Popular Astronomy.

REVISION OF WOLF'S SUNSPOT RELATIVE NUMBERS.*

PROFESSOR A. WOLFER.

The next number (XCIII) of *Astronomische Mittheilungen* will contain a new edition of Wolf's table of relative numbers, in which not only will some inaccuracies of the earlier tables—partly errors of computation, partly typographical errors—be expunged but those older observations that have come to light since the tables were compiled, but have not yet been worked up, will be used in the revision. For the most part these new observations were made at Kremsmünster during the years 1802-1830 and have furnished a very valuable addition to the record of sunspots. I now send you a copy of this corrected and completed series (Table I), entitled "Observed Sunspot Relative Numbers." This table, extending from 1749-1901, replaces that published by Wolf in 1880 in No. L of the *Astronomische Mittheilungen*, as well as the various copies which afterwards appeared in the *Meteorologische Zeitschrift* and in the *Memorie della Società degli Spettroscopisti Italiani*. [It also replaces the table on pages 505-506, *Monthly Weather Review*, November 1901]. It contains no error, and is now to be regarded as definitive so long as the complete new reduction of the whole amount of observational material is not executed, for which the preliminary work is now going on; this will, however, apparently require five or six years more.

Those numbers in the above-mentioned table that are entered in heavy-faced type depend wholly on actual observations; those in light-faced type depend in great part upon actual observations, yet also have in part been obtained by means of graphic interpolations between the days of any month that contained observations with considerable gaps between them; the interpolated

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* From the *Monthly Weather Review*, April 1, 1902, edited by Professor Cleveland Abbey, Washington, D. C. Professor Wolfer of France has recently completed a revision of the sunspot relative numbers which replaces all previous publications. Professor Abbey put these numbers in graphic form which we have reproduced in cuts on pages 454 to 457. Tables of these numbers referred to in the above article are omitted because they are too bulky for our space.

numbers were combined with the observed numbers in the computation of the monthly means. Only a very few monthly means, in the eighteenth century, depend entirely upon interpolations; by far the larger number are based upon actual observations. But every monthly mean in which even a single interpolated number has been used is shown by light-faced type; in this respect the distinction may have been too rigorous rather than indulgent, and the light-faced type are, therefore, in no sense to be regarded as an indication of the unreliability of the corresponding numbers.

I have thought that it would perhaps be agreeable to you also to possess new editions of the two other tables that are based upon the preceding, namely, the table of "Smoothed relative numbers" and that of the "Epochs of maxima and minima," which are directly deduced from the preceding. In No. XLII of his *Astronomische Mittheilungen* Wolf has published the smoothed numbers up to 1876, inclusive, and reproductions of these are found in various periodicals and other publications. But there are some errors of computation in this table, and numerous typographical errors occur in the reproductions.

The smoothed relative numbers of Table 2 present the mean course of the spot phenomena; that is to say, without the numerous secondary short periodical variations that really occur in addition to the 11-year variation. Investigations into the general course of the phenomena and the periods of a higher order should therefore, be based upon these smoothed numbers and not on those observed. The method of formation of these numbers has been explained by Wolf, in No. XLII of his *Astronomische Mittheilungen*. The mean of every twelve consecutive observed monthly relative numbers is taken, and every pair of two consecutive means is again united into one mean value according to the following scheme:

$$1/12 \ (\text{I} + \text{II} + \text{III}\ldots\ldots\ldots+ \text{XII}) = n_1, \text{ for epoch July 1.}$$
$$1/12 \ (\text{II} + \text{III} + \text{IV}\ldots\ldots\ldots+ \text{XII} + \text{I}) = n_2, \text{ for epoch Aug. 1.}$$
$$1/2 \ (n_1 + n_2) = r, \text{ which is the smoothed number for mid-July.}$$

This method of smoothing is conformable to that which Wolf has used for eliminating the annual period of the variations of magnetic declination when comparing the latter with the solar spots. This consideration was not necessary for the relative numbers but the combination of twelve months into one mean has been adopted in order to secure a uniform method of treating both phenomena. Table 2, which contains these smoothed

values, has been newly computed from beginning to end and is entirely free from error.

