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which agree in a very satisfactory manner with that which I have found from the Internal Contacts, viz.:

$$\pi = 8''{\cdot}850 \pm 0{\cdot}022.$$

The results given by the other contacts observed in 1882 agree within the probable errors of their determinations with those obtained from the Internal Contacts.

On the Relation between Diameter of Image, Duration of Exposure, and Brightness of Objects in Photographs of Stars taken at the Royal Observatory, Greenwich. By W. H. M. Christie, M.A., F.R.S., Astronomer Royal.

Since the 13-inch photographic equatoreal was mounted and brought into working order, a number of experimental photographs have been taken at Greenwich with a view to determining the increase of diameter of star disc with exposure, and the relation between the brightness of a star and the exposure required to photograph it. Thanks to the fine definition and excellent clock-work of Sir Howard Grubb's instrument, the star images on these photographs are as a rule remarkably sharp, and very small measurable discs have been obtained in the case of faint stars.

Photographs of selected regions, for which Professor Pritchard has determined with the wedge photometer the magnitudes of certain stars of 9th and 11th magnitude, have been taken with a graduated series of exposures, and the diameters of the star discs on certain of these photographs have been measured under a microscope with a filar micrometer. The microscope has a magnifying power of about 15, referred to a distance of 10 inches. The star discs measured were scattered over the field up to a distance of 60′ from the centre, the plates being placed about 1^{mm} within the focus for the centre, *i.e.* focussed on a point about 52′ from the centre, so as to equalise the definition over the field.

I.—*The Relation between Diameter of Image and Duration of Exposure.*

Two distinct formulæ have been proposed to express this relation, both of which are, from the nature of the case, empirical:—

1. The diameter varies as a power of the exposure, *i.e.*,

$$\frac{d}{d_0} = \left(\frac{t}{t_0}\right)^p \quad \text{or} \quad \log d - \log d_0 = p\,(\log t - \log t_0)$$

where d, d_0 are the diameters of the images of the same star corresponding to exposures of t^s and t^s_0 respectively under similar

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conditions, and p should be a constant; p has been found by Professor Pritchard, M. Charlier, and others to be one-quarter, but Bond found it to be one-half, and Mr. Turner found that it varied from less than one-third for small diameters to more than one-third for large diameters, tending towards the square root for the former and towards the fourth root for the latter.

2. The diameter increases as the logarithm of the exposure, *i.e.*,

$$d - d_0 = m\,(\log t - \log t_0)$$

where m should be a constant.

It will be shown that neither of these formulæ satisfactorily represents the measures of diameters on the Greenwich photographs, the discussion of which leads to the conclusion that the square root of the diameter increases as the logarithm of the exposure, *i.e.*,

$$\sqrt{d} - \sqrt{d_0} = n\,(\log t - \log t_0),$$

n being very nearly unity when d is expressed in seconds of arc.

The following tables give the results of measures of diameters of star images with various exposures, on six photographs, the exposure being made in the order stated under each photograph. The total number of measures here discussed is about 2220, 830 images of 153 stars having been measured, each at least twice, and usually by two or more measurers. The initials of the Measurers, A. E., A. R., E. R., and W. C., are those of Miss Everett, Miss Russell, Miss Rix, and Mr. Christie. Care was taken to make the measures on the same system throughout, the diameter measured being intermediate between that of the extreme penumbra and of the nucleus, so that it should represent fairly the total photographic action in the case of small faint images as well as of the denser images. In a few cases, where a line of the *réseau* or a defect on the film interfered with the image, it has been necessary, in taking means for groups, to supply a value of diameter by comparison with other measures, without, however, introducing any appreciable uncertainty in the result.

In the several columns of the tables are given:—1. Duration of exposure in seconds; 2, its logarithm; 3, diameter in seconds of arc; 4, its square root; 5, its logarithm; 6, 7, and 8, the resulting values for m, n, and p in the three formulæ given above, formed by dividing the respective differences of consecutive values of d, $\sqrt{d}$ and log d by the differences of log t. The progression in the values of m and p is evident, and it is clear that the value 0·25 for p will not represent the smaller diameters. The magnitudes given for stars of 9 and 11 magnitudes (marked photom.) are taken from the lists recently issued from the Oxford University Observatory for selected fields, which have proved of the greatest service in the inquiry. The numeration of the stars (1 to 12, 9 mag., 13 to 24, 11 mag.) has also been taken

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from these lists. Argelander's magnitudes from the Bonn Durchmusterung are given for the brighter stars.

Photo. 128. λ *Serpentis.* 1891 July 17.

Exposures: 12ˢ, 15ˢ, 20ˢ, 30ˢ, 40ˢ, 250ˢ, 187½ˢ, 125ˢ, 94ˢ, 75ˢ.

Photographer: Mr. Criswick.

Measurers: A. E., E. R., W. C. Each star was measured by 3 observers, 5 measures in all being made in most cases.

I. λ *Serpentis*; 4·5 mag. (Arg.); 4·7 (Uran. Oxon.).

t	log t	d	$\sqrt{d}$	log d	m	n	p
s		″	″				
250	2·398	19·70	4·44	1·295	12·1	1·44	·28
187½	2·273	18·19	4·26	1·260	11·6	1·36	·30
125	2·097	16·15	4·02	1·208	5·4	0·73	·14
94	1·973	15·48	3·93	1·190	7·3	0·92	·21
75	1·875	14·76	3·84	1·169	3·6	0·48	·11
40	1·602	13·78	3·71	1·139	7·9	1·04	·26
30	1·477	12·79	3·58	1·107	7·5	1·08	·27
20	1·301	11·47	3·39	1·060	4·2	0·64	·17
15	1·176	10·94	3·31	1·039	4·4	0·72	·18
12	1·079	10·51	3·24	1·022			

II. 5 stars. 8·8 to 9·2 mag.

Mean mag. 9·06 photom. (9·04 Arg.)

t	log t	d	$\sqrt{d}$	log d	m	n	p
		″	″				
250	2·398	6·41	2·53	0·807	5·0	1·04	·36
187½	2·273	5·78	2·40	0·762	2·6	0·51	·20
125	2·097	5·33	2·31	0·727	5·6	1·29	·48
94	1·973	4·63	2·15	0·666	5·4	1·33	·54
75	1·875	4·10	2·02	0·613	3·2	0·84	·39
40	1·602	3·22	1·79	0·508	2·9	0·80	·42
30	1·477	2·86	1·69	0·456	2·7	0·85	·45
20	1·301	2·38	1·54	0·377	2·2	0·72	·41
15	1·176	2·11	1·45	0·324	2·7	0·93	·61
12	1·079	1·85	1·36	0·267			

III. 9 stars. 10·7 to 11·2 mag.

Mean mag. 10·96 photom.

t	log t	d	$\sqrt{d}$	log d	m	n	p
s		″	″				
250	2·398	2·96	1·72	0·471	2·9	0·88	·45
187½	2·273	2·60	1·61	0·415	3·4	1·14	·65
125	2·097	2·00	1·41	0·301	1·1	0·32	·24
94	1·973	1·87	1·37	0·272			

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Photo. 174. ω^2 *Cygni.* 1891 October 7.

