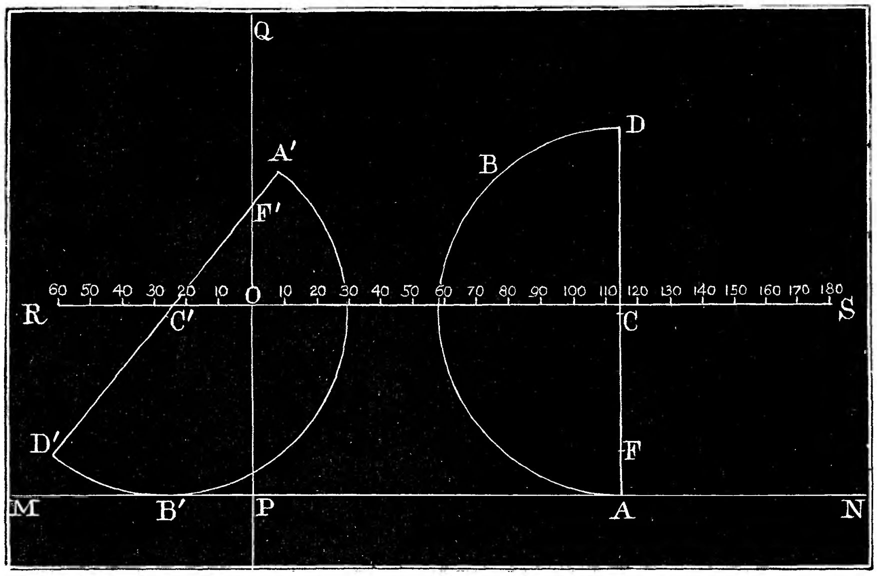
positions of a piece of stout cardboard, carefully cut in the form of a semicircle of 200^{mm} diameter, the radius CA being



Kepler's Equation.

graduated in millimetres. A point F is taken on CA, so that $\frac{CF}{CA}=e$.

To use the apparatus, all that is necessary is to set AD at right angles to RS (which is easily done by the lines on the paper) at a distance from PQ equal to the mean anomaly (nt), and to roll the semicircular card along the ruler until the point F lies on the line PQ. Then the distance OC′ is equal to $u-nt$.

It only remains, therefore, to add OC′, which is obtained in degrees directly from the scale RS, to nt in order to obtain u in degrees.

In the example illustrated in the figure, I have taken nt equal to twice the angular unit $=114^\circ \cdot 6$ and $e=0 \cdot 75$. By rolling the card I find

$$OC' = 26^\circ \cdot 8\,;$$

whence

$$u = 114^\circ \cdot 6 + 26^\circ \cdot 8 = 141^\circ \cdot 4.$$

On the Parallax of Double Stars. By Arthur A. Rambaut, M.A.

(*Communicated by Sir R. S. Ball, LL.D., F.R.S.*)

In a paper published in the *Proceedings of the Royal Irish Academy* (2nd ser., vol. iv., No. 6), I pointed out the relation connecting the parallax and the relative velocity of the components of a double star with the period and angular elements of its orbit, and discussed the possibility of determining the distance by means of spectroscopic observations of this velocity.

At the time my paper was read (May 1886), my examination of the question seemed to point to the conclusion that the velocities to be expected were too small to be measured with any degree of certainty, and although the photographic method even then wore a very promising appearance, it seemed that, for some time to come, at least, there was but little prospect of success in this direction.

Since that date, however, the splendid progress made by Professor Pickering and others in the art of photographing stellar spectra has altered the whole aspect of the question, and in particular the magnificent series of results, lately published in the *Astronomische Nachrichten* by Professor Vogel and Dr. Scheiner, seems to demonstrate the possibility of applying this method with success to the determination of parallax.

It is in the hope of inducing astronomers who may be engaged on this kind of observation, or who have the requisite instruments at their disposal, to take up what, under present circumstances, appears to me a promising field of work that I venture once more to direct attention to the subject.

In my paper before referred to (p. 666) I have shown that if

ϕ	denotes	the	angle between the tangent to the orbit and the major axis,	
λ	,,	,,	angle between the line of nodes and the major axis,	
γ	,,	,,	inclination	of the orbit,
a	,,	,,	semi-axis-major	
e	,,	,,	eccentricity	
r	,,	,,	radius vector	
P	,,	,,	period,	
π	,,	,,	parallax in secs. of arc	of the star,
V	,,	,,	velocity in miles per sec. in the line of sight	
l	,,	,,	mean motion of the Earth in miles per second,	

then

$$\pi V = \frac{la^2 \sqrt{1-e^2} \sin(\phi - \lambda) \sin \gamma}{Pr \sqrt{1 - e^2 \cos^2 \phi}}.$$

This equation gives us a relation between π and V depending only on the period and angular elements of the orbit, by means of which, if V has been determined by observation, π can be immediately deduced.

