this law, the mutual shocks of the molecules, by modifying the magnitude and the direction of their velocities, tends to bring them back promptly. The physicists have tried, not without success, to explain in this manner the experimental properties of gases, for instance the law of Mariotte.

Let us now consider the Milky Way; there also we see countless dust, only the grains of this dust are no longer atoms, they are stars; these grains also move with great swiftness; they act upon each other at a distance, but this action is so small at a great distance that their trajectories are rectilinear; and yet, from time to time, two of them may approach each other enough to be swerved from their route, like a comet which has passed too near to Jupiter. In a word, to the eyes of a giant for whom our Suns would be what atoms are for us, the Milky Way would seem only a bubble of gas.

Such was the leading idea of Lord Kelvin. What can we conclude from this comparison? To what extent is it exact? That is what we shall try to find out together; but before reaching a definite conclusion and without wishing to prejudge it, we foresee that the kinetic theory of gases will be for the astronomer a model which he ought not to follow blindly, but with which he can usefully inspire himself. Up to the present, celestial mechanics has attacked only the solar system or a few double star systems. In face of this unity presented by the Milky Way, or the star clusters, or the resolvable nebulae, it drew back, because it saw there nothing but chaos. But the Milky Way is not more complicated than a gas; the static methods founded on a calculation of probabilities applicable to the one are also applicable to the other. First of all it is important to note the resemblance of the two cases and their difference.

Lord Kelvin strove to determine by this means the dimensions of the Milky Way; for that, one is reduced to counting the stars visible in our telescope; but we are not sure that, beyond the stars which we see there are not others which we do not see; so that what we should measure in this manner would be not the size of the Milky Way but the power of our instruments. The new theory will offer us other resources. We are indeed acquainted with the movements of the stars nearest to us and we form an idea of the greatness and the direction of their particular velocities. If the ideas expressed above are correct, these velocities should follow Maxwell's law, and their average value will let us know, so to speak, that which corresponds to the temperature of our imaginary gas. But this temperature itself depends on the dimensions of our gaseous bubble. In fact, how will a gaseous mass abandoned in space act, if its elements attract each other according to the law of Newton? It will take the spherical form; moreover, on account of gravitation, its density will be greater in the center, the pressure also will increase from the surface to the center because of the weight of the exterior parts attracted toward the center; finally the temperature will increase toward the center: the temperature and the pressure being connected by the law called adiabatic, as happens in the successive layers of our atmosphere. At the very surface,

FIGURE 2. THE MILKY WAY IN THE SOUTHERN HEMISPHERE.

the pressure will be zero, and it will be the same with the absolute temperature, i.e. with the velocity of the molecules.

Here a question presents itself: I have spoken of the adiabatic law, but this law is not the same for all gases, since it depends on the relation of their two specific heats; for the air and analogous gases this relation is 1.42; but ought the Milky Way to be likened to the air? Evidently not; it should be considered as a monatomic gas, like the vapor of mercury, like argon, like helium, that is to say, the relation of their specific heats should be considered equal to 1.66. In fact, one of our molecules would be, e.g., the solar system; but the planets are only very small personages, the Sun alone counts, so that our molecule is truly monatomic. Even if we take a double star, it is probable that the action of a strange star, which might happen to come near it, would become perceptible enough to deflect the general movement of transfer of the system long before being capable of disturbing the relative orbits of the two components; in a word, the double star would act like an indivisible atom. However that may be, the pressure and consequently the temperature at the center of the gaseous sphere would be correspondingly greater as the sphere was larger, since the pressure increases with the weight of all the overlying layers. We can suppose that we are nearly at the center of the Milky Way, and in observing the average velocity proper to the stars we shall learn what corresponds to the central temperature of our gaseous sphere and we shall determine its radius.

