THE ASTROPHYSICAL JOURNAL, **268**:L27–L32, 1983 May 1

THE DISCOVERY OF NONRADIAL INSTABILITY STRIPS FOR HOT, EVOLVED STARS[1]

SUMNER G. STARRFIELD,[2,3] ARTHUR N. COX,[2] STEPHEN W. HODSON,[4] AND W. D. PESNELL[2,5]
Received 1982 September 20; accepted 1983 January 7

ABSTRACT

We have performed radial and nonradial, linear, nonadiabatic pulsation analyses of model stellar envelopes in the effective temperature range from 8×10^4 K to 1.5×10^5 K. These models have total masses of 0.6 $M_\odot$ and radii chosen so that they line up along a pre–white dwarf cooling curve computed by Schönberner. We use three different interior compositions: (1) 100% carbon; (2) 50% ^{12}C and 50% ^{16}O (by mass); and (3) 10% ^{12}C and 90% ^{16}O (by mass). We find nonradial and radial instability strips for each composition caused by the partial ionization of carbon or oxygen or both. Our objective is to find the cause of the observed pulsations of PG 1159−035, a hot, evolved star with $T_e \geq 10^5$ K. While this star may be too hot to lie within any of our new instability strips, the closest agreement comes from models with significant amounts of ^{16}O near the stellar surface. If correct, this result implies that helium burning in evolved stars produces much more ^{16}O than heretofore believed.

Subject headings: stars: evolution — stars: interiors — stars: pulsation — stars: variables

I. INTRODUCTION

This investigation is motivated by the discovery on 1979 April 29, of a (so-far) unique, hot, pulsating variable star (McGraw *et al.* 1979*a*, *b*). Designated as PG 1159−035 (hereafter PG), this star was originally detected as a 14.5 mag blue object in Green's (1977) QSO survey. Its spectrum shows no evidence for hydrogen being present in the atmosphere, and the spectral region around λ4640 and λ4686 is extremely complex with λ4686 of He II in emission (Green and Liebert 1979; Wesemael *et al.* 1982). Optical spectra photometric data, *International Ultraviolet Explorer* data, and *Voyager 1* and *Voyager 2* observations imply that the effective temperature of this star certainly exceeds 10^5 K, is probably around 1.2×10^5 K, and could be as large as 1.5×10^5 K (McGraw 1982). Its surface gravity is uncertain, but analysis of a broad depression around λ4686 of He II implies a value exceeding 10^7–10^8 cm s^{-2} (Wesemael *et al.*).

Although these data are already unusual, the most exciting discovery was that this star was oscillating with two periods of ~ 539 s and ~ 460 s (McGraw *et al.* 1979*a*, *b*). We realized immediately that this star fell near no known instability strip and began searching for a region of instability in the H-R diagram based on the κ mechanism for partial ionization of helium, carbon, or mixtures of carbon and oxygen. We were successful for low-order *radial* modes at effective temperatures of ~ 8 $\times 10^4$ K and luminosities of ~ 10^4 $L_\odot$ (Starrfield, Cox, and Hodson 1979, hereafter Paper I). However, further observations (McGraw 1982) have shown that T_e is much larger, and the light variations of this star are extremely complex with many periods, quite reminiscent of ZZ Ceti variables which are thought to be oscillating in *nonradial* *g*-modes (cf. Winget *et al.* 1982).

II. METHODS AND MODEL ENVELOPES

Observations of DA white dwarfs at significantly cooler temperatures than PG imply masses of ~ 0.6 $M_\odot$ (cf. Weidemann 1979). PG is not a DA dwarf, and there are theoretical reasons for suspecting that cool DB's may have masses appreciably lower than the DA's (Alcock 1979). Nevertheless, PG is hot, and its surface composition appears to be mainly helium and carbon, indicating that it has probably undergone asymptotic giant branch helium-shell flash evolution and must have a mass exceeding 0.5 $M_\odot$. Therefore, we assume that it has a mass of 0.6 $M_\odot$ and lies on an evolutionary track computed by Schönberner (1979) that includes mass loss. Earlier we chose point E, after all nuclear burning, on his evolutionary track which is reproduced in Paper I. The new data imply that PG is much more evolved and lies somewhere around point F on his track (see Fig. 1 of Schönberner).

