ON THE PERIODIC CHANGES OF THE VARIABLE STAR Z HERCULIS.

By N. C. Dunér.

Toward the end of July, 1894, Mr. Chandler discovered that the star BD. + 15°3311, whose place for 1900.0 is

$$\alpha = 17^{\mathrm{h}}\ 53^{\mathrm{m}}\ 36^{\mathrm{s}}.06, \quad \delta = +15^\circ\ 8'\ 47''.2,$$

is a variable of the Algol type, and he found its period to be $3^{\mathrm{d}}\ 23^{\mathrm{h}}\ 50^{\mathrm{m}}$. Circumstances prevented him, however, from observing the star except when its light was increasing, and he therefore communicated his discovery to a number of European astronomers, among them the writer, in order that if possible more complete observations might be obtained. I received Mr. Chandler's ephemeris on September 12, but on account of unfavorable weather I was unable to observe a minimum until September 18. In the mean time (on the 15th) a telegram arrived from Professor Hartwig, announcing that he had discovered the variability of the same star, and that its period was $1^{\mathrm{d}}\ 23^{\mathrm{h}}\ 55^{\mathrm{m}}\ 40^{\mathrm{s}}$. This simultaneous, independent discovery by two different persons would certainly have been very surprising, if both of them had not mentioned that their attention had been directed to this star by a note in the admirable Photometric Durchmusterung of Messrs. Müller and Kempf, in which it is stated (p. 482) that the star is of a suspicious character and requires further watching.

In consequence of Professor Hartwig's telegram I resumed observation of the star on September 20, when the sky was again clear and, in fact, determined a minimum epoch. But while the star sank on the 18th more than a whole magnitude below its usual brightness, on the 20th the reduction of light at minimum was not half a magnitude; and instead of occurring a few minutes earlier than on the 18th, as the information received from both discoverers led me to expect, the minimum was early by a whole hour. On the strength of this observation I sent the following telegram to Professor Krüger on February 21:

"The new variable is probably of the Y Cygni type, with unequally bright components. Faint and very bright minima alternate; periods 47 and 49 hours."

Further observations have shown, as I will explain fully below, that these words still represent the facts, as far as they are known, with very considerable exactness.

On September 20 and 21 Professor Hartwig wrote to Professor Krüger with reference to his discovery of the variability of the star and his researches on its periodic changes. Hartwig, like myself, found an analogy with Y Cygni, but it was evident that he had not obtained complete observations of the uneven minima, and hence regarded the analogy as perfect; *i. e.*, he believed the components to be equally bright, and hence was led to assume that the uneven minima occurred 52 hours before the even ones. My observations of September 20 and 24 show, however, that my views as stated above are correct. On all three days at the beginning of observation (*i. e.*, fully three hours before daylight, when the even minima occurred) I have seen Z Herculis brighter than BD. + 14° 3378 and found that its light was diminishing toward a minimum two hours later. If the secondary minimum occurs four hours earlier than the principal minimum, Z Herculis should have been only a little brighter than BD. + 15° 3301, or perhaps about as bright as BD. + 15° 3309, at the time when my observations began.

Some time after this Herr Lindemann, of Pulkowa, announced his opinion that the principal minimum is double, so that two minima, separated by a minimum of less pronounced character, occur in the course of an hour. I have carefully studied the observations of Herr Lindemann, but I cannot share his opinion, for the quite numerous observations which have been made by several European astronomers according to Argelander's method do not show any such depression in the light curve, and Lindemann's observations themselves by no means support this hypothesis in an unqualified manner. Thus, if simple curves of the ordinary form are drawn, it will be found that they represent all of the two days' observations quite well, with the exception of a single measurement (the next to the last on the second day), which shows a discrepancy of 0.4 magnitude. No other observation differs from the curve by more than 0.2 magnitude, and even such discrepancies as this occur very rarely and are not at all systematically distributed.

The minima which I have myself been able to observe are as follows:

Epoch	Minimum, Greenwich M. T.	Brightness	Remarks
0	Sept. 18^d 8^h 44^m	7.0	Good
1	" 20 7 18	14.0	Very good
3	" 24 7 5	15.0	Good
6	" 30 7 59	6.0	Very good
8	Oct. 4 8 32	4.6	Good
12	" 12 7 54	....	Quite uncertain
18	" 24 7 9	4.0	Rather uncertain
36	Nov. 29 5 22	4.0	Fairly certain
37	Dec. 1 4 33	13.8	Quite good

The extremely bad weather in October and November interfered very greatly with the observations.