We can form some idea of the result from the following considerations. Let us take a simpler hypothesis: the Milky Way is spherical and its masses are distributed in a homogeneous manner; the result is that the stars within it describe ellipses having the same center. If we suppose that the velocity becomes zero at the surface, we can calculate this velocity at the center, by the equation of living forces. We thus find that this velocity is proportional to the radius of the sphere and to the square root of its density. If the mass of this sphere were that of the Sun and its radius that of the terrestrial orbit, it is easy to see that this velocity would be that of the Earth in its orbit. In the case that we have supposed, the mass of the Sun should be distributed in a sphere with a radius one million times larger, this radius being the distance of the nearest stars; the density is then 10^{18} times less; now the velocities are of the same order, hence it must be that the radius is 10^9 times greater, that is one thousand times the distance of the nearest stars, which would make about one thousand millions of stars in the Milky Way.

You will say that these hypotheses diverge widely from reality; in the first place the Milky Way is not spherical and we shall soon return to this point; and then the kinetic theory of gases is not compatible with the hypothesis of a homogeneous sphere. But in making the exact calculation according to this theory one would find no doubt a different result but one of the same order;

now in such a problem the data are so uncertain that the order of dimension is the only end at which we are to aim.

Here a first remark presents itself; the result of Lord Kelvin which I have just verified by an approximate calculation obviously agrees with the estimates which the observers have been able to make with their telescopes; so that we must conclude that we have very nearly pierced the Milky Way. That

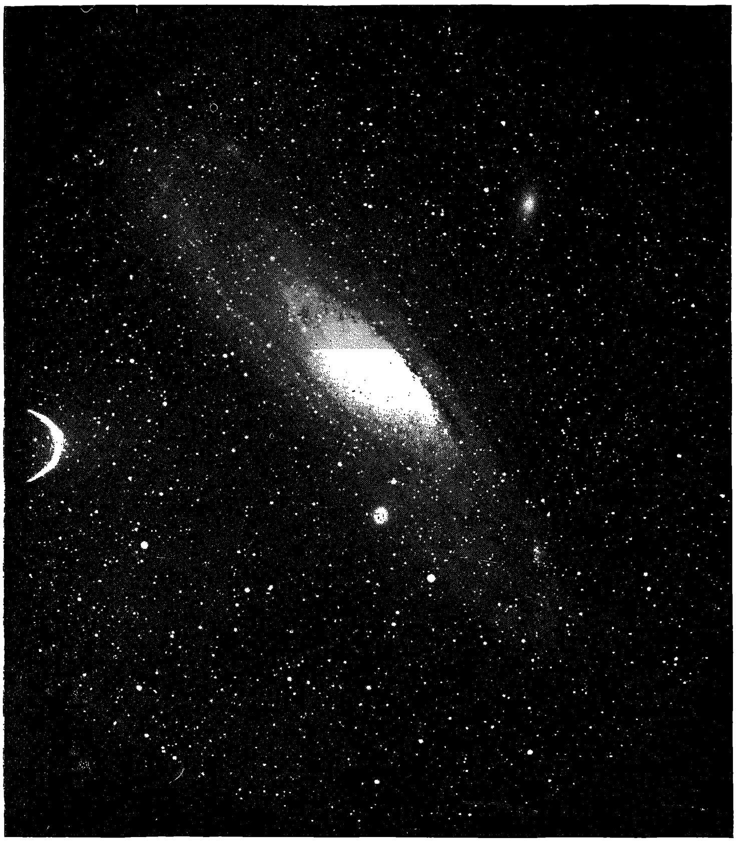

FIGURE 3. THE GREAT ANDROMEDA NEBULA.
Photographed by H. C. Wilson.

enables us to decide another question. There are the stars which we see because they shine; but might there not be obscure stars which circulate in the interstellar spaces and whose existence might long remain unknown? Very well then, that which Lord Kelvin's method would give us would be the total number of stars including the dark ones; since his number is comparable to that which the telescope gives, then there is no dark matter, or at least not so much as there is of shining matter.