Exposures: 5^s, 10^s, 20^s, $40_1{}^s$, $80_1{}^s$, 640^s, 320^s, 160^s, $80_2{}^s$, $40_2{}^s$.

Photographer: Miss Rix.

Measurers: A. E., W. C., E. R., 2 measures by each; $40_1{}^s$ and $40_2{}^s$, $80_1{}^s$ and $80_2{}^s$, were measured separately, and as there appeared to be no systematic difference between $40_1{}^s$ and $40_2{}^s$, and between $80_1{}^s$ and $80_2{}^s$, the means of each pair were taken.

a. ω^2 *Cygni.* 4·9 mag. (Arg.), 5·0 (Uran. Oxon.).

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	31·07	5·58	1·493	14·2	1·33	·22
320	2·505	26·80	5·18	1·428	16·2	1·67	·29
160	2·204	21·94	4·68	1·341	11·4	1·23	·24
80	1·903	18·50	4·31	1·268	6·8	0·83	·17
40	1·602	16·46	4·06	1·217	8·6	1·10	·25
20	1·301	13·86	3·73	1·142	5·1	0·74	·17
10	1·000	12·32	3·51	1·091	9·3	1·41	·37
5	0·699	9·53	3·09	0·979			

b. B.D. + 48°, 3154; 5·9 mag. (Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	16·56	4·07	1·219	8·5	1·11	·24
320	2·505	14·00	3·74	1·146	7·2	1·01	·24
160	2·204	11·84	3·44	1·073	5·3	0·80	·27
80	1·903	10·24	3·20	1·010	6·2	1·03	·29
40	1·602	8·38	2·89	0·923	7·9	1·47	·49
20	1·301	5·99	2·45	0·777	7·8	1·80	·73
10	1·000	3·64	1·91	0·561	5·9	1·77	·94
	0·699	1·90	1·38	0·279			

c. B.D. + 48°, 3148; 7·5 mag. (Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	18·33	4·28	1·263	9·2	1·11	·24
320	2·505	15·56	3·95	1·192	8·6	1·16	·26
160	2·204	12·98	3·60	1·113	6·8	0·97	·25
80	1·903	10·94	3·31	1·039	4·5	0·74	·19
40	1·602	9·58	3·09	0·981	7·1	1·21	·37
20	1·301	7·43	2·73	0·871	4·5	0·90	·29
10	1·000	6·07	2·46	0·783	8·4	1·91	·77
5	0·699	3·55	1·89	0·550			

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d. B.D. + 48°, 3128; 6·0 mag. (Arg.); 5·64 mag. (Harv. Photom.).

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	19·41	4·41	1·288	8·4	1·00	·23
320	2·505	16·87	4·11	1·227	10·0	1·31	·28
160	2·204	13·87	3·72	1·142	8·4	1·16	·29
80	1·903	11·33	3·37	1·054	4·3	0·67	·18
40	1·602	10·03	3·17	1·001	5·8	0·96	·27
20	1·301	8·30	2·88	0·919	4·0	0·73	·23
10	1·000	7·10	2·66	0·851	7·1	1·43	·51
5	0·699	4·99	2·23	0·698			

e. B.D. + 49°, 3310; 6·7 mag. (Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	13·78	3·71	1·139	5·4	0·73	·18
320	2·505	12·16	3·49	1·085	7·1	1·07	·28
160	2·204	10·03	3·17	1·001	6·8	1·13	·33
80	1·903	7·98	2·83	0·902	2·1	0·40	·12
40	1·602	7·35	2·71	0·866	5·6	1·11	·37
20	1·301	5·68	2·38	0·754	2·8	0·60	·23
10	1·000	4·83	2·20	0·684	3·9	0·93	·40
5	0·699	3·67	1·92	0·564			

g. B.D. + 49°, 3298; 6·9 mag. (Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	12·71	3·57	1·104	4·0	0·60	0·33
320	2·505	11·51	3·39	1·004	7·9	1·23	0·14
160	2·204	9·14	3·02	0·961	5·6	0·97	0·30
80	1·903	7·45	2·73	0·872	3·8	0·73	0·24
40	1·602	6·30	2·51	0·799	6·0	1·29	0·48
20	1·301	4·51	2·12	0·654	2·6	0·64	0·28
10	1·000	3·73	1·93	0·571	5·4	1·60	0·82
5	0·699	2·10	1·45	0·323			

2. B.D. + 48°, 3145; 9·0 mag. (Arg.); 9·02 mag. (photom.)

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
s		″	″				
640	2·806	9·51	3·08	0·978	7·2	1·23	0·38
320	2·505	7·33	2·71	0·865	4·9	0·98	0·33
160	2·204	5·86	2·42	0·768	8·1	1·87	0·77
80	1·903	3·45	1·86	0·538	2·6	0·74	0·37
40	1·602	2·68	1·64	0·428			

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Photo. 178. 36 *Pegasi.* 1891 October 9.

Exposures: 5^s, 10^s, 20^s, 40_1^s, 80_1^s, 640^s, 320^s, 160^s, 80_2^s, 40_2^s, 10_2^s.

Photographer: Mr. Criswick.

Measurers: A. E, A. R.

40_1^s and 40_2^s, 80_1^s and 80_2^s, were measured separately and the means of each pair taken.

I. 5 stars. Nos. 1–5. 8·6 to 9·0 mag. photom.

Mean mag. 8·90 photom. (9·08 Arg.)

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s 640	2·806	8″·02	2″·83	0·905	6·3	1·16	·39
320	2·505	6·14	2·48	0·789	3·6	0·77	·28
160	2·204	5·06	2·25	0·705	6·0	1·50	·64
80	1·903	3·25	1·80	0·513	3·7	1·13	·62
40	1·602	2·12	1·46	0·328	2·4	0·93	·60
20	1·301	1·40	1·18	0·148			

II. 4 stars. Nos. 8, 9, 11, 12. 9·2 to 9·4 mag. photom.

Mean mag. 9·28 photom. (9·25 Arg.)

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s 160	2·204	3″·43	1″·85	0·536	4·1	1·20	·64
80	1·903	2·21	1·49	0·345	2·8	1·06	·68
40	1·602	1·38	1·17	0·141			

III. 7 stars. Nos. 13–19. 10·8 to 11·0 mag. photom.

Mean mag. 10·87 photom.

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s 640	2·806	3″·74	1″·93	0·574	4·4	1·26	·64
320	2·505	2·41	1·55	0·383	3·8	1·43	·94
160	2·204	1·26	1·12	0·101			

IV. 5 stars. Nos. 20–24. 11·0 to 11·3 mag. photom.

Mean mag. 11·16 photom.