In order to derive elements from all the observations so far published, I first of all determined the following approximate elements from my own observations of the even epochs:

$$\text{Min.} = 1894.0 + 261^d.37 + 3^d.992\ \frac{E}{2}\,.$$

I then expressed all observations of the even epochs in days and fractions of a day, reduced them to the Sun and compared them with the above elements. In doing this I was, however, unable to avail myself of Chandler's observations, as they have not yet been published. The following results were obtained:

E.	Greenwich M. T.	Obs.—Comp.	Observer
− 2	$257^d.372$	$-0^d.006$	H.
0	261 .342	− 0 .028	H.
0	261 .360	− 0 .010	Pann.
0	261 .360	− 0 .010	Prag
0	261 .364	− 0 .006	D.
2	265 .355	− 0 .007	Pann.
4	269 .354	0 .000	H.
6	273 .348	+ 0 .002	H.
6	273 .332	− 0 .014	D.
6	273 .333	− 0 .013	Pl.
6	273 .323	− 0 .023	Li.
8	277 .355	+ 0 .017	D.
12	285 .327	+ 0 .005	D.
16	293 .279	− 0 .027	Li.
18	297 .296	− 0 .002	D.
36	333 .220	− 0 .006	D.

In the column "Observer," D. stands for Dunér, H. for Hartwig, Li. for Lindemann, Pann. for Pannekock, Pl. for Plassmann, and Prag for the observers Gruss and Láska at that place. The above differences between observation and computation have been combined to form normal differences in the manner indicated by the horizontal lines; but in so doing Hartwig's second observation was rejected, as Hartwig himself notes that the atmospheric conditions on that day were extremely bad. No considerable effect on the result would be produced, however, if this observation were included. I have given my observations of the 12th and 18th minima only one-fourth the weight of Lindemann's, since both were observed on only one side of the light curve, and the first, in particular, was very uncertain. In this way the following equations of condition were obtained:

$$\begin{aligned} x \qquad &= -0.008 \\ x + 3y &= -0.001 \\ x + 8y &= -0.017 \\ x + 18y &= -0.006 \end{aligned}$$

From these equations I obtained the following corrected elements:

$$\textit{Even Epochs.} \quad \text{Min.} = 1894.0 + 261^d.361 + 3^d.99201\ \frac{E}{2}.$$

The residual errors are exhibited below:

E.	Obs.—Comp.	Observer	E.	Obs.—Comp.	Observer
−2	$+0^d.003$	H.	6	$-0^d.005$	D.
0	$-0\ .019$	H.	6	$-0\ .004$	Pl.
0	$-0\ .001$	Pann.	6	$-0\ .014$	Li.
0	$-0\ .001$	Prag	8	$+0\ .026$	D.
0	$+0\ .003$	D.	12	$+0\ .014$	D.
2	$+0\ .002$	Pann.	16	$-0\ .018$	Li.
4	$+0\ .009$	H.	18	$+0\ .007$	D.
6	$+0\ .011$	H.	36	$+0\ .003$	D.

Only three uneven minima have so far been observed, all of them by me. I have not used them for determining the period, partly on this account, and partly because the small amount of the whole change during these minima renders the determination of the epochs quite uncertain. The following elements are therefore determined with reference to the period which was found for the even epochs:

Uneven Epochs. $\text{Min.} = 1894.0 + 263^d.312 + 3^d.99201\ \frac{E-1}{2}$.

The observations are thus represented:

E.	Min.	Obs.—Comp.
1	$263^d.304$	$-0^d.008$
3	$267\ .295$	$-0\ .009$
37	$335\ .186$	$+0\ .018$

From the two systems of elements I have computed the following ephemeris:

EPHEMERIS OF Z HERCULIS.

E.	Even Epochs.					E.	Uneven Epochs.				
100	1895,	April	5^d	23^h	4^m	101	1895,	April	7^d	21^h	45^m
120		May	15	21	10	121		May	17	19	51
140		June	24	19	15	141		June	26	17	56
160		August	3	17	19	161		August	5	16	1
180		September	12	15	24	181		September	14	14	6
200		October	22	13	29	201		October	24	12	10

In the above ephemeris, as elsewhere in this article, Greenwich mean time is understood.

It will be seen at once that the star cannot be observed in Europe in 1895. On the other hand, observations can be made under favorable circumstances in America—at the California observatories as early as the beginning of spring. The star is one of the very greatest interest, and it is greatly to be wished that it may be diligently observed, all the more because in 1896 observations can probably be made only in Asia and Australia.