We use the Los Alamos stellar envelope code to integrate inward to more than 96% of the mass in all cases. We assume a surface composition of half helium

[1]Supported in part by NSF grants AST 79-21073 and AST 81-17177 to Arizona State University and by the US Department of Energy.
[2]Theoretical Division, Los Alamos National Laboratory.
[3]Department of Physics, Arizona State University.
[4]Earth and Space Science Division, Los Alamos National Laboratory.
[5]Department of Physics, University of Florida.

and half carbon by mass, and at a temperature of $\sim 2 \times 10^5$ K (in most cases) we switch to one of three compositions which then remains unchanged throughout the rest of the envelope. These three compositions are: pure carbon; half carbon, half oxygen (by mass) (C5O5); or 10% carbon, 90% oxygen (by mass) (C1O9). We switch to these three compositions at 2×10^5 K (typically zone 300 at 10^{-13} $M_\star$ deep) because, if we allow helium to extend deeper into the envelope, it "poisons" carbon or oxygen driving at these effective temperatures. The nonradial blue edge for helium driving is much cooler (Winget 1981), and therefore the helium layer must be very thin.

Characteristics of our envelopes will be described more fully in another publication (see also Starrfield *et al.* 1982, 1983). Here we briefly mention that all of our envelopes have a region of convective instability, and we treat this region with a standard mixing length formulation assuming $l/H_p = 1$ (Cox and Giuli 1968). For our stability analysis we make the standard assumption of frozen-in convection so that regions of strong convection have neutral stability and can neither drive nor damp pulsations.

Our nonradial analyses are done with a computer code based on the method of Saio and Cox (1980) with some additions from Winget (1981) and Winget *et al.* (1982). A comparison of our nonradial results for the ZZ Ceti variables with those of Winget *et al.* (1982) appears in Starrfield *et al.* (1982). We used 329 zones and a central mass "ball" for our PG models. We restricted our study to *g*-modes in the period range ~ 130 s $< \Pi < \sim 650$ s so as to include the observed periods for PG.

III. NONRADIAL RESULTS

In Table 1 we tabulate, for the three compositions, the first and last pulsationally unstable mode, period, growth rate, and kinetic energy for those modes. We do this only for the lowest three values of l ($l = 1, 2, 3$) since Dziembowski (1977) has shown that higher values of l are unobservable.

The results for the carbon models show that the blue edge for periods near those of PG is 8×10^4 K. We have not, as yet, identified the red edge, but some models that we have computed indicate that it extends to effective temperatures cooler than 7×10^4 K. We also performed a radial, linear nonadiabatic study at some of the tabulated effective temperatures, and there also exists a carbon instability strip for them. However, the radial periods are ~ 10 s to ~ 20 s, and the growth rates, ΔKE/KE per period, are two to three orders of magnitude smaller than the nonradial growth rates given in Table 1.

Figure 1 shows the nonradial work function ($\Delta \mathrm{KE} = \oint P\, dV$ per period) for the $l = 2$, g_{17}, g_{16}, and g_{15} modes. We find a very broad region of driving in these models: from temperatures of $\sim 4.6 \times 10^5$ K (surface mass fraction M_s of $\sim 4.5 \times 10^{-11}$ $M_\star$, zone 260, $x = 0.9985$) to temperatures of $\sim 1.7 \times 10^6$ K ($M_s \sim 6 \times 10^{-10}$ $M_\star$, zone 215 $x = 0.9941$). This region is much broader than we found for the ZZ Ceti variables (Starrfield *et al.* 1982) and indicates the depth of the carbon partial ionization region. Nevertheless, this region is quite close to the surface and, as the effective temperature increases, it becomes so shallow that it can no longer cause an instability in a large enough mass.

In order to find an instability at higher temperatures, we must include in our mixture an element that ionizes at even higher temperatures. The obvious choice is oxygen since we can expect it to be produced, at some level, by helium burning in the late stages of stellar evolution. Since we do not know the original mass of this star nor do we know how much mass was lost as it evolved to its present location in the H-R diagram, we chose two different values for the carbon to oxygen ratio.