In my previous paper I took unity as the critical value of πV for this reason, that if πV is at any time equal to unity, we know either that π is not less than one-tenth of a second of arc, or that V is not less than ten miles per second—ten miles appearing to me at that time as being about the limit of velocity which could with certainty be detected.

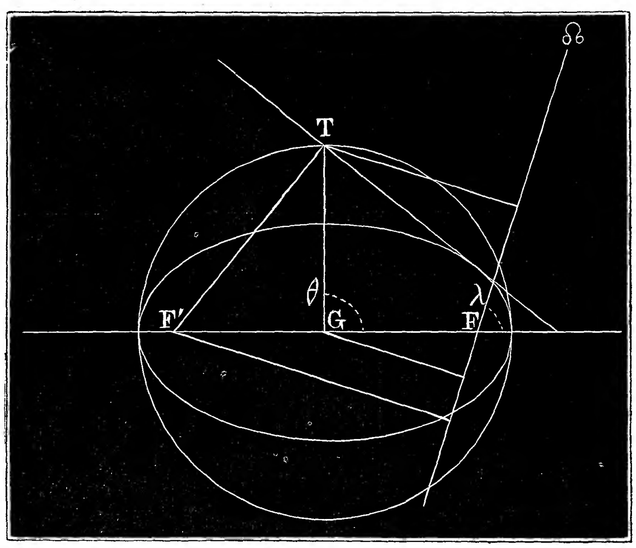
A perusal of Professor Vogel's paper in Nos. 2896 and 2897 of the *Astronomische Nachrichten* shows, however, that this value may now be very much reduced. He there gives, amongst other results, ten different values for the motion of *α Aurigæ* in the line of sight, obtained on different nights, all lying between $+3{\cdot}1$ and $+4{\cdot}0$ German geographical miles, while his results for *Venus* agree within a tenth of a geographical mile with the calculated values. If, then, we assume that where the components are fairly equal in brightness, a velocity of one English mile per second is not quite beyond the capabilities of the spectrographic method, we shall find that a number of double stars whose orbits have been determined are within measurable distance either by the trigonometrical or spectrographic method, so far, at least, as the values of π and V are concerned.

This being the case it seems to me desirable to reduce the formulæ to a more convenient shape, and to compute πV for some other stars not included in my previous paper.

I have shown (p. 667) that we may write

$$\pi V = \frac{lap' \sin(\phi - \lambda) \sin \gamma}{Pb},$$

where b is the semi-axis-minor and p' the perpendicular from the



Parallax of Double Stars.

empty focus on the tangent to the orbit. Also $p' \sin(\phi - \lambda)$ is the orthogonal projection of p' on the line of nodes, and this (see figure) is equal to the sum of the projections of F′G and GT, where F′G is half the distance between the foci, and GT the line joining the foot of p' with the centre of the ellipse.

But the projection of F′G is equal to $ae \cos \lambda$, and that of GT is equal to $a \cos(\theta - \lambda)$, θ being the true anomaly, so that we have

$$\pi V = \frac{la^2 \sin \gamma}{Pb} [e \cos \lambda + \cos (\theta - \lambda)],$$

$$\left.\begin{aligned} \text{or } \pi V &= A + B \cos (\theta - \lambda) \\ \text{where } A &= \frac{lae \sin \gamma \cos \lambda}{P \sqrt{1 - e^2}} \\ \text{and } B &= \frac{la \sin \gamma}{P \sqrt{1 - e^2}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

A and B being constants depending only on the elements of the orbit.

If we wish to know at what time the value of πV reaches its maximum value for any particular binary pair we have to make

$$A + B \cos (\theta - \lambda) \text{ a maximum,}$$

or

$$\sin (\theta - \lambda) = 0.$$

Hence we see that, if we disregard the sign of V, the value of πV will be greatest when $\theta = \lambda$ or $\pi + \lambda$, or, as I showed otherwise in my previous paper, the relative velocity in the line of sight is a maximum when the body is passing through the line of nodes, and the first of equations (a) shows that its greatest value is $\pm A + B$, the upper or lower sign being taken according as λ is in the first or fourth, or in the second or third quadrants.