Here I might close this article; but the temptation to enter into some speculations on the probable constitution of the system of bodies which compose the star is too strong to be resisted. Z Herculis occupies a unique position among the stars of the Algol type, inasmuch as its minima are of different degrees of brightness, regularly alternating between faint and very bright. As we now know that the interval between a bright and a faint minimum is greater than that between the latter and the next bright minimum, there can be no doubt that Z Herculis belongs to the Y Cygni type, and that the system does not, like Algol, consist of a bright and a dark body, but of two bodies, both of which are bright. But contrary to the case of Y Cygni, one component must be brighter than the other. The observations which are available suffice not only to determine the relative magnitudes of the two components, but also their relative brightness per unit of surface.

Herr Lindemann determined the magnitude of the star at its usual brightness and also at the time of a principal minimum, and found it to be

At maximum $= 6^{m}.89$,
At principal minimum $= 8\ .05$,

and with these data I find that the magnitude is

At secondary minimum $= 7^{m}.35$.

The relative degrees of brightness at maximum, uneven minimum and even minimum, are as

$$1 : 2/3 : 1/3.$$

If we let

$A =$ the surface of the brighter star,
$xA =$ the surface of the fainter star,
$1 =$ the brightness of the unit surface of the first star,
$y =$ the brightness of the unit surface of the second star,

the total brightness of Z Herculis at maximum, uneven and even minima respectively, will be represented by the following equations:

$$A+Axy = 1,$$
$$A = 2/3,$$
$$A-Ax+Axy = 1/3.$$

Subtracting the third from the first equation we obtain

$$Ax = 2/3,$$

and combining this with the second equation,

$$x = 1,$$
$$y = 1/2.$$

Hence Z Herculis consists of two components of equal size, one of which is twice as bright as the other. It is here assumed that the mutual eclipses are central, or nearly so; still I have not been able to hit upon any other assumption that represents the observations satisfactorily.

Still further conclusions can be drawn from my observations. Thus, observations extending through a whole period show that near the time of a principal minimum the variation of light extends over 6.6 hours, while near the time of a secondary minimum the entire variation requires only 4.0 hours. The orbits of the stars must therefore be considerably eccentric, and the bright minimum falls nearer to the perihelion, the faint minimum nearer to the aphelion.

If we assume that the bright minimum falls exactly at perihelion, and the faint minimum exactly at aphelion, we can determine the eccentricity of the orbit; for we have for perihelion and aphelion respectively,

$$r_1 dv = \frac{a \cos\phi}{1-e} dM,$$

$$r_2 dv_2 = \frac{a \cos\phi}{1+e} dM,$$

or,

$$\frac{r_1 dv_1}{r_2 dv_2} = \frac{1+e}{1-e}.$$

Now $\frac{r_1 dv_1}{r_2 dv_2}$ is the ratio of the arcs described by the two bodies in a unit of time, or, under the assumption which has been made, the ratio of the durations of eclipse at the even and uneven minima. Hence,

$$\frac{1+e}{1-e} = \frac{6.6}{4.0},$$

from which $e=0.245$, this value of e being the lower limit of eccentricity.

According to our elements the minima of the epochs 0, 1, 2 occur at the following times:

E.	Min.	Diff.
0	$261^d.361$	
		$+1^d.951$
1	263.312	
		$+2.041$
2	265.353	

It is therefore clear that the minima do not fall exactly at the perihelion and aphelion points, as in that case the above differences would be equal, but it is also clear that they must fall very close to these points. With the aid of the above differences the angle between the line of apsides and the line of sight can be found tentatively. After several trials I have assumed that

$$e=0.2475.$$

As the line of sight must evidently pass through both stars at the epochs of both minima, the true anomalies must then be respectively

$$v_1 \text{ and } v_2=180^\circ+v_1.$$

Hence, giving v_1 some hypothetical value, the mean anomalies M_1 and M_2 can be determined by the formulæ

$$\tan \tfrac{1}{2} E = \tan \tfrac{1}{2} v \sqrt{\frac{1-e}{1+e}},$$
$$M=E-e \sin E.$$

Now, since the interval between the epochs 0 and 1 is 1.951 days, and that between 1 and 2 is 2.041 days, it is evident that aphelion occurs during the latter interval. Hence M_2 must be numerically greater than M_1, and v_2 numerically greater than v_1. We have then

$$M_2-M_1=\frac{2.041}{1.951+2.041}.360^\circ=184^\circ.06.$$

The assumed value v_1 is correct if the values of M_2 and M_1 found by it differ by $184^\circ.06$. After several trials I found:

$$v_1=3^\circ.98, \qquad v_2=183^\circ.98,$$
$$E_1=3.09, \qquad E_2=185.02,$$
$$M_1=2.33, \qquad M_2=186.39,$$

which exactly fulfil the required conditions.