One composition that was available to us consists of half carbon and half oxygen by mass. As can be seen in Table 1, the presence of oxygen in the envelope produced an instability strip extending from at least 8×10^4 K to $\sim 1.20 \times 10^5$ K, in much better agreement with the observations of PG. However, the blue edge for the periods that are observed is $\sim 1.0 \times 10^5$ K some 2×10^4 K cooler than the most likely effective temperature of PG.

At 10^5 K at $l = 2$, we find that the strongest driving occurs between temperatures of $\sim 1 \times 10^6$ K ($M_s \sim 5 \times 10^{-11}$ $M_\star$) and $\sim 2 \times 10^6$ K ($M_s \sim 8 \times 10^{-10}$ $M_\star$) somewhat deeper and hotter than we found for the pure carbon envelope with $T_e \approx 8 \times 10^4$ K. Since we would have expected carbon driving to occur closer to the surface as the effective temperature increased, most of the excitation in this model must be coming from the partial ionization of oxygen; carbon serves only to dilute the driving due to the decrease in $d \ln \kappa / d \ln T$. We also find that the maximum instability (largest value of η and smallest value of the kinetic energy) occurs at temperatures of $\sim 9 \times 10^4$ K for the $l = 1$ modes, at temperatures of $\sim 10^5$ K for the $l = 2$ modes, and also at $\sim 10^5$ K for the $l = 3$ modes. The peak value of η (ΔKE/KE per period) reaches $\sim 5 \times 10^{-3}$ for the g_{18}^2 mode with a period of 570 s. While this occurs in the model with $T_e = 9 \times 10^4$ K, it is more than 10^5 times larger than any of the unstable radial periods.

Given the success of this composition, we decided to increase the mass fraction of oxygen present in the envelopes to ~ 90% with the rest carbon. We assumed that carbon was poisoning the excitation by reducing the oxygen opacity bump at temperatures of 10^6 K to $\sim 3 \times 10^6$ K. We realize that present stellar evolution calculations using currently accepted values of θ_α^2 (cf. Fowler, Caughlan, and Zimmerman 1975) do not allow

1983ApJ...268L..27S

TABLE 1

PERIODS, GROWTH RATES, AND KINETIC ENERGIES FOR THE UNSTABLE NONRADIAL g-MODES

PARAMETER	T_e(K) 8×10^4	9×10^4	1.0×10^5	1.1×10^5	1.2×10^5	1.5×10^5
$l = 2$; ^{12}C = 1.0 (by mass)						
Mode	9	S	S	S	S	...
Π(s)	255	S	S	S	S	...
η	2.9E-6	S	S	S	S	...
KE(ergs)	1.5E46	S	S	S	S	...
Mode	21	S	S	S	S	...
Π(s)	560	S	S	S	S	...
η	3.2E-4	S	S	S	S	...
KE(ergs)	1.7E44	S	S	S	S	...
$l = 3$; ^{12}C = 1.0 (by mass)						
Mode	13	15	6	S	S	...
Π(s)	258	307	129	S	S	...
η	1.5E-4	8.2E-5	1.4E-9	S	S	...
KE(ergs)	1.7E45	2.4E44	6.7E46	S	S	...
Mode	27	15	10	S	S	...
Π(s)	521	307	204	S	S	...
η	2.2E-4	8.2E-5	8.3E-7	S	S	...
KE(ergs)	1.6E44	2.4E44	3.4E45	S	S	...
$l = 1$; ^{12}C = 0.5, ^{16}O = 0.5 (by mass)						
Mode	4	4	4	5	S	S
Π(s)	290	281	278	341	S	S
η	1.3E-7	1.8E-7	1.4E-7	4.2E-7	S	S
KE(ergs)	7.8E46	6.6E46	6.6E46	2.3E46	S	S
Mode	11	10	10	7	S	S
Π(s)	616	565	607	436	S	S
η	1.8E-4	3.4E-4	2.0E-4	3.0E-6	S	S
KE(ergs)	3.9E45	1.1E45	5.5E44	3.3E45	S	S
$l = 2$; ^{12}C = 0.5, ^{16}O = 0.5 (by mass)						
Mode	7	10	9	9	10	S
Π(s)	259	345	324	311	316	S
η	1.1E-5	3.5E-5	2.0E-4	1.3E-4	4.9E-5	S
KE(ergs)	6.6E45	2.1E46	1.5E45	1.2E45	1.2E45	S
Mode	19	18	16	12	11	S
Π(s)	624	570	546	404	347	S
η	6.0E-3	5.0E-3	8.8E-4	4.6E-4	6.4E-5	S
KE(ergs)	8.6E44	4.2E44	3.1E44	4.1E44	4.9E44	S