I may take this opportunity of saying that the method which computers of double-star orbits adopt of placing ☊ (on which the quadrant of λ depends) in all four quadrants is entirely misleading, as it is calculated to give the impression that there is some means of determining which is the ascending node from micrometric observations alone; whereas the fact is that we cannot from these data decide at which node the companion is receding from or approaching the Sun, nor is there even any convention as to which direction of motion should be implied by the term *ascending*. We can, of course, from micrometric measures fix the position of the *line* of nodes, but the longitude of the *ascending* node must always be doubtful to the extent of 180°, unless the spectroscope be employed and the resolved part of the velocity be sufficiently large to become sensible.

With regard to this point Villarceau, in his paper on double-star orbits, in the *Connaissance des Temps*, 1877, remarks:—"L'angle ☊ sera indéterminée à 180 degrés près. Cela doit être, puisque il est impossible de distinguer si le satellite en traversant l'un des nœuds s'écarte ou se rapproche de notre système solaire." Lower down he says: "Le signe de I" (the inclination of the orbit) "reste arbitraire. La longitude ☊, étant celle du nœud ascendant, est ambiguë, comme il vient d'être dit, et l'on peut toujours prendre I positif et compris entre 0 et 180 degrés. . . . L'hypothèse de I négatif change effectivement le nœud ascendant en nœud descendant." And, again, "La valeur de I . . . . restera affectée du double signe qu'on ne peut éviter."

Now, although this ambiguity is of no importance when we are concerned with only one particular binary system, and aim only at predicting the distance and position of the pair for any given time, yet if we wish to draw general conclusions from the situations of a large number of groups, such, for the sake of illustration, as Mädler's conception of a "stellar equatorial plane," to which he supposes the planes of the double-star orbits to be approximately parallel, this ambiguity meets us at the very outset. If, for instance, we take the star *p Eridani*, and inquire how the plane of its orbit is situated with regard to the plane passing through *Procyon*, *Mira Ceti*, and the Sun, we shall find that we cannot tell whether it is approximately perpendicular or approximately parallel to it. Or if we compare the plane of the orbit of δ *Cygni* with that of the Earth's equator, we cannot determine whether it is inclined to it at an angle of 7 or of 82 degrees. The advantages, therefore, to be expected from spectroscopic, or rather spectrographic, observations of double stars for which πV is greater than 0·1, are

(1) An independent check on the parallax where this has been determined trigonometrically.

(2) A determination of the parallax where, owing to its smallness, the trigonometrical method fails.

(3) A determination of the sign of the inclination which will remove the ambiguity attaching to the situation of the orbit.

In Table I. below I give the elements of a number of orbits, and the values of A and B as computed from these elements. These quantities will enable us to compute the value of πV corresponding to any given time with the least possible labour, as it is only necessary to compute the value of θ for the given time, and to substitute the value thus found in the first of equations (α). In the last column of this table is given the greatest value to which πV can attain; and if it is required to ascertain at what epoch πV reaches its maximum value, it is only necessary to find the date corresponding to $\theta = \lambda$ or $\theta = \pi + \lambda$, according to the quadrant in which λ lies, as explained above. In Table II. will be found the values of πV for some of the most remarkable pairs, corresponding to the date 1891·0, and in those cases where the parallax has been determined I have added the value of V in the third column. It will be noticed that in the case of these stars, as might have been expected in consequence of their comparative nearness to us, the velocities are for the most part very small and such as direct eye-observations must fail to deal with. The success of Professor Vogel's method, however, and of the Henry Draper Memorial photographs, seems to hold out the hope that even velocities such as these may become sensible on the plates, and, in view of the very striking observations on ζ *Ursæ Majoris* lately published by Professor Pickering, it is not improbable that the highest velocities will be found, not in the wide and easy doubles, but in those close pairs such as δ *Equulei*, where, owing either to their remoteness or the smallness of the linear dimensions of their orbits, it is extremely difficult to divide the discs optically or to separate the two spectra on the photographic plate.

TABLE I.