Finally, we must ascertain whether the assumed value of e is sufficiently accurate. For this purpose we first compute, with the aid of the values of M_1 and M_2 just found and the durations of the two minima, the mean anomalies M_1', M_1'' respectively at the beginning and at the end of the even minima, and the corresponding values M_2', M_2'' for the uneven minima. From these we deduce the corresponding true anomalies v_1', v_1'', v_2', v_2'', and finally the linear values, corresponding to these anomalies, of

$$r_1' \sin(v_1 - v_1'), \quad r_1'' \sin(v_1'' - v_1), \quad r_2' \sin(v_2 - v_2'), \quad r_2'' \sin(v_2'' - v_2).$$

Then, if the assumed e is correct, these last four values must be equal, since the stars are practically at an infinite distance.

The durations of the minima found above were

For even minima, $6^h\ 36^m = 0^d.27500$,
For uneven minima, $4\quad 0\ = 0\ .16667$.

Hence

$$M_1'' - M_1' = \frac{0.16667}{3.99201} \cdot 360^\circ = 15^\circ.03,$$

$$M_2'' - M_2' = \frac{0.27500}{3.99201} \cdot 360^\circ = 24^\circ.80\,;$$

$$M_1' = -5^\circ.18 \qquad M_1'' = 9^\circ.85$$
$$M_2' = 173.99 \qquad M_2'' = 198.79$$

from which we obtain:

$v_1' = -8^\circ.85$, $v_1'' = +16^\circ.76$, $\log r_1' = 9.87753$, $\log r_1'' = 9.88018$,
$v_2' = 176.26$, $v_2'' = 191.76$, $\log r_2' = 0.09574$, $\log r_2'' = 0.09306$,
$r_1' \sin(v_1 - v_1') = 0.1675$, $r_1'' \sin(v_1'' - v_1) = 0.1679$,
$r_2' \sin(v_2 - v_2') = 0.1675$, $r_2'' \sin(v_2'' - v_2) = 0.1677$.

This agreement is far more complete than is necessary, since even the third decimal may be uncertain by several units. The eccentricity which we have found is somewhat uncertain in consequence of this fact, but an eccentricity less than 0.2 or greater than 0.3, does not seem to me to harmonize with my observations.

The linear values derived above are evidently equal to the diameter of the stars expressed in terms of the major axis of the orbit.

Collecting what has been ascertained in the preceding investigation with respect to the constitution of Z Herculis, we have the following result:

Z Herculis consists of two stars of equal size, one of which is twice as bright as the other. These stars revolve around their center of gravity in an elliptical orbit whose semi-axis major is six times the diameter of the stars.[1] *The plane of the orbit passes through the Sun, the eccentricity is 0.2475, and the line of apsides is inclined at an angle of 4° to the line of sight. The stars revolve in this orbit in 3 days, 23 hours, 48 minutes, 30 seconds.*

Hence Z Herculis stands in an isolated position among stars of the Algol type, or rather it forms a hitherto missing link between the stars of the pure Algol type and Y Cygni. As I have already remarked, it is highly desirable that the star should be diligently observed by American astronomers during the present year. The last part of the above investigation shows, moreover, that it is not enough to make a large number of observations for determining the minimum epochs; the duration of the light-changes is of equal importance and should be determined as sharply as possible. Hence, when circumstances permit, the observations should begin while the star is still at its full brightness, and continue until full brightness is again restored. Accurate photometric measurements should also be made of the magnitude at maximum, as well as at both minimum epochs. The above investigation shows how much respecting the nature of the star such observations would reveal; at the same time I would be the first to acknowledge that the numerical data I have employed are greatly in need of revision, and that it will be necessary to wait for observations made under favorable conditions[2] before we can draw trustworthy conclusions as to the numerical relations of the system. These observations should moreover be made very soon. At present the angle between the line of apsides and the line of sight is small. But judging by the case of Y Cygni, we may expect that this angle is subject to rapid changes. No opportunity should therefore be neglected to determine this element as soon and as well as possible.

[1] It is here assumed that one star remains fixed in the focus of the ellipse.

[2] The conditions were very unfavorable here in 1894.