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s 640	2·806	2″·47	1″·57	0·394	4·0	1·46	·95
320	2·505	1·28	1·13	0·109			

Photo. 180. 36 *Pegasi.* 1891 October 12.

Exposures: 631^s, 398^s, 251^s, 159^s, 100^s, 632^s, 10^s, 16^s, 25^s, 40^s, 633^s.

Photographer: Miss Everett.

Measurers: E. R., A. E.

632^s and 633^s were measured for Group I. and the means taken.

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I. 3 stars. 36 *Pegasi*, B.D. + 8°·5880, B.D. + 8°·5870.

Mags. 5·8, 7·7, 7·8 Arg.

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		″	″				
398	2·600	12·90	3·59	1·111	5·7	0·80	·20
251	2·400	11·76	3·43	1·071	7·2	1·10	·29
159	2·201	10·32	3·21	1·014	5·7	0·90	·26
100	2·000	9·18	3·03	0·963	5·7	0·95	·29
63	1·799	8·04	2·84	0·905	4·8	0·90	·28
40	1·602	7·08	2·66	0·850	3·9	0·75	·26
25	1·398	6·30	2·51	0·799	5·1	1·05	·38
16	1·204	5·28	2·30	0·723	7·2	1·70	·70
10	1·000	3·84	1·96	0·584			

II. 3 stars. Nos. 1, 5, 6. 8·6 to 9·1 mag.

Mean mag. 8·90 photom. (9·03 Arg.)

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		″	″				
398	2·600	6·54	2·56	0·816	4·5	0·95	·33
251	2·400	5·64	2·37	0·751	5·4	1·15	·46
159	2·201	4·56	2·14	0·659	3·6	0·90	·38
100	2·000	3·84	1·96	0·584	2·4	0·60	·29
63_2	1·799	3·36	1·84	0·526	3 0	0·90	·43
40	1·602	2·76	1 66	0·441	3·3	1·05	·60
25	1·398	2·10	1·45	0·322	2·7	1·00	·65
16	1·204	1·56	1·25	0·193			

III. 6 stars. Nos. 1–6. 8·6 to 9·1 mag.

Mean mag. 8·93 photom. (9·12 Arg.)

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		″	″				
63_2	1·799	3·48	1·87	0·542	3·6	1·05	·51
40	1·602	2·76	1·66	0·441	2·1	0·65	·36
25	1·398	2·34	1·53	0·369	3·6	1·30	·79
16	1·204	1·62	1·27	0·210			

IV. 6 stars. Nos. 7–12. 9·1 to 9·4 mag.

Mean mag. 9·25 photom. (9·20 Arg.)

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		″	″				
63_2	1·799	2·46	1·57	0·391	2·1	0·70	·41
40	1·6c2	2·04	1·43	0·310	2·4	0·90	·59
25	1·398	1·56	1·25	0·193			

V. 8 stars. Nos. 13–20. 10·8 to 11·0 mag.

Mean mag. 10·89 photom.

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		″	″				
398	2·600	3·00	1·73	0·477	3·0	0·90	0·49
251	2·400	2·40	1·55	0·380	1·8	0·60	0·35
159	2·201	2·04	1·43	0·310	2·4	0·90	0·59
100	2 000	1·56	1·25	0·193			

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VI. 4 stars. Nos. 21–24. 11·1 to 11·3 mag.
Mean mag. 11·20 photom.

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		"	"				
398	2·600	2·40	1·55	0·380	4·8	1·75	1·11
251	2·400	1·44	1·20	0·158	1·2	0·50	0·40
159	2·201	1·20	1·10	0·079			

Photo. 181. 36 *Pegasi.* 1891 October 12.

Exposures: 100_1^s, 631^s, 398^s, 100_2^s, 16^s, 25^s, 40^s, 40^s, 63^s, 100_3^s.

Photographer: Miss Everett.

Measurers: A. E., A. R.

It was intended also to have exposures of 251^s and 159^s, but, as there is only one image corresponding, it has been thought better not to use the measures of this image, there being no means of determining which exposure it represents. There seems to be no doubt as to the long exposures of 631^s and 398^s, the clock times of beginning and ending having been recorded. The exposure 100_2^s is checked by the measures for 100_3^s. The sky was not quite free from cloud.

I. 3 stars. 36 *Pegasi*, B.D. + 8°·5880, B.D. + 8°·5870. Mags. 5·8, 7·7, 7·8 Arg.

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		"	"				
631	2·800	14·72	3·84	1·168	5·9	0·80	0·18
398	2·600	13·54	3·68	1·132	4·7	0·67	0·17
100_2	2·000	10·76	3·28	1·032			
100_3	2·000	10·68	3·27	1·029	10·6	1·75	0·48
63	1·799	8·60	2·93	0·935	6·3	1·10	0·35
40	1·602	7·35	2·71	0·866	6·5	1·25	0·42
25	1·398	6·06	2·46	0·783	6·4	1·35	0·52
16	1·204	4·78	2·19	0·679			

II. 3 stars, Nos. 1, 5, 6. 8·6 to 9·1 mag.
Mean mag. 8·90 photom. (9·03 Arg.)

t	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
s		"	"				
631	2·800	7·36	2·71	0·867	4·5	0·85	0·29
398	2·600	6·46	2·54	0·810	3·5	0·73	0·28
100_2	2·000	4·32	2·08	0·636			
100_3	2·000	4·46	2·11	0·649	6·7	1·75	0·79
63	1·799	3·06	1·75	0·486	2·1	0·65	0·32
40	1·602	2·64	1·62	0·422	3·2	1·05	0·61
25	1·398	2·00	1·41	0·301	3·1	1·20	0·81
16	1·204	1·38	1·17	0·140			

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III. 6 stars, Nos. 1–6. 8·6 to 9·1 mag.
Mean mag. 8·93 photom. (9·12 Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *i*	*m*	*n*	*p*
s		″	″				
100_2	2·000	4·76	2·18	0·678			
100_3	2·000	4·68	2·16	0·670	7·0	1·75	0·77
63	1·799	3·32	1·82	0·521	3·0	0·85	0·43
40	1·602	2·72	1·65	0·435	3·5	1·15	0·65
25	1·398	2·02	1·42	0·305	3·1	1·20	0·79
16	1·204	1·40	1·18	0·146			

IV. 6 stars, Nos. 7–12. 9·1 to 9·4 mag.
Mean mag. 9·25 photom. (9·20 Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *i*	*m*	*n*	*p*
s		″	″				
100_2	2·000	3·83	1·96	0·583			
100_3	2·000	3·56	1·89	0·551	6·3	1·85	0·90
63	1·799	2·44	1·56	0·387	3·6	1·25	0·76
40	1·602	1·72	1·31	0·236	1·4	0·55	0·39
25	1·398	1·44	1·20	0·158			

V. 8 stars, Nos. 13–20. 10·8 to 11·0 mag.
Mean mag. 10·89 photom.

t	log *t*	*d*	$\sqrt{d}$	log *i*	*m*	*n*	*p*
s		″	″				
631	2·800	3·98	1·99	0·601	5·3	1·40	·68
398	2·600	2·92	1·71	0·465	1·8	0·57	·33
100_2	2·000	1·71	1·33	0·232			
100_3	2·000	1·97	1·40	0·295	2·6	1·10	·72
63	1·799	1·32	1·15	0·121			

VI. 4 stars, Nos. 21–24. 11·1 to 11·3 mag.
Mean mag. 11·20 photom.

t	log *t*	*d*	$\sqrt{d}$	log *i*	*m*	*n*	*p*
s		″	″				
631	2·800	3·01	1·73	0·477	3·3	1·00	·53
398	2·600	2·35	1·53	0·371	1·4	0·50	·33
100_2	2·000	1·55	1·24	0·190			
100_3	2·000	1·49	1·22	0·173			

Photo. 185. ω^2 *Cygni.* 1891 October 14.