PARAMETER	T_e(K) 8×10^4	9×10^4	1.0×10^5	1.1×10^5	1.2×10^5	1.5×10^5
$l = 3$; ^{12}C = 0.5, ^{16}O = 0.5 (by mass)						
Mode	10	10	13	13	14	S
Π(s)	255	251	316	308	308	S
η	7.0E-5	2.2E-4	1.9E-3	2.0E-3	1.0E-3	S
KE(ergs)	4.7E45	2.0E45	5.4E44	2.6E44	2.2E44	S
Mode	27	26	21	17	15	S
Π(s)	650	589	502	399	333	S
η	2.8E-3	4.4E-3	1.3E-3	7.0E-4	4.9E-4	S
KE(ergs)	2.1E46	7.5E44	1.6E44	1.7E44	2.5E44	S
$l = 1$; ^{12}C = 0.1, ^{16}O = 0.9 (by mass)						
Mode	3	5	5	5	S	S
Π(s)	254	357	354	338	S	S
η	8.2E-9	3.0E-6	2.7E-6	3.7E-7	S	S
KE(ergs)	3.6E47	1.6E46	1.5E46	2.5E46	S	S
Mode	10	9	9	9	S	S
Π(s)	448	557	563	534	S	S
η	1.2E-5	1.7E-4	1.4E-4	2.4E-5	S	S
KE(ergs)	8.4E45	3.2E45	1.8E45	1.4E45	S	S
$l = 2$; ^{12}C = 0.1, ^{16}O = 0.9 (by mass)						
Mode	7	5	9	9	10	9
Π(s)	260	207	326	309	318	314
η	7.1E-6	2.1E-6	1.7E-4	9.2E-5	2.7E-5	1.5E-5
KE(ergs)	9.1E45	1.5E46	2.1E45	1.5E45	1.6E45	1.6E45
Mode	17	18	17	13	12	11
Π(s)	603	571	578	435	381	372
η	3.6E-4	2.7E-3	3.2E-4	1.5E-5	1.7E-6	1.7E-6
KE(ergs)	1.3E46	1.5E45	8.2E44	3.8E44	4.6E45	4.6E45
$l = 3$; ^{12}C = 0.1, ^{16}O = 0.9 (by mass)						
Mode	10	11	13	13	14	9
Π(s)	259	269	319	308	311	222
η	3.3E-5	4.3E-4	1.2E-3	1.1E-3	2.1E-4	2.4E-5
KE(ergs)	8.9E45	1.3E45	1.1E45	4.2E44	1.6E45	
Mode	25	27	25	17	14	12
Π(s)	651	645	582	401	311	287
η	3.1E-4	3.8E-3	3.2E-3	4.1E-4	2.1E-4	6.5E-5
KE(ergs)	3.7E46	3.3E45	2.9E44	3.6E44	3.8E44	3.8E44

NOTE.—S = stable.