Name of Star.	Magnitudes of Components.	Ω	γ	λ	e	a	P	T	Authority.	A	B	±A+B
		° ′	° ′	° ′		″	years.					
η Cassiopeiæ	4, 7½	33 20	48 18	196 7	0·624	8·639	+195·2	1902·0	Gruber	−0·469	0·782	1·251
36 Andromedæ	6, 7	57 54	41 39	142 19	·634	1·54	+349·1	1798·8	Doberck	−0·035	0·070	0·105
p Eridani	7, 7	81 42	44 40	327 15	·378	3·82	−117·5	1817·5	„	+0·145	0·456	0·601
40 Eridani (B.C.)	9, 11	146 20	76 20	354 23	·136	5·99	−139·0	1863·9	Gore	+0·105	0·781	0·886
α Canis Majoris	1, 12½	45 27	58 37	136 39	·591	8·53	− 44	1889·4	Pritchard	−1·629	3·788	5·417
Σ 1037	7, 7	156 58	68 17	273 27	·632	0·182	− 15·0	1827·7	Mädler	+0·010	0·269	0·279
α Geminorum	2½, 3½	31 58	42 5	294 1	·344	7·5375	−996·9	1750·3	Thiele	+0·014	0·100	0·114
ζ Cancri	5, 5½	81 33	15 32	109 44	·391	0·853	− 60·3	1866·0	Seeliger	−0·010	0·076	0·086
Σ 3121	7½, 8	16 30	74 12	149 30	·26	[0·71]	+ 37·0	1842·8	Doberck	−0·079	0·353	0·432
ω Leonis	6, 7	148 46	64 5	121 4	·536	0·890	+110·8	1841·8	„	−0·044	0·158	0·202
φ Ursæ Maj. = O.Σ.208	6, 7	105 18	57 57	72 7	·788	0·54	+115·4	1877·1	Casey	+0·029	0·119	0·148
γ Leonis	2, 3½	111 34	43 6	195 22	·733	1·98	+407·0	1741·0	Doberck	−0·066	0·090	0·156
ξ Ursæ Majoris	4, 5	100 13	56 40	125 0	·416	2·580	− 60·8	1814·5	Pritchard	−0·172	0·720	0·892
O.Σ. 235	6, 7	96 17	60 13	129 55	·587	1·066	+ 94·4	1839·1	Doberck	−0·084	0·224	0·308
γ Virginis	3, 3	33 35	35 6	283 44	·896	3·97	−185·0	1836·7	Thiele	+0·109	0·513	0·622

Name of Star.	Magnitudes of Components.	Ω	γ	λ	e	a	P	T	Authority.	A	B	±A+B
42 Comæ Ber.	6, 6	10° 30′	90° 0′	99° 11′	·480	0″·657	years. + 25·7	1859·9	O.Σ. and Dubjago	−0·041	0·538	0·579
Σ 1757	8, 9	120 40	49 25	140 37	·623	2·13	+291·8	1792·65	Casey	−0·063	0·131	0·194
Σ 1819	8, 8	156 25	37 31	348 56	·305	1·46	−340·1	1797·0	,,	+0·015	0·051	0·066
α Centauri	1, 4	25 47	79 32	54 47	·526	17·50	+ 77·4	1876·0	Elkin	+1·462	4·827	6·289
ξ Boötis	4½, 6½	26 22	36 55	117 46	·708	4·86	−127·35	1770·7	Doberck	−0·198	0·600	0·798
44 i Boötis	5, 6	65 29	70 5	1 18	·71	3·093	+261·1	1783·0	,,	+0·207	0·292	0·499
η Coronæ Borealis	5, 5½	26 42	57 59	215 35	·2625	0·827	+ 41·6	1850·3	Dunér	−0·069	0·323	0·392
μ² Boötis	7, 8	166 7	35 12	40 54	·567	1·057	−266·0	1862·55	Pritchard	+0·022	0·051	0·073
O.Σ. 298	7, 8	14 38	56 10	342 31	·487	0·886	+ 68·8	1813·0	Doberck	+0·105	0·226	0·331
γ Coronæ Borealis	4, 7	110 24	85 12	233 30	·350	0·70	− 95·5	1843·7	,,	−0·030	0·144	0·174
51 ξ Libræ	5, 5	12 15	68 42	89 16	·077	1·26	+ 95·9	1859·6	,,	0·000	0·227	0·227
σ Coronæ Borealis	5, 6	16 27	31 56	73 51	·7515	5·885	+845·9	1826·9	,,	+0·022	0·103	0·125
λ Ophiuchi	4, 6	88 40	21 27	151 0	·493	1·19	+240·95	1800·5	,,	−0·017	0·038	0·055
ζ Herculis	5, 5½	41 44	43 14	252 45	·463	1·284	− 34·4	1864·8	,,	−0·073	0·533	0·606
Σ 2130 = μ Draconis	5, 5	Indeterminate	0 0	Indeterminate	·493	3·38	−648·0	1940·35	Berberich	0·000	0·000	0·000*

* The position-angle of periastron is 275° 26′.