Exposures: $41^m\ 52^s$, 251^s, 40^s, 25^s, 159^s, 100^s, 16^s.

Photographer: Miss Everett.

Measurers: A. E., A. R. Ten measures of ω^2 *Cygni* by A. E. and A. R. and 2 measures of each of the other stars by A. E. or A. R. (alternately).

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I. ω^2 *Cygni.* 4.9 mag. (Arg.) 5.0 (*Uran. Oxon.*).

t (m s)	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
41 52	3.400	45″.62	6″.75	1.659	18.10	1.50	0.22
251	2.400	27.52	5.25	1.440	14.7	1.47	.25
159	2.201	24.60	4.96	1.391	11.0	1.69	.30
100	2.000	21.38	4.62	1.330	11.0	1.26	.25
40	1.602	17.00	4.12	1.230	8.6	1.08	.23
25	1.398	15.24	3.90	1.183	6.9	0.88	.21
16	1.204	13.90	3.73	1.143			

II. 8 stars. 5.9 to 7.9 mag. Arg.

Mean mag. 7.08 Arg.

t (m s)	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
41 52	3.400	25″.49	5″.05	1.406	11.1	1.25	.25
251	2.400	14.42	3.80	1.159	8.9	1.12	.29
159	2.201	12.65	3.56	1.102	7.6	1.15	.28
100	2.000	11.12	3.33	1.046	5.2	0.80	.22
40	1.602	9.07	3.01	0.958	5.8	1.00	.30
25	1.398	7.88	2.81	0.897	6.3	1.19	.38
16	1.204	6.66	2.58	0.824			

III. 6 stars. 6.8 to 8.0 mag. Arg.

Mean mag. 7.35 Arg.

t (m s)	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
41 52	3.400	17″.08	4″.13	1.232	7.9	1.11	.27
251	2.400	9.14	3.02	0.961	6.0	1.01	.31
159	2.201	7.94	2.82	0.900	5.7	1.05	.33
100	2.000	6.80	2.61	0.833	4.0	0.83	.29
40	1.602	5.20	2.28	0.716	5.5	1.28	.52
25	1.398	4.08	2.02	0.611	3.8	0.98	.45
16	1.204	3.34	1.83	0.524			

IV. 3 stars. 8.64 to 9.02 mag. photom.

Mean mag. 8.86 photom. (9.07 Arg.)

t (m s)	log *t*	*d*	√*d*	log *d*	*m*	*n*	*p*
41 52	3.400	16″.18	4″.02	1.209	7.7	1.11	.28
251	2.400	8.44	2.91	0.926	4.8	0.91	.26
159	2.201	7.48	2.73	0.874	6.6	1.25	.42
100	2.000	6.16	2.48	0.790	4.6	1.00	.39
40	1.602	4.32	2.08	0.636	2.4	0.59	.26
25	1.398	3.84	1.96	0.584	4.4	1.19	.57
16	1.204	2.98	1.73	0.474			

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V. 9 stars. 8·99 to 9·39 mag. photom.

Mean mag. 9·17 photom. (9·32 Arg.)

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
m s		"	"				
41 52	3·400	11·94	3·46	1·077	6·4	1·10	·33
251	2·400	5·56	2·36	0·745	6·2	1·40	·54
159	2·201	4·33	2·08	0·637	4·6	1·15	·52
100	2·000	3·41	1·85	0·532	2·4	0·71	·35
40	1·602	2·47	1·57	0·393	3·1	1·03	·63
25	1·398	1·84	1·36	0·265	3·0	1·24	·85
16	1·204	1·26	1·12	0·100			

VI. 12 stars. 10·7 to 11·2 mag. photom.

Mean mag. 10·93 photom.

t	log *t*	*d*	$\sqrt{d}$	log *d*	*m*	*n*	*p*
m s		"	"				
41 52	3·400	6·24	2·50	0·795	3·6	0·89	·38
251	2·400	2·58	1·61	0·412	3·0	1·01	·57
159	2·201	1·99	1·41	0·299	1·8	·65	·43
100	2·000	1·63	1·28	0·212			

The values of *p* and *n* have been grouped, taking log. diameter as argument, and the means of the groups, with the number of values in each group, are exhibited in the following table, which shows clearly the progression in the values for *p*, while *n* is sensibly constant. The values corresponding to the 5^s exposure in Photo. 174 have not been used in forming these means, as there might be a sensible error in estimating the duration of such a short exposure. For simplicity of computation the values of *p* and *n* have been treated as of equal weight in forming the means.

Means of Groups.

Log. Diam. Limits.	Log. Diam. Mean.	No. in Group.	Diam.	$\sqrt{\text{Diam.}}$	*p*	*n*	Discordance from Mean.
			"	"			
0·12–0·30	0·24	19	1·74	1·32	·657	0·996	– 039
0·31–0·41	0·36	12	2·29	1·51	·530	0·910	–·125
0·42–0·53	0·46	16	2·88	1·70	·526	1·039	+·004
0·56–0·66	0·61	15	4·07	2·02	·508	1·185	+·150
0·67–0·78	0·72	12	5·25	2·29	·403	1·060	+·025
0·78–0·88	0·83	14	6·76	2·60	·351	1·047	+·012
0·88–0·98	0·92	15	8·32	2·88	·301	1·017	–·018
0·98–1·08	1·03	16	10·72	3·27	·242	0·924	–·111
1·08–1·18	1·13	15	13·49	3·67	·220	0·922	–·113
1·18–1·55	1·30	17	19·95	4·47	·240	1·246	+·211
	Total ...	151			Mean ...	1·0346	± ·085

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The discordances from the mean in the separate values of n (given in the last column) appear to be accidental, and we may conclude that the formula—

$$\sqrt{d} - \sqrt{d_0} = 1 \cdot 03 \, (\log t - \log t_0),$$

or

$$\sqrt{\text{diam.}} = 1 \cdot 03 \times \log \text{exposure} + \text{const.}$$

(the diameter being expressed in seconds of arc) represents the measures satisfactorily, through the range 1″·3 to 35″·5 in the diameters, corresponding to a range of 8 magnitudes, or to a ratio of 1 to 1600 in the limiting light-intensities.