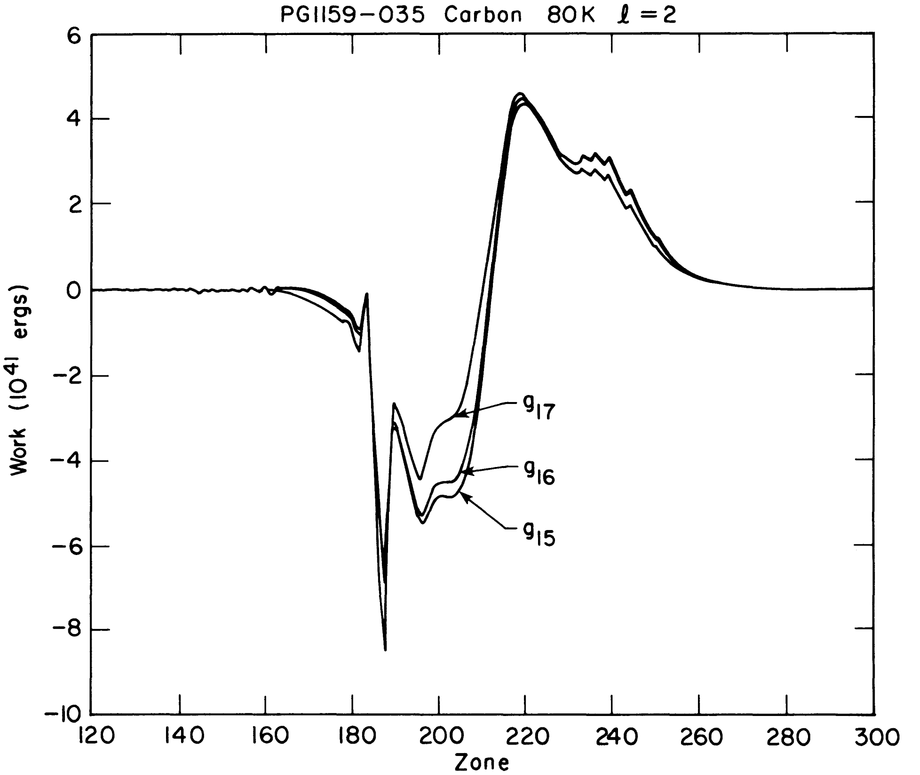

FIG. 1.—The work done per cycle for the driving and damping zones is shown for g_{15}, g_{16}, and g_{17} modes. The g_{17} mode has the smallest amplitude due to its larger number of nodes and generally smaller eigenvector amplitude. For these plots, $l = 2$ in the carbon model for an effective temperature of 80,000 K.

such a high percentage of oxygen to be produced so near the stellar surface (Becker 1982). Nevertheless, we still had not obtained an instability strip with PG placed within it, and this composition seemed a reasonable extrapolation of our partial oxygen composition. Neon is not produced in enough abundance to be important for our pulsation calculations.

The results for this composition can also be found in Table 1. The eigenfunction and work plots for the $l = 2$ g_{17} (578 s), g_{16} (551 s), and g_{15} (516 s) modes can be found in Figure 2.

Oxygen driving occurs over a broader range of effective temperature in these envelopes, and the low-order modes remain unstable to effective temperature exceeding 1.5×10^5 K. However, none are at the correct periods for PG. The reason for this very broad instability strip can be seen in the work plot for the model with $T_e = 10^5$ K. There are two peaks, one at about zone 208 and one at about zone 238. The broad region, closer to the surface, occurs between temperatures of $\sim 1 \times 10^6$ K ($M_s \sim 6 \times 10^{-11}$ $M_\star$) and $\sim 2 \times 10^6$ K ($M_s \sim 9 \times 10^{-10}$ $M_\star$) and is caused by a combination of carbon and oxygen ionization. The inner peak occurs at a temperature of $\sim 3.3 \times 10^6$ K ($M_s \sim 6 \times 10^{-9}$ $M_\odot$) and is due to pure oxygen driving. As the effective temperature increases, the outer peak becomes less important and the inner peak increases in importance. However, because it is located deeper in the star, it is only capable of exciting the lower order, shorter period modes that have an appreciable eigenfunction amplitude in this region. Because of these two peaks, the nonradial instability strip for this mixture extends for more than 7×10^4 K and is the widest instability strip so far discovered.

One problem, which also exists for the nonradial studies of the ZZ Ceti variables (cf. Winget *et al.* 1982; Winget 1981), is identifying the modes that are actually being excited in the star. We have a number of different models where periods around 540 s and 460 s are unstable with lower kinetic energies than for lower g-modes, but in no case are they the only unstable modes nor are they the most unstable modes. This is true for all three compositions. In addition, as has also been found for the ZZ Ceti variables (Starrfield *et al.* 1982, 1983), we have also found short-period, unstable radial modes in these envelopes.