The epochs of maximum and minimum, as deduced from the new Table 2, are given in Table 3; they differ from those published by Wolf (No. LVI of his *Mittheilungen*) only in respect to the maximum of 1805, which, by reason of the newly added observations at Kremsmünster, is changed from 1804.2 to 1805.2. On the other hand, in the later reproductions of Wolf's table there are some typographical errors even in that which Wolf himself published in the year 1890, in the first volume of his Handbook of Astronomy. The present Table 3 is for the moment to be considered as definitive and free from error. I have attributed definite weights to the individual epochs, in order to indicate to those who make use of them some idea as to their reliability. The epochs since 1823, namely, since Schwabe's observations began, all receive the weight 10; for epochs preceding these I have again carefully checked the observational material on which they are based and compared them as to completeness and uniformity, and I believe that the estimate of relative reliability of the individual epochs which I have attained is not far from the truth. With the epochs and weights of Table 3, I have also newly computed the mean length of the 11-year period and the normal epochs of maximum and minimum, and find the values given in Table 3; these differ only very slightly from Professor Newcomb's results[*], although the epochs used by him differ partly from the new ones, being older values such as Wolf had published in 1872, but had afterwards changed.

As concerns the 35-year period discovered by Dr. Lockyer, it is quite certain that the interval between each minimum and the following maximum, or the so-called duration of increase, as well as the duration of decrease of the sunspot curve, shows variations in the different periods that are not accidental and that most probably have a periodic character. But I do not believe that the material used by Dr. Lockyer is sufficient to give the length of the greater period with any certainty, because the whole interval of time that Dr. Lockyer takes into consideration covered only one and a half of these greater periods of thirty-five years each. As soon as I received the memoir of Dr. Lockyer I at once investigated whether the existence of this suspected period could be traced backward. This attempt was thoroughly justified, because at least since 1750, the epochs of maximum and

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[*] *Astrophysical Journal*, XIII, No. 1.

minimum are relatively well ascertained and are much more re-
liable than Dr. Lockyer seems inclined to allow. With the excep-
tion of the years 1784 and 1788, which have the weight of 4
ascribed to them in Table 3, there is no other epoch whose un-
certainty amounts to one-half year. In Table 4 you will find the
result to which I at that time arrived; column *a* contains the
interval of time elapsing between a minimum and the following
maximum; these are therefore the numbers which, according to
Dr. Lockyer, should follow a 35-year period; these are displayed
graphically in curve I, fig. 1, and leave no doubt as to their
variability; in fact this variability seems to be much stronger
than Dr. Lockyer found from the periods used by him. Appar-
ently, they vary to and fro, but a constant period can not be
found therein. The great variation between 1780 and 1835
would in fact rather point to the period of fifty-five years, which
was previously suspected by Wolf, and would in no case make a
35-year period probable.

Dr. Halm of Edinburg* has also drawn attention to the varia-
bility of the interval between minima and maxima, and in fact,
this variability is still more plainly brought out by the method
of treatment adopted by him, in that he takes account of the
ratio of the ascending to the descending part of the 11-year spot
period, instead of considering the absolute duration of the as-
cending portion, as Dr. Lockyer has done. The accompanying
Table 4 contains, the interval of time, *b*, between the maximum
and the following minimum, as also the ratio *a/b*; the latter is
also shown graphically in curve II, fig. 1. The variability of the
ratio *a/b* is very striking, and shows in general the same course
as in curve I, but there is no indication of a regular recurring
period, and especially one of thirty-five years.

Only one result which Dr. Lockyer has brought out is certain,
and one which was also demonstrated by Dr. Halm, both from
the observed facts and as a consequence of his new theory of the
periodicity of the solar phenomena: namely, that the just men-
tioned variations of *a* and of the ratio *a/b* proceed parallel with
the intensity of the development of the spots through each
11-year period in that a maximum follows a minimum quicker in
proportion as the maximum is higher, or as Dr. Halm expresses
it—"in the individual spot periods the maximum occurs earlier in
proportion as the development of the spots is more rapid." Dr.
Halm has compared the ratio *a/b* with the absolute altitude of

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* *Astronomische Nachrichten*, No. 3723.