The following are the probable errors of a determination of n in the tables, inferred from the discordances from the mean in the several groups:

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	Mean
·259	·196	·191	·337	·264	·143	·156	·177	·158	·186	·207

Each of these values represents the probable error of a determination of the quantity $\frac{\sqrt{d} - \sqrt{d'}}{\log t - \log t'}$. The mean value of $\log t - \log t' = 0 \cdot 226$, whence the probable error of a determination of $\sqrt{d} - \sqrt{d'}$ is 0·047, corresponding to 0·117 magnitude, since 0·4 in $\log t - \log t'$ or in $\sqrt{d} - \sqrt{d'}$ represents 1 magnitude. The probable error of a result for $\sqrt{d}$ in the above tables is therefore $\frac{0 \cdot 047}{\sqrt{2}} = 0 \cdot 033$, corresponding to 0·083 magnitude. Each of these results for $\sqrt{d}$ is the mean of about eleven measures, either of one star or of a small group, all taken with the same exposure, there being 2220 measures for 197 values of $\sqrt{d}$. In this probable error is included the systematic error depending on the state of the sky and other causes incidental to the particular exposure, this systematic error being probably an important element in the probable error inferred from the discordances in the values of n.

The probable error for a determination of n in the tables appears to be approximately the same for each of the ten groups, though it tends to become slightly smaller for the groups with larger diameter and longer exposure.

As the value of n for each group is the mean of about fifteen separate values (on the average), its probable error (considering all the groups as of approximately the same weight) would be 0·053, and the mean error 0·063; whereas the mean discordance from the formula is 0·085, a quantity which so slightly exceeds the computed mean error, that the formula may safely be taken as representing the observations within the limits of accidental error.

The accompanying curves exhibit the results graphically.

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Fig. 1 shows the value of p as ordinate, with log. diameter as abscissa, the curve being drawn freely by hand among the dots.

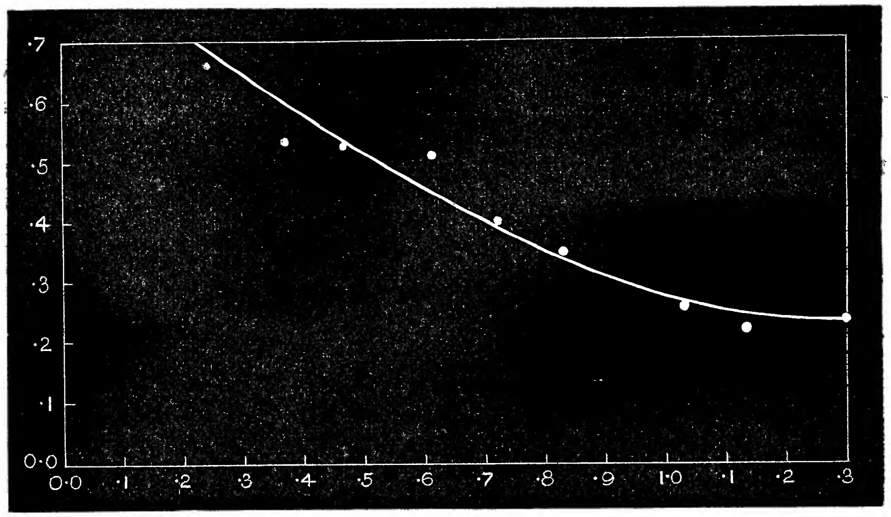
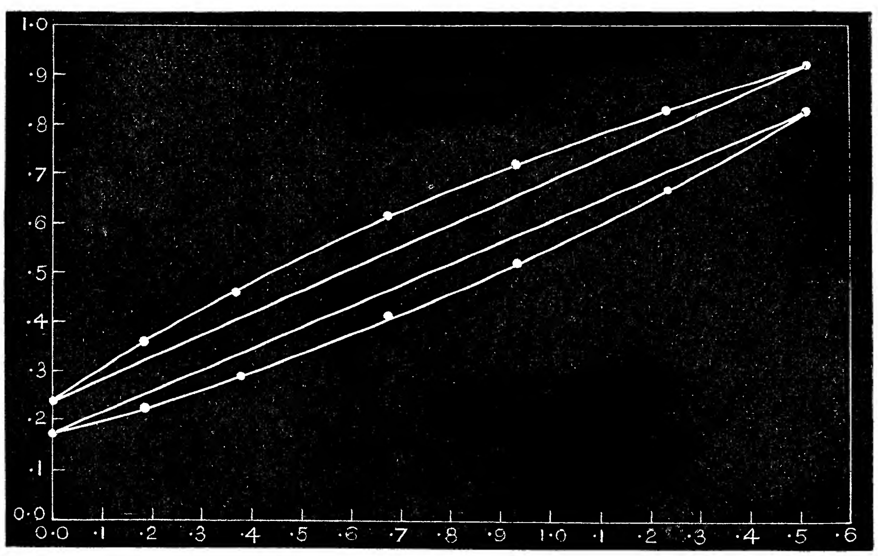


Fig. 2.—The upper curve represents the log. diameter (as ordinate), and the lower the diameter (as ordinate), with the log. duration of exposure (diminished by a constant) as abscissa, the abscissa being deduced from the log. diameter by summation, taking the value of p from the curve in fig. 1. The straight lines represent the formulæ

$$\log d - \log d_0 = p\ (\log t - \log t_0),$$

and

$$d - d_0 = m\ (\log t - \log t_0),$$



where m and p are constants deduced from the extreme values. The latter represents the measures a little better than the former, the truth lying between the two.

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Fig. 3 represents the square root of the diameter as ordinate with the log. duration of exposure (diminished by a constant) as abscissa, deduced as for fig. 2. It will be seen that the dots thus deduced lie sensibly on a straight line.