Name of Star.	Magnitudes of Components.	Ω	γ	λ	e	a	P	T	Authority.	A	B	±A+B
		° ′	° ′	° ′		″	years.					
Σ 2173	6, 6	152 39	80 53	7 16	·135	1·009	− 45·4	1872·9	Dunér	+0·055	0·409	0·464
μ^2 Herculis	9½, 10½	57 57	60 43	156 21	·302	1·46	+ 54·25	1877·1	Doberck	−0·126	0·455	0·581
τ Ophiuchi	5, 5½	67 1	46 8	36 26	·6055	1·193	+217·9	1818·5	"	+0·045	0·092	0·137
70 *p* Ophiuchi	4, 6	127 23	58 5	151 55	·467	4·790	− 94·4	1808·9	Pritchard	−0·370	0·899	1·269
ζ Sagittarii	6, 9	83 22	58 48	263 21	·170	0·53	− 18·7	1882·9	Gore	−0·009	0·455	0·464
γ Coronæ Australis	5½, 5½	49 9	68 38	255 24	·699	2·400	− 55·6	1882·8	Schiaparelli	−0·183	1·039	1·222
δ Cygni	3, 8	91 8	37 46	203 2	·286	2·310	−415·1	1904·1	Behrmann	−0·017	0·066	0·083
β Delphini	3, 11	10 56	61 35	220 57	·096	0·460	+ 16·955	1868·85	Celoria	−0·040	0·556	0·596
λ Cygni	5, 6	105 18	58 48	139 6	·602	0·602	− 93·4	1926·9	Glasenapp	−0·058	0·128	0·186
4 Aquarii	6, 7	340 14	56 37	235 0	·461	0·717	−129·8	1752·0	Doberck	−0·025	0·096	0·121
61 Cygni	5, 6	340 3	71 25	271 33	·534	30·42	+523	1834·8	Peters (Provisional)	+0·017	1·204	1·221
"	5, 6	341 4	63 55	278 16	·174	29·48	+782·6	1834·8	" (Final)	+0·016	0·635	0·651
δ Equulei	4½, 5	24 3	81 45	74 1	·201	0·406	− 11·478	1892·0	Glasenapp	+0·037	0·660	0·697
τ Cygni	5, 8	83 0	44 40	205 26	·3475	1·19	− 53·9	1864·0	Gore	−0·096	0·306	0·402
ζ Aquarii	4, 4	140 51	44 42	134 40	·652	7·64	−1578·8	1924·15	Doberck	−0·038	0·083	0·121
Σ 3062	7, 8	38 35	32 11	92 7	·461	1·27	+104·4	1834·9	"	0·002	0·135	0·137

TABLE II.

Values for 1891·0.

Name.	πV	V	π and Authority.
η Cassiopeiæ	−0·348	2·3	0″·15 O.Σ.
36 Andromedæ	+0·034		
p Eridani	−0·305		
α Canis Maj.	−5·326	13·4	0·39 Gill and Elkin.
Σ 1037	+0·151		
α Geminorum	+0·112	0·6	0·20 Johnson.
ξ Ursæ Maj.	−0·294	7·4	0·04 Klinkerfues.
γ Virginis	+0·076		
α Centauri	+2·741	3·7	0·74 Gill and Elkin.
ξ Boötis	+0·372		
44 *i* Boötis	−0·082		
η Coronæ Bor.	−0·352		
51 ξ Libræ	+0·184		
ζ Herculis	−0·325		
70 *p* Ophiuchi	+0·197	1·2	0·16 Krueger.
γ Coronæ Austr.	+0·698		
Σ 2173	−0·327		
O.Σ. 235	+0·076		
61 Cygni	−1·173	2·5	0·47 Ball.
,,	−0·285	0·7	,, ,,
ζ Aquarii	+0·041		

On some Celestial Photographs made with a large Portrait Lens at the Lick Observatory. By E. E. Barnard, M.A.

Professor Holden having secured for the Lick Observatory a very large portrait lens (maker, Willard, New York, 1859), I have taken advantage of the opportunity to make with it some experimental long-exposure negatives of the Milky Way, the great nebula of *Andromeda*, and the *Pleiades*. This lens, with the diaphragm removed, has an equivalent aperture of 5·9 inches, and a focal length of 31 inches.

The lens was mounted temporarily in a rough wooden box, and before beginning work the stellar focus was accurately determined by a series of exposures on *Polaris*, &c.

The camera was then strapped firmly upon the tube of the 6½-inch equatoreal, which, with a high power, was used as a following telescope. This instrument, having slow-motion rods in right ascension and declination at the eye-end, was specially