Before going further we ought to look at the problem from another side. Is the Milky Way constituted of a gas, properly so-called? It is known that Crookes introduced the notion of a fourth state of matter in which gases having become too rarefied are no longer true gases and they become what is called radiant matter. The Milky Way—considering the smallness of its density, will it be like gaseous matter or like radiant matter? It is the consideration of what is called the *libre parcours* (free course) which will furnish us the answer.

The trajectory of a gaseous molecule may be regarded as formed of rectilinear segments joined by very small arcs corresponding to successive shocks. The length of each one of these segments is what is called the free course; this length of course, is not the same for all the segments and for all the molecules; but we can take an average; that is what is called the mean course (*parcours moyen*); this increases as the density of the gas diminishes. The matter will be radiant if the average course is greater than the dimensions of the vessel where the gas is enclosed, so that a molecule may chance to travel through the entire vessel without suffering any shock; in the other case it remains gaseous. From this it results that the same fluid may be radiant in a small vessel and gaseous in a large vessel; it is perhaps on account of this that, in one of Crookes' tubes, the larger the tube is the farther one must force the vacuum.

What happens, then, for the Milky Way? It is a mass of gas whose density is very low but whose dimensions are very great; does a star have chances to cross it without suffering a shock, i.e. without passing near enough to another star to be perceptibly swerved from its course? What do we understand by *near enough*? That is necessarily a little arbitrary; let us say that it is the distance from the Sun to Neptune, which would represent a deviation of about ten degrees; then let us suppose each one of our stars enveloped in a protecting sphere of this radius; will a straight line be able to pass between these spheres? At the average distance of the stars of the Milky Way the radius of one of these spheres will be seen at an angle of about one-tenth of a second; and we have a thousand million stars! Let us place on the celestial sphere a thousand million little circles with a radius of a tenth of a second. Is there any chance that these circles will cover the celestial sphere a great number of times? Far from it; they will only cover the sixteen-thousandth part. Thus the Milky

Way is not the image of gaseous matter but of the radiant matter of Crookes. Nevertheless, since our preceding conclusions are fortunately not very precise, we do not have to modify them perceptibly.

There is another difficulty; the Milky Way is not spherical, and up to this time we have reasoned as though it were, since that is the form of equilibrium which a gas isolated in space would take. In support of this theory there exist star clusters whose form is globular and to which what has just been said applies better. Herschel had already endeavored to explain their remarkable appearance. He supposed that the stars of the clusters

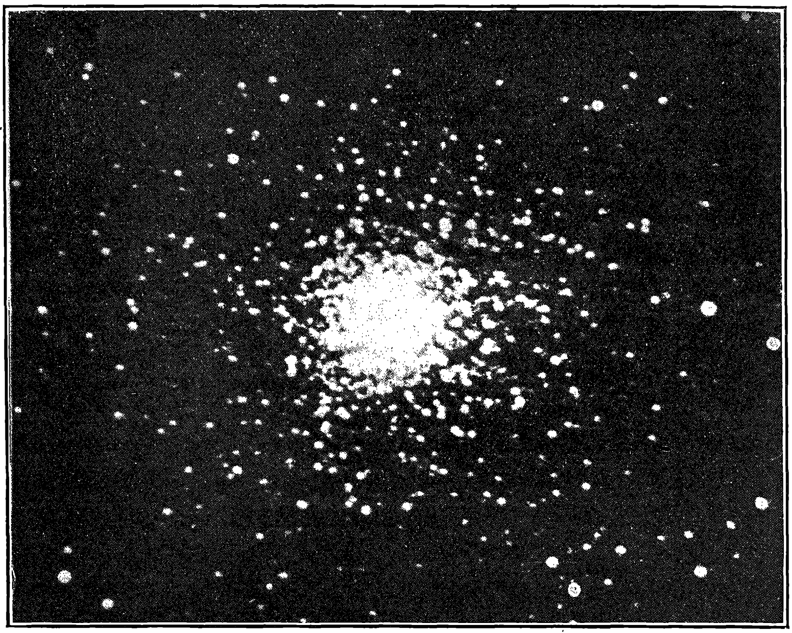

FIGURE 4. TYPE OF GLOBULAR CLUSTER: CLUSTER IN HERCULES.
Photographed by Isaac Roberts.