$$\sqrt{d} - \sqrt{d_0} = 1{\cdot}03\,(\log t - \log t_0).$$

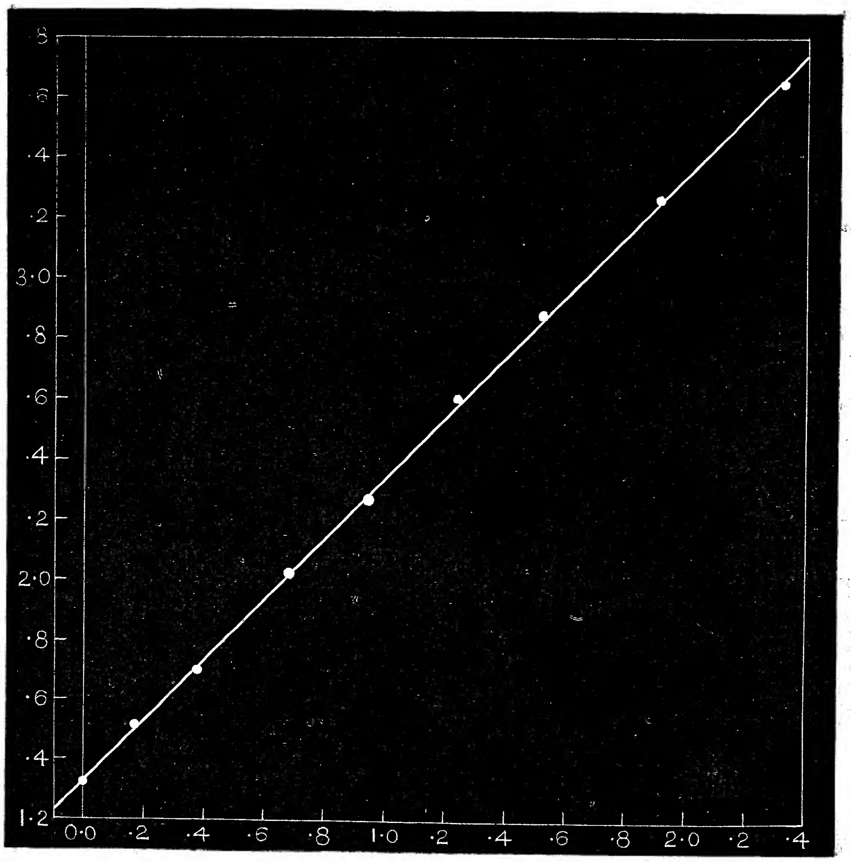

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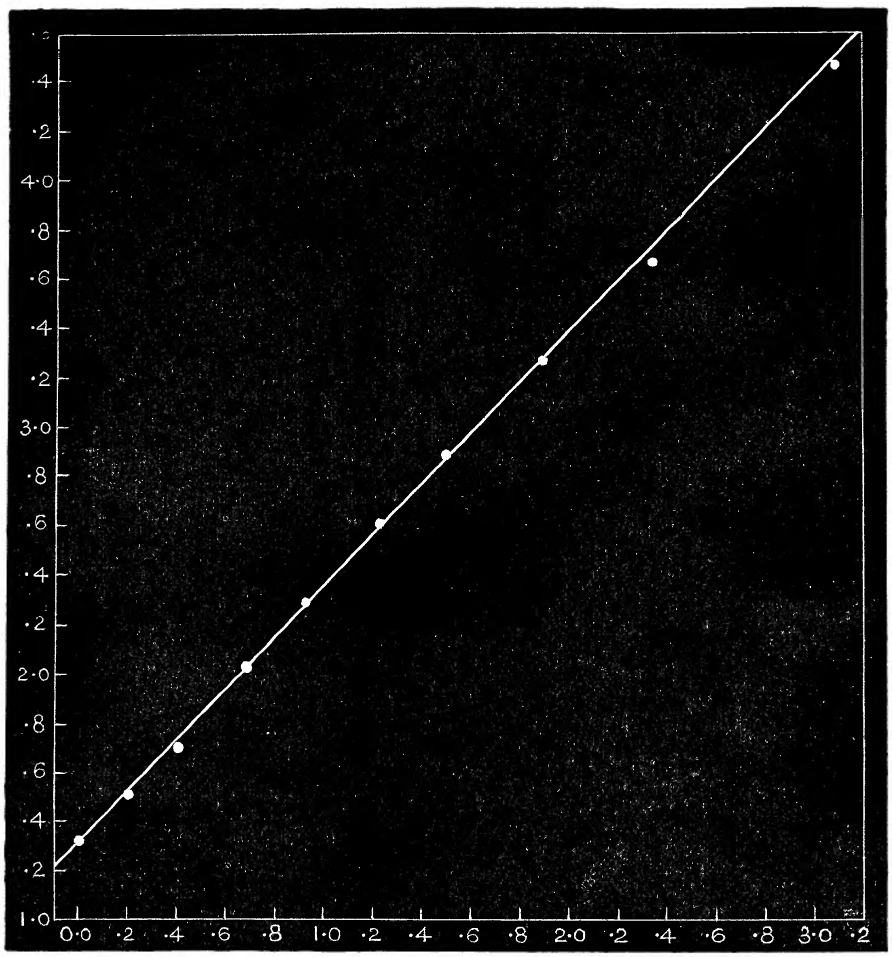
Fig. 4.—The dots exhibit the relation between $\sqrt{\text{diameter}}$ as ordinate and log. duration of exposure (diminished by a constant) as abscissa, the latter being obtained directly from the values for n in the table (Means of Groups). The straight line represents the formula

$$\sqrt{d} - \sqrt{d_0} = 1{\cdot}03 (\log t - \log t_0).$$



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The following table gives the values of $y = \sqrt{\text{diam.}}$ and of $x = \log t - \log t_0$, from which the dots have been plotted, the values of x being obtained by summation of the quantities $\delta x = \frac{\delta y}{n}$, where n is the mean of the preceding and following observed values applicable to the interval δy. The third column shows the values of $x = \frac{y - y_0}{1{\cdot}03}$ from the formula, and the fourth the discordance, calculated − observed, in log t, converted into discordance in magnitude in the last column, by dividing by 0·4.

$\sqrt{\text{Diam.}}$ y	$\log t - \log t_0$ x	$\frac{y-y_0}{1\cdot03}$	Discordance C−O. in log t	in mag.
1·32″	0·000	0·000		
1·51	0·199	0·184	−·015	−·038
1·70	0·394	0·369	−·025	−·062
2·02	0·682	0·680	−·002	−·005
2·29	0·922	0·942	+·020	+·050
2·60	1·216	1·243	+·027	+·068
2·88	1·487	1·515	+·028	+·070
3·27	1·889	1·893	+·004	+·010
3·67	2·322	2·282	−·040	−·100
4·47	3·060	3·058	−·002	−·005
		Mean Discordance	±·018	±·045

Thus the mean discordance between the abscissæ deduced directly from the observations, and those given by the formula, corresponds to less than one-twentieth of a magnitude through a range of eight magnitudes.

II.—*The Relation between Duration of Exposure and Brightness of Star photographed.*

A discussion of the measures of diameters given in the preceding tables compared with the valuable photometric determinations of magnitudes of the 9th and 11th magnitude stars, made at the Oxford University Observatory with Professor Pritchard's wedge photometer, and with Argelander's magnitudes for stars of about the 7th and 9th magnitudes, gives data for determining whether the law that, for equal photographic effects,

$$\text{Duration of exposure} \times \text{brightness of object} = \text{const.}$$

holds satisfactorily under the conditions and within the limits here considered.