the individual maxima, as expressed by Wolf's relative numbers; on the other hand, Dr. Lockyer bases his position on the total sum of the spotted area for each individual period. We attain to the same results if, for comparison, we use the mean annual relative number for a single 11-year period, namely, the sum total of the annual relative numbers for the whole period divided by the length of the period, therefore, $r_{mean} = \dfrac{\Sigma r}{P}$. The curves III and IV fig. 1, present the values of r_{max}. and r_{mean}. As we see, the agree-
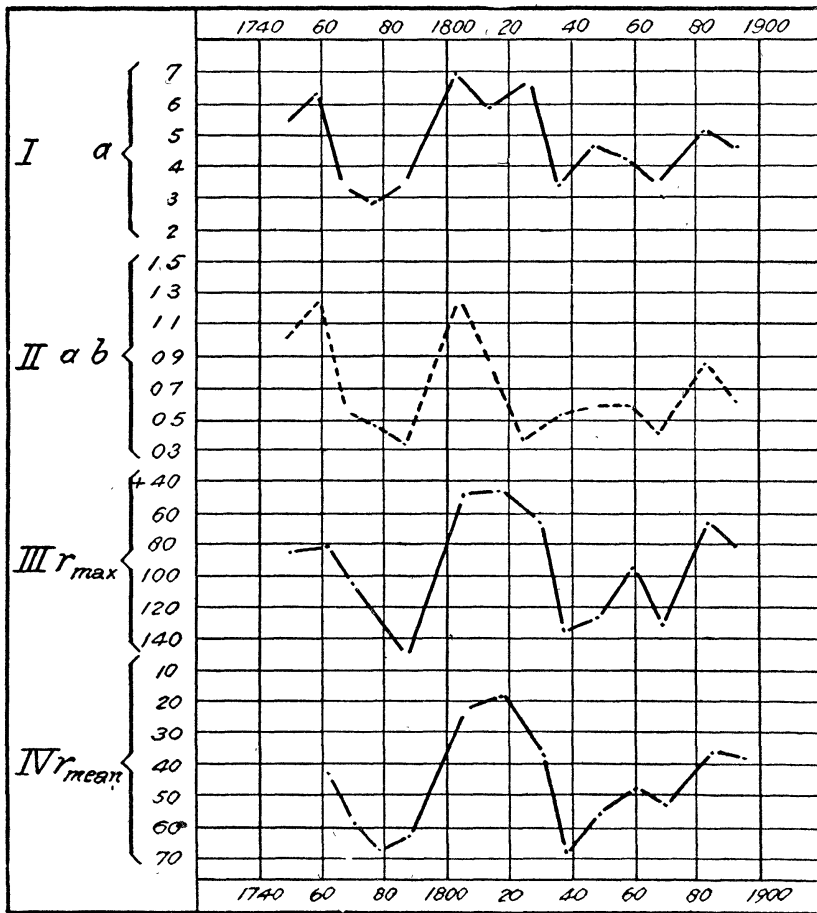

FIGURE 1.