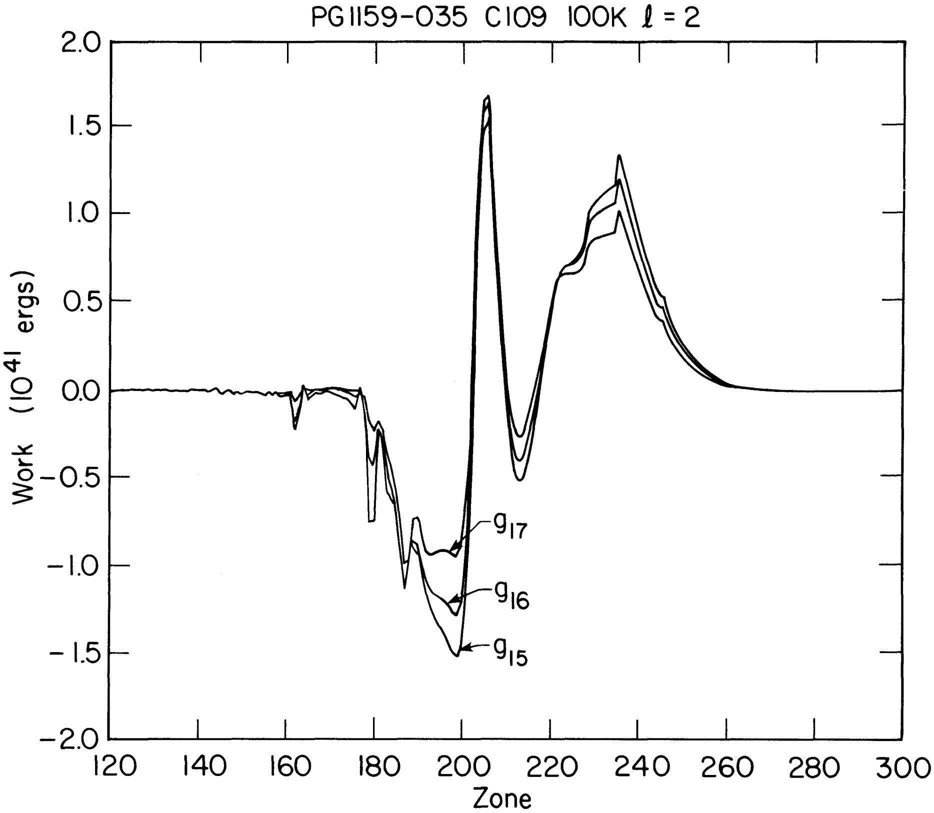

FIG. 2.—The work done per cycle for the driving and damping zones is shown for the g_{15}, g_{16}, and g_{17} modes. The g_{17} mode has the smallest amplitude due to its larger number of nodes and generally smaller eigenvector amplitude. For these plots, $l = 2$ in the C109 model for an effective temperature of 100,000 K.

IV. SUMMARY AND DISCUSSION

Our reason for initiating this investigation was the discovery of PG, a hot, hydrogen-poor star pulsating in nonradial g (presumably) modes analogous to the ZZ Ceti variables. However, its observed effective temperature is ~ 1.2×10^5 K which places it to the blue of our oxygen-rich model nonradial blue edges *for its observed periods*. A similar situation exists for the ZZ Ceti variables which now appear to lie to the blue of the theoretical nonradial blue edge (Greenstein 1982). The cause of this discrepancy is not known.

Nevertheless, our results show quite conclusively that, in order for PG both to be pulsating and have an effective temperature exceeding 10^5 K, it must have a significant amount of oxygen in layers close to the surface ($M_s \sim 10^{-10}\ M_\star$). Since PG is evolved, the significant amounts of oxygen that we require must have been produced by the $^{12}C(\alpha, \gamma)^{16}O$ reaction during helium shell burning on the asymptotic giant branch. Inasmuch as the amount of ^{16}O produced depends on the value of θ_α^2, the reduced width for the ~ 7.1 MeV level in ^{16}O (Fowler, Caughlan, and Zimmerman 1975), it may be necessary to redetermine the value of this parameter. It may also be possible that the ^{16}O abundance in degenerate cores is much greater than currently believed which would strongly affect models for Type I supernovae. However, it should also be pointed out that, if too much ^{16}O is produced during these stages of evolution, it might not be possible to make carbon stars.