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It will be remembered that at the meeting of the Comité de la Carte du Ciel at Paris, in 1889, it was decided that, in order to photograph the 11th magnitude stars, the exposure should be that required for Argelander's 9th magnitude, multiplied by 6·25 (or, rather, 6·31, as it should have been), the intention being to prolong Argelander's scale from mag. 9·0 to mag. 11·0. Since then doubts have been raised as to the validity of this procedure, it being considered that the exposure must be increased in a much higher ratio than 1 to 6·31 to photograph stars fainter by 2·0 magnitudes. The present discussion is directed principally to this point, the photographic images of stars of the 9th and 11th magnitudes being compared with the Oxford photometric magnitudes. As a further check, the photographic images of stars of about the 7th and 9th magnitudes have been compared with Argelander's visual magnitudes, but difficulty has been experienced in obtaining a sufficiency of such stars to eliminate the large accidental errors arising from differences between visual and photographic brightness.

In the following comparison of diameters, the first column indicates the two results compared, denoted by the exposure, and the group in the preceding tables; thus 12_2^s denotes the 12^s exposure in Group II., and similarly in other cases. In the second column the difference of $\sqrt{\text{diameter}}$ for the two exposures compared is given, the values being taken from those for the groups in the above-mentioned tables. The third column gives the log. factor by which the exposure should be multiplied to equalise the two photographic images. Since $\delta(\sqrt{\text{diam.}}) = \delta(\log t)$ very nearly, the log. factor is obtained simply by adding to the difference of the logarithms of exposures the quantities in the second column. The log. factor for 1·0 magnitude is obtained at once by dividing by the difference of magnitudes of the groups compared (nearly 2·0 magnitudes in all cases), and the resulting factor for 1 magnitude is given in the last column, this being the factor by which the duration of exposure should be multiplied to obtain equal photographic effect from a star fainter by 1 magnitude.

Comparison of Diameters of 9th and 11th mag. Stars (Oxford Photometric Magnitudes).

Photo. 128.

s s	$\delta(\sqrt{d})$	Log. factor for 1 90 mag.	Log. factor for 1 mag.	Factor for 1 mag.
$12_2 - 94_3$	−·01	0·88	0·463	2·90
$15_2 - 125_3$	+·04	0·96	0·505	3·20
$20_2 - 187_3$	−·07	0·90	0·474	2·98
$30_2 - 250_3$	− 03	0·89	0·468	2 94
			Mean	3·00

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Photo. 178.

s s	$\delta(\sqrt{d})$	Log. factor for 1·93 mag.	Log. factor for 1 mag.	Factor for 1 mag.
$20_1 - 160_3$	+·06	0·96	0·497	3·14
$40_2 - 320_4$	+·04	0·94	0·487	3·07
$40_1 - 320_3$	−·09	0·81	0·420	2·63
$80_2 - 640_4$	−·08	0·82	0·425	2·66
$80_1 - 640_3$	−·13	0·77	0·399	2·51
				Mean 2·80

Photo. 180.

s s		for 1·96 mag.		
$16_3 - 100_5$	+·02	0·82	0·418	2·62
$25_4 - 159_6$	+·15	0·95	0·485	3·05
$25_3 - 159_5$	+·10	0·90	0·459	2·88
$40_4 - 251_6$	+·23	0·03	0·525	3·35
$40_3 - 251_5$	+·11	0·91	0·464	2·72
$63_4 - 398_6$	+·02	0·82	0·418	2·62
$63_3 - 398_5$	+·14	0·94	0·480	3·02
				Mean 2·89

Photo. 181.

s s		for 1·96 mag.		
$16_3 - 63_5$	+·03	0·63	0·321	2·09
$25_4 - 100_6$	−·03	0·57	0·291	1·95
$25_3 - 100_5$	+·05	0·65	0·332	2·15
$40_4 - 100_6$	+·08	0·48	0·245	1·76
$40_4 - 398_6$	−·28	0·72	0·367	2·33
				Mean 2·06
s s				
$40_3 - 100_5$	+·28	0·68	0·347	2·22
$40_3 - 398_5$	−·06	0·94	0·480	3·02
$63_4 - 398_6$	+·03	0·83	0·423	2·65
$63_3 - 398_5$	+·11	0·91	0·464	2·91
$100_4 - 631_6$	+·20	1·00	0·510	3·24
$100_3 - 631_5$	+·18	0·98	0·500	3·16
				Mean 2·87

Photo. 185.

s s		for 1·96 mag.		
$16_5 - 100_6$	−·16	0·64	0·327	2·12
$25_5 - 159_6$	−·05	0·75	0·383	2·42
$40_5 - 251_6$	−·04	0·76	0·388	2·44
s m s				
$251_5 - 41{\cdot}52_6$	−·14	0·86	0·439	2·75
				Mean 2·43

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Comparison of Diameters of 7th and 9th mag. Stars (Argelander's Magnitudes).

Photo. 185.

Groups II. and IV. consist of Stars photographically bright as compared with their visual magnitudes.

s s	$\delta(\sqrt{d})$	Log. factor for 1·78 mag.	Log. factor for 1 mag.	Factor for 1 mag.
$16_2 - 100_4$	+ ·10	0·90	0·506	3·21
$25_2 - 159_4$	+ ·08	0·88	0·494	3·12
$40_2 - 251_4$	+ ·10	0·90	0·506	3·21
m s $251_2 - 41{\cdot}52_4$	− ·22	0·78	0·438	2·74
				Mean 3·07
		for 1·97 mag.		
$16_3 - 100_5$	− ·02	0·78	0·396	2·49
$25_3 - 159_5$	− ·06	0·74	0·376	2·38
$40_3 - 251_5$	− ·08	0·72	0·365	2·32
m s $251_3 - 41{\cdot}52_5$	− ·44	0·56	0·284	1·92
				Mean 2·28

Taking the means for each photograph (181 being divided into two groups of diameters under and over 2″·1 respectively) we have

Photo.	Factor for 1 magnitude. Oxford photom.	Argelander	Diff. of Mag. for Factor 2·512 Oxford photom. mag.	Argelander mag.
128	3·00	...	0·839	...
178	2·80	...	0·895	...
180	2·89	...	0·868	...
181_1	2·06	...	1·274	...
181_2	2·87	...	0·873	...
185	2·43	3·07	1·036	0·821
185		2·28		1·117
Means	2·675	2·675	0·964	0·969

It appears that whether we take the Oxford photometric magnitudes or Argelander's, the factor 2·512 for the exposure gives 1·0 magnitude within the limits of accidental error.

Considering the large differences between photographic and visual brightness in the case of individual stars (amounting probably in some instances to nearly ± 1 magnitude) and the accidental variations in atmospheric or instrumental tremors, in the state of the sky, in incipient dew on the object glass, and in sensibility or development of different parts of the photographic film, the discordances in the individual results for the factor are not surprising, and the mean value agrees quite as closely with the law

$$\text{Exposure} \times \text{brightness} = \text{const.}$$

as could be expected.

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It is curious, however, that Photo. 181 gives a markedly large factor for the smaller diameters, indicating that the first faint images of 11th magnitude stars were more readily obtained relatively to those of the 9th magnitude stars than the denser images, and Photo. 185 to a certain extent gives a similar result. Both these photographs, which, it may be remarked, were taken in moonlight, show small sharp images even of the faintest stars.