are uniformly distributed in such a manner that one cluster would be a homogeneous sphere; each star then would describe an ellipse and all these orbits would be traveled over in the same time, so that at the end of a period the cluster would find again its primitive configuration and that this form would be stable. Unfortunately the clusters do not appear homogeneous; one observes condensation at the center, one would observe it even if the sphere were homogeneous, since it is thicker at the center; but it would not be so marked. We can then rather compare a cluster to a gas in adiabatic equilibrium and which takes the spherical form because that is the form of equilibrium of a gaseous mass.

But, you will say, these clusters are much smaller than the Milky Way of which they probably even form a part, and although they are denser they will still give us something more analogous to radiant matter; now gases reach their adiabatic equilibrium only by a succession of innumerable shocks of the molecules. There might be some way of arranging that. Let us suppose that the stars of the cluster have just enough energy so that their velocity is annulled when they reach the surface; then they will be able to pass through the mass without shock, but, having reached the surface they will turn backward and cross it again; after a great number of crossings they will end by being turned aside by a shock; in these conditions we should still have a matter that could be considered as gaseous; if by chance there had been in the cluster stars whose velocity was greater, they have left it long ago, they have gone away to return there no more. For all these reasons it would be interesting to examine the known clusters, to try to explain the law of their densities and to see if it is the adiabatic law of gases.

But let us return to the Milky Way; it is not spherical, but is rather to be described as a flattened disk. It is plain then, that a body leaving the surface without velocity will arrive at the center with different velocities, according as it leaves the surface in the neighborhood of the middle of the disk or at its edge; the velocity would be considerably greater in the latter case.

Now up to this point we have admitted that the real velocities of the stars, those which we observe, should be comparable to those which similar masses would attain; this leads to some difficulty. We have given above a value for the dimensions of the Milky Way, and we have deduced it from the observed real velocities which are of the same order of magnitude as that of the Earth in its orbit; but what is the dimension that we have thus measured? Is it the thickness? Is it the radius of the disk? It is doubtless something intermediate; but what can we say then of the thickness itself or of the radius of the disk? Data fail me for making this calculation; I limit myself to giving you a glimpse of the possibility of basing a valuation at least approximate on a thorough discussion of the proper motions.

We find ourselves then in the presence of two hypotheses: either the stars of the Milky Way are animated with velocities which are for the most part parallel to this galactic plane, but otherwise distributed uniformly in all directions parallel to this plane. If it is so, the observing of the proper motions ought to reveal to us a preponderance of components parallel to the Milky Way; this is yet to be determined, for I do not know whether a systematic discussion has been made from this point of view. On the other hand, such an equilibrium could be only temporary, for in consequence of the shocks, the molecules, that is to say the stars, will in time reach remarkable velocities in the direction perpendicular to the Milky Way and they will end by quitting its plane, so that the system will tend toward the spherical form, the only form of equilibrium of an isolated gaseous mass. Or else, the entire system is animated by a common rotation, and it is for this reason that it is flattened like the Earth, like Jupiter, like all bodies which revolve. Only, as the flattening is considerable it must be that the rotation is rapid; rapid indeed, but we must agree on the meaning of this word. The density of the Milky Way is 10^{25} times less than that of the Sun; a velocity of rotation which is $\sqrt{10^{25}}$ times less than that of the Sun would then be its equivalent from the point of view of the flattening; a velocity 10^{12} times slower than that of the Earth, that is a thirtieth of a second of an arc in a century, would be a very rapid rotation, almost too rapid for stable equilibrium to be possible.