ment with I and II is quite remarkable. That the interval between minima and maxima depends intimately upon the intensity of development of the spots during that period would seem therefore to be quite certain, according to Lockyer's and Halm's investigations; both these quantities are subject to variations of an apparently periodic character; but it seems to me that the material actually available does not indicate a somewhat regular periodicity; and that the continued existence of a 35-year period is not yet demonstrated.
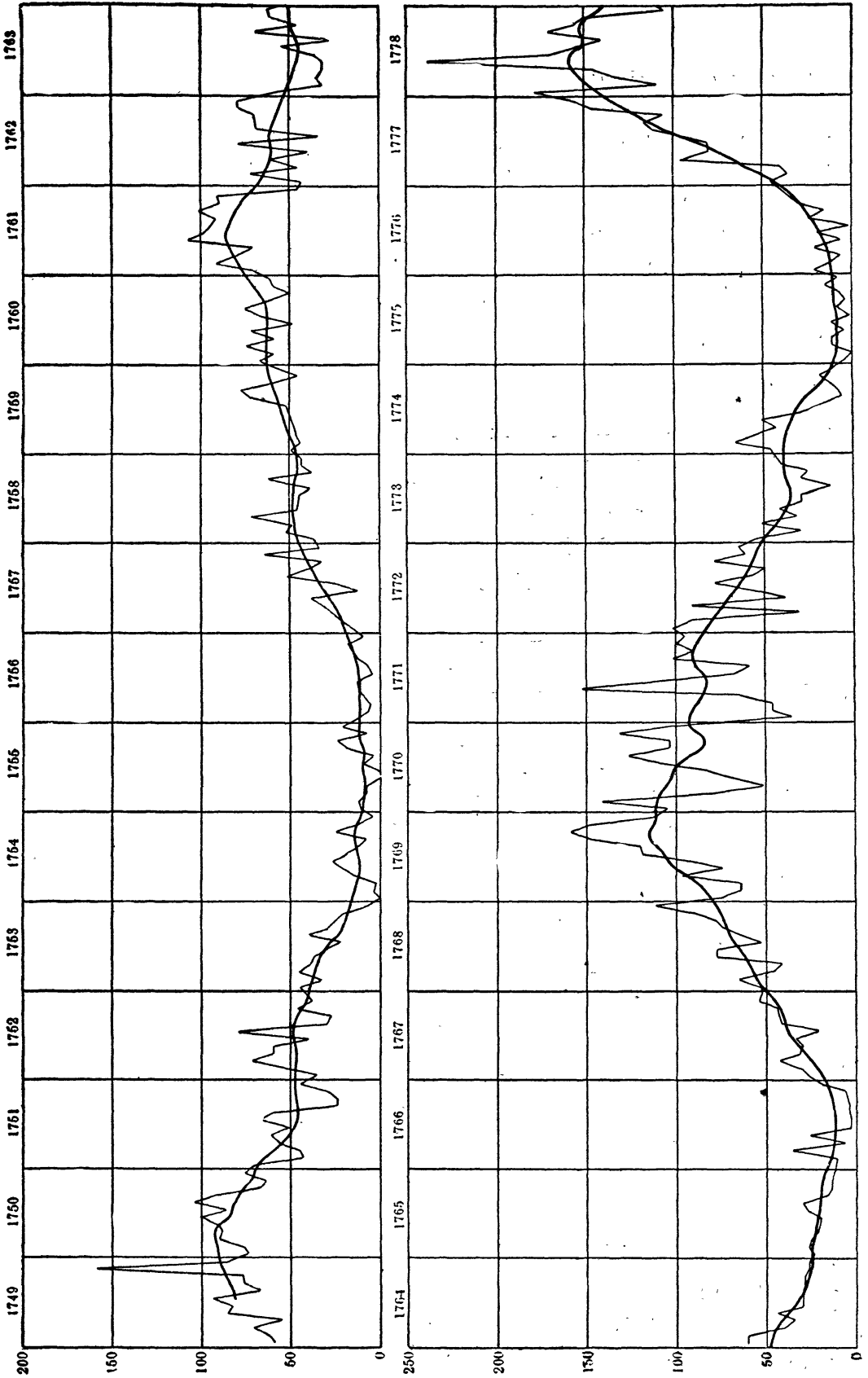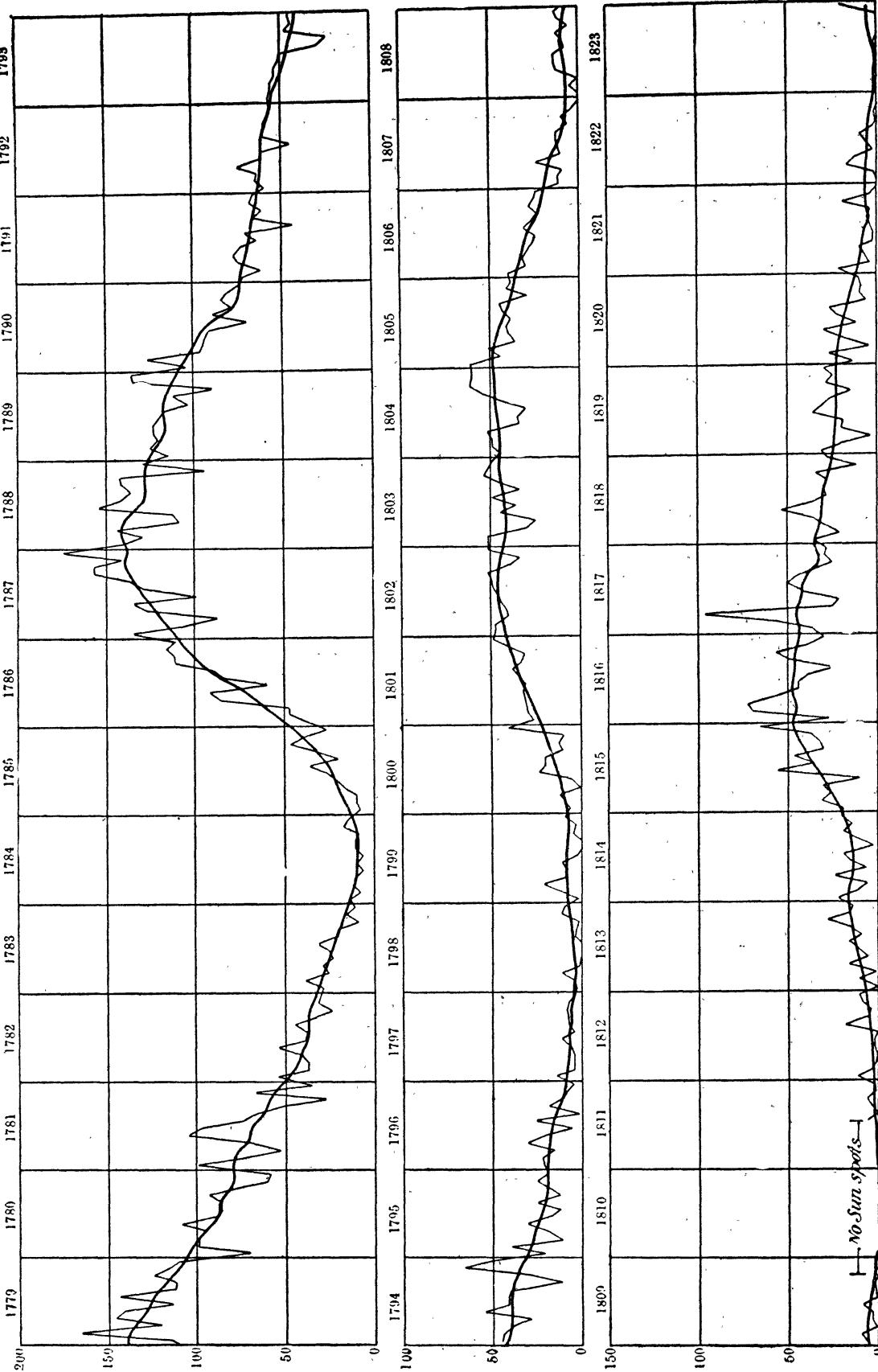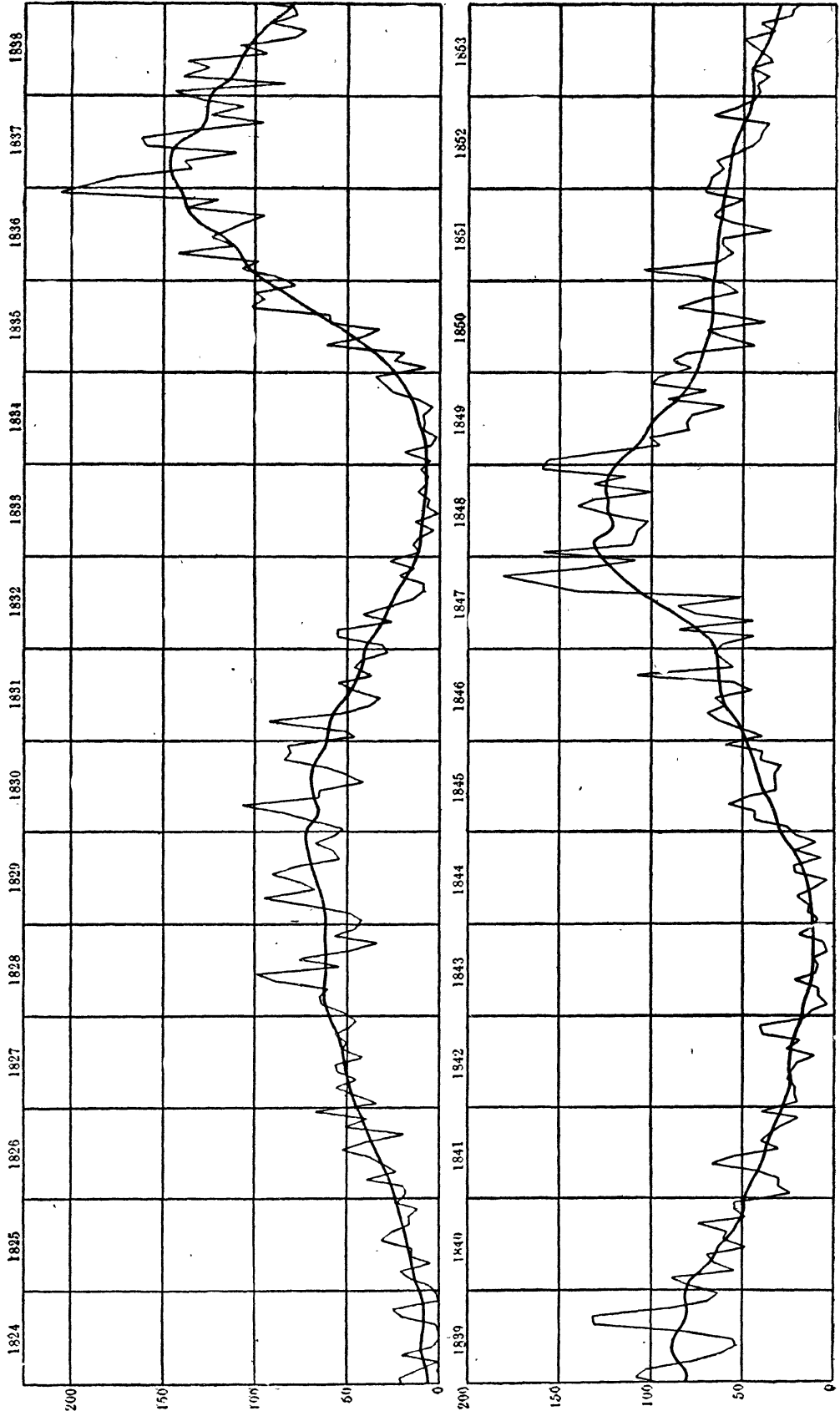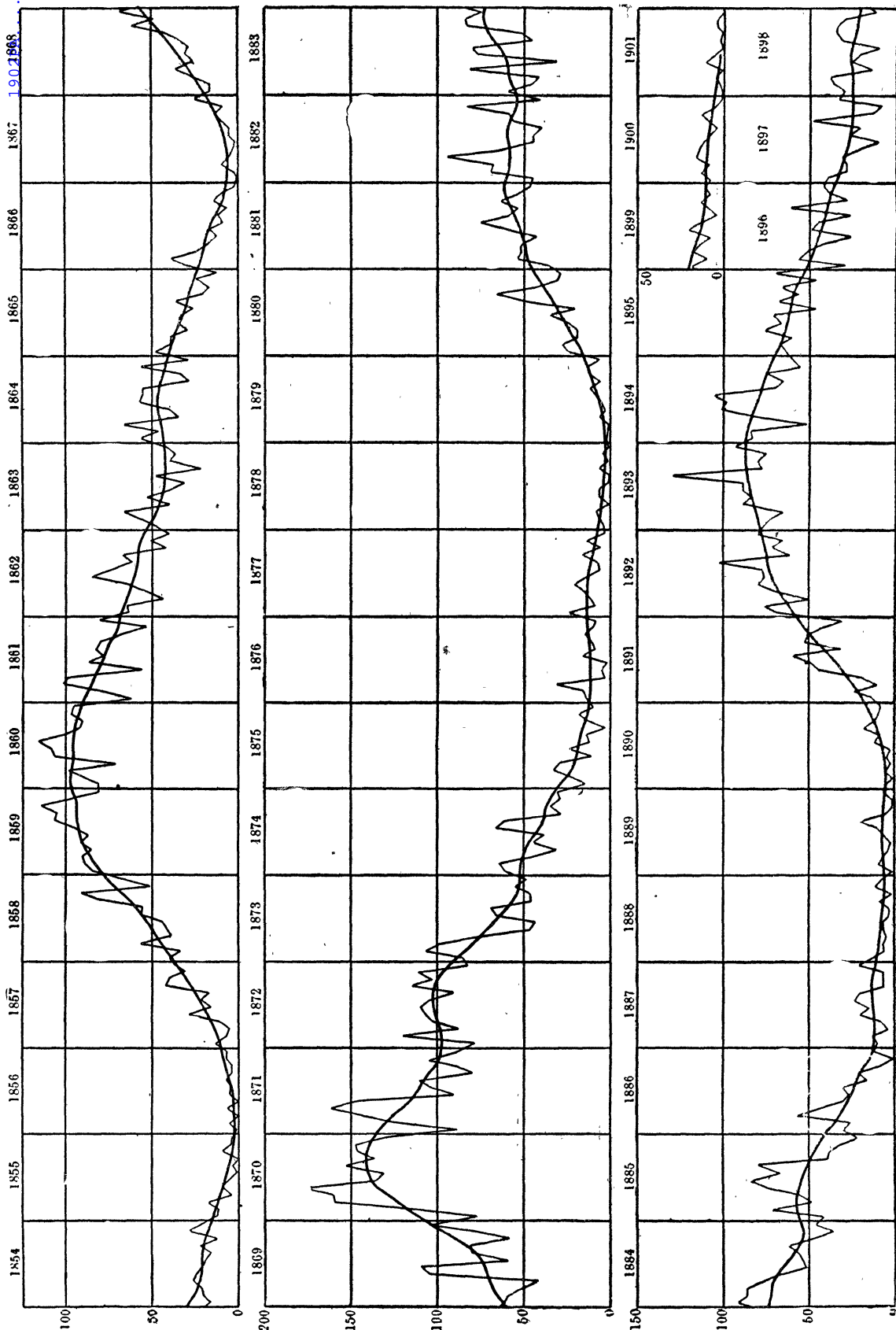
TABLE 3.—*Epochs of Sunspot Maxima aud Minima.*