We are grateful to Drs. S. A. Becker, J. P. Cox, C. J. Hansen, J. Liebert, J. T. McGraw, D. E. Winget, and H. M. Van Horn for many useful discussions during the progress of this work. We also thank J. Liebert and J. T. McGraw for continuously updating our knowledge of PG 1159–035 in advance of publication and contributing to this new confrontation of pulsation theory with evolution theory and observations. S. G. Starrfield and W. D. Pesnell are grateful to Drs. George Bell and Stirling Colgate for the hospitality of the Los Alamos National Laboratory and a generous allotment of computer time.

REFERENCES

Alcock, C. 1979, in *IAU Colloquium 53*, *White Dwarfs and Variable Degenerate Stars*, ed. H. M. Van Horn and V. Weidemann (New York: University of Rochester Press), p. 202.

Becker, S. 1982, private communication.

Cox, J. P., and Giuli, R. T. 1968, *Principles of Stellar Structure*, (New York: Gordon & Breach).

Dziembowski W. 1977, *Acta Astr.*, **27**, 203.

Fowler, W. A., Caughlan, G. W., and Zimmerman, B. A. 1975, *Ann. Rev. Astr. Ap.*, **13**, 69.

Green, R. 1977, Ph.D. thesis, California Institute of Technology.

Green, R. F., and Leibert, J. W. 1979, in *IAU Colloquium 53*, *White Dwarfs and Variable Degenerate Stars*, ed. H. M. Van Horn and V. Weidemann (New York: University of Rochester Press), p. 118.

Greenstein, J. L. 1982, *Ap. J.*, **258**, 661.

McGraw, J. T. 1982, private communication.

McGraw, J. T., Starrfield, S. G., Angel, J. R. P., and Carlton, N. P., 1979*a*, *Smithsonian Ap. Obs. Spec. Rept.*, **385**, 125.

McGraw, J. T., Starrfield, S. G., Leibert, J., and Green, R., 1979*b*, in *IAU Colloquium 53*, *White Dwarfs and Variable Degenerate Stars*, ed. H. M. Van Horn and V. Weidemann (New York: University of Rochester Press), p. 377.

Saio, H., and Cox, J. P. 1980, *Ap. J.*, **236**, 549.

Schonberner, D. 1979, *Astr. Ap.*, **79**, 108.

Starrfield, S. G., Cox, A. N., and Hodson, S. W. 1979, in *IAU Colloquium 53*, *White Dwarfs and Variable Degenerate Stars*, ed. H. M. Van Horn and V. Weidemann (New York: University of Rochester Press), p. 382 (Paper I).

Starrfield, S. G., Cox, A. N., Hodson, S. W., and Pesnell, W. D. 1982, "Proceedings of Pulsations of Classical and Cataclysmic Variable Stars" (Boulder: University of Colorado and National Bureau of Standards), p. 78.

Starrfield, S. G., Cox, A. N., and Hodson, S. W., and Clancy, S. P. 1983, *Ap. J.*, **269**, in press.

Weidemann, V. 1979, in *IAU Colloquium 53*, *White Dwarf and Variable Degenerate Stars*, ed. H. M. Van Horn and V. Weidemann (New York: University of Rochester Press), p. 206.

Wesemael, F., Liebert J., and Green, R. 1982, preprint.

Winget, D. E. 1981, Ph.D. thesis, University of Rochester.

Winget, D. E., Van Horn, H. M., Tassoul, M., Hansen, C. J., Fontaine, G., and Carroll, B. W. 1982, *Ap. J.* (*Letters*), **252**, 165.

ARTHUR N. COX: T-6, MS B288, Los Alamos National Laboratory, Los Alamos, NM 87545

STEPHEN N. HODSON: ESS5, MS F665, Los Alamos National Laboratory, Los Alamos, NM 87545

W. D. PESNELL: Department of Physics, University of Florida, Gainesville, FL 32611

SUMNER G. STARRFIELD: Department of Physics—UPH, Arizona State University, Tempe, AZ 85287