In this hypothesis the observable proper motions will appear to us uniformly distributed and there will no longer be any preponderance for the components parallel to the galactic plane. They will tell us nothing about the rotation itself since we are a part of the revolving system. If the spiral nebulae are other Milky Ways, strangers to our own, they are not drawn into this rotation and their own motions could be studied. It is true that they are very distant; if a nebula has the dimensions of the Milky Way and if its apparent radius is e.g. 20″, its distance is 10,000 times the radius of the Milky Way. But that will not matter, since it is not about the translation of our system that we are seeking information but about its rotation. The fixed stars by their apparent motion, plainly reveal to us the diurnal rotation of the Earth although their distance is immense. Unfortunately the possible rotation of the Milky Way, however swift it may be relatively, is very slow from the absolute point of view, and besides the pointings on the nebulae can not be made very exact; so that thousands of years of observations would be necessary in order to learn anything.

However that may be, in this second hypothesis the figure of the Milky Way would be a figure of definitive equilibrium. I will no longer discuss the relative value of these two hypotheses for there is a third which is perhaps more probable. We know that among the irresolvable nebulae several families can be distinguished; the irregular nebulae, like that of Orion, the planetary and ring nebulae, the spiral nebulae. The spectra of the first two families have been determined, they are discontinuous; these nebulae are then not formed of stars; moreover, their distribution over the heavens seems to depend upon the Milky Way, whether they have a tendency to go away from it, or on the other hand to approach it, they form a part of the system. On the contrary the spiral nebulae are generally considered as independent of the Milky Way; it is admitted that they are, like it formed of a mul-

FIGURE 5. THE SPIRAL NEBULA OF CANUM VENATICORUM.
Photographed by Isaac Roberts.

titude of stars, that they are, in a word, other Milky Ways very distant from our own. The recent works of Stratonoff tend to make us consider the Milky Way itself as a spiral nebula, and that is the third hypothesis of which I wished to speak to you.

How shall we explain the very singular appearances presented by the spiral nebulae and which are too regular and too constant to be due to chance?

In the first place a glance at one of these pictures suffices to show that the mass is in rotation; even the direction of the rotation, can be seen; all the spiral radii are curved in the the same direction; it is evident that it is the *advancing wing* which lags on the pivot and that determines the direction of rotation. But that is not all; it is clear that these nebulae can not be likened to a gas in repose, nor even to a gas in relative equilibrium under the influence of a uniform rotation; they must be compared to a gas in permanent motion in which internal currents rule.

Let us suppose, for example that the rotation of the central nucleus be rapid (you know what I mean by that word), too rapid for stable equilibrium; then at the equator the centrifugal force will surpass the attraction, and the stars will have a tendency to escape along the equator and will form divergent currents; but in receding, since their moment of rotation remains constant and the radius vector increases, their angular velocity will diminish, and that is why the advancing wing seems to lag behind. In this way of looking at it there would be no real permanent motion, the central nucleus would constantly lose matter which would go away to return no more and would gradually exhaust itself. But we can modify the hypothesis. In proportion as it recedes the star loses its velocity and finally stops; at this moment attraction seizes it again and brings it back toward the nucleus; there would then be centripetal currents. It must be admitted that centripetal currents are of first rank and centrifugal currents of second rank, if we take up again the comparison with a troop in battle executing a wheel; and in fact the component centrifugal force must be compensated by the attraction exercised by the central layers of the swarm upon the extreme outer layers.

Moreover, at the end of a certain time, a permanent order is established; the mass having curved itself, the attraction exercised on the pivot by the advancing wing tends to delay the pivot and that of the pivot on the advancing wing tends to accelerate the advance of this wing which no longer increases its lagging, so that finally all the radii end by turning with a uniform velocity. It may be admitted nevertheless that the rotation of the nucleus is more rapid than that of the radii.

There is one question; why do these centripetal and centrifugal swarms tend to concentrate themselves in radii instead of scattering themselves in all directions? Why do these radii divide regularly? If the swarms concentrate themselves, it is because of the attraction exercised by the swarms already existing upon the stars which come out of the nucleus, in their neighborhood. As soon as an inequality is produced it tends to accentuate itself on this account.