Minima.	Weight.	Maxima.	Weight.	Minima.	Weight.	Maxima.	Weight.
1610.8	5	1615.5	2	1755.2	9	1761.5	7
1619.0	1	1626.0	5	1766.5	5	1769.7	8
1634.0	2	1639.5	2	1775.5	7	1778.4	5
1645.0	5	1649.0	1	1784.7	4	1788.1	4
1655.0	1	1660.0	1	1798.3	9	1805.2	5
1666.0	2	1675.0	2	1810.6	8	1816.4	8
1679.5	2	1685.0	2	1823.3	10	1829.9	10
1689.5	2	1693.0	1	1833.9	10	1837.2	10
1698.0	1	1705.5	4	1843.5	10	1848.1	10
1712.0	3	1718.2	6	1856.0	10	1860.1	10
1723.5	2	1727.5	4	1867.2	10	1870.6	10
1734.0	2	1738.7	2	1878.9	10	1883.9	10
1745.0	2	1750.3	7	1889.6	10	1894.1	10

Year.
Mean date of minima...1744.21 $\pm$ 0.30
Mean date of maxima..:.............1749.37 $\pm$ 0.43
Length of the period from the minima.................................= 11.141 $\pm$ 0.036
Length of the period from the maxima................................= 11.091 $\pm$ 0.053
Length of the period in general, or 1/3 (max. + 2 min.)......= 11.124 $\pm$ 0.030