Similar conclusions result from an examination of the images visible on the photographs, made by myself with an achromatic magnifying lens of about 1-inch focus, and tabulated before the plates were measured. In the following table the images visible on Photos. 178, 180, and 181 are indicated by the corresponding exposure, in italics when the image was only faintly visible, in ordinary type when it was fairly distinct, and in heavy type when it was unmistakeable, at a glance, and accurately measurable for diameter. The images with longer exposures were, of course, also visible, but there is no occasion to note them.

Exposures giving Images of 9th and 11th mag. Stars on Photographs of the District round 36 Pegasi.

Star's No.	Magnitude Photom.	Magnitude Arg.	Photo. 178.		Photo. 180.			Photo. 181.		
			s	s	s	s	s	s	s	s
1	8·6	8·7	...	**20**	...	10	**16**	...	...	**16**
2	8·9	9·2	20	**40**	...	10	**16**	...	...	**16**
3	9·0	9·3	20	**40**	...	*10*	**16**	...	...	**16**
4	9·0	9·1	...	**20**	...	10	**16**	...	...	**16**
5	9·0	9·1	...	**20**	...	10	**16**	...	...	**16**
6	9·1	9·3	*20*	**40**	...	...	**16**	...	...	**16**
7	9·1	9·0	...	**40**	...	16	**25**	...	16	**25**
8	9·2	9·1	*40*	**80**	...	16	**25**	*16*	*25*	**40**
9	9·2	9·4	80	...	...	25	**40**	...	*16*	**25**
10	9·3	9·2	80	...	...	16	**25**	...	...	**40**
11	9·3	9·2	...	**40**	*16*	25	**40**	...	...	**16**
12	9·4	9·3	*40*	**80**	...	40	**63**	...	25	**40**
13	10·8	...	*80*	**160**	...	*63*	**100**	*40*	63	**100**
14	10·8	...	...	**160**	...	*63*	**100**	...	40	**63**
15	10·8	...	*160*	**320**	...	*63*	**100**	*40*	*63*	**100**
16	10·9	...	80	**160**	...	63	**100**	...	63	**100**
17	10·9	...	...	**320**	...	*63*	**100**	*40*	63	**100**
18	10·9	...	*160*	**320**	...	63	**100**	...	63	**100**
19	11·0	...	*80*	**160**	...	*63*	**100**	*40*	63	**100**
20	11·0	...	*160*	**320**	...	*63*	**100**	...	*63*	**100**
21	11·1	...	*160*	**320**	...	**100**	**159**	...	...	**100**
22	11·2	...	...	**320**	...	100	159	...	*63*	**100**
23	11·2	...	...	**320**	...	100	**159**	...	*63*	**100**
24	11·3	...	160	**320**	...	*159*	**251**	...	*100*	**159**

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It thus appears that this independent examination fully confirms the results deduced from the measures, particularly in the case of Photo. 181. It is remarkable that on this photograph (taken in moonlight) some 11th magnitude stars were impressed faintly in 40^s, and nearly all in 63^s. Measurable images of all stars down to 11·0 magnitude were obtained in 100^s, both on this photograph and on No. 180. On the other hand, about three times the exposure was required on Photo. 178, a fact which may be explained by a slight dimming of the object glass by dew, noticed by the observer a little time after this photograph was taken.

The results of this discussion may be thus summarised:—

Putting d=diameter (in seconds of arc) of the image of a star of magnitude m (Pogson's scale) with the exposure t (in seconds of time), we have—

I. For the same star with different exposures,

$$\sqrt{d} = 1{\cdot}03 \log t + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

an empirical formula which represents the observations through a range of eight magnitudes with a mean apparent error of only $\pm$ 0·018 corresponding to $\pm$ ·045 mag.

II. For equal diameters

$$\text{Exposure} \times \text{brightness} = \text{const.}$$

or

$$0{\cdot}4 \times m = \log t + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The relation actually found is

$$0{\cdot}4 \times m = 0{\cdot}97 \log t + \text{const.},$$

which is sensibly the same as (2).

III. From (1) and (2) it follows that, for the same exposure, the relation between diameter of image and magnitude of star is

$$0{\cdot}4 \times m = \text{const.} - \frac{\sqrt{d}}{1{\cdot}03}$$

or

$$m = \text{const.} - 2{\cdot}43 \sqrt{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

that is,

$$\text{Magnitude of star} = \text{const.} - 2{\cdot}43 \times \sqrt{\text{diameter.}}$$

The constants are, of course, different in the different cases. In (1) the constant would be different for stars of different magnitudes; in (2) for stars of different diameter; and in (3) for stars with different exposures. They would also, of course, be slightly different for different plates.

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Combining (1), (2), *and* (3) *in one formula, we have*

$$m = 2{\cdot}5\ (\log t - 0{\cdot}97\sqrt{d}) + \text{const.} \quad . \quad . \quad . \quad . \quad (4)$$

which may be considered as replacing the approximate empirical formula given in the "Monthly Notices" for March, 1891, *vol. li. p.* 284.

Royal Observatory, Greenwich:
1892 *January* 8.

On the Observations for Coincidence of the Collimators through the Cube of the Transit Circle at the Royal Observatory, Greenwich (II). By H. H. Turner, M.A., B.Sc.

In a former paper (*M.N.* vol. xlvi. p. 329) I referred to the fact that when observations for coincidence of the collimators at the Royal Observatory were made by viewing the South Collimator with the North through eight holes of sector form cut in the central cube of the transit circle, the readings were found to differ systematically from those obtained when the view was unobstructed; and I detailed a series of simple experiments which seemed to show that this phenomenon was purely optical. This conclusion has been confirmed by an interesting and elaborate series of experiments by Dr. Wislicenus at Strasburg. (*Ast. Nach.* No. 3067). He states his chief conclusions as follows:—

1. The systematic differences found at Greenwich are purely of an optical nature, and can be obviated by making the hole in cube circular and larger than the object-glasses of the collimators.

2. If the object-glass of a telescope be obscured by concentric discs or rings quite symmetrically, or by symmetrical screens of other forms, such as the radial bars at Greenwich, not only is the character of the focal image altered, but it *may* be displaced laterally.

With regard to the excellent though somewhat obvious suggestion in (1), it may be remarked that there are practical difficulties in the way of adopting it in the case of the Greenwich transit circle; but the point should not be lost sight of in designing a new instrument.

With regard to conclusion (2), Dr. Wislicenus lays some stress on the word *may*. He is inclined to attribute such lateral displacements to residual chromatic and spherical aberrations, which make the actual focus of an object-glass somewhat indefinite. Instead of a bright point we get in fact a short line of light along the optical axis, the brightest point of which is selected for focussing. Cutting off some of the rays, however symmetrically, will cause a different point of this line to be selected, and for all but central pencils we shall thus get a paral-

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