Why do the radii divide regularly? This is a more delicate question. Suppose there is no rotation, that all the stars are in two rectangular planes in such manner that their distribution is symmetrical with relation to these two planes. In such symmetry, there would be no reason for their leaving these planes nor for the symmetry changing. This configuration would then give us equilibrium but *it would be an unstable equilibrium.*

On the contrary, if there is rotation we shall find a configuration of analogous equilibrium with four bent radii equal to each other and cutting each other at 90°, and if the rotation is rapid enough this equilibrium may be stable.

I am not prepared to state more exactly: it is enough for me to let you see that these spiral forms may perhaps be explained some day by simply calling in the law of gravitation and static considerations recalling those of the theory of gases.

What I have just said to you of internal currents shows you that there may be some interest in discussing systematically all available proper motions; that can be undertaken in a hundred years, when the second edition of the chart of the heavens will be compared with the first which we are now making.

But in closing I should like to call your attention to one question, that of the age of the Milky Way or of the nebulae. If what we believe we have seen should be confirmed, we can form some idea about this. This kind of static equilibrium of which gases give us the model can establish itself only after a great number of shocks. If these shocks are rare it can be produced only after a very long time; if the Milky Way (or at least the clusters which are a part of it), if the nebulae have really attained the equilibrium, it is because they are very old, and we shall have an inferior limit of their age. We would also have a superior limit; this equilibrium is not definitive and can not last forever.

Our spiral nebulae would be comparable to gases animated with permanent motions; but gases in motion are viscous and their velocities are finally exhausted. What corresponds here to viscosity (and which depends on the chances of shock to the molecules) is exceedingly weak, so that the present system may endure for an extremely long time, yet not always; so that our Milky Ways can not exist eternally nor become infinitely old.

And that is not all. Let us consider our atmosphere: at its surface must reign an infinitely small temperature and the velocity of the molecules is nearly zero. But it is only a question of mean velocity; in consequence of the shocks one of these molecules may gain an enormous velocity (rarely it is true) and then it will depart from the atmosphere, and once gone it will never enter it again; our atmosphere is thus exhausting itself with extreme slowness. The Milky Way too will from time to time lose a star by the same mechanism and that also limits its duration.

Very well then, it is certain that if we compute in this manner the age of the Milky Way we shall find some enormous figures. But here is another difficulty. Certain physicists, relying upon other considerations, estimate that the suns can have only an ephemeral existence, about fifty millions of years; our minimum would be greater than that by far. Must we believe that the evolution of the Milky Way began when matter was still dark? But how did the stars which compose it reach the adult age all at the same time, an age which is to last so brief a space? Or are they rather to reach it successively and are those that we see only a small minority compared with those which are extinct or which will one day shine out? But how shall we reconcile that with what we have said above about the absence of dark matter in considerable proportion? Should we abandon one of the two hypotheses, and which one? I content myself with pointing out the difficulty without pretending to solve it; I will close then with a great interrogation point. The more so as it is interesting to state problems even when the solution of them seems very far distant.

[Translated by Miss Isabella Watson, Northfield, Minn.]

OBSERVATIONS OF RED STARS FOR COLOR EFFECT IN PARALLAX WORK.

In the TRANSACTIONS OF THE ASTRONOMICAL OBSERVATORY OF YALE UNIVERSITY, Vol. 2 pt. 1., is found on page 184, an article showing the effect of the red color of some stars on the observations of those stars for parallax.

We give below a large part of that article that our readers may see the relation between theory and observation in regard to this interesting question.

"The work of the preceding pages, in which every precaution to eliminate known sources of error was employed, appears to us to be free from all systematic error, except perhaps one due to the star's color. Any perceptible difference in the mean refrangibility of the light of two stars might possibly produce an effect upon the measured distance between them which would be a function of the hour angle, and hence affect their apparent rela-