TABLE 4 —*Mutual Relations of Maxima and Minima.*

Date of		Interval		Ratio $a:b$.	r_{max}.	Σr, min. to min.	Length of period P, min. to min.	$r_{mean} = \dfrac{\Sigma r}{P}$
Min.	Max.	Max. — min. = a	Min. — max. = b					
		Years	Years				Years	
1745.0	1750.3	5.3	4.9	1.08	83	......	......	......
1755.2	1761.5	6.3	5.0	1.26	80	471.6	11.3	42
1766.5	1769.7	3.2	5.8	0.55	103	533.0	9.0	59
1775.5	1778.4	2.9	6.3	0.46	151	615.8	9.2	67
1784.7	1788.1	3.4	10.2	0.33	133	840.9	13.6	62
1798.3	1805.2	6.9	5.4	1.28	47	281.6	12.3	23
1810.6	1816.4	5.8	6.9	0.84	46	238.0	12.7	19
1823.3	1829.9	6.6	4.0	1.65	67	394.7	10.6	37
1833.9	1837.2	3.3	6.3	0.52	137	653.9	9.6	68
1843.5	1848.1	4.6	7.9	0.58	125	691.8	12.5	55
1856.0	1860.1	4.1	7.1	0.58	95	548.7	11.2	49
1867.2	1870.6	3.4	8.3	0.41	132	617.3	11.7	53
1878.9	1883.9	5.0	5.7	0.88	65	382.8	10.7	36
1889.6	1894.1	4.5	*7.4	*0.61	84	*458.8	*11.9	*39

* These figures depend on assumed values of the date and the r for the minimum of 